M U L T I P L E   C R I T E R I A   D E C I S I O N   M A K I N G
Vol. 10 2015

Joanna Siwek*

SINGLE GOOD EXCHANGE MODEL WITH CHANGEABLE PREFERENCES GIVEN AS A TWO-SIDED MATCHING

Abstract

Markets are usually considered as strongly efficient – each investor is said to have the same information at the same time. But due to incomplete, false or vague information on the market, significant data have become an expensive good. Thus, the accessibility to it may vary.

In the following paper a behavioural approach to decision-making is presented. An investor's decision to enter a trade is based on multiple criteria such as knowledge, personal experience, investing history and individual characteristics. All those factors are reflected in individual investor's preference toward a short or long position in a trade of good.

In the paper we present two exchange models of an arbitrary good, where information about the market is reflected in investors' preferences. A two-sided matching approach for choosing contract sides is given. Simulations of market dynamics, including asymmetry and changeability of information, are performed and a possible equilibrium is discussed. The main idea of this paper is to research possible states of market equilibrium on the basis of behavioural factors and describe its usefulness for modelling market dynamics.

Keywords: two-sided matching, exchange model, game theory.

1 Introduction

The main problem researched in this paper is the influence of asymmetric information on investors' decisions, which is reflected in contracts made on the market. The behavioural factors represented in investing preferences are included in the model beside the economic laws.

* Poznań University of Economics and Business, Faculty of Informatics and Electronic Economy, Department of Operations Research, Al. Niepodległości 10, 60-875 Poznań, Poland, e-mail: joanna.siwek@ue.poznan.pl.

We assume here that differences in accessibility to data do exist. The reasoning is that information is quite an expensive good. With money, investor may get access to good market brokers, faster and better equipment, business partners with more experience. All of this may result in investors' different knowledge about the market, which influence their decisions and reactions to the same market factors.

By the knowledge of the market we understand not only raw data, but also methods of processing and conclusions based on it. The term also includes personal characteristics that allow investors to successfully operate on the market. Examples may be connections, back office, risk aversion and education.

The main motivation for the research conducted is the assumption of market effectiveness in most of existing models. Furthermore, many models do not include behavioural factors or the possibility that the information will change over time or will be updated.

In this article, simulations of the market are performed, using the Visual Basic for Applications language. The sets of initial preferences are created randomly. The preferences are changing with every iteration step, on the basis of the investing history. Each step corresponds to the time required to finalize a contract. The simulation ends if a certain equilibrium is reached. This will be explained further in the paper.

There are many models of market exchange, encompassing, for example, vague information or behavioural aspects of decision (Kunreuther, Pauly, 1985). In some of them the market is considered quantitatively, in others the behavioural aspects are formulated in terms of fuzzy numbers (Piasecki, Witoch, 2014). There is also a game-theoretical approach in which the market is represented by a game, with investors being players (Shapley, Shubik 1969).

2 Theoretical assumptions

The main idea of this paper is to represent the market as a simple exchange model, based on Two-Sided Matching theory. In this approach we consider only four investors, willing to take short or long position in a contract, without possibility of not investing. The short side of a contract sells a particular good and the investor on the long side buys it. Each of the investors has a set of preferences toward a bargain with the remaining ones. These are based on his or her market knowledge, personal characteristics and experience, and may change over time.

In general, in Two-Sided Matching problems there are two disjoint sets U and W. Each agent from U and W submits a list of acceptable participants from the other set, which may be ranked in order of preference. We say that any two participants and $n_j \in W$, where $i, j \in \{1,2, \dots, N\}$, find each other acceptable if both

m_i and n_j rank each other on their respective preference lists. A matching M is a set of disjoint pairs (m, n) such that $m \in U, n \in W$, where for each pair m and n find each other acceptable, and M satisfies certain assumptions, that is specific capacity constraints (Abraham, 2003). We will denote the pair (m, n) from a particular matching M, by $M(m, n)$.

A matching M is called unstable if there are pairs $(m_i, n_j) \in M$ and $(m_s, n_t) \in M,\ i, j, s, t \in N\backslash\{\infty\}$, such that m_i prefers n_t to n_j and n_j prefers m_s to m_i (Gale, Shapley, 1962). The pair $(m_i, n_j) \in M$ is called a blocking pair.

A matching M is stable if it cannot be improved upon by any individual or any pair of agents (Roth, Sotomayor, 1992).

Let us now focus on the definition of preferences. We will denote by P the set of all preference lists, $\mathrm{P} = \{P(m_1), \dots, P(m_t), P(n_1), P(n_2), \dots, P(n_t)\}$, one for each agent from each side of the contract. A specific market is denoted by the triple $(U, W; P)$. We will write $m_i >_{n_k} m_j$ to mean that n_k strictly prefers m_i to m_j. Analogously, $m_i \geq_{n_k} m_j$ means that n_k prefers m_i at least as well as m_j. Similarly with $n_i >_{m_k} n_j$ and $n_i \geq_{m_k} n_j$ (Roth, Sotomayor, 1992).

Also, if m_i is the best possible partner for n_j, then n_j is the worst possible partner for m_i (Biro, 2007), which means that in the market model, agents forming a contract have opposite interest over the possible outcome. It follows that the matchings do not treat both sides of the trade equally. The side that is first to pick the partner is the favoured one. In the remainder of the paper we will assume that the favoured side is the short side. The interpretation is that if an agent sells goods on the market it means that he has already entered the market and has a knowledge about it or has means and knowledge to enter it with a product to sell, and that knowledge gives him an advantage over the agent on the long side.

Apart from Game Theory, in the paper we also consider aspects of market equilibrium and its stability. In the dynamic concept of market we have that partial equilibrium is reached if for a certain good in a particular moment there exist a vector of prices that global demand equals global supply. Global equilibrium exists when this situation arises for every good on the market (Arrow, Hurwicz, 1958; Malaga, 2012).

Regarding the asymmetry of information, we will assume that there is a certain amount of shared information, interpreted as official information about an agent, that is accessible to every other agent. However, preferences of a particular entrepreneur are based not only on that information, but also on his individual interpretation of other data, given to him, for example, by different brokers.

The information possessed is reflected directly in the agent's preferences regarding other agents. If the data about target companies is vast and useful, the entrepreneur has a chance of using it to achieve an income in a trade with another agent. That is, if information is unambiguous and complete, the agent it refers to will be more preferred by the entrepreneur possessing the information. Otherwise, if the information is vague, trade with the agent is risky and less preferred.

In addition, we may deduce that if the preferences differ from one agent to another, there may be some kind of instability in the company, which is reflected in increased risk, there is an information chaos on the market or the entrepreneur is investing based strictly on his behavioural decisions. On the other hand, if the preferences are similar, the situation may suggest perfect information on the market, stability of the companies and lack of behavioural factors in decision making.

While studying numerical examples we may encounter two scenarios. First, the case may end in a cycle: that is, the preferences of each agent will be the same as at some point in the past. Taking into account that the probability of a certain side winning will be computed on the basis of preferences, the repetition of preferences clearly indicate a cycle of investment decisions on the market. We will interpret this as the information being explicit and complete. That is, all the investors had opportunities to make decisions based on fair information and the market appears to be stable – we may predict what will happen next, and the investment risk has decreased.

The other case is when all the probabilities of a certain side winning take the same values. There would be the same chance for every agent to win, which we interpret as an information chaos. Because everything may happen, the risk of the investment is high. The market is not stable, we cannot assume that at some moment in the future a cycle will appear and an opportunity to predict future market conditions will arise. The computations will be performed until one of these situations occurs.

3 Matching models

The simplest exchange model includes four agents willing to buy or sell a contract for some kind of asset. In this case investors do not specify which position – long or short – they want to take. From the information given they choose an agent they want to trade with, and they choose to buy or sell depending on the chance of achieving a revenue.

The first set of investors' preferences reflects the situation on the market. From those, the probability of each entrepreneur's winning is calculated as follows:

Procedure 1
Weights are assigned to the preferred matchings – if an agent was first on the investor's preference list he gets weight 3, if second weight 2, else 1.

For every matching, the weighted number of investors willing to trade with the given agent is calculated (let us call it the sum of revenues and denote by sr). That is, if we denote by w_{ij} the weight of a matching between agents W_i on the long side and W_j on the short side, we have:

$$sr_j = \sum_{i=1}^{n} w_{ij}$$

The probability of agent's W_j winning is computed as follows:

$$p_j = \frac{\sum_{i=1}^{n} w_{ij}}{\sum_{k=1}^{n}\sum_{i=1}^{n} w_{ik}} = \frac{sr_j}{\sum_{k=1}^{n} sr_k}$$

where *n* is the number of agents on the market.

The next step is to choose pairs for the trades. In this situation we will assume that the trades favour the short side. Following that, the first two agents selling the asset are determined by taking the maximal value of the winning probability. The interpretation may be the following: a new market is being created, and only trustworthy (and well informed) companies are allowed to sell their assets. In order to choose long sides of the trades, we take the short side with the maximal probability and assign to them the first agent from their preference list who is not on the short side of the other contract. The other investor takes the long side of the second contract.

While modelling the formation of the contracts we assume that the agents change their investing positions, that is, if an agent was selling the contract then in the next iteration he will be buying and vice versa. Thus, we take the agents from the long position in the previous iteration and assign them to the short position. Long positions in the contract are then calculated as in the first case.

With each contract executed, the preferences of the investors change. If an agent was on the winning side, in the next contract his preferences are exactly the same as in the previous one. If he was on the losing side, the agent he has lost to will now be last on his preference list. We interpret it that the investor has lost and he assumes that his information about the market was wrong or incomplete. He now wishes to invest in the agent he thought was second-best to trade with. The agent he had lost with is treated as suspicious and falls to the last place of his preference list. The iterations are performed until we encounter a cycle or all the probabilities have the same value.

The second model of an exchange presented here includes separate preferences and blocking pairs. We assume now that the agents have two sets of preferences: one for taking the short side in a trade and another one for the long side. By a blocking pair we mean a pair for which there exists another matching with a higher revenue than the one considered.

Procedure 2

First, the tables of preferences are created for short (S) and long (L) sides with the following probabilities:

$$p_j^L = \frac{\sum_{i=1}^{n} w_{ij}{}^S}{\sum_{k=1}^{n}\sum_{i=1}^{n} w_{ik}{}^S} = \frac{sr_j{}^S}{\sum_{k=1}^{n} sr_k{}^S}$$

$$p_j^S = \frac{\sum_{i=1}^{n} w_{ij}{}^L}{\sum_{k=1}^{n}\sum_{i=1}^{n} w_{ik}{}^L} = \frac{sr_j{}^L}{\sum_{k=1}^{n} sr_k{}^L}$$

A table containing the sums of pairs $w_{ij}^S + w_{ij}^L$ is created and the maximal sum is found $\underset{i,j}{max}{}^1\,(w_{ij}^S + w_{ij}^L)$.

If the indices for the maximal value are k and l, we find second-to-maximal value, that is $\underset{i\neq k, j\neq l}{max}{}^1\,(w_{ij}^S + w_{ij}^L)$.

To find a blocking pair, another table of revenues is created.

Using the second table, we calculate $\underset{i,j}{max}{}^2\,(w_{ij}^S + w_{ij}^L)$ and, assuming that the indices for the last value were s and t, we set $\underset{i\neq s, j\neq t}{max}{}^2\,(w_{ij}^S + w_{ij}^L)$.

If we have:

$$w_{kl}^S + w_{kl}^L + \underset{i\neq k, j\neq l}{max}{}^1\,(w_{ij}^S + w_{ij}^L) \geq w_{st}^S + w_{st}^L + \underset{i\neq s, j\neq t}{max}{}^2\,(w_{ij}^S + w_{ij}^L)$$

then we choose $w_{kl}^S + w_{kl}^L$ and $\underset{i,j}{max}{}^1\,(w_{ij}^S + w_{ij}^L)$ as the revenues characterizing the first and second contracts, respectively. Otherwise we choose the values $w_{st}^S + w_{st}^L$ and $\underset{i,j}{max}{}^2\,(w_{ij}^S + w_{ij}^L)$.

To sum up, the introduction of blocking pairs we ensures that the contracts created on the market were optimal. That is, if the first two contracts were to be set up, and the total revenue of some other matching was higher, then the other contracts based on the other matching would be eventually formed. The pairs introducing higher revenue to the second matching are called the blocking pairs.

4 Empirical examples

The results of the computations for the exchange models are given in the Appendix. From the initial random preferences (Iteration 0) we may conclude that the information on the market is quite clear and complete. The argument may be that the data regarding agent 3 must be satisfying, since two of three agents want to trade with him, and the last one has agent 3 second of his preference list. Similarly, agent 4 must not be a good partner for business, because two of three agents prefer him least. Thus, we can conclude that the initial market information level was high.

The table of preference shows the computation of the sum of revenues (sr_j) and the probability of winning (p_j). The 'Contracts' table shows formed contracts and their winners, who are determined based on the probability of achieving success.

When moving to net iteration, investors change contract sides. Those who won retain their previous preference while for those who lost, the last trade partner falls to the last place on their preference list. The procedure then continues.

In Iteration 8, the preferences are exactly the same as those in Iteration 4, which means that we have encountered a cycle. An interesting issue is that of possible connection between high information level (similar initial preferences) and cycle generation.

For the model with blocking pairs 38 iterations were necessary to obtain a cycle. From the preference table we can obtain detailed information about W1 and vague information about other agents. That is, the information about the market is not perfect.

Now, we have both short and long preferences for every investor. In the preference table we compute the sums of revenues from a possible contract for each investor (${sr_j}^S$, ${sr_j}^L$) and probability of winning when taking a side in a contract (p_j^S, p_j^L). In the table 'Contracts, we designate initial trades to be made, check if there exist a blocking pair with higher revenue under some other matching and create final contracts. As a last step we indicate winners and change preferences of those who lost.

It took 38 iterations to find a cycle, while in other observed models, even with same initial preferences, fewer were necessary. Therefore it is possible that the blocking pairs influence the market and make it harder to find a cycle, hence a kind of stability.

5 Conclusion

We consider the market as stable when during a cycle, because we are able to predict what will happen in the next moment (here represented as an iteration step). We associate this situation with complete or nearly perfect initial information about the market, which allows investors to act only on rational premises and optimization.

On the other hand, we find the market to be in chaos, if the probabilities of each investor to win in a contract are equal, everything is possible, hence the information about the market must be incomplete or vague. That means that no investor has information that will give him an advantage over other investors, reflected in his probability of winning.

The innovative aspect of this paper is use of two-sided matching theory in market simulation. Furthermore, the assumption of general market information being encompassed in changeable investors' preferences which are the only incentive for a decision has not been yet fully explained. The idea of market equilibrium given as a cyclic set of preferences and market instability as a set of equal probabilities for all decision alternatives have not yet been researched, either.

One of the disadvantages of this two models is that there is no way to indicate which information or behaviour influences the investors' preferences. The information is taken as a whole, showing only the general state of the market. Moreover, the simulations are computed ceteris paribus – no other factors than the preferences change. No price factor or market broker's fee is taken into account.

Another problem is the size of the model – the simple version includes only four entrepreneurs and needs to be generalized for an arbitrary number of them. Furthermore, the model does not reflect the type of the instrument being traded, although its type greatly influences the way the contracts are being made, which requires creating submodels.

The advantages, on the other hand, are that the given model includes behavioural aspects of decision-making and different information existing on a market. Also, the model is quite universal for different types of goods and trades and also allows more sides of a contract to be introduced. What is more, the model allows to simulate tones of the market and thus, future decisions that will be made by the investors relying on their market knowledge.

As for the further research, the main idea is to bring the model closer to the reality of market dynamics, to make it possible to predict future markets behaviours. To do so, we need to introduce more trading agents, include trading fee, allow investors to exit the market and let new investors enter it. All of these modifications are possible, but require more complex computation techniques.

We may consider a third party entering the market (e.g. a broker facilitating the conclusion of a contract). In this case Three Sided Matching theory can be used (Biro, McDermid, 2010; Eriksson, Sjostrand, Strimling, 2006). The third party might be a solution to the problem of equal probabilities in some contracts. Preferences of the third party may be represented by a fee level. If the fifth agent enters the market, we may need to introduce the procedure for the possibility of leaving the market because the contracts require an even number of agents.

Even though it is a possible to generalize the model for n agents, a few problems arise. First of all, the tables of preferences for a large number of agents are immense, because for both short and long position they have the size $n \times n$. Second, the more contracts there are, the longer the blocking pairs procedure is. With each two new agents, another step in the blocking pairs procedure is necessary. Also, when facing a possibility of a draw, the more agents tie, the less the model reflects the information about the market.

Appendix

i=0

Preferences			
w1:	3	2	4
w2:	3	1	4
w3:	2	4	1
w4:	1	3	2

table of preference					
	1	2	3	4	
1	x	2	3	1	
2	2	x	3	1	
3	1	3	x	2	
4	3	1	2	x	
sr	6	6	8	4	24
p	0,25	0,25	0,33	0,17	

Contracts

Position:	Short		Long	winner	loser
Agent no.	3	vs	4	3	4
p	0,33		0,17		
Agent no.	2	vs	1	1	2
p	0,25		0,25		

i=1

Preferences			
w1:	3	2	4
w2:	3	4	1
w3:	2	4	1
w4:	1	2	3

table of preference					
	1	2	3	4	
1	x	2	3	1	
2	1	x	3	2	
3	1	3	x	2	
4	3	2	1	x	
sr	5	7	7	5	24
p	0,21	0,29	0,29	0,21	

Contracts

Position:	Short		Long	winner	loser
Agent no.	4	vs	2	2	4
p	0,21		0,29		
Agent no.	1	vs	3	3	1
p	0,21		0,29		

i=2

Preferences			
w1:	2	4	3
w2:	3	4	1
w3:	2	4	1
w4:	1	3	2

table of preference					
	1	2	3	4	
1	x	3	1	2	
2	1	x	3	2	
3	1	3	x	2	
4	3	1	2	x	
sr	5	7	6	6	24
p	0,21	0,29	0,25	0,25	

Contracts

Position:	Short		Long	winner	loser
Agent no.	2	vs	4	2	4
p	0,29		0,25		
Agent no.	3	vs	1	3	1
p	0,25		0,21		

i=4

Preferences			
w1:	4	3	2
w2:	3	4	1
w3:	2	1	4
w4:	1	3	2

table of preference					
	1	2	3	4	
1	x	1	2	3	
2	1	x	3	2	
3	2	3	x	1	
4	3	1	2	x	
sum	6	5	7	6	24
prob.	0,25	0,21	0,29	0,25	

Contracts

Position:	Short		Long	winner	loser
Agent no.	3	vs	1	3	1
prob.	0,29		0,25		
Agent no.	2	vs	4	4	2
prob.	0,21		0,25		

i=8

Preferences			
w1:	4	3	2
w2:	3	4	1
w3:	2	1	4
w4:	1	3	2

table of preference					
	1	2	3	4	
1	x	1	2	3	
2	1	x	3	2	
3	2	3	x	1	
4	3	1	2	x	
sum	6	5	7	6	24
prob.	0,25	0,21	0,29	0,25	

Contracts

Position:	Short		Long	winner	loser
Agent no.	3	vs	1	3	1
prob.	0,29		0,25		
Agent no.	2	vs	4	4	2
prob.	0,21		0,25		

Figure 1. Tables of preferences and contracts for a simple exchange model, iterations 0-8

i=0

SHORT	Preferences		
w1:	3	2	4
w2:	4	1	3
w3:	2	1	4
w4:	3	1	2

LONG	Preferences		
w1:	4	3	2
w2:	3	4	1
w3:	2	4	1
w4:	2	3	1

preference table

short\long	1		2		3		4		sum
1	0	0	2	1	3	1	1	1	
2	2	1	0	0	1	3	3	3	
3	2	2	3	3	0	0	1	2	
4	2	3	1	2	3	2	0	0	
sum S	3		7		7		7		24
sum L		6		6		7		5	24
probability S	0,1		0,3		0,3		0,3		
probability L		0,25		0,3		0		0,21	

Contracts

Table of revenues for each possible pairing				
short\long	1	2	3	4
1	0	3	4	2
2	3	0	4	6
3	4	6	0	3
4	5	3	5	0
max revenue for first pair	short	long	winner	
	2	4	2	
Sum of revenues			10	

Table of revenues for each possible pairing after removing first pair				
	1	2	3	4
1	0	0	4	0
2	0	0	0	0
3	4	0	0	0
4	0	0	0	0
remaining max revenue	short	long	winner	
	1	3	3	

Revenues for choosing first blocking pair				
	1	2	3	4
1	0	3	0	2
2	3	0	4	0
3	4	6	0	3
4	5	3	5	0
max revenue for alternative first pair	short	long	winner	
	3	2	3	
Summed revenues of			11	

Revenues for choosing second blocking pair				
	1	2	3	4
1	0	0	0	2
2	0	0	0	0
3	0	0	0	0
4	5	0	0	0
Second blocking pair	short	long	winner	
	4	1	1	

Final pairing		
contract	1st	2nd
winner	3	1
position	short	long
loser	2	4
position	long	short

Figure 2. Tables of preferences and contracts for a simple exchange model with blocking pairs, iteration 0

References

Abraham D.J. (2003), *Algorithmics of Two-Sided Matching Problems*, Master Thesis, University of Glasgow, http://www.dcs.gla.ac.uk/research/algorithms/ASMPI/DJA-thesis.pdf (16.06.2014, 14:53).

Andrews M.J., Bradley S., Upward R. (2001), *Estimating the Probability of a Match Using Microeconomic Data for the Youth Labour Market*, Labour Economics, Vol. 8.

Alkan A., Gale D. (2003), *Stable Schedule Matching under Revealed Preference*, Journal of Economic Theory, Vol. 112.

Alcalde J. (1995), *Exchange-proofness or Divorce-proofness? Stability in One-sided Matching Markets*, Economic Design, Vol. 1, Iss. 1.

Arrow K.J., Hurwicz L. (1958), *On the Stability of the Competitive Equilibrium I*, Econometrica, Vol. 26, No. 4.

Biro P., McDermid E. (2010), *Three-sided Stable Matchings with Cyclic Preferences*, Algorithmica, Vol. 58.

Biro P. (2007), *The Stable Matching Problem and Its Generalizations: An Algorithmic and Game Theoretical Approach*, PhD Thesis, University of Debrecen, http://www.researchgate.net/publication/242391326_The_stable_matching_problem_and_its_generalizations_an_algorithmic_and_game_theoretical_approach (16.06.2014).

Echenique F., Oviedo J. (2006), *A Theory of Stability in Many-to-many Matching markets*, Theoretical Economics, Vol. 1.

Eriksson K., Sjostrand J., Strimling P. (2006), *Three-dimensional Stable Matching with Cycle Preferences*, Mathematical Social Sciences, Vol. 52.

Gale D., Shapley L.S. (1962), *College Admissions and the Stability of Marriage*, The American Mathematical Monthly, Vol. 69, No. 1.

Irving R.W. (1994), *Stable Marriage and Indifference*, Discrete Applied Mathematics, Vol. 48.

Iwama K., Miyazaki S. (2008), *A Survey of the Stable Marriage Problem and Its Variants*, International Conference on Informatics Education and Research For Knowledge – Circulating Society, IEEE.

Kojima F., Pathak P. (2009), *Incentives and Stability in Large Two-Sided Matching Markets*, American Economic Review, Vol. 99, No. 3.

Kunreuther H., Pauly M. (1985), *Market Equilibrium with Private Knowledge*, Journal of Public Economics, Vol. 26.

Malaga K. (2012), *Mikroekonomia. Oswajanie z matematyką*, 2nd ed., Wydawnictwo C.H. Beck, Warszawa.

Pais J. (2006), *Random Matching in College Admissions Problem*, Economic Theory, Vol. 35, Iss. 1.

Peck J., Shell K., Spear S.E. (1992), *The Market Game: Existence and Structure of Equilibrium*, Journal of Mathematical Economics, Vol. 21.

Piasecki K., Witoch J. (2014), *Behavioral Present Value Defined as a Fuzzy Number – A New Approach*, submitted: http://www.researchgate.net/publication/262117073_Behavioural_present_value_defined_as_fuzzy_number_-_a_new_approach (22.06.2014).

Roth A.E., Sotomayor M. (1992), *Two-sided Matching* [in:] *Handbook of Game Theory*, eds. R.J. Aumann, S. Hart, Vol. 1, Elsevier.

Shapley L.S., Shubik M. (1969), *On Market Games*, Journal of Economic Theory, Vol. 1.