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Multi-objective data envelopment analysis: A game of multiple attribute decision-making

Accepted by Editor Ewa Ziemia | Received: April 15, 2019 | Revised: June 13, 2019 | Accepted: June 17, 2019.

Abstract

Aim/purpose – The traditional data envelopment analysis (DEA) is popularly used to evaluate the relative efficiency among public or private firms by maximising each firm's efficiency: the decision maker only considers one decision-making unit (DMU) at one time; thus, if there are n firms for computing efficiency scores, the resolution of n similar problems is necessary. Therefore, the multi-objective linear programming (MOLP) problem is used to simplify the complexity.

Design/methodology/approach – According to the similarity between the DEA and the multiple attribute decision making (MADM), a game of MADM is proposed to solve the DEA problem. Related definitions and proofs are provided to clarify this particular approach.

Findings – The multi-objective DEA is validated to be a unique MADM problem in this study: the MADM game for DEA is eventually identical to the weighting multi-objective DEA. This MADM game for DEA is used to rank ten LCD companies in Taiwan for their research and development (R&D) efficiencies to show its practical application.

Research implications/limitations – The main advantage of using an MADM game on the weighting multi-objective DEA is that the decision maker does not need to worry how to set these weights among DMUs/objectives, this MADM game will decide the weights among DMUs by the game theory. However, various DEA models are eventually evaluation tools. No one can guarantee us with 100% confidence that their evaluated results of DEA could be the absolute standard. Readers should analyse the results with care.

Originality/value/contribution – A unique link between the multi-objective CCR DEA and the MADM game for DEA is established and validated in this study. Previous scholars seldom explored and developed this breathtaking view before.

Keywords: Multi-Objective Linear Programming (MOLP), Data Envelopment Analysis (DEA), Multiple Attribute Decision Making (MADM), Research and Development (R&D) efficiency.

JEL Classification: C44, C57.

1. Introduction

Past research has shown that data envelopment analysis (DEA) defines the mathematical programming of output/input ratio as the index of production efficiency, as developed by Charnes, Cooper, & Rhodes (1978), and followed by many authors, for example, Banker, Charnes, & Cooper (1984), Golany (1988), Kao (1994) etc.

Some scholars supported the appropriateness of using multi-objective linear programming (MOLP) in a DEA model by the concept of Pareto efficiency/frontier (Chen, Larbani, & Chang, 2009; Stewart, 1996). DEA and MOLP both search for a set of non-inferior solutions (Steuer, 1986; Zeleny, 1973); thus, solving the DEA problem by a multi-objective programming aspect is natural, reasonable and appropriate (Li & Reeves, 1999). The first work integrating DEA and MOLP is due to Golany (1988), who proposed an interactive multi-objective procedure (IMOLP – interactive MOLP) to determine efficient output levels. The algorithm consists of sequential solutions to a set of related linear programming problems, in which the objective function is to maximise a weighted sum of the former objectives. Li & Reeves (1999) presented a multi-objective model that considers two additional efficiency measures: the minimisation of the sum of the DMU distances to the frontier (minisum) and the minimisation of the most significant distance (minimax), in addition to the maximisation of the classical efficiency in DEA. Kornbluth (1991) noticed that the DEA model could be identical to a multi-objective linear fractional programming problem. The objective function of the model has the same expression as in the original model by Charnes et al. (1978), which is abbreviated as the CCR model, but applied to maximise the efficiency of every DMU, instead of one at a time, the restrictions remaining unchanged. Also, Joro (1998) made an extension of the Value Efficiency Analysis method to determine targets for inefficient DMUs. Joro et al. (1998) observing the problem of characterizing efficient facets, made a structural

comparison of the CCR (Charnes et al., 1978) and BCC (Banker et al., 1984) models with the reference point approach to solving multi-objective problems.

Various DEA models are eventually evaluation tools. No one can guarantee us with 100% confidence that their evaluated results of DEA could be the absolute standard. The Multiple Objective Linear Programming (MOLP) problem is commonly used to find the trade-off boundary among many conflicting objectives in the real world (Cohon, 1978), and of course, the trade-off boundary surely defines the core of MOLP. This paper focuses on an interesting issue: the weighting method for the multi-objective DEA model and the game of multiple attribute decision making: MADM game (Chen, 2004, 2006). In the weighting approach for MOLP, the weighting method guarantees the weighting Pareto optimum among all DMUs. Coincidentally, if we view the DEA problem from the MADM game perspective, this game leads to the same weighting concept for MOLP. This particular game also reduces numerous computations of traditional DEA models: the weight for each DMU/objective could be achieved at the same time by the two-person zero-sum game theory. Chen & Larbani (2006) once mentioned the use of two-person zero-sum game on the multiple attribute decision making (MADM) problems: they call it an MADM game, some scholars also extended this idea in various forms (Kacher & Larbani, 2008; Larbani, 2009). In this study, according to the characteristics of multiple attributes in DEA, it is interesting that we can compute the DEA weights from the MADM game perspective, and this new view will be available in this paper. The weighting multi-objective DEA and the MADM game of DEA will lead to the same results: the weighting Pareto equilibrium.

This paper is organised as follows: the MADM game, traditional CCR DEA and weighting MOLP are reviewed in Section 2. A modified DEA model: the multi-objective DEA model with its slack analysis is proposed in Section 3. In Section 4 an actual example is used to validate this interesting idea. This example is a case study of the R&D efficiency of Taiwan high-tech industry. Finally, conclusions and recommendations are given in Section 5.

2. Literature review

In this section, we will review the basic concepts of MADM game, CCR DEA with applications and MOLP.

2.1. MADM game

Multiple Attribute Decision Making (MADM) is a management science technique, which is popularly used to rank the priority of alternatives for their competing attributes (Goodwin & Wright, 2004; Hwang & Yoon, 1981). Weights are the core of MADM and DEA: it is evident that different weights lead to different evaluation results and decisions. Several approaches are valuable for assessing the weights of MADM problems, e.g. the eigenvector method, ELECTRE, and TOPSIS. However, very few scholars had ever explored the two-person zero-sum game on MADM issues until Chen & Larbani (2006). The brief introduction of MADM game begins from a simple two-person zero-sum game as follows.

2.1.1. Fundamentals of two-person zero-sum game

A two-person zero-sum game is the simplest case of game theory with two players only. A decision maker resolves such a game by assuming that both players propose pure (discrete), mixed (probability), or continuous strategies. Cooperation may exist in games, but in most cases, non-cooperation is more attractive because it is more realistic, especially in the presence of competition between players. Only the non-cooperative case is discussed in this study. A two-person zero-sum game is eventually a matrix:

$$D = \begin{bmatrix} a_{11} & a_{12} \dots & a_{1n} \\ a_{21} & a_{22} \dots & a_{2n} \\ \dots & & \\ a_{m1} & a_{m2} \dots & a_{mn} \end{bmatrix} \quad (1)$$

where D represents the pay-off matrix of player A (the first player). Each element in D is initially assumed to be a positive real number in (1); if each element is negative, we need to change the following inequalities accordingly. The pay-off matrix of the second player B is $-D$; thus, the sum of A's pay-off matrix and B's pay-off matrix is exactly zero (zero-sum).

Definition 2.1. A vector x in IR^m is said to be a mixed strategy of player A if it satisfies the following probability condition:

$$x^t e_m = 1 \quad (2)$$

where the components of $x = [x_1, x_2, \dots, x_m]^t$ are greater than or equal to zero; e_m is a $m \times 1$ vector, where each element is equal to 1 in e_m . The pay-off of the first player A is defined by D . The pay-off matrix of the second player B is $-D$; thus, the sum of A's pay-off matrix and B's pay-off matrix is precisely zero (zero-sum); e_n is a $n \times 1$ vector, where each component is equal to 1 in e_n . Similarly, a mixed strategy of player B is defined by $y = [y_1, y_2, \dots, y_n]^t$ and $y^t e_n = 1$.

Remark 2.1. Please note an interesting observation: if x and y are the normalised weight vectors. However, this view, i.e. the weight point, is seldom considered in any game book.

Definition 2.2. If players A and B propose the mixed strategies, respectively, then the expected pay-off of player A is defined by:

$$x^t D y = \sum_{j=1}^n \sum_{i=1}^m a_{ij} x_i y_j. \quad (3)$$

Since we deal with a zero-sum game, the expected pay-off of player B is $-x^t D y$. Based on (3), the optimal strategies (Nash equilibrium) of players are defined as follows.

Definition 2.3. Player A's mixed strategy x^* and player B's mixed strategy y^* are said to be optimal strategies under the Nash equilibrium in the game (1) (Larbani, 2009) if $x^t D y^* \leq x^{*t} D y^*$ and $x^{*t} D y \leq x^{*t} D y^*$, for any mixed strategies x and y . Player A's objective is maximising his pay-off over all possible x when player B chooses his best strategy y^* ; on the contrary, Player B's objective is minimising his pay-off over all possible y when player A chooses his best strategy x^* .

Remark 2.3. Various definitions of equilibrium inevitably lead to multiple mathematical conditions of x and y ; however, only the Nash equilibrium is used in this study. The Nash equilibrium is: whether it is from the view of Player A or Player B, no one can propose better (x^{**}, y^{**}) , which are not (x^*, y^*) , but satisfies $x^{*t} D y^{**} \leq x^{**t} D y^{**}$ and $x^{**t} D y^* \leq x^{**t} D y^{**}$. In other words, no player is willing to drop the current x^* and y^* to find another better x^{**} and y^{**} because their pay-offs by x^{**} and y^{**} are worse than those of equilibrium by x^* and y^* .

Lemma 2.1. Consider the two-person zero-sum game (1). Any solution x^* to the following optimisation problem:

$$\text{Max}_x \{ \text{Min} \{ \sum_{i=1}^m a_{i1} x_i, \sum_{i=1}^m a_{i2} x_i, \dots, \sum_{i=1}^m a_{in} x_i \} \} \quad (4)$$

$$\text{st } x' e_m = 1$$

$$x \geq 0$$

is an optimal strategy of player A. Here $x \geq 0$ means that all components of x are greater than or equal to zero. While any solution y^* of the following optimization problem:

$$\text{Min}_y \{ \text{Max} \{ \sum_{j=1}^n a_{1j} y_j, \sum_{j=1}^n a_{2j} y_j, \dots, \sum_{j=1}^n a_{mj} y_j \} \} \quad (5)$$

$$\text{st } y' e_n = 1$$

$$y \geq 0$$

is an optimal strategy of player B.

Remark 2.4. Player A's decision is using the Max-Min principle for his strategy x , i.e. player A first minimises his expected pay-off according to a different strategy y of B, and then maximises his minimal expected pay-off by x . On the contrary, player B's decision is using the Min-Max principle for his strategy y , i.e. player B first maximises his expected pay-off according to a different strategy x of A, and then maximises his maximal expected pay-off by y .

Finding optimal strategies of players by solving problems (4) and (5) is a common task in the game theory: Theorem 2.1 helps us to find the optimal strategies of players (Osborne & Rubinstein, 1994).

Theorem 2.1. Given a pay-off matrix D , the optimal strategies defined via (4) and (5) are solved by the following problems:

$$\text{Min}_{x'} \quad x'' e_m \quad (6)$$

$$\text{st } x'' D \geq e_n'$$

$$x' \geq 0$$

and

$$\text{Max}_{y'} \quad y'' e_n \quad (7)$$

$$\text{st } Dy' \leq e_m$$

$$y' \geq 0.$$

2.1.2. MADM game

The idea of the MADM game was initially proposed by Chen (2004, 2006) and completed in the work of Chen & Larbani (2006). This concept is straight-forward: let’s view the game (1) as an MADM problem and the pay-off matrix as the decision matrix. Since no relevant literature of MADM game was proposed before, the following idea strictly follows previous papers and assumes that the MADM process is a two-person zero-sum game, which is played by the decision maker and Nature. Nature is not rational, not malicious, and does not have any idea for the decision maker’s preference on each alternative. The new approach presented in the paper is supported by the decision maker’s conservative strategy. Why and how such an interesting thought comes? It is easy to observe that every manager makes several decisions every day against, e.g. limited time, limited information, limited resources, etc. But each manager still wants a better decision, although everything is limited, i.e. against him. Therefore, this two-person zero-sum game is undoubtedly appropriate for resolving the conflicts of a decision maker, and it can be solved by Theorem 2.1 (Osborne & Rubinstein, 1994).

Definition 2.4. Suppose a decision maker has m alternatives ($Alt_i, i = 1, 2, \dots, m$) concerning n attributes ($C_j, j = 1, 2, \dots, n$) in an MADM process, then his crisp decision matrix D is defined as follows:

$$D = \begin{matrix} & C_1 & C_2 & \dots & C_n \\ \begin{matrix} Alt_1 \\ Alt_2 \\ \dots \\ Alt_m \end{matrix} & \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \end{matrix} \quad (8)$$

where $a_{ij} \geq 0$ represents the evaluation of alternative i with respect to attribute j ; $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$. Similarly, if $a_{ij} \leq 0$, we just need to simply change the inequalities of problems (6)-(7) accordingly.

Definition 2.5. For a crisp D , the expected score of alternative i : $ES(Alt_i)$ is defined as follows:

$$ES(Alt_i) = x_i^* \sum_{j=1}^n a_{ij} y_j^* \quad (9)$$

where x_i^*, y_j^* are the components of optimally mixed strategies x^*, y^* in Definition 2.1, x_i^*, y_j^* can be easily found by Theorem 2.1.

Remark 2.1. This definition shows that the performance of an alternative in MADM is computed just like computing the probable outcome in a two-person zero-sum game when player A decides his final strategy. The higher the $ES(Alt_i)$, the more the alternative Alt_i is preferred. Although the MADM game is quite simple, there is an interesting link between the MADM game and the weighting multi-objective DEA, this will be explored and validated later.

2.2. CCR DEA model with applications

The DEA model, developed by Charnes et al. (1978), is the mathematical programming model that modifies defects of the Farrell model, which is unable to handle several inputs and outputs. Within this model, it is assumed that there are n decision-making units (DMUs), with m inputs and p outputs, while the efficiency evaluation model of k^{th} DMU can be defined as in Equation (10).

$$\begin{aligned} \text{Max } f_k &= \frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} & (10) \\ \text{st } \frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} &\leq 1, k=1, 2, \dots, n; \\ u_r &\geq \varepsilon, r=1, 2, \dots, p; \\ v_i &\geq \varepsilon, i=1, 2, \dots, m. \end{aligned}$$

where:

- x_{ik} – the i^{th} input value for k^{th} DMU,
- y_{rk} – the r^{th} output value for the k^{th} DMU,
- u_r – the weight values of the output,
- v_i – the weight values of the input,
- ε – a small positive value.

It is difficult to obtain the solution from Equation (10) because it is a non-linear programming problem. Charnes et al. (1978) transformed the Equation (10) into a linear programming problem in Equation (11), by which a decision maker can more easily obtain a solution.

$$\begin{aligned}
 \text{Max } \theta_k &= \sum_{r=1}^p u_r y_{rk} & (11) \\
 \text{st } \sum_{i=1}^m v_i x_{ik} &= 1 \\
 \sum_{r=1}^p u_r y_{rk} - \sum_{i=1}^m v_i x_{ik} &\leq 0, \quad k = 1, 2, \dots, n; \\
 u_r &\geq \varepsilon, \quad r = 1, 2, \dots, p; \\
 v_i &\geq \varepsilon, \quad i = 1, 2, \dots, m.
 \end{aligned}$$

where θ_k is the efficiency value for the k^{th} DMU, θ_k is a crisp number under x_{ik} and y_{rk} . The traditional CCR DEA model is useful for computing each DMU's efficiency from one by one, then decide the overall efficiency for each DMU. Suppose there are n DMUs for efficiency evaluation, the problem (11) must be solved n times. If a decision maker applies the multi-objective approach to the CCR DEA model, it is valuable to reduce the n computations to just one time. This multi-objective DEA problem is presented in Section 3.1.

There are still many applications of CCR DEA model after 2010. For example, Chen & Jia (2017) introspected China's rapid economic growth, which led to tremendous pressure on natural resources. They used the CCR DEA method with slack analysis to evaluate the environmental efficiencies of China's industry from 2008 to 2012. Yang, Ouyang, Fang, Ye, & Zhang (2015) measured the ecological efficiencies in China during the period of 2000-2010, and concluded that environmental efficiencies across 30 provinces show regional disparities. Barros & Athanassiou (2015) compared the seaport efficiency between Greece and Portugal, they tried to find out those best practices that will lead to improved performance in the context of European seaport policy. Yang, Wu, Liang, Bi, & Wu (2011) proposed the production possibility set, and a supply chain of CCR DEA model to appraise the overall technical efficiency of supply chains. Mousavi-Avval, Rafiee, Jafari, & Mohammadi (2011) used the CCR DEA to estimate the energy efficiencies of soybean producers based on eight energy inputs including human labour, diesel fuel, machinery, fertilisers, chemi-

cal, water for irrigation, electricity and seed energy, and single output of grain yield. They ranked efficient and inefficient farmers, then identified optimal energy requirement and wasteful uses of energy.

2.3. Multi-objective programming problem

We introduce the traditional linear programming with q linear objective functions as follows (Steuer, 1986; Stewart, 1996; Zeleny, 1973):

$$\begin{aligned} \text{Max } z(\mathbf{x}) &= (z_1(\mathbf{x}), z_2(\mathbf{x}), \dots, z_q(\mathbf{x}))^T & (12) \\ \text{s.t. } \mathbf{Ax} &\leq \mathbf{b}, \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where:

$z_k(\mathbf{x})$ – an objective function for the k -th objective, $k=1,2,\dots,q$,

\mathbf{x} – the decision variable vector, $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$,

\mathbf{b} – the Right Hand Side (RHS) vector, $\mathbf{b} = (b_1, b_2, \dots, b_m)^T$,

\mathbf{A} – the coefficient matrix, $\mathbf{A} = [a_{ij}]_{m \times n}$.

There are also many resolution approaches for the problem (12), e.g. the weighting method or the distance method (Steuer, 1986). The simple weighting approach is used in this study. That is, once the weight for the k -th objective is determined as w_k and $\sum_{k=1}^q w_k = 1$, then the problem (12) can be resolved by the following weighting method:

$$\begin{aligned} \text{Max } z(\mathbf{x}) &= \sum_{k=1}^q w_k z_k(\mathbf{x}) & (13) \\ \text{s.t. } \mathbf{Ax} &\leq \mathbf{b}, \mathbf{x} \geq \mathbf{0} \end{aligned}$$

3. Research methodology

Here, the exciting link between the MADM game and the weighting multi-objective DEA will be explored and validated.

3.1. Multi-objective DEA

Let us consider the Eq. (10) again for a given DMU k :

$$\begin{aligned} \text{Max } f_k &= \frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} & (14) \\ \text{st } \frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} &\leq 1, \quad k = 1, 2, \dots, n; \\ u_r &\geq \varepsilon, \quad r = 1, 2, \dots, p; \\ v_i &\geq \varepsilon, \quad i = 1, 2, \dots, m. \end{aligned}$$

Now, we drop the assumption $\sum_{i=1}^m v_i x_{ik} = 1$, normalising the weighted input of the k -th DMU in (14).

We introduce our multi-objective approach for DEA as follows:

a) since $f_k = \frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}}$ and $\frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \leq 1$ are assumed in the CCR DEA model,

then $f_k = \frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \leq 1$ for a given k -th DMU;

b) we use the multi-objective approach here to maximise the efficiency of all DMUs at the same time; and the referred standard of efficiency will be a trade-off boundary among the performances ($f_k, k = 1, 2, \dots, n$) of n DMUs;

c) since $f_k = \frac{\sum_{r=1}^p u_r y_{rk}}{\sum_{i=1}^m v_i x_{ik}} \leq 1$ in (a), it is easy to show that $g_k = \sum_{i=1}^m v_i x_{ik} - \sum_{r=1}^p u_r y_{rk} \geq 0$

for each DMU, and the ideal optimum of g_k should be zero.

Here $g_k = \sum_{i=1}^m v_i x_{ik} - \sum_{r=1}^p u_r y_{rk}$ is now used to replace those above f_k ; thus, we can deduce the multi-objective CCR DEA model by (b) as follows:

$$\text{Min } g_1 = \sum_{i=1}^m v_i x_{i1} - \sum_{r=1}^p u_r y_{r1} \quad (15)$$

$$\text{Min } g_2 = \sum_{i=1}^m v_i x_{i2} - \sum_{r=1}^p u_r y_{r2}$$

$$\text{Min } g_n = \sum_{i=1}^m v_i x_{in} - \sum_{r=1}^p u_r y_{rn}$$

$$\text{st } \sum_{r=1}^p u_r y_{rk} - \sum_{i=1}^m v_i x_{ik} \leq 0, k=1,2,\dots,n;$$

$$u_r \geq \varepsilon, r=1,2,\dots,p;$$

$$v_i \geq \varepsilon, i=1,2,\dots,m.$$

When g_k is zero, this also implies that the k -th DMU satisfies $\sum_{r=1}^p u_r y_{rk} = \sum_{i=1}^m v_i x_{ik}$.

Now, we introduce the weighting method of MOLP from Eq. (13) to minimise the gap between the ideal vector of $(g_1^*, g_2^*, \dots, g_n^*) = (0, 0, \dots, 0)$, then the problem (15) can be rewritten as:

$$\text{Min } \sum_{k=1}^n w_k g_k \quad (16)$$

$$\text{st } \sum_{r=1}^p u_r y_{rk} - \sum_{i=1}^m v_i x_{ik} \leq 0, k=1,2,\dots,n;$$

$$u_r \geq \varepsilon, r=1,2,\dots,p;$$

$$v_i \geq \varepsilon, i=1,2,\dots,m.$$

where $g_k = \sum_{i=1}^m v_i x_{ik} - \sum_{r=1}^p u_r y_{rk}$, $k=1,2,\dots,n$.

Here w_k represents the weight of the k -th DMU. If we introduce a dual variable for each constraint, λ_k, s_{1s} and s_{2r} , we get the following slack analysis problem once the w_k is determined:

$$\begin{aligned}
 & \text{Max } \varepsilon \left(\sum_{r=1}^p s_{2r} + \sum_{i=1}^m s_{1i} \right) \tag{17} \\
 & \text{st } s_{2r} - \sum_{k=1}^n \lambda_k y_{rk} \leq - \sum_{l=1}^n w_k y_{rk}, \quad r=1,2,\dots,p; \\
 & \quad s_{1i} + \sum_{k=1}^n \lambda_k x_{ik} \leq \sum_{k=1}^n w_k x_{ik}, \quad i=1,2,\dots,m; \\
 & \quad \lambda_k, s_{2r}, s_{1i} \geq 0.
 \end{aligned}$$

According to the MOLP theory, in general, there could be many points in the trade-off boundary (efficient frontier), when we set the unique w_k . The problem (17) is used for slack analysis by referring the given w_k to show the inefficient DMUs how to adjust their input/output to increase their efficiencies. Since the model (16) is valuable in measuring global efficiency, it is used for the later analysis of R&D efficiency example. The remained difficulty for the problem (16) is how can we decide the weight w_k ? The MADM game for DEA in the next section seems to be a right answer.

3.2. MADM game for DEA

Now, let's revisit the MADM game, which has the following special decision matrix:

$$D = \begin{matrix} & v'_1 & v'_2 & \dots & v'_m & u'_1 & u'_2 & \dots & u'_p \\ \begin{matrix} w_1, \text{DMU}_1 \\ w_2, \text{DMU}_2 \\ \dots \\ w_n, \text{DMU}_n \end{matrix} & \begin{bmatrix} -x_{11}, -x_{21}, \dots, -x_{m1}, y_{11}, y_{21}, \dots, y_{p1} \\ -x_{12}, -x_{22}, \dots, -x_{m2}, y_{12}, y_{22}, \dots, y_{p2} \\ \dots \\ -x_{1n}, -x_{2n}, \dots, -x_{mn}, y_{1n}, y_{2n}, \dots, y_{pn} \end{bmatrix} \end{matrix} \tag{18}$$

Here w_k also represents the normalised weight of the k -th DMU, where $d = \sum_{i=1}^m v_i + \sum_{r=1}^p u_r$, then $v'_i = \frac{v_i}{d}$ represents the normalised DEA weight of the i -th

input, and $u'_r = \frac{u_r}{d}$ represents the normalised DEA weight of the r -th output.

The following propositions show an interesting link between the two-person zero-sum game (18) and the problem (16).

Proposition 3.1. Consider the MADM game (18), if the w_k^* satisfies the equilibrium of game (18), then this w_k^* automatically satisfies the weighting objective optimum (weighting Pareto optimum) of the problem (16): i.e. $\sum_{k=1}^n w_k^* g_k \geq \sum_{k=1}^n w_k^* g_k^*$.

Here $g_k^* = \sum_{i=1}^m v_i^* x_{ik} - \sum_{r=1}^p u_r^* y_{rk}$.

Proof. This proof is simple: let's first assume the w_k^* satisfies the equilibrium of MADM game (20). And please recall the equilibrium of game (1) is $x^t D y^* \leq x^{*t} D y^*$ and $x^* D y \leq x^* D y^*$. Consider the two-person zero-sum game (18) for MADM, now the decision matrix D is exactly in (18), if we replace the x vector by $w = [w_1, w_2, \dots, w_n]$ and replace the y vector by $[v'_1, v'_2, \dots, v'_m, u'_1, u'_2, \dots, u'_p]_{m+p,1}$; thus, the following conditions are held for each DMU:

$$w_k \left(\sum_{k=1}^n u'_{rk} y_{rk} - \sum_{k=1}^n v'_{ik} x_{ik} \right) \leq w_k^* \left(\sum_{k=1}^n u'_{rk} y_{rk} - \sum_{k=1}^n v'_{ik} x_{ik} \right), \forall k \equiv w_k g_k^* \geq w_k^* g_k^*, \forall k \quad (19)$$

and

$$w_k^* \left(\sum_{k=1}^n u'_{rk} y_{rk} - \sum_{k=1}^n v'_{ik} x_{ik} \right) \leq w_k^* \left(\sum_{k=1}^n u'_{rk} y_{rk} - \sum_{k=1}^n v'_{ik} x_{ik}^* \right), \forall i \equiv w_k^* g_k \geq w_k^* g_k^*, \forall k \quad (20)$$

Then summing up the Eq.(20) for $k = 1, 2, \dots, n$, it is clear we get: $\sum_{k=1}^n w_k^* g_k \geq \sum_{k=1}^n w_k^* g_k^*$. This means the w_k^* generated in the game (18) is identical to aggregate the multiple objectives of the problem (16) by weighting concept.

Proposition 3.2. Consider the game (18), which additionally includes three constraints: (a) $\sum_{r=1}^p u_r y_{rk} - \sum_{i=1}^m v_i x_{ik} \leq 0$, $k = 1, 2, \dots, n$, (b) $u_r \geq \varepsilon$, $r = 1, 2, \dots, p$; and (c) $v_i \geq \varepsilon$, $i = 1, 2, \dots, m$. Then such a modified game is a weighting multi-objective

DEA model, which is identical to solving the problem (16) with an additional constraint: $\sum_{i=1}^m v_i + \sum_{r=1}^p u_r = d$ by the w_k^* weighting.

Proof. If we want to conclude that two optimising problems are synonymous, we can compare their objectives and constraints between them simultaneously. The Proposition 3.1 already validates the w_k^* in (18) is identical to the weighting concept to aggregate the multiple objectives in (16). Now, the remained part is to check the constraints. We introduce the synonymous problems of Theorem 2.1:

$$\text{Max}_x \quad v_A \tag{21}$$

$$\text{st } x^t D \geq v_A e_n^t$$

$$x^t e_m = 1, \quad x \geq 0.$$

$$\text{Min}_y \quad v_B \tag{22}$$

$$\text{st } Dy \leq v_B e_m$$

$$y^t e_n = 1, \quad y \geq 0.$$

Here $v_A = \text{Min} \left\{ \sum_{i=1}^m a_{i1}x_i, \sum_{i=1}^m a_{i2}x_i, \dots, \sum_{i=1}^m a_{in}x_i \right\}$ and $v_B = \text{Max} \left\{ \sum_{j=1}^n a_{1j}y_j, \sum_{j=1}^n a_{2j}y_j, \dots, \sum_{j=1}^n a_{mj}y_j \right\}$. In addition, $x' = \frac{x}{v_A}$ and $y' = \frac{y}{v_B}$, if we replace the $v_A (v_B)$ by d , replace x vector by w and replace the y vector by $[v', u']_{m+p,1} = [v'_1, v'_2, \dots, v'_m, u'_1, u'_2, \dots, u'_p]_{m+p,1}$, then the problems (21)-(22) are deduced to the game (18), which can be solved by the problems (23)-(24):

$$\text{Max}_x \quad d \tag{23}$$

$$\text{st } w^t D \geq d e_n^t$$

$$w^t e_m = 1, \quad w \geq 0.$$

$$\text{Min}_y \quad d \tag{24}$$

$$\text{st } D[v', u']_{m+p,1} \leq d e_m$$

$$[v', u']_{m+p,1}^t e_n = 1, \quad [v', u']_{m+p,1} \geq 0.$$

The problem (23) decides the w_k^* , and the problem (24) determines the (u_r^*, v_i^*) . But when readers compare the problem (24) with the multi-objective DEA problem (16), it is obvious that the problem (24) lacks the constraints:

$$(a) \sum_{r=1}^p u_r y_{rk} - \sum_{i=1}^m v_i x_{ik} \leq 0, \quad k = 1, 2, \dots, n, \quad (b) \quad u_r \geq \varepsilon, \quad r = 1, 2, \dots, p, \quad \text{and} \quad (c) \quad v_i \geq \varepsilon,$$

$i = 1, 2, \dots, m$. On the contrary, if we compare the problem (16) with the problem (24), it is clear that the problem (16) lacks the normalisation condition:

$$\sum_{i=1}^m v_i + \sum_{r=1}^p u_r = d.$$

Thus, this study proposes the new multi-objective DEA to completely bridge the gap between the problem (24) and the problem (16). That is, we present a new problem: Eq. (24) combining with (a), (b) and (c) is sufficient to represent the new multi-objective DEA problem: Eq. (16) combining with

$$\text{an additional constraint: } \sum_{i=1}^m v_i + \sum_{r=1}^p u_r = d.$$

4. Research findings and discussions

We compute the scores of R&D efficiencies from ten liquid-crystal display (LCD) companies in Taiwan as an actual application. First of all, the earlier literature of this industry (Chan, 2003; Khalil, 2000; Murphy, Trailer, & Hill, 1996; Naik & Chakravarty, 1992; Tseng, Chiu, & Chen, 2009) is reviewed to define appropriate input and output variables. Three input variables are defined as the operational cost, the number of R&D faculty, and the R&D expense. The output variables are defined as the net profit and the number of patents. The normalisation process of the actual data of these companies is dividing each column element by the maximal value in its corresponding column. The normalised value will be ranging from 0 to 1 in Table 1.

Table 1. Normalised R&D data of ten LCD companies

Company	Inputs			Outputs	
	Operational cost	Number of R&D faculty	R&D expense	Net profit	Number of patents
<i>l</i>	<i>2</i>	<i>3</i>	<i>4</i>	<i>5</i>	<i>6</i>
A	1.000	1.000	1.000	1.000	1.000
B	0.701	0.193	0.835	0.703	0.403
C	0.420	0.415	0.714	0.361	0.210
D	0.343	0.081	0.307	0.284	0.081
E	0.350	0.060	0.329	0.287	0.133
F	0.058	0.202	0.088	0.053	0.036

Table 1 cont.

<i>l</i>	2	3	4	5	6
G	0.241	0.023	0.164	0.239	0.250
H	0.258	0.088	0.167	0.237	0.274
I	0.122	0.085	0.282	0.101	0.238
J	0.018	0.007	0.030	0.015	0.044

When ε is set to 0.001, we resolve the modified MADM game for d and (u_r^*, v_i^*) by Eq. (23)-(24), then from the MADM game perspective for DEA, we found $d^* = 1$, only the DMU₁ has the $w_1^* = 1$ (DMU₁ is the referred efficient point for the other DMUs) and the $w_k^* = 0$ for the other DMUs, $k = 2, 3, \dots, 10$; furthermore, $v_1^* = 0.489$, $v_2^* = 0.001$, $v_3^* = 0.009$, $u_1^* = 0.499$, $u_2^* = 0.001$. Thus, these results of Table 2 are obtained by applying the equation (u_r^*, v_i^*) above. The problem above (u_r^*, v_i^*) form the MADM game for DEA, is identical to resolving the multi-objective DEA problem (16) by using only $w_1^* = 1$ and adding the constraint: $\sum_{i=1}^m v_i + \sum_{r=1}^p u_r = 1$ into Eq. (16). Given Table 2, Company G's and B's global efficiencies are the best, but Company I's and J's global efficiencies are the worst. Suggested improvements for each DMU achieving higher scores are summarized in Table 3 by the slack analysis problem (17).

Table 2. R&D efficiency of ten LCD companies

Company	Global Efficiency Score
A	0.984
B	0.985
C	0.835
D	0.818
E	0.808
F	0.887
G	0.985
H	0.910
I	0.798
J	0.784

Table 3. Suggested improvements of ten LCD companies

Company	Inputs			Outputs	
	Operational cost	Number of R&D faculty	R&D expense	Net profit	Number of patents
<i>l</i>	2	3	4	5	6
A	0	0	0	0	0
B	0	0	0	0	0
C	0	-1,627	-2,081,094	+12,059,880	+56

Table 3 cont.

<i>I</i>	2	3	4	5	6
D	0	-208	-354,870	+12,168,527	+68
E	0	-112	-441,400	+13,037,708	+57
F	0	-852	-233,339	+977,828	+6
G	0	0	0	0	0
H	-12,144	-1	0	+6,953	0
I	0	-217	-587,236	+1,607,984	0
J	0	0	-340	+2,390	0

Note:

1. The profit/cost are displayed by the unit of a thousand Taiwanese Dollar.
2. Here “+” represents the increment; on the contrary, “-” denotes the decrement.

This paper proposes a new model which differs from traditional and existing multi-objective DEA models in that its objective function is the difference between inputs and outputs instead of the ratio of outputs/inputs. Nevertheless, the linear programming problem simplifies the ration difficulty in traditional DEA problems. Then an MOLP problem is formulated for the computation of common weights for all DMUs by game theory. To be precise, the modified MADM game is used to generate standard weights. The dual problem of this model is also investigated. Finally, the MADM game for DEA is eventually synonymous to the weighting multi-objective DEA according to our new formulations in this study.

5. Conclusions and recommendations

This section is arranged into three parts: research contribution, research implication, research limitation and future works.

5.1. Research contribution

This study successfully extends the traditional CCR DEA model by the concept of multi-objective linear programming (MOLP) problem, which is synonymous to an MADM game in this study. This new approach is quite different from the earlier multi-objective DEA approaches. The advantage of using this new model reduces the computation times from n to 1 because the referred DMU is automatically decided. Moreover, considering the resolution of multi-objective programming problem for DEA here, the decision maker does not need to worry how to set these weights among DMUs/objectives, this MADM game will decide the weights among DMUs and the weights of (u_r^*, v_i^*) simultaneously.

Furthermore, the w_k^* also clearly shows us what the referred efficient DMU is and which DMU is seen by how much.

This newly multi-objective vision is useful, straightforward, and simplifies the problem of selecting the referred DMU and the computational complexity in the traditional CCR DEA model. Also, a new link between the multi-objective CCR DEA and the MADM game is established and validated in this study.

According to the computational example, this new model performs well and finds two companies: I and J which have poor scores. Moreover, the slack analysis helps us to find the suggested improvements in Table 3. Generally speaking, high R&D expense and low net profit lead to inefficiency of most companies.

5.2. Research implication

First, from the theoretical perspective, previous scholars from the field of multiple criteria decision making (MCDM) pointed to the fascinating relationship between the MOLP problem and the DEA problem. They explored the common weight approach to DEA based on MOLP. The common weight method eventually came from the idea of compromised programming in MCDM. This study explores further based on the past efforts. Therefore, a new linear programming problem for computing the efficiency of a decision-making unit (DMU) by the two-person zero-sum game theory is initially introduced here. The proposed model is very different from traditional and existing multi-objective DEA models since its objective function is the difference between inputs and outputs instead of using the ratio. The dual problem of slack analysis for the new model is also investigated.

Secondly, from the perspective of practitioners, most of the Taiwan business operate in a medium or small scale: they possess the characteristics of Original Equipment Manufacturer (OEM). Low pricing, high responsiveness and mass production are their key competitiveness factors (Khalil, 2000; Naik & Chakravarty, 1992). Since OEM companies manufacture by orders, it is really difficult to foresee the actual demand in the market. They must continuously seek to enlarge the market share by mass production, high responsiveness, and low pricing. In simple terms, these companies produce fashioned electronics competing with time. Besides, most of Taiwan's LCD technologies are initially imported from Japan or Korea. These Taiwan companies must pay the high R&D expenses to get the privilege of using or extending these technologies in

production. Thus, the R&D faculty is under very high pressure to fully and efficiently utilise the intellectual properties (IP) because the technology life cycle is concise nowadays (Khalil, 2000). That is why most of the companies are suggested to reduce the related expenses from R&D in Table 3. In summary, these companies are not able to produce adequate quantity of LCD (economics of scale) under such high R&D expenses and short product life cycle.

5.3. Research limitations and future works

This paper proposes and validates an exciting link between (MOLP) DEA problem and an MADM game when they are used for computing the common weights. Although the model proposed in this study may be unique and strict, this could be an innovative and interesting attempt to bridge the gap between the weighing MOLP of DEA and the MADM game of DEA. Interested readers could explore more issues shortly. For example, since the transformation proposed here is unique, scholars need to explore its general form and implication. However, there are still few papers exploring the MADA game and its extension and adoption for broader uses.

Moreover, we still need to analyse the computational results with care because of the selected inputs and outputs. The more detailed research for relevant variables of input and output for precise and correct measure the R&D efficiencies of companies is required. Finally, some non-linear production functions can also be explored in future studies by non-linear games with multi-objective optimisation (Miettinen, 1999) when re-solving the DEA weights.

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