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AN EXTENSION OF THE CODAS METHOD
BASED ON INTERVAL ROUGH NUMBERS
FOR MULTI-CRITERIA GROUP DECISION MAKING

Abstract

This study aims to develop a new Interval Rough COmbinative Distance-based Assessment (IR CODAS) method for handling multiple criteria group decision making problems using linguistic terms. A single decision maker is unable to express his opinions or preferences on multiple criteria decisions, while a Multi-Criteria Group Decision Making MCGDM process ensures successful outcomes when handling greater imprecision and vagueness information. A real-life case study of risk assessment is investigated using our proposed IR-CODAS method to test and validate its application; a sensitivity analysis is also performed.

Keywords: Interval Rough Numbers, group decision making, IR-CODAS method, risk assessment.

1 Introduction

The decision making process is characterized by uncertainty and subjectivity; decision makers (DMs) are often faced with a dilemma while assigning a decision to certain criteria and they evaluate the alternatives in different uncertain decision making situations. Indeed, uncertainties are generally handled using the application of Rough Set Theory (RST), especially Interval Rough Numbers IRNs.

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RST has been successfully applied in a good number of MCDM studies. For instance, Song and Cao (2017) presented a rough approach based on DEMATEL to assess the interaction between requirements of Product-Service System (PSS). A rough Technique for Ordering Preference by Similarity to Ideal Solution (TOPSIS) approach is also proposed by Song et al. (2014) to improve the effectiveness of failure mode and effect analysis technique.

Some researchers have studied IRNs. For instance, Lu, Huang and He (2011) developed a fuzzy linear programming method, based on rough intervals, to generate simultaneous water allocation strategies in agricultural irrigation systems. In turn, to solve the multi-objective hub location and hub network design problem, Niakan, Vahdani and Mohammadi (2015) used a hybrid solution, based on inexact programming, interval-valued fuzzy programming and rough interval programming.

Regarding the hybridization of extensions of rough sets, a number of approaches have been proposed, such as the hybrid DEMATEL-ANP-MAIRCA model where Pamucar et al. (2017) developed a new approach for dealing with uncertainty based on IRNs. In addition, Pamucar, Petrovic and Cirovic (2018) modified the BWM (Best-Worst Method) and MABAC (Multi-Attributive Border Approximation area Comparison) methods by integrating fuzzy rough numbers per interval. To process the uncertainty contained in group decision making, Pamucar, Edmundas and Zavadskas (2018) integrated IRNs within the MABAC and AHP methods for rating university web pages. Also, the Normalized Weighted Geometric Bonferroni Mean (NWGBM) operator of the IRNs is used by Pamucar, Božanić et al. (2018) and is applied to the DEMATEL and COPRAS model to solve the problem of selecting an optimal direction for making a temporary military route. Moreover, Pamucar, Chatterjee and Zavadskas (2019) integrated IRNs into the Best Worst Method (BWM) and Weighted Aggregated Sum Product Assessment (WASPAS) method along Multi--Attributive Border Approximation area Comparison (MABAC) to evaluate third-party logistics (3PL) providers. As the Internet of Things (IoT) technology has rapidly developed, Kao, Nawata and Huang (2019) proposed a novel Hybrid method BR-DEMATEL that integrates Bayesian theory, interval rough number, and DEMATEL for Systemic Factor Evaluation-based Technological Innovation System (TIS) for the Sustainability of IoT in the Manufacturing Industry. We can see that many researchers have studied the combination of interval rough theory and Multi-Criteria Decision Making (MCDM) methods for different decision making problems which shows the importance of using interval rough MCDM approaches.

The MCDM tackles four types of problems: ranking, sorting, choice and description. In recent years, a new ranking MCDM method has been proposed, namely COmbinative Distance-based Assessment (CODAS), developed by Keshavarz Ghorabaee et al. (2016). The ranking of alternatives is determined using two measures: The main and primary measure uses the Euclidean distance of alternatives from the Negative Ideal Solution, while the secondary measure is the Taxicab distance

Lately, CODAS has been successfully applied in Group Decision Making (GDM) in various fields. For instance, Keshavarz Ghorabaee et al. (2017) solved group decision problems using a combination of trapezoidal fuzzy numbers and the CODAS method for market segment evaluation. Moreover, Yeni and Özçelik (2019) presented the Interval-Valued Atanassov Intuitionistic Fuzzy CODAS (IVAIF-CODAS) method and applied it to a personnel selection problem. To handle uncertainty, Pamucar et al. (2018) employed integrated MCDM framework using Linguistic Neutrosophic Numbers (LNN) and the CODAS method to select the optimal Power-Generation Technology (PGT). Furthermore, Roy et al. (2019) presented an extension of the CODAS approach using Interval--Valued Intuitionistic Fuzzy Sets (IVIFS) to select the best sustainable material for the automotive instrument panel. Based on 2-tuple Linguistic Pythagorean Fuzzy Sets (2TLPFSs), He et al. (2020) developed a novel CODAS model. Remadi and Frikha (2020) developed new methodologies in group decision making where triangular intuitionistic fuzzy numbers (TIFNs) are integrated into the CODAS method to solve the green supplier selection problem. In turn, Wang et al. (2020) presented the 2-tuple linguistic neutrosophic CODAS model. CODAS has also been expanded by Lan et al. (2021) to solve multiple attribute group decision making (MAGDM) issues with Interval-valued bipolar uncertain linguistic numbers (IVBULNs) on the basis of two kinds of distance measures and aggregating operators for risk assessment of mergers and acquisitions of Chinese enterprises.

Furthermore, in real-life problems, complex decision making situations with multiple and often conflicting objectives occur. In addition, the CODAS method is a new evaluation tool and has been proved to be efficient in dealing with MCDM problems. It has a systematic and simple computation procedure. Moreover, it can be assumed that a single decision maker is unable to express their opinions or preferences regarding multiple criteria decisions. On the other hand, in many situations, the DMs are unable to provide precise values and their information is vague and cannot be evaluated exactly in numerical values. This implies that Multi-Criteria Group Decision Making can be beneficial for selecting the optimal solution. Indeed, due to a greater imprecision and

vagueness of Group Decision Makers information, we suggest integrating rough set theory into CODAS. As mentioned above, a DMs' information cannot be evaluated exactly in numerical values for risk evaluation are usually uncertain, we choose to treat subjectivity and uncertainty in a group MCDM process through IRNs. We can see that although there exist papers that use IRNs in ranking methods and the aggregation operators, there has been no study on developing the CODAS method to solve multicriteria group decision making problems with IRNs. Therefore, in this paper we will approach Multi-Criteria Group Decision Making (MCGDM) problems to expand the CODAS method within Interval Rough Numbers to deal with imprecision and to develop a novel MCGDM method.

The structure of the rest of this paper is organized as follows. In Section 2, a general overview of the rough set approach as well as some fundamental concepts of Interval Rough Numbers will be presented. In Section 3, we will describe the proposed method based on IR-CODAS. In Section 4, the suggested approach will be applied to a case study of risk evaluation and a sensitivity analysis of the proposed IR-CODAS method will be performed. Finally, conclusions and suggestions will be presented.

2   Preliminaries

2.1 Rough set theory

RST is a mathematical formalism proposed in 1982 by Zdzisław Pawlak to support decision making processes. It generalizes classical set theory. A rough set is an important mathematical tool for dealing with imprecise, inconsistent and incomplete information and knowledge. This concept was introduced by Pawlak (1982).

The basic notions of RST are as follows: Indistinguishable relation on the set of actions (the objects of the decision), lower and upper approximation of a subset or of a partition of U, dependence and reduction of attributes from the set of attributes and decision rules identified with the decision classes.

For algorithmic reasons, the information about the objects is provided in the form of a data table, composed of a set of actions (alternatives) A (in rows) described by a set of attributes (criteria) R (in column). Each cell in this table indicates an assessment (quantitative or qualitative) of the object in that row using the attribute of the corresponding column. Formally, the data table can be defined by an information system S expressed by the 4-tuple S = {U, R, V, f}, R = C ∪ D, where U is a finite non-empty set of objects (called the universe), R is a finite nonempty set of attributes, the subsets C and D are called condition

attribute set and decision attribute set, respectively. $V = \bigcup_{a \in R} V_a$ where V_a is the set of values of attribute a and card(V_a) > 1, and $f : R \to V$ is an information or a description (Zhang, Xie and Wang, 2016).

Definition 1: Indiscernible relation (Zhang, Xie and Wang, 2016)

Indiscernibility arises when it is not possible to distinguish between elements of the same set. Given a subset of the attribute set B $\subseteq$ R, an indiscernible relation ind(B) on the universe U can be defined as follows:

$$ind(B) = \{(x, y)|\ (x, y) \in U^2, \forall_{b \in B}(b(x) = b(y))\} \tag{1}$$

Definition 2: Upper and lower approximation sets (Zhang, Xie and Wang, 2016)

Given an information system S = ⟨U; R; V; f⟩, for a subset X $\subseteq$ U, its lower and upper approximation sets are defined, respectively, by:

$$\overline{apr}(X) = \bigcup_{E_i \cap A \neq \varnothing} E_i = \{x \in U | [x] \cap X\} \neq \varnothing \tag{2}$$

$$\underline{apr}(X) = \bigcup_{E_i \subseteq A} E_i = \{x \in U | [x] \subseteq X\} \tag{3}$$

where $[x]$ denotes the equivalence class of x. The upper approximation $\overline{apr}(X)$ is the union of all elementary sets which have a nonempty intersection with A, while the lower approximation $\underline{apr}(X)$ is the union of all elementary sets which are subsets of A. In other words, the lower approximation contains the objects definitively belonging to the set, while the upper approximation contains the objects that can belong to the set. In fact, $\overline{apr}(X)$ is the largest compound set containing X, while $\underline{apr}(X)$ is the least compound set containing X.

For all the subsets $X, Y \subseteq$ U, the upper and lower approximations $\overline{apr}(X)$ and $\underline{apr}(X)$ satisfy the following properties (Pawlak, 1982):

(P1) $\underline{apr}(X) \subseteq X \subseteq \overline{apr}(X)$,

(P2) $\underline{apr}(\varnothing) = \overline{apr}(\varnothing) = \varnothing$,

(P3) $\underline{apr}(U) = \overline{apr}(U) = U$,

(P4) $\underline{apr}(X \cap Y) = \underline{apr}(X) \cap \underline{apr}(Y)$,

(P5) $\overline{apr}(X \cap Y) \subseteq \overline{apr}(X) \cap \overline{apr}(Y)$,

(P6) $\underline{apr}(X \cup Y) \supseteq \underline{apr}(X) \cup \underline{apr}(Y)$,

(P7) $\overline{apr}(X \cup Y) = \overline{apr}(X) \cup \overline{apr}(Y)$,

(P8) $\underline{apr}(X) = (\overline{apr}(X^c))^c$; $\overline{apr}(X) = (\underline{apr}(X^c))^c$,

(P9) $\underline{apr}(X) = \underline{apr}(\underline{apr}(X)) = \overline{apr}(\underline{apr}(X))$,

(P10) $\overline{apr}(X) = \overline{apr}(\overline{apr}(X)), = \underline{apr}(\overline{apr}(X))$,

where X^c = U – X denotes the complement of A.

The property (P1) says that the two operators determine a range in which the given set falls. The properties (P2) and (P3) are the conditions that the operators must satisfy at the two extreme points: Ø, or the minimum element and U, or the maximum element. The properties (P4)-(P7) describe weak distributivity and distributivity of the operators $\overline{apr}$ and $\underline{apr}$. The property (P8) states that the operator pair is double. Properties (P9) and (P10) state that the result of a double application of the new operators is identical to that of a single application. It is important to note that these properties are not independent.

The universe can be divided into three disjoint regions: the positive *POS(X)*, the bounded *BRN(X)* and the negative *NEG(X)* regions of *X* which are constructed from the equivalence classes:

$$POS(X) = \underline{apr}(X) \tag{4}$$
$$BRN(X) = \overline{apr}(X) - \underline{apr}(X) \tag{5}$$
$$NEG(X) = U - \overline{apr}(X) \tag{6}$$

If x ∈ *POS(X)*, then x belongs to the target set *X*.

If x ∈ *BRN(X)*, then x does not belong to the target set *X*.

If x ∈ *NEG(X)*, it cannot be determined whether x belongs to the target set *X* or not.

Definition 3: Definable sets (Zhang et al., 2016)

The empty set and the union of elementary sets are called compound or definable sets. Given an information system S = {U, R, V, f}, for any target subset $X \subseteq U$ and attribute subset $B \subseteq R$, if and only if $\underline{apr}(X) = \overline{apr}(X)$ (i.e. the bounded region BRN(X) = ∅), then *X* is called a definable set with respect to B.

Definition 4: Rough Sets (Zhang, Xie and Wang, 2016)

Given an information system S = {U, R, V, f}, for any target subset $X \subseteq U$ and attribute subset $B \subseteq R$, if and only if $\overline{apr}(X) \neq \underline{apr}(X)$ (i.e. the bounded region BRN(X) ≠ ∅), then X is called a rough set with respect to B, defined by $\left[\underline{apr}(X), \overline{apr}(X)\right]$.

2.2 Interval rough numbers

Suppose we have: a set of k classes representing the preferences of the decision maker DM, $P = (J_1, J_2, ..., J_k)$, which satisfies the condition $J_1 < J_2 < , ..., < J_k$ and another set of z classes that also represent the DM's preferences defined in the universe U, $P^* = (I_1, I_2, ..., I_z)$. Suppose that all the

objects recorded in an information table are defined in U and are linked to the preferences of the DM. In P*, each class of objects is represented by an interval $I_j = \{I_{ij}, I_{sj}\}$, provided that $I_{ij} \leq I_{sj}$ ($1 \leq j \leq m$) and $I_{ij} \leq I_{sj} \in P$, such that I_{ij} is the lower interval bound, while I_{sj} is the upper interval bound of the jth object class. Suppose U is the universe and let Y be an arbitrary element of U. If the upper and lower bounds of the object class are sorted, so that $I_{i1}^* < I_{i2}^* < \ldots < I_{ih}^*$ and $I_{s1}^* < I_{s2}^* < \ldots < I_{sk}^*$ ($1 \leq h, \ k \leq m$), then two new sets containing the lower object class $P_i^* = (I_{i1}^*, I_{i2}^*, \ldots, I_{ih}^*)$ and the upper objects class $P_s^* = (I_{s1}^*, I_{s2}^*, \ldots, I_{sk}^*)$ are defined. Then, for any class of objects $I_{ij}^* \in P$ with ($1 \leq j \leq h$) and $I_{sj}^* \in P$ with ($1 \leq j \leq k$), the lower approximations of I_{ij}^* and I_{sj}^* are defined as follows (Wang and Tang; 2011):

$$\underline{Apr}(I_{ij}^*) = \cup\, Y \in U/P_i^*(Y) \leq I_{ij}^* \tag{7}$$

$$\underline{Apr}(I_{sj}^*) = \bigcup Y \in U/P_s^*(Y) \leq I_{sj}^* \tag{8}$$

The upper approximations of I_{ij}^* and I_{sj}^* are defined by the following equations:

$$\overline{Apr}(I_{ij}^*) = \bigcup Y \in U/P_i^*(Y) \leq I_{ij}^* \tag{9}$$

$$\overline{Apr}(I_{sj}^*) = \bigcup Y \in U/P_s^*(Y) \leq I_{sj}^* \tag{10}$$

So both the lower class I_{ij}^* and the upper class I_{sj}^* are defined by their lower limits $\underline{Lim}(I_{ij}^*)$ and $\underline{Lim}(I_{sj}^*)$ and their upper limits $\overline{Lim}(I_{ij}^*)$ and $\overline{Lim}(I_{sj}^*)$:

$$\underline{Lim}(I_{ij}^*) = \frac{1}{M_I} \sum P_i^*(Y) | Y \in \underline{Apr}(I_{ij}^*) \tag{11}$$

$$\underline{Lim}(I_{sj}^*) = \frac{1}{M_I^*} \sum P_s^*(Y) | Y \in \underline{Apr}(I_{sj}^*) \tag{12}$$

where M_I and M_I^* are the sum of the objects in the lower approximation of the object classes I_{ij}^* and I_{sj}^*, respectively. The upper limits $\overline{Lim}(I_{ij}^*)$ and $\overline{Lim}(I_{sj}^*)$ are defined by:

$$\overline{Lim}(I_{ij}^*) = \frac{1}{M_S} \sum P_i^*(Y) | Y \in \overline{Apr}(I_{ij}^*) \tag{13}$$

$$\overline{Lim}(I_{sj}^*) = \frac{1}{M_S^*} \sum P_s^*(Y) | Y \in \overline{Apr}(I_{sj}^*) \tag{14}$$

where M_s et M_s^* are the sum of the objects in the upper approximation of the object classes I_{ij}^* and I_{sj}^*, respectively.

For the lower class, the rough boundary interval of I_{ij}^* is an interval between its lower and upper limits, denoted by $BR(I_{ij}^*)$, while for the upper class, the rough boundary interval of I_{sj}^* is $BR(I_{sj}^*)$:

$$BR(I_{ij}^*) = \overline{Lim}(I_{ij}^*) - \underline{Lim}(I_{ij}^*) \tag{15}$$

$$BR(I_{sj}^*) = \overline{Lim}(I_{sj}^*) - \underline{Lim}(I_{sj}^*) \tag{16}$$

Then, the uncertain class of objects I_{ij}^* and I_{sj}^* can be defined using their lower and upper limits:

$$R\big(I_{ij}^*\big) = \big[\underline{Lim}(I_{ij}^*), \overline{Lim}(I_{ij}^*)\big] \tag{17}$$

$$R\big(I_{sj}^*\big) = \big[\underline{Lim}(I_{sj}^*), \overline{Lim}(I_{sj}^*)\big] \tag{18}$$

As we can see, each class of objects is defined by its lower and upper limits that represent the interval rough number, which is defined as:

$$IR\big(I_j^*\big) = \big[R\big(I_{ij}^*\big), R\big(I_{sj}^*\big)\big] \tag{19}$$

Definition 5: The distance between two IRNs (Wang et al., 2016):

Let $A_1 = ([a_1,b_1][c_1,d_1])$ and $A_2 = ([a_2,b_2][c_2,d_2])$ be two IRNs. The distance between them can be defined as:

$$d(A_1, A_2) = \frac{|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |d_1 - d_2|}{4} \tag{20}$$

This satisfies the properties of distance measures, which are: $d(A_1, A_2) \geq 0$ and $d(A_1, A_2) = d(A_2, A_1)$.

Definition 6: Arithmetic Operations of IRNs (Wang et al., 2016):

Let $A_1 = ([a_1,b_1][c_1,d_1])$ and $A_2 = ([a_2,b_2][c_2,d_2])$ be two IRNs. We define:

$$A_1 + A_2 = ([a_1, b_1][c_1, d_1]) + ([a_2, b_2][c_2, d_2]) = \tag{21}$$
$$= ([a_1 + a_2, b_1 + b_2][c_1 + c_2, d_1 + d_2])$$

$$A_1 - A_2 = ([a_1, b_1][c_1, d_1]) - ([a_2, b_2][c_2, d_2]) = \tag{22}$$
$$= ([a_1 - a_2, b_1 - b_2][c_1 - c_2, d_1 - d_2])$$

$$A_1 \times A_2 = ([a_1, b_1][c_1, d_1]) \times ([a_2, b_2][c_2, d_2]) = \tag{23}$$
$$= ([a_1 \times a_2, b_1 \times b_2][c_1 \times c_2, d_1 \times d_2])$$

$$\frac{A_1}{A_2} = \frac{([a_1, b_1][c_1, d_1])}{([a_2, b_2][c_2, d_2])} = \left(\left[\frac{a_1}{a_2}, \frac{b_1}{b_2}\right]\left[\frac{c_1}{c_2}, \frac{d_1}{d_2}\right]\right) \tag{24}$$

$$k \times A_1 = k \times ([a_1, b_1][c_1, d_1]) =$$
$$= \begin{cases} ([k \times a_1, k \times b_1][k \times c_1, k \times d_1]) & if \ k > 0 \\ ([k \times b_1, k \times a_1][k \times d_1, k \times c_1]) & if \ k < 0 \end{cases} \quad (25)$$

3   The IR-CODAS method

IR-CODAS is our proposed approach integrating IRNs into the CODAS multicriteria method. It allows modeling imprecision and fuzziness of the information provided.

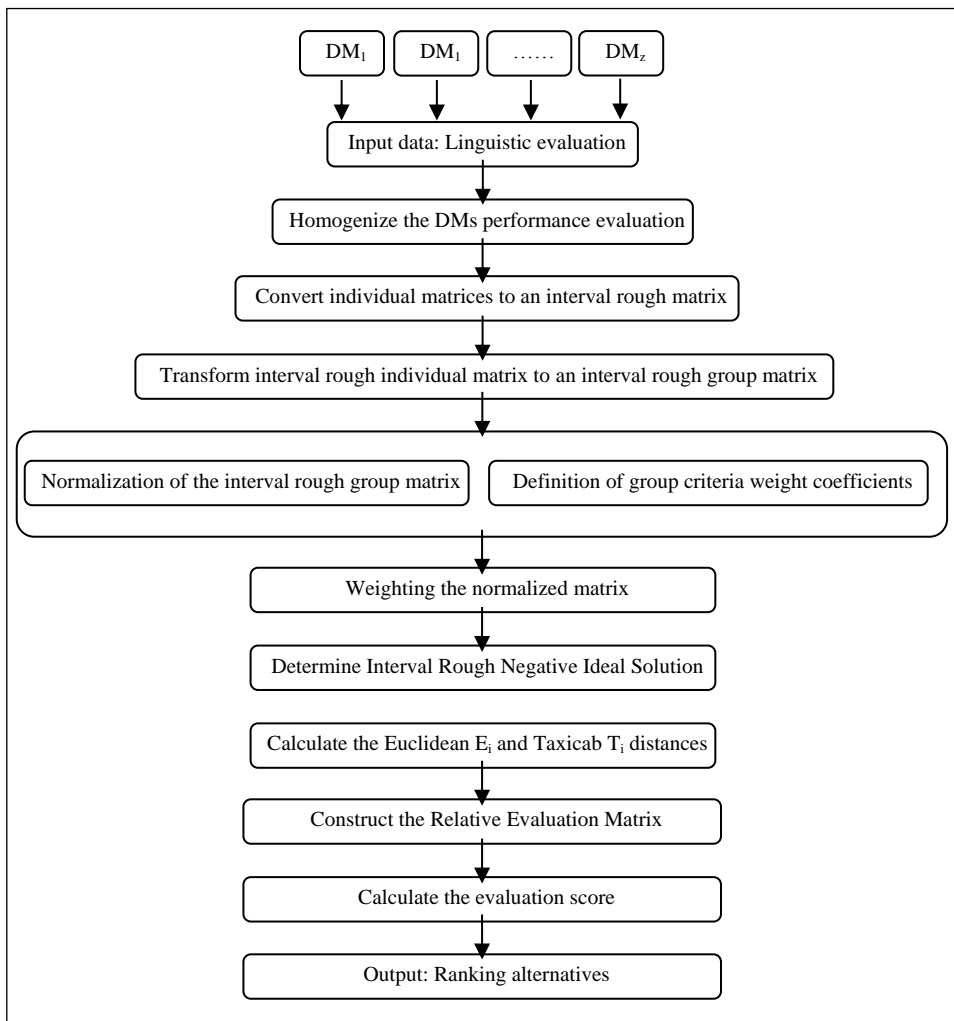As presented in Figure 1, IR-CODAS consists of the following steps:

Figure 1: The structure of the proposed IR-CODAS method for an MCGDM problem

Step 1: Define a multi-criteria decision making model that consists of m alternatives $A_i(i = 1, 2, …, m)$, n criteria C_j ($j = 1, 2, …, n$) and a team of k DMs, who evaluate alternatives according to all criteria. Every p^{th} DM presents his evaluation in the following matrix:

$$X^k = \left[x_{ij}^p; x_{ij}^{p*}\right]_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ … \\ A_m \end{array} \overset{\begin{array}{cccc} C_1 & C_2 & … & C_n \end{array}}{\left[\begin{array}{cccc} x_{11}^p; x_{11}^{p*} & x_{12}^p; x_{12}^{p*} & … & x_{1n}^p; x_{1n}^{p*} \\ x_{21}^p; x_{21}^{p*} & x_{22}^p; x_{22}^{p*} & … & x_{2n}^p; x_{2n}^{p*} \\ … & … & … & … \\ x_{m1}^p; x_{m1}^{p*} & x_{m2}^p; x_{m2}^{p*} & … & x_{mn}^p; x_{mn}^{p*} \end{array}\right]_{m \times n}} \quad (26)$$

where x_{ij}^p and x_{ij}^{p*} are the linguistic variables of the pth DM ($p \in \{1, 2, …, z\}$) for the ith alternative ($i \in \{1, 2, …, m\}$) according to j^{th} criterion ($j \in \{1, 2, …, n\}$). Thus, matrices $X^1, X^2, …., X^p, .., X^k$ are obtained using performance rating for m alternatives on n criteria provided by p DMs.

Step 2: Homogenize the performance evaluations of the DMs. For each DM, matrix X^k is determined by DMs' evaluations and qualitative criterion evaluates alternatives using the following linguistic expressions provided by the group of DMs, taking into account the type of criteria (benefit or cost). As in Stevic et al. (2017), we use linguistic terms where the value of each pair x_{ij}^p is converted to an integer, as shown in Table 1.

Table 1: Linguistic scale for evaluating the alternatives

Linguistic terms	Benefit Criteria (Max)	Cost Criteria (Min)
Very Poor (VP)	1	9
Poor (P)	3	7
Medium (M)	5	5
Good (G)	7	3
Very Good (VG)	9	1

Step 3: Using equations (1-12) we convert the individual matrices to an interval rough matrix $Z^p = \left[IR\left(x_{ij}^p\right)\right]_{m \times n} \forall\ p = 1, …, z$:

$$Z^p = \begin{array}{c} \\ A_1 \\ A_2 \\ … \\ A_m \end{array} \overset{\begin{array}{cccc} C_1 & C_2 & … & C_n \end{array}}{\left[\begin{array}{cccc} IR\left(x_{11}^p\right) & IR\left(x_{12}^p\right) & … & IR\left(x_{1n}^p\right) \\ IR\left(x_{21}^p\right) & IR\left(x_{22}^p\right) & … & IR\left(x_{2n}^p\right) \\ … & … & … & … \\ IR\left(x_{m1}^p\right) & IR\left(x_{m2}^p\right) & … & IR\left(x_{mn}^p\right) \end{array}\right]_{m \times n}} \quad (27)$$

Step 4: Transform the individual interval rough matrix Z^p to a group interval rough matrix $Z = \left[IRG(x_{ij})\right]_{m \times n} \ \forall\, i = 1, \ldots, m$ and $\forall\, j = 1, \ldots, n$:

$$IRG(x_{ij}) = \frac{1}{z} \sum_{p=1}^{k} IR(x_{ij}^p) \tag{28}$$

where z is the total number of DMs.

Step 5: Normalize the elements of the group interval rough matrix Z using equation (29):

$$IR(t_{ij}) = \left(\left[t_{ij}^i, t_{ij}^s\right], \left[t_{ij}'^i, t_{ij}'^s\right]\right) =$$

$$= \begin{cases} \left(\left[\dfrac{x_{ij}^i}{\max\limits_i x_{ij}'^s}, \dfrac{x_{ij}^s}{\max\limits_i x_{ij}'^i}\right], \left[\dfrac{x_{ij}'^i}{\max\limits_i x_{ij}^s}, \dfrac{x_{ij}'^s}{\max\limits_i x_{ij}^i}\right]\right) & if \ j \in N_b \\[2ex] \left(\left[\dfrac{\min\limits_i x_{ij}^i}{x_{ij}'^s}, \dfrac{\min\limits_i x_{ij}^s}{x_{ij}'^i}\right], \left[\dfrac{\min\limits_i x_{ij}'^i}{x_{ij}^s}, \dfrac{\min\limits_i x_{ij}'^s}{x_{ij}^i}\right]\right) & if \ j \in N_c \end{cases} \tag{29}$$

where N_b and N_c are the sets of profit and cost criteria, respectively. In addition, $\min_i x_{ij}$ and $\max_i x_{ij}$ are the minimum and maximum values of the bounded approximate interval of the criteria, respectively.

The elements $IR(t_{ij})$ of the normalized matrix (N) are:

$$N = \left[IR(t_{ij})\right]_{m \times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ \ldots \\ A_m \end{array} \overset{\begin{array}{cccc} C_1 & C_2 & \ldots & C_n \end{array}}{\begin{bmatrix} IR(t_{11}) & IR(t_{12}) & \ldots & IR(t_{1n}) \\ IR(t_{21}) & IR(t_{22}) & \ldots & IR(t_{2n}) \\ \ldots & \ldots & \ldots & \ldots \\ IR(t_{m1}) & IR(t_{m2}) & \ldots & IR(t_{mn}) \end{bmatrix}_{m \times n}} \tag{30}$$

Step 6: Definition of group criteria weight coefficients:

$$w_j = \frac{1}{z} \sum_{p=1}^{k} w_j^p \tag{31}$$

where w_j^p is the importance of jth criterion ($j \in \{1, 2, \ldots, n\}$) provided by the pth DM ($p \in \{1, 2, \ldots, z\}$).

Step 7: Weighting the previous normalized group interval rough matrix R by multiplying the obtained matrix with weighted values of the criteria:

$$IR(r_{ij}) = w_j \times IR(t_{ij}) = \left(\left[r_{ij}^i, r_{ij}^s\right], \left[r_{ij}'^i, r_{ij}'^s\right]\right)_{m \times n} =$$
$$= \left[w_j t_{ij}^i, w_j t_{ij}^s\right], \left[w_j t_{ij}'^i, w_j t_{ij}'^s\right] \tag{32}$$

where w_j is the importance of jth criterion.

We obtain the following weighted normalized group interval rough matrix:

$$R = \begin{bmatrix} ([r_{11}^i, r_{11}^s], [r_{11}'^i, r_{11}'^s]) & ([r_{12}^i, r_{12}^s], [r_{12}'^i, r_{12}'^s]) & \cdots & ([r_{1n}^i, r_{1n}^s], [r_{1n}'^i, r_{1n}'^s]) \\ ([r_{21}^i, r_{21}^s], [r_{21}'^i, r_{21}'^s]) & ([r_{22}^i, r_{22}^s], [r_{22}'^i, r_{22}'^s]) & \cdots & ([r_{2n}^i, r_{2n}^s], [r_{2n}'^i, r_{2n}'^s]) \\ \cdots & \cdots & \cdots & \cdots \\ ([r_{m1}^i, r_{m1}^s], [r_{m1}'^i, r_{m1}'^s]) & ([r_{m2}^i, r_{m2}^s], [r_{m2}'^i, r_{m2}'^s]) & \cdots & ([r_{mn}^i, r_{mn}^s], [r_{mn}'^i, r_{mn}'^s]) \end{bmatrix}_{m \times n} \tag{33}$$

Step 8: Determine the Interval Rough negative ideal solution $IR(NIS_j)$ $(j \in \{1, 2, \ldots, n\})$:

$$IR(NIS_j) = [NIS_j]_{1 \times m} \tag{34}$$

$$NIS_j = \min_i IR(r_{ij}) = [NIS_j^i, NIS_j^s][NIS_j'^i, NIS_j'^s] =$$
$$= \left[\min_i r_{ij}^i, \min_i r_{ij}^s\right]\left[\min_i r_{ij}'^i, \min_i r_{ij}'^s\right] \tag{35}$$

Step 9: Calculate the Euclidean E_i and Taxicab T_i distances of alternatives i $(i \in \{1, \ldots, m\})$ from the $IR(NIS_j)$ as follows:

$$E_i = \sqrt{\frac{\sum_{j=1}^m [(r_{ij}^i - NIS_j^i)^2 + (r_{ij}^s - NIS_j^s)^2 + (r_{ij}'^i - NIS_j'^i)^2 + (r_{ij}'^s - NIS_j'^s)^2]}{4}} \tag{36}$$

$$T_i = \frac{\sum_{j=1}^m [|r_{ij}^i - NIS_j^i| + |r_{ij}^s - NIS_j^s| + |r_{ij}^i - NIS_j^i| + |r_{ij}^i - NIS_j^i|]}{4} \tag{37}$$

The Euclidean and Taxicab distances are converted from IRNs to crisp numbers.

Step 10: Construct the Relative Evaluation Matrix Re:

$$Re = [h_{ik}]_{n \times n} \tag{38}$$

$$h_{ik} = (E_i - E_k) + \left(\psi(E_i - E_k) \times (T_i - T_k)\right) \tag{39}$$

where $k \in \{1, 2, \ldots, n\}$ and ψ is a threshold function to determine the equality of the Euclidean distances of two alternatives, defined as follows:

$$\psi(E_i - E_k) = \begin{cases} 1 & if \ |E_i - E_k| \geq \tau \\ 0 & if \ |E_i - E_k| < \tau \end{cases} \tag{40}$$

In this function, τ is the threshold parameter that can be set by the decision maker. It is suggested to set this parameter at a value between 0.01 and 0.05.

Step 11: Calculate the evaluation score H_i of each alternative i ($i \in \{1, 2, …, m\}$):

$$H_i = \sum_{k=1}^{n} h_{ik} \tag{41}$$

Step 12: Rank the alternatives according to the decreasing values of evaluation score H_i. The alternative with the highest evaluation score is the most desirable alternative.

4 Application of the IR-CODAS model for risk assessment

The Sfax "Hannibal" gas processing plant produces natural gas, diesel fuel, hydrogen sulfide, sulfuric acid, potassium hydrate, etc. The Sulfox unit of the Hannibal British Gas industry is focused on energy recovery, specifically the transfer of hydrogen sulfide gas H_2S to sulfuric acid H_2SO_4.

The gas treatment process generates several risks. Thus, the need to assess the risks and to know the most important of them in order to take the necessary precautions is essential to prevent them. For this reason, we test the applicability of the proposed IR-CODAS model under uncertain environment for MCGDM to the risk assessment problem. After a preliminary screening, we established that there are five types of risks of H_2S gas emissions into the atmosphere: Explosion (A_1), Fire (A_2), Leak (A_3), Respiratory fatigue (A_4) and Dysfunction of control devices (A_5). These risks are the alternatives of our model and they are evaluated by a committee of three decision makers (DM) according to four criteria: Security (C_1), Frequency of exposure (C_2), Degree of severity (C_3) and Environmental impact (C_4), where C_1 is a benefit criterion and the others are cost criteria.

Step 1: After the DMs' evaluation of criteria, the study consider four criteria that are evaluated by a linguistic scale in three matrices (Table 2).

Table 2: Linguistic Assessing Matrix by three DMs

	DM$_1$				DM$_2$				DM$_3$			
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
A_1	G, VG	P, VP	VG, VG	G, G	M, G	VP, VP	G, VG	VG, G	G, G	P, M	VG, VG	VG, VG
A_2	G, VG	P, VP	VG, VG	G, VG	VG, VG	VP, VP	G, G	M, G	G, VG	P, P	M, VG	G,VG
A_3	G, VG	G, M	G, VG	M, G	G,VG	G, P	G, VG	M, G	VG, VG	G, VG	M, VG	P, M
A_4	M, G	G, M	M, G	VP, VP	G , G	M, M	G, G	VP, VP	M ,VG	G, M	M, G	VP, P
A_5	M, G	VG, M	M, G	P, M	G, VG	VG, M	G, VG	P, P	M, G	VG, G	M, VG	VP, P

Step 2: Using Table 1, we transformed the linguistic input values, which are recorded in Table 2, into integer data shown in Table 3.

Table 3: Evaluation of alternatives by three DMs

	DM$_1$				DM$_2$				DM$_3$			
	C$_1$	**C$_2$**	**C$_3$**	**C$_4$**	**C$_1$**	**C$_2$**	**C$_3$**	**C$_4$**	**C$_1$**	**C$_2$**	**C$_3$**	**C$_4$**
A$_1$	7, 9	7, 9	1, 1	3, 3	5, 7	9, 9	1, 3	1, 3	7, 7	5, 7	1, 1	1, 1
A$_2$	7, 9	7, 9	1, 1	1, 3	9, 9	9, 9	3, 3	3, 5	7, 9	7, 7	1, 5	1, 3
A$_3$	7, 9	3, 5	1, 3	3, 5	7, 9	3, 7	1, 3	1, 5	9, 9	1, 3	1, 5	5, 7
A$_4$	5, 7	3, 5	3, 5	9, 9	7, 7	5, 5	3, 3	9, 9	5, 9	5, 7	3, 5	7, 9
A$_5$	5, 7	1, 5	3, 5	5, 7	7, 9	3, 5	1, 3	7, 7	5, 7	1, 3	1, 1	7, 9

Step 3: According to Table 3 and Equations (1-12), we convert the individual matrices to an interval rough matrix.

As an example of calculating the evaluation for the position A$_4$-C$_1$, we select the object classes x_{41}^p and x_{ij}^{p*}. Each class contains three elements:

$$x_{41}^p = \{\,5; 7; 5\}$$
$$x_{41}^{p*} = \{7; 7; 9\}$$

By applying expressions (7-14), we form rough sequences for each object class.

For the first object class we get:

$\underline{Lim}(5) = 5 \qquad \overline{Lim}(5) = \frac{1}{3}(5 + 7 + 5) = 5.67 \qquad RN(5) = [5; 5.67]$

$\underline{Lim}(7) = \frac{1}{3}(5 + 7 + 5) = 5.67 \qquad \overline{Lim}(7) = 7 \qquad RN(7) = [5.67; 7]$

For the second object class we get:

$\underline{Lim}(7) = 7 \qquad \overline{Lim}(7) = \frac{1}{3}(7 + 7 + 9) = 7.67 \qquad RN(7) = [7; 7.67]$

$\underline{Lim}(9) = \frac{1}{3}(7 + 7 + 9) = 7.67 \qquad \overline{Lim}(9) = 9 \qquad RN(9) = [7.67; 9]$

On the basis of rough sequences, we obtain for each DM the following interval rough numbers:

IRN(DM$_1$) = [5; 5.67] [7; 7.67],
IRN(DM$_2$) = [5.67; 7] [7; 7.67],
IRN(DM$_3$) = [5; 5.67] [7.67; 9].

In our case study, the evaluation of alternatives by three decision makers has been performed using interval rough numbers as shown in Table 4.

Table 4: Initial interval rough matrix for three DMs

DM$_1$				
	C$_1$	C$_2$	C$_3$	C$_4$
A$_1$	[6.33; 7][7.66; 9]	[6; 8][8.33; 9]	[1; 1][1; 1.67]	[1.67; 3][2.33; 3]
A$_2$	[7; 7.67][9; 9]	[7; 7.67][8.33; 9]	[1; 1.67][1; 2.33]	[1; 1.67][3; 3.67]
A$_3$	[7; 7.66][9; 9]	[2.33; 3][5; 7]	[1; 1][3; 3.67]	[2; 3][5; 5]
A$_4$	[5; 5.67][7; 7.66]	[3; 4.33][5; 5.66]	[3; 3][4.33; 5]	[8.33; 9][9; 9]
A$_5$	[5; 5.67][7; 7.66]	[1; 1.67][4.33; 5]	[1.67; 3][3; 5]	[5; 6.33][7; 7.67]
DM$_2$				
A$_1$	[5; 6.33][7; 7.66]	[7; 9][8.33; 9]	[1; 1][1.67; 3]	[1; 1.67][2.33; 3]
A$_2$	[7.67; 9][9; 9]	[7.67; 9][8.33; 9]	[1.67; 3][2.33; 3]	[1.67; 3][3.67; 5]
A$_3$	[7; 7.66][9; 9]	[2.33; 3][5; 7]	[1; 1][3; 3.67]	[1; 3.67][5; 5]
A$_4$	[5.67; 7][7; 7.67]	[4.33; 5][5; 5.66]	[3; 3][3; 4.33]	[8.33; 9][9; 9]
A$_5$	[5.67; 7][7.66; 9]	[1.67; 3][4.33; 5]	[1; 1.67][2; 4]	[6.33; 7][7; 7.67]
DM$_3$				
A$_1$	[6.33; 7][7; 7.66]	[5.7; 7][8.33]	[1; 1][1; 1.67]	[1; 1.67][1; 2.33]
A$_2$	[7.67; 9][9; 9]	[7; 7.67][7; 8.33]	[1; 1][3.67; 5]	[1; 1.67][3; 3.67]
A$_3$	[7.66; 9][9; 9]	[1; 2.33][3; 5]	[5; 6.33][9; 9]	[3; 4.33][5; 7]
A$_4$	[5; 5.67][7.67; 9]	[4.33; 5][5.66; 7]	[3; 3][4.33; 5]	[7; 8.33][9; 9]
A$_5$	[5; 5.67][7; 7.66]	[1; 1.67][3; 4.33]	[1; 1.67][1; 3]	[6.33; 7][7.67; 9]

Step 4: In this step, the DMs' individual evaluations can be fused into the group assessing matrix with IRNs using Equation (28). So, for the sequence x_{41} we obtain:

$$IRG(x_{41}^i) = \frac{x_{41}^{i1} + x_{41}^{i2} + x_{41}^{i3}}{z} = \frac{5 + 5.67 + 5}{3} = 5.22$$

$$IRG(x_{41}^s) = \frac{x_{41}^{s1} + x_{41}^{s2} + x_{41}^{s3}}{z} = \frac{5.67 + 7 + 5.67}{3} = 6.11$$

$$IRG(x_{41}^{\prime i}) = \frac{x_{41}^{\prime i1} + x_{41}^{\prime i2} + x_{41}^{\prime i3}}{z} = \frac{7 + 7 + 7.67}{3} = 7.22$$

$$IRG(x_{41}^{\prime s}) = \frac{x_{41}^{\prime s1} + x_{41}^{\prime s2} + x_{41}^{\prime s3}}{z} = \frac{7.67 + 7.67 + 9}{3} = 8.11$$

Then $IRG(x_{41}) = [5.22; 6.11][7.22; 8.11]$.

For our case, using Table 4 and Equation (28), we transform individual interval rough matrix to a group interval rough matrix shown in Table 5.

Table 5: Interval Rough Group Matrix

GIR	C_1	C_2	C_3	C_4
A_1	[5.89; 6.78][7.22; 8.11]	[6; 8][7.89; 8.78]	[1; 1][1.22; 2.11]	[1.22; 2.11][1.89; 2.78]
A_2	[7.45; 8.56][9; 9]	[7.22; 8.11][7.89; 8.78]	[1.22; 2.11][1.89; 2.78]	[1.22; 2.11][3.22; 4.11]
A_3	[7.22; 8.11][9; 9]	[1.89; 2.78][4; 6]	[1; 1][3.22; 4.11]	[2.22; 4.56][5; 5]
A_4	[5.22; 6.11][7.22; 8.1]	[3.89; 4.78][5.22; 6.11]	[3; 3][3.89; 4.78]	[7.89; 8.78][9; 9]
A_5	[5.22; 6.11][7.22; 8.11]	[1.22; 2.11][3.89; 4.78]	[1.22; 2.11][2; 4]	[5.89; 6.78][7.22; 8.11]

Step 5: Equation (29) is applied to normalize the Interval Rough Group Matrix (Table 5) and we obtain the results listed in Table 6.

An example of calculating a normalized matrix for the cost criteria C_2:

$$IRG(x_{42}) = \left[\frac{\min_i x_{i2}^i}{x_{42}^{\prime s}}; \frac{\min_i x_{i2}^s}{x_{42}^{\prime i}}\right]\left[\frac{\min_i x_{i2}^{\prime i}}{x_{42}^s}; \frac{\min_i x_{i2}^{\prime s}}{x_{42}^i}\right] = \left[\frac{1.22}{6.11}; \frac{2.11}{5.22}\right]\left[\frac{3.89}{4.78}; \frac{4.78}{3.89}\right]$$
$$= [0.2; 0.4][0.81; 1.23]$$

An example of calculating a normalized matrix for the benefit criteria C_1:

$$IRG(x_{31}) = \left[\frac{x_{31}^i}{\max_i x_{i1}^{\prime s}}; \frac{x_{31}^s}{\max_i x_{i1}^{\prime i}}\right]\left[\frac{x_{31}^{\prime i}}{\max_i x_{i1}^s}; \frac{x_{31}^{\prime s}}{\max_i x_{i1}^i}\right]$$
$$= \left[\frac{7.22}{9}; \frac{8.11}{9}\right]\left[\frac{9}{8.56}; \frac{9}{7.45}\right] = [0.8; 0.9][1.05; 1.2]$$

Table 6: Normalized Interval Rough Matrix

	C_1	C_2	C_3	C_4
A_1	[0,65; 0.75][0.84; 1.09]	[0,14; 0.27][0.49; 0.8]	[0,47; 0.82][1.22; 2.11]	[0,44; 1.12][0.9; 2.28]
A_2	[0,83; 0.95][1.05; 1.21]	[0,14; 0.27][0.48; 0.66]	[0,36; 0.53][0.58; 1.73]	[0,3; 0.66][0.9; 2.28]
A_3	[0,8; 0.9][1.05; 1.21]	[0,2; 0.53][1.4; 2.53]	[0,21; 0.26][0.41; 0.7]	[0,14; 0.23][0.22; 0.35]
A_4	[0,58; 0.68][0.84; 1.09]	[0,2; 0.4][0.81; 1.23]	[0,13; 0.26][0.41; 0.7]	[0,14; 0.23][0.22; 0.35]
A_5	[0.58; 0.68][0.84; 1.09]	[0,26; 0.54][1.84; 3.92]	[0,25; 0.5][0.58; 1.59]	[0,15; 0.29][0.28; 0.47]

Step 6: The relative importance weights of the four criteria provided by the DMs are assumed to be crisp numbers which are presented in Table 7. Then we define the group criteria weight coefficient using Equation (31).

Table 7: The relative importance weights of the four criteria by the three DMs

Criterion	DM_1	DM_2	DM_3
C_1	0,48	0,4	0,3
C_2	0,01	0,1	0,2
C_3	0,47	0,3	0,2
C_4	0,04	0,2	0,3

Step 7: Weighting the previous normalized group interval rough matrix (Table 6) by Equation (32).

Step 8 and Step 9: After normalizing and calculating the weighted normalized matrix, we determine the *IR(NIS)*, the Euclidean and Taxicab distances of alternatives given in Table 8.

Table 8: Weighted Normalized Group Interval Rough Matrix

	C_1	C_2	C_3	C_4	E_i	T_i
A_1	[0,26; 0,3][0,33; 0,43]	[0.01; 0.03][0.05; 0.08]	[0.15; 0.27][0.4;0.68]	[0.08; 0.2][0.16; 0.41]	0,42	0,44
A_2	[0,33; 0,38][0,41; 0,48]	[0.01; 0.03][0.05; 0.07]	[0.12; 0.17][0.19;0.56]	[0.05; 0.12][0.16; 0.41]	0,36	0,33
A_3	[0,32; 0,36][0,41; 0,48]	[0.02; 0.05][0.14; 0.26]	[0.07; 0.08][0.132; 0.23]	[0.02; 0.04][0.04; 0.06]	0,12	0,13
A_4	[0,23; 0,27][0,33; 0,43]	[0.02; 0.04][0.08; 0.13]	[0.04; 0.08][0.13; 0.23]	[0.02; 0.04][0.04; 0.06]	0,03	0,03
A_5	[0,23; 0,27][0,33; 0,43]	[0.03; 0.06][0.19; 0.4]	[0.08; 0.16][0.19; 0.51]	[0.03; 0.05][0.05; 0.08]	0,25	0,25
IR(NIS)	[0,23; 0,27][0,33; 0,43]	[0.01; 0.03][0.05; 0.07]	[0.04; 0.08][0.13; 0.23]	[0.02; 0.04][0.04; 0.06]		

Step 10: Construct the Relative Evaluation Matrix Re by using Table 8 and Equations (38-40) with the threshold parameter τ set to 0.03 (Table 9).

Table 9: Relative Evaluation matrix

	A_1	A_2	A_3	A_4	A_5	H_i	Rank
A_1	0,00	0,17	0,61	0,80	0,36	1,94	1
A_2	−0,17	0,00	0,44	0,63	0,19	1,08	2
A_3	−0,61	−0,44	0,00	0,19	−0,25	−1,10	4
A_4	−0,80	−0,63	−0,19	0,00	−0,44	−2,06	5
A_5	−0,36	−0,19	0,25	0,44	0,00	0,13	3

Step 11: We compute the value of the evaluation score of each alternative using Table 9 and Equation (41):

$$H_1 = 1.94; H_2 = 1.08; H_3 = -1.10; H_4 = -2.06; H_5 = 0.13$$

Step 12: We rank the alternatives in decreasing order. Evidently, the order is A_1-A_2-A_5-A_3-A_4 and from the above findings it follows that A_1 is the most dangerous risk among the five alternatives in this case study.

The traditional crisp CODAS method evaluates alternatives using crisp numbers. Indeed, crisp values as input data are insufficient to model real-life situations and complex concepts with multiple and often conflicting objectives which frequently occur in multicriteria decision aid. For instance, in risks assessment some criteria are considered very important and the way of indicating their importance needs to be more flexible. The linguistic term "very good" can be preferably expressed an IRN rather than a single crisp number. However, in this paper, we use IRNs to assess the risks, since DMs can flexibly express their opinions using linguistic terms.

On the other hand, the proposed distance-based IR-CODAS method used two types of distance in evaluation process: Euclidean distance and Taxicab distance which helps to increase the precision of ranking results in group decision making process (which is accompanied by a great amount of uncertainty and subjectivity). An interval structure can be used to synthesize the decision rules provided by the DMs. Thus, in this study, we have introduced the theory of rough sets, an approach based on IRNs for representing uncertainty in group decision making. So, IR-CODAS transforms individual linguistic matrices into interval rough matrices with different size of interval to capture preference uncertainty of the DMs.

Furthermore, a sensitivity analysis is performed to determine the effect of the different threshold parameters on the rankings. According to step 10, the Relative Evaluation Matrix Re depends on the threshold parameter τ that denotes the degree of closeness of the Euclidean distances of two alternatives.

Table 10: Difference of Euclidean distance

	A_1	A_2	A_3	A_4	A_5
A_1	0,00	0,06	0,30	0,39	0,17
A_2	−0,06	0,00	0,23	0,32	0,11
A_3	−0,30	−0,23	0,00	0,09	−0,12
A_4	−0,39	−0,32	−0,09	0,00	−0,21
A_5	−0,17	−0,11	0,12	0,21	0,00

From the absolute value of the difference of Euclidean distance given in Table 10, it can be seen that all differences exceed 0.05. Hence, the evaluation score H_i of each alternative is the same. Even if we increase the value of τ and disregard the condition $0.01 \leq \tau \leq 0.05$, Table 11 shows that there are no changes in the rankings despite the differences in the threshold function values.

Table 11: Evaluation score H_i and ranking results with different values of τ

Alternatives	$\tau = 0,03$		$\tau = 0,07$		$\tau = 0,1$	
	H_i	Rank	H_i	Rank	H_i	Rank
A_1	1,94	1	1,84	1	1,84	1
A_2	1,08	2	1,08	2	1,08	2
A_3	−1,10	4	−1,10	4	−1,20	4
A_4	−2,06	5	−2,06	5	−1,96	5
A_5	0,13	3	0,13	3	0,13	3

However, the weight coefficients of the evaluation criteria have a great influence on the results. Hence, we compute the final ranking of the alternatives by replacing the group procedure by the individual procedure, i.e. we omit step 6 and keep the importance coefficients provided by each DM; at the end, the DMs' individual evaluations scores H_i^p can be fused into the collective evaluation score GH_i for each alternative. The final ranking orders of alternatives is shown in Table 12.

Table 12: Individual and group evaluations score matrix

Alternatives	H_i^1	H_i^2	H_i^3	GH_i	Rank
A_1	0,26	1,19	0,23	0,56	3
A_2	0,90	0,92	−0,10	0,58	2
A_3	0,31	−0,30	0,28	0,10	4
A_4	−0,94	−2,70	−3,17	−2,27	5
A_5	−0,54	0,88	2,75	1,03	1

As can be seen, dysfunction of control devices risk (A_5) is the most dangerous. Clearly, changes in the procedure of calculating criteria weights leads to a change in the ranks of individual alternatives, which confirms that the model is sensitive to changes in weight coefficients. Compared to the previous results, we can notice that the first ranked alternatives (A_1, A_2 and A_5) are the most important and it is necessary to take essential precautions to prevent them.

5   Conclusion

The CODAS method is a simple and easily applicable multi-criteria decision making method. To handle uncertainty, it is impossible to provide data with crisp numbers in an adequate way. Therefore, we propose to develop a subjective model using linguistic evaluation. Since the group decision making process proceeds in an uncertain environment, this assessment is complex. Thus, our proposed approach IR-CODAS refers to the integration of the interval rough numbers into the CODAS methods to solve group decision making problems under uncertainty.

The applicability of the proposed model is validated through a real-life case study of the gas processing industry in Sfax. Namely, our IR-CODAS approach was applied to select the most important risks in order to take the necessary precautions to prevent them. A sensitivity analysis was conducted, confirming the validity of the final results. We changed the threshold parameters values which do not influence the ranking of alternatives. Furthermore, we choose to test the final ranking using the individual procedure of each DM.

Future research intends to develop a preference disaggregation approach deducing criteria weight values and threshold parameters from the information provided by the DMs. As well, we aim to integrate interval rough numbers into other methods and develop new MCDM methods.

References

He T., Zhang S., Wei G., Wang R., Wu J., Wei C. (2020), *CODAS Method for 2-tuple Linguistic Pythagorean Fuzzy Multiple Attribute Group Decision Making and Its Application to Financial Management Performance Assessment*, Technological and Economic, Development of Economy, 26(4), 920-932.

Kao Y.-S., Nawata K., Huang C.-Y. (2019), *Systemic Functions Evaluation Based Technological Innovation System for the Sustainability of IoT in the Manufacturing Industry*, Sustainability, 11(8), 2342, https://doi.org/10.3390/su11082342.

Keshavarz Ghorabaee M., Amiri M., Zavadskas E.K., Hooshmand R., Antucheviciene J. (2017), *Fuzzy Extension of the CODAS Method for Multi-criteria Market Segment Evaluation*, Journal of Business Economics and Management, 18, 1-19.

Keshavarz Ghorabaee M., Zavadskas E.K., Turskis Z., Antucheviciene J. (2016), *A New Combinative Distance-based Assessment (CODAS) Method for Multi-criteria Decision--making*, Economic Computation & Economic Cybernetics Studies & Research, 50(3), 25-44.

Lan J., Wu J., Guo Y., Wei C., Wei G., Gao H. (2021), *CODAS Methods for Multiple Attribute Group Decision Making with Interval-valued Bipolar Uncertain Linguistic Information and Their Application to Risk Assessment of Chinese Enterprises' Overseas Mergers and Acquisitions*, Economic Research-Ekonomska Istraživanja, https://doi.org/10.1080/1331677X.2020.1868323.

Lu H.W., Huang G.H., He L. (2011), *An Inexact Rough-Interval Fuzzy Linear Programming Method for Generating Conjunctive Water-Allocation Strategies to Agricultural Irrigation Systems*, Applied Mathematical Modelling, 35, 4330-4340.

Niakan F., Vahdani B., Mohammadi M. (2015), *A Multi-objective Optimization Model for Hub Network Design under Uncertainty: An Inexact Rough-interval Fuzzy Approach*, Engineering Optimization, 47(12), 1670-1688.

Pamucar D., Badi I., Sanja K., Obradovic R. (2018), *A Novel Approach for the Selection of Power Generation Technology Using a Linguistic Neutrosophic CODAS Method: A Case Study in Libya*, Energies, 11(9), 2489, https://doi.org/10.3390/en11092489.

Pamucar D., Božanić D., Lukovac V., Komazec N. (2018), *Normalized Weighted Geometric Bonferroni Mean Operator of Interval Rough Numbers − Application in Interval Rough DEMATEL-COPRAS Model*, Facta Universitatis, Series: Mechanical Engineering, 16(2), 171-191.

Pamucar D., Chatterjee K., Zavadskas E.K. (2019), *Assessment of Third-party Logistics Provider Using Multi-criteria Decision-making Approach Based on Interval Rough Numbers*, Computers & Industrial Engineering, 127, 383-407.

Pamucar D., Edmundas S.Z., Zavadskas K. (2018), *Integration of Interval Rough AHP and Interval Rough MABAC Methods for Evaluating University Web Pages*, Appl. Soft Comput., 67, 141-163.

Pamucar D., Mihajlovic M., Obradovic R., Atanaskovic P. (2017), *Novel Approach to Group Multi-Criteria Decision Making Based on Interval Rough Numbers: Hybrid DEMATEL-ANP--MAIRCA Model*, Expert Systems with Applications, 88, 58-80.

Pamucar D., Petrovic I., Cirovic G. (2018), *Modification of the Best-Worst and MABAC Methods: A Novel Approach Based on Interval-valued Fuzzy-rough Numbers*, Expert Systems with Applications, 91, 89-106.

Pawlak Z. (1982), *Rough Sets*, International Journal of Parallel Programming, 11, 341-356.

Remadi F.D., Frikha H.M. (2020), *The Triangular Intuitionistic Fuzzy Numbers CODAS Method for Solving Green Material Selection Problem*, 2020 International Multi-Conference on Organization of Knowledge and Advanced Technologies (OCTA).

Roy J., Das S., Kar S., Pamučar D. (2019), *An Extension of the CODAS Approach Using Interval--Valued Intuitionistic Fuzzy Set for Sustainable Material Selection in Construction Projects with Incomplete Weight Information*, Symmetry, 11, 393.

Song W., Cao J. (2017), *A Rough DEMATEL-based Approach for Evaluating Interaction between Requirements of Product-service System*, Computers & Industrial Engineering, 110, 353-363.

Song W., Ming X., Wu Z., Zhu B. (2014), *A Rough TOPSIS Approach for Failure Mode and Effects Analysis in Uncertain Environments*, Qual. Reliab. Eng. Int., 30, 473-486.

Stevic´ Ž., Pamucˇar D., Zavadskas E.K., C´irovic´ G., Prentkovskis O. (2017), *The Selection of Wagons for the Internal Transport of a Logistics Company: A Novel Approach Based on Rough BWM and Rough SAW Methods,* Symmetry, 9, 264.

Wang J., Tang P. (2011), *A Rough Random Multiple Criteria Decision-making Method Based on Interval Rough Operator*, Control and Decision Making, 26(7), 1056-1059.

Wang J.Q., Kuang J.J., Wang J., Zhang, H.Y. (2016), *An Extended Outranking Approach to Rough Stochastic Multi-criteria Decision-making Problems*, Cognitive Computation, 8(6), 1144-1160.

Wang P., Wang J., Wei G., Wu J., Wei C., Wei Y. (2020), *CODAS Method for Multiple Attribute Group Decision Making Under 2-Tuple Linguistic Neutrosophic Environment*, Informatica, 31(1), 161-184.

Yeni F.B., Özçelik G. (2019), *Interval-Valued Atanassov Intuitionistic Fuzzy CODAS Method for Multi Criteria Group Decision Making Problems*, Group Decision and Negotiation, 28, 433-452.

Zhang Q., Xie Q., Wang G. (2016), *A Survey on Rough Set Theory and Its Applications*, CAAI Transactions on Intelligence Technology, 1(4), 323-333.