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**REVIEW OF SELECTED METHODS FOR PORTFOLIO  
OPTIMIZATION OF AND IRREVERSIBLE  
INVESTMENT IN POWER GENERATION  
ASSETS UNDER UNCERTAINTY**

**Summary:** In this paper, we present some examples to illustrate the use of selected financial methods for the portfolio optimization of power generation assets. We start with classical MV theory, followed by dynamic variants to MV portfolio optimization, and eventually how fuzzy set theory can be used in portfolio optimization of power generation assets. In light of the ongoing liberalization process of the energy markets, and the risks and uncertainties created by rising shares of renewable electricity in the spot market, we present real option model specifications that can be used by energy companies to run these assets profitably and to make rational new investments in conventional power plants. Such models can help to determine the optimal timing to invest (or disinvest) in power plants, and can thus be powerful and useful tools for decision-makers.

**Keywords:** portfolio optimization, fuzzy set theory, real options analysis.

**Introduction**

The liberalization of electricity markets in many countries increased the competition for market shares amongst electricity producers and retailers alike, as well as raised the types and levels of risk and uncertainty in the investment process. Higher risk and uncertainty require some upgrading in the “toolbox” used for decision-making in the energy business. Moreover, regarding the increase of the market risk (with respect to the development of the future electric-

ity demand and supply as well as electricity and fuel prices), regulatory risks (related to uncertainty in environmental and energy regulations, and market design), and the use of renewable energy technologies, it was necessary to apply more sophisticated methods in the investment process. Indeed, these facts affected the application of financial mathematical methods to the valuation process of electricity generation assets.

An advanced and robust analytical framework increasingly applied also in the energy domain is Mean-Variance Portfolio (MVP) analysis. This theory provides a mathematical framework that enables to identify the set of efficient portfolios (in the sense of return maximization for a given expected risk or risk minimization for a demanded expected return). Moreover, specific financial risks related to various technologies as well as the technical, economic and societal aspects of the plants can explicitly be considered. Nevertheless, the classic MVP theory is a static approach, which requires some modifications, especially when technical change, learning effects, and dynamics in the generation mix shall additionally be taken into account.

Regarding the evaluation of investment decisions under uncertain future market conditions traditionally the net present value (NPV) criterion is typically used but in many cases totally inadequate. In order to capture unexpected market developments, more powerful approaches such as real options analysis (ROA) can be applied. Based on option pricing theory (in financial securities), ROA uses continuous time stochastic processes and enables the modeling of uncertainty in the economic variables (e.g., in terms of cash flows, prices, returns, or asset values) and managerial flexibility (e.g., in terms of the optimal timing of the (dis-)investment) in the decision-making process.

In this paper, we present the results obtained from using MVP and ROA for decision-making in the energy sector, conducted by the authors in three different projects over the last six years. The remainder of this article is structured as follows. In section 1, we give a briefly literature overview regarding the use of MVP and ROA in the energy domain. Section 2 presents different MVP optimization model specifications, whose application is illustrated with some results from our own studies. The application of ROA in decision-making processes in the energy sector, with some examples from conducted research studies, is in Section 3. The last Section concludes.

## 1. State of the art of MVP and ROA applied to the energy sector

Mean variance portfolio theory has been successfully applied to the energy sector for determining the optimal power generation mix. The first application to the energy sector in the US was presented by Bar-Lev and Katz [1976], and for the European Union by Awerbuch and Berger [2003]. Further applications of MVP for the valuation of power generation assets can be found, e.g., in the reviews by Bazilian and Roques [2008] and Madlener [2012], respectively. The reviews provide several different approaches concerning the application of MVP theory on power generation portfolios. The studies initially started with power generation costs as a proxy for the profitability of the assets, later refined by the explicit inclusion of revenues from electricity production in the portfolio analysis. Moreover, such studies incorporate both econometric analysis as well as methods of different kinds from the finance literature (especially regarding the definition of risk) to improve the technologies' representation in power generation mix.

For investment decisions under uncertainty, ROA has become increasingly popular in applications in different sectors of the economy since the early 1990s. Fernandes et al. [2011] provide a comprehensive review of the current state-of-the-art in the application of ROA in the energy sector, for non-renewable as well as for renewable energy technologies. They introduce the basic principles of the real options theory and define common types of real options to their valuation methods. Considering different studies from the end of the 1980s till 2010, they describe various applications of ROA in the oil industry, the power generation sector, energy commodity markets, as well as in carbon emission mitigation policy, with different solution methods applied (such as partial differential equation modeling, binomial option valuation, Monte Carlo simulation, and dynamic programming).

In the context of our research (here: the use of ROA applied to gas-fired power plants), and depending on the models which have to be developed, different studies have provided a variety of important new insights. Regarding the optimal timing to disinvest, which so far has only found attention in a few studies (mainly dealing with the agricultural and dairy industry, and production planning issues), the standard solution methods presented by e.g. Mun [2006] can be applied. However, the applications presented by Näsäkkälä and Fleten [2005] and Fleten and Näsäkkälä [2010], as well as modeling ideas proposed by Deng et al. [2001], and Deng and Oren [2003], are important new developments in terms of applying real options modeling and analysis to investments in flexibility measures.

## 2. Portfolio theory and its application to power generation assets

The main purpose of portfolio analysis is to find a portfolio that is best suited to the investor's objective. The two most common dimensions to all investors are return and risk. Nevertheless, when considering the application of portfolio optimization to power generation assets, the first step is the definition of an adequate return measure as a portfolio selection criterion for real assets, in our case typically power plants. Many previous studies have simply used costs as a proxy variable for determining optimal portfolios. In contrast, we proposed two alternative measures: annual return (which is more static, and can be better applied by the analysis of technologies already in use) and present value. Based on the NPV principle, the present value of the  $i$ -power plant ( $PV_i$ ) is defined as follows (cf. Madlener et al. [2009] and Madlener et al. [2010]):

$$PV_i = \sum_{t=0}^T \frac{r - c_f - c_{CO_2} - c_{O\&M} - c_c - \delta}{(WACC)^t} \quad (1)$$

with revenues,  $r$ , fuel costs,  $c_f$ , carbon dioxide mitigation costs,  $c_{CO_2}$ , operation and maintenance costs,  $c_{O\&M}$ , cost of capital,  $c_c$ , and capital depreciation,  $\delta$ . This proposed definition of return as a measure of a power plant's economic viability was used in all most of the portfolio selection models considered and analyzed by us so far.

In several of our studies, the major electricity generation assets operated by the energy company E.ON in Germany, Sweden, and the United Kingdom were analyzed. Moreover, to capture the dynamics and in particular the changes in the generation mix, planned and actually realized new investments in different power technologies were taken into consideration. The input data regarding economic as well as technical parameters necessary for the calculations can be found in Madlener et al. [2009, 2010] and Madlener and Glensk [2010], respectively.

### 2.1. Mean-variance portfolio theory

The application of the standard MVP selection model of Markowitz [Markowitz, 1952] provides the basis for the allocation of different generation technologies in efficient portfolios. Regarding the trade-off between return (defined as the present value of the power plant, cf. eq. (1)) and risk (represented by the standard deviation  $V(R_p)$ ) the portfolio selection model is defined as follows:

$$V(R_p) \rightarrow \min \quad (2)$$

subject to:

$$E(R_p) = \sum_{i=1}^n E(PV_i)x_i \quad (3)$$

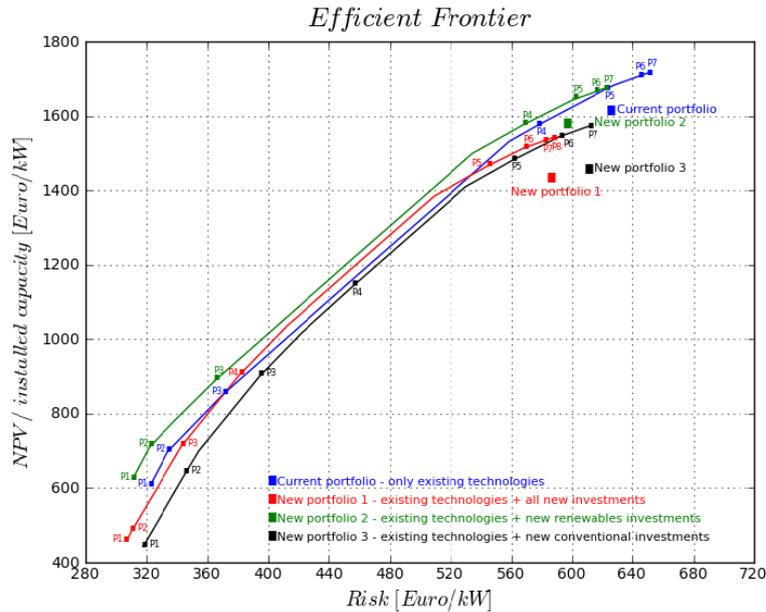
$$0 \leq x_i \leq x_{i,max} \quad (4)$$

$$\sum_{i=1}^n x_i = 1 \quad (5)$$

where  $E(R_p)$  denotes the portfolio's expected return,  $E(PV_i)$  the expected return of the individual assets (power plant),  $x_i$  and  $x_{i,max}$  the share (weight) and maximal share, respectively, of asset  $i$ , in the portfolio, and  $V(R_p)$  the portfolio risk. The latter is given by the following equation:

$$V(R_p) = \sqrt{\sum_{i=1}^n x_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n x_i x_j \sigma_i \sigma_j \rho_{ij}}, \quad (6)$$

where  $\sigma_i^2$  is the variance of component asset  $i$ ,  $\sigma_i$  the volatility of asset  $i$ , and  $\rho_{ij}$  the correlation coefficient between  $i$  and  $j$ . Using this optimization model for the set of existing and new power generation technologies of E.ON in Germany yields the efficient frontiers depicted in Figure 1.



Note: "existing technologies" means "technologies already in use".

**Fig. 1.** Efficient frontiers of E.ON's current power generation mix and new investments

Source: Madlener et al. [2010].

We observe here, that current and new portfolios of E.ON in Germany (the colored bigger squares) are located way off the efficient frontiers, indicating some scope for improvement of the portfolio. Moreover, new portfolios of E.ON in Germany (i.e. where new investments were considered) have smaller present values but also lower risks in comparison to the current portfolio consisting of already existing and operating technologies only. Furthermore, regarding new investments it can be noticed that all new investments do have a positive impact on the present value (red line between P4-P5). The highest positive impact on the present value results from adding new renewable energy technologies to the existing generation mix (green line).

## 2.2. Fuzzy semi-mean absolute deviation portfolio selection model

Taking the Markowitz approach into account, other researchers developed models for portfolio optimization using different functions for measuring the performance of a portfolio. One of these alternative approaches uses the semi-mean absolute deviation as a risk measure [Konno and Yamazaki, 1991]. The proposed model results in the linearization of the portfolio optimization model. According to Konno and Koshizuka [2005], the semi-mean absolute deviation (SMAD) model for portfolio selection is specified as:

$$\frac{1}{T} \sum_{t=1}^T d_t \rightarrow \min \quad (7)$$

$$\sum_{i=1}^n E(PV_i)x_i \rightarrow \max \quad (8)$$

subject to:

$$d_t \geq - \sum_{i=1}^n (PV_{it} - E(PV_i))x_i, t = 1, 2, \dots, T \quad (9)$$

$$d_t \geq 0 \quad (10)$$

$$\sum_{i=1}^n x_i = 1 \quad (11)$$

$$0 \leq x_i \leq x_{i,max} \quad (12)$$

where  $PV_{it}$  is the present value (return) of portfolio share (asset)  $i$  in period  $t$  and  $d_t$  refers to the negative deviation between the realization of the portfolio's present value (return) and its expected value at time  $t$  over a time span  $T$ .

The imperfect knowledge about returns, and the uncertain environment make the investment process more complex. To capture this complex reality of decision-making problems, fuzzy set theory is proposed as an alternative to the standard MVP approach (see e.g. Fang et al. [2008]). One of proposed models which regard to the investor's aspiration levels of return and risk is the fuzzy semi-mean absolute deviation (FSMAD) portfolio selection model defines as:

$$\Lambda \rightarrow \max \quad (13)$$

subject to:

$$\alpha_R \sum_{i=1}^n PV_i x_i - \Lambda \geq \alpha_R R_M \quad (14)$$

$$\alpha_w \frac{1}{T} \sum_{t=1}^T d_t + \Lambda \geq \alpha_w w_M \quad (15)$$

$$d_t + \sum_{i=1}^n (PV_{it} - PV_i) x_i \geq 0 \quad (16)$$

$$d_t \geq 0 \quad (17)$$

$$\sum_{i=1}^n x_i = 1 \quad (18)$$

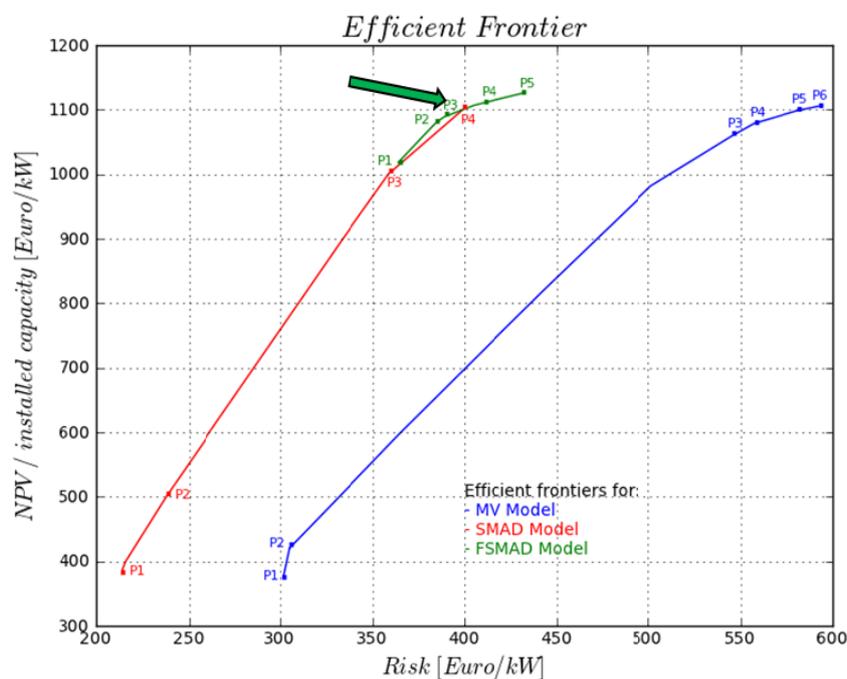
$$0 \leq x_i \leq x_{i,max} \quad (19)$$

$$\Lambda \geq 0 \quad (20)$$

where  $\Lambda = \log \frac{\lambda}{1-\lambda}$  ( $\lambda$  denotes the value of membership function for expected return and risk<sup>1</sup>). Parameters  $R_M$  and  $w_M$  are the mid-points where the membership function value  $\lambda$  is equal to 0.5, whereas the parameters  $\alpha_R$  and  $\alpha_w$  determine the shape of these membership functions. The variable  $d_t$  is a time variable connected with the semi-mean absolute deviation<sup>2</sup>.

<sup>1</sup> The formal presentation of these membership functions can be seen in Glensk and Madlener [2010].

<sup>2</sup> More information about this model and other proposed and analyzed fuzzy portfolio optimization problems can be found in Glensk and Madlener [2010].

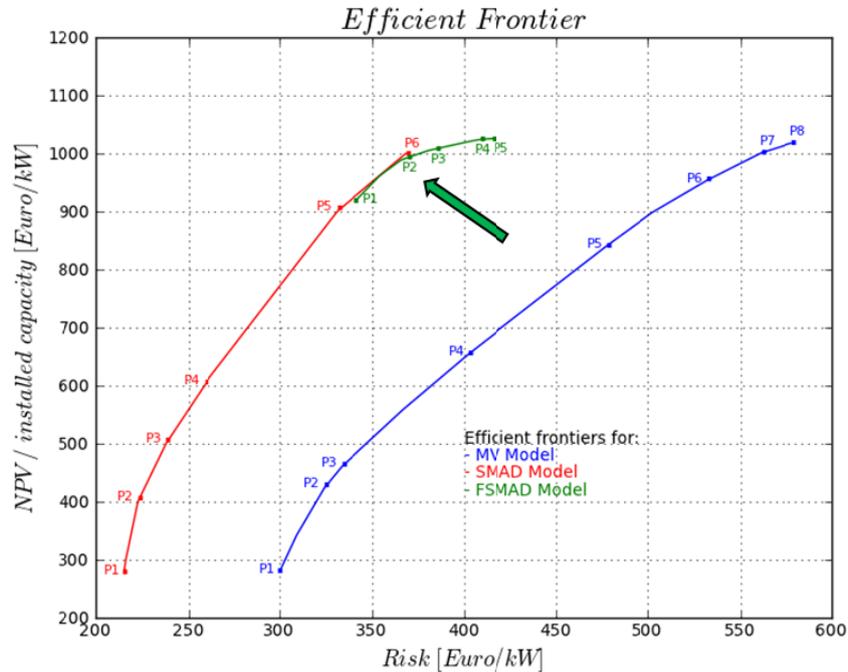


**Fig. 2.** Efficient frontiers for technologies already in use only, using MV, SMAD, and FSMAD model specifications

Source: Glensk and Madlener [2010, 2014].

The empirical analysis conducted in the mentioned projects focused on technologies already in use as well as new investments in power generation assets, and compared the efficient portfolios obtained from using MVP, SMAD, and FSMAD models (cf. Figure 2). When regarding only technologies already in use the shift in the scale of risk can be observed by applying the SMAD (Figure 2, red line) model in comparison to the MVP model. Moreover, the incorporation of the decision-maker's objectives and aspirations in the FSMAD optimization model results in a smaller set of decision alternatives (Figure 2, green line). Concluding, a better portfolios – in terms of risk and return combinations – was found for the FSMAD model compared to the MVP and SMAD models.

Almost the same findings can be observed when technologies already in use and new investments are analyzed together (Figure 3).

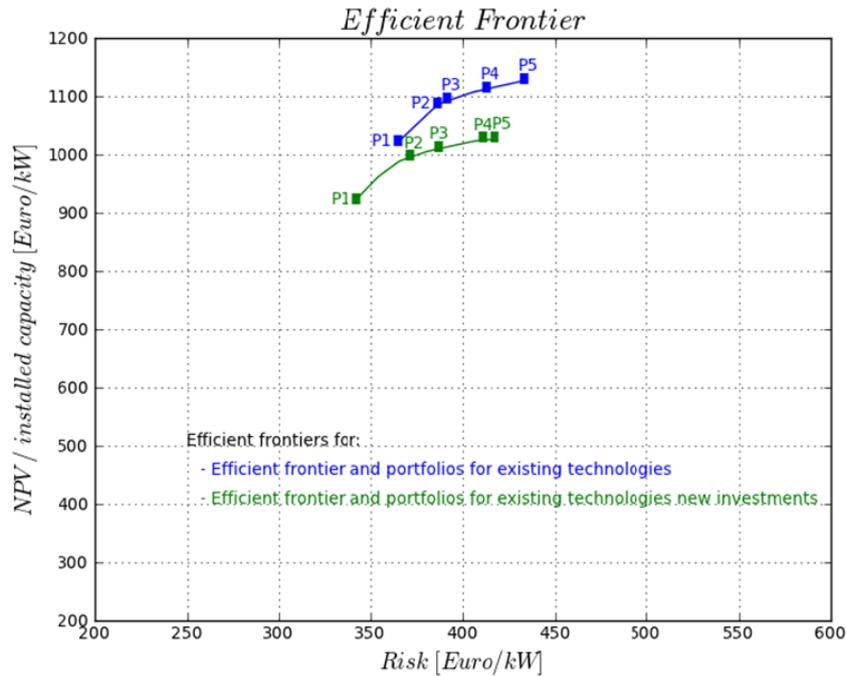


**Fig. 3.** Efficient frontier of already used and new technologies, using MV, SMAD and FSMAD model specifications

Source: Glensk and Madlener [2010].

Comparing the situation without new investments (Figure 2) with the situation where investments in new assets are included (Figure 3), it can be noticed that the efficient frontier for the FSMAD model partly coincides with the efficient frontier derived from using the SMAD model (Figure 3).

Finally, we compared the FSMAD model for technologies already in use and the FSMAD model for both technologies in use and new investments (Figure 4). Here, efficient portfolios with new investments have smaller present values but also smaller risk levels compared to the situation without new investments. This can be attributed to the fact that the investment costs have to be included in the calculation process of the present value.



Note: “existing technologies” means “technologies already in use”.

**Fig. 4.** Efficient frontier of current (blue line) and prospective (green line) power generation mixes

Source: Glensk and Madlener [2010].

The application of the FSMAD model for the special case of onshore wind power plants in Germany can be found in Madlener et al. [2011].

### 2.3. Multi-period portfolio selection model

Although standard MVP analysis has been successfully applied to real assets (power plants), the approach suffers from several important limitations. One of them is its single-period character, which does not adequately capture the investor’s long-term investment goals. On the contrary, the multi-period models, properly formulated, can solve these limitations and take advantage of volatility by rebalancing the asset mix. The proposed model is based on the multi-stage stochastic optimization problem introduced by Mulvey et al. [1997], defined as follows:

$$\alpha R_{pT} - (1 - \alpha) \text{Var}(R_{pT}) \rightarrow \max \quad (21)$$

subject to:

$$\sum_{i=1}^n x_{it}^s = 1 \quad s \in S, t = 1, \dots, T \quad (22)$$

$$x_{it}^s = x_{it-1}^s + p_{it}^s - d_{it}^s \quad s \in S, t = 1, \dots, T, i = 1, \dots, n \quad (23)$$

$$0 \leq x_{it}^s \leq x_{imax} \quad s \in S, t = 1, \dots, T, i = 1, \dots, n \quad (24)$$

$$x_{it}^s = x_{it}^{s'} \quad s \in S, t = 1, \dots, T, i = 1, \dots, n \quad (25)$$

for all scenarios  $s$  and  $s'$  with identical past up to time  $t$ , where:

$$R_{pT} = \sum_{s=1}^S q^s R_{pT}^s \quad \text{defines portfolio return,}$$

$$R_{pT}^s = \prod_{t=0}^T \sum_{i=1}^n PV_{it}^s x_{it}^s \quad \text{defines portfolio return for scenario } s,$$

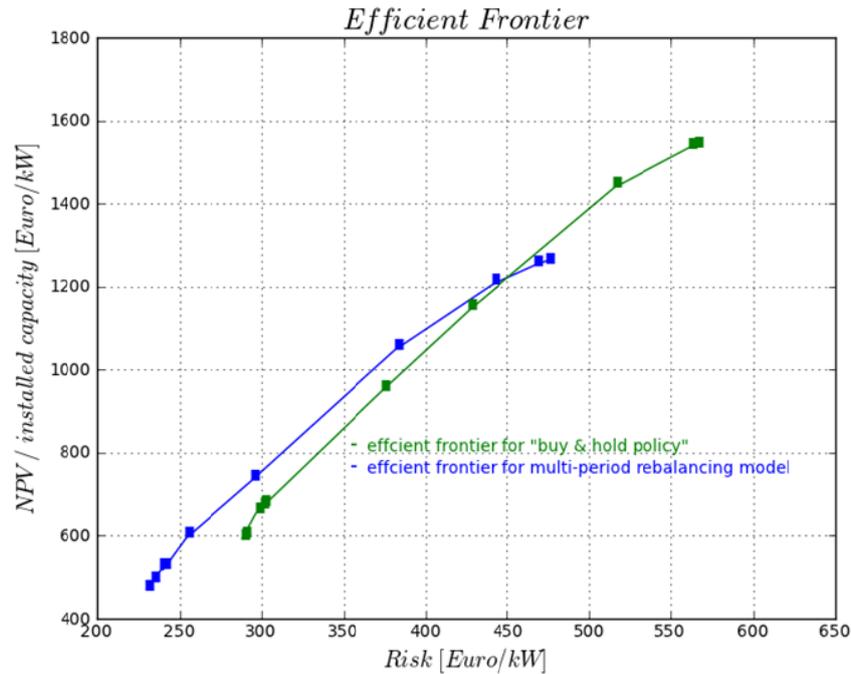
$$Var(R_{pT}) = \sum_{s=1}^S q^s (R_{pT}^s - R_{pT})^2, \quad (26)$$

and  $PV_{it}^s$  is the uncertain return (present value) of technology  $i$  in period  $t$ , under the scenario  $s$ ,  $\alpha$  a parameter indicating the relative importance of variance as compared to the expected value  $\alpha \in (0, 1)$ ,  $q^s$  the probability that scenario  $s$  occurs ( $\sum_{s=1}^S q^s = 1$ ),  $x_{it}^s$  the percentage of technology  $i$  in time  $t$  under the scenario  $s$ ,

and  $p_{it}^s, d_{it}^s$  are the proportions of technology  $i$  added into/deleted from portfolio in time  $t$  under the scenario  $s$ .

The presented model was applied for power generation assets owned by E.ON in Germany and compared with the so-called “buy-and-hold strategy”, which corresponds to the single-period portfolio selection problem where the investor makes the decision at the beginning of the investment period and holds this portfolio during the entire period of time considered. For more information see Mulvey et al. [2003, 2004]. The efficient portfolios obtained for the multi-period portfolio selection model shift in both their risk and return levels, and the portfolios are characterized by smaller risk and return levels (Figure 5, blue line).

More information regarding this kind of analysis and the results obtained can be found in Glensk and Madlener [2011a, 2011b, 2013, 2014].



**Fig. 5.** Efficient frontiers of technologies already in use and new investments (offshore wind power and concentrated solar power) for the multi-period portfolio selection model and the “buy & hold policy” (4-years decision-making period)

Source: Glensk and Madlener [2011b].

### 3. Real options theory and its application to investments in the energy sector

In investment decision-making processes, the use of NPV and simple spreadsheet calculations dominate in practice and the energy sector is certainly no exception in this respect. Nevertheless, the NPV rule does not always seem to be adequate if the energy market environment is complex and subject to various risks and uncertainties. ROA offers decision-makers more flexibility regarding the type of decision and option valuation (e.g. options to invest, abandon, expand, contract, shut down, grow etc.) and also concerning the inclusion of market uncertainties. The applied here valuation method depends on the real option model formulation.

In light of the often unprofitable operation of conventional power plants due to the low variable costs and increased contribution of renewable energy technologies, the owners of such unprofitable conventional power plants ask the question of whether to shut down or to continue the operation of the power plant.

If a plant is to be shut down, then the second question arises when it is the optimal time to do so. A third question comes up on how to make existing (e.g. gas-fired) power plants more profitable by flexibility enhancements. And two further questions are on which technical measures (retrofit measures) can render a gas-fired power plant more profitable (enhanced flexibility) and when to incorporate new (flexibility) measures to raise the profitability of the power plant. Based on these questions, we developed real options models that enable to study the disinvestment and the investment in flexibility-enhancing components. The investigations were made for a highly energy-efficient and recently built combined-cycle gas-fired power plant in Germany (for more information see e.g. Glensk and Madlener [2015], Glensk et al. [2015a, 2015b]).

### 3.1. Real options model for disinvestment

Assuming, that the power plant's capacity factor (in such kind of model the so-called "underlying asset") is a stochastic variable, normally distributed, and approximated by a binomial distribution, the binomial lattice approach can be applied in a real options model for the optimal disinvestment in a power plant.  $t$  allows to determine the "up" and "down" movements as follows:

$$up = e^{(\sigma\sqrt{\Delta t})} \text{ and } down = e^{(-\sigma\sqrt{\Delta t})}$$

subsequently used to set up the binomial tree, where  $\sigma$  is the associated volatility, and  $\Delta t$  is the time step.

Using dynamic programming, the optimal project value as a function of the capacity factor  $CF_{i,t}$  is given by:

$$PV_{i,t}(CF_{i,t}) = \max \left\{ PCF_{i,t} + \frac{RV_t + \alpha \cdot PV_{i,t+1}(CF_{i,t+1}) + (1 - \alpha) \cdot PV_{i+1,t+1}(CF_{i+1,t+1})}{1 + r_f} \right\} \quad (27)$$

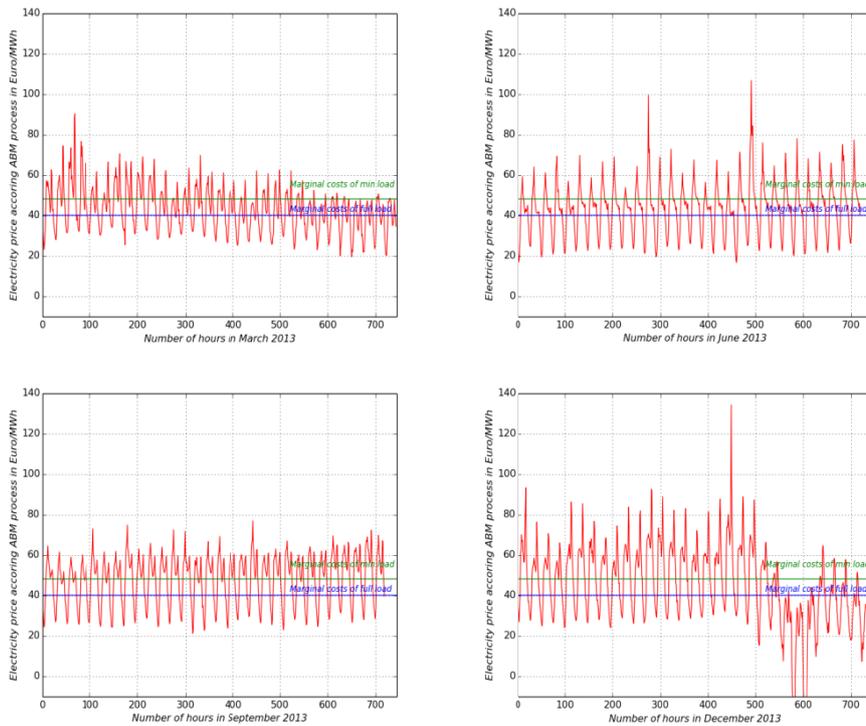
where  $RV_t$  denotes the residual value,  $PCF_{i,t}$  the project cash flow for the  $i$ th "down" move at the current time period  $t$ ,  $\alpha$  defines the probability of an "up" movement,  $CF_{i,t}$  denotes the capacity factor for the  $i$ th "down" move at time  $t$ ,  $r_f$  the risk-free rate, and  $i$  the number of "down" movements (for more description of the model see Glensk et al. [2015a, 2015b]). Finally, as soon as the project value is equal to the residual value, the power plant should be shut down; otherwise, the operation is continued. The calculation procedure proposed can easily be solved and visualized in Microsoft Excel (see Figure 6).

	0	1	2	3	4	5	6	18	19	20	21	22	23	24	25	26
0.1400	0.1400	0.1393	0.1379	0.1359	0.1329	0.1290	0.1233	0.1150	0.1034	0.8947	0.7894	0.6947	0.6094	0.5347	0.4694	0.4147
0.1427	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9053	0.8000	0.7053	0.6200	0.5447	0.4794	0.4247	0.3700
0.1454	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9159	0.8106	0.7159	0.6306	0.5553	0.4900	0.4353	0.3806
0.1481	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9265	0.8212	0.7265	0.6412	0.5659	0.5006	0.4459	0.3912
0.1508	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9371	0.8318	0.7371	0.6518	0.5765	0.5112	0.4565	0.4018
0.1535	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9477	0.8424	0.7477	0.6624	0.5871	0.5218	0.4671	0.4124
0.1562	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9583	0.8530	0.7583	0.6730	0.5977	0.5324	0.4777	0.4230
0.1589	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9689	0.8636	0.7689	0.6836	0.6083	0.5430	0.4883	0.4336
0.1616	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9795	0.8742	0.7795	0.6942	0.6189	0.5536	0.4989	0.4442
0.1643	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	0.9901	0.8848	0.7901	0.7048	0.6295	0.5642	0.5095	0.4548
0.1670	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0007	0.8954	0.8007	0.7154	0.6401	0.5748	0.5201	0.4654
0.1697	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0113	0.9060	0.8113	0.7260	0.6507	0.5854	0.5307	0.4760
0.1724	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0219	0.9166	0.8219	0.7366	0.6613	0.5960	0.5413	0.4866
0.1751	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0325	0.9272	0.8325	0.7472	0.6719	0.6066	0.5519	0.4972
0.1778	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0431	0.9378	0.8431	0.7578	0.6825	0.6172	0.5625	0.5078
0.1805	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0537	0.9484	0.8537	0.7684	0.6931	0.6278	0.5731	0.5184
0.1832	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0643	0.9590	0.8643	0.7790	0.7037	0.6384	0.5837	0.5290
0.1859	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0749	0.9696	0.8749	0.7896	0.7143	0.6490	0.5943	0.5396
0.1886	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0855	0.9802	0.8855	0.8002	0.7249	0.6596	0.6049	0.5502
0.1913	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.0961	0.9908	0.8961	0.8108	0.7355	0.6702	0.6155	0.5608
0.1940	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1067	1.0014	0.9067	0.8214	0.7461	0.6808	0.6261	0.5714
0.1967	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1173	1.0120	0.9173	0.8320	0.7567	0.6914	0.6367	0.5820
0.1994	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1279	1.0226	0.9279	0.8426	0.7673	0.7020	0.6473	0.5926
0.2021	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1385	1.0332	0.9385	0.8532	0.7779	0.7126	0.6579	0.6032
0.2048	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1491	1.0438	0.9491	0.8638	0.7885	0.7232	0.6685	0.6138
0.2075	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1597	1.0544	0.9597	0.8744	0.7991	0.7338	0.6791	0.6244
0.2102	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1703	1.0650	0.9703	0.8850	0.8097	0.7444	0.6897	0.6350
0.2129	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1809	1.0756	0.9809	0.8956	0.8203	0.7550	0.7003	0.6456
0.2156	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.1915	1.0862	0.9915	0.9062	0.8309	0.7656	0.7109	0.6562
0.2183	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2021	1.0968	1.0021	0.9168	0.8415	0.7762	0.7215	0.6672
0.2210	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2127	1.1074	1.0127	0.9274	0.8521	0.7868	0.7321	0.6774
0.2237	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2233	1.1180	1.0233	0.9380	0.8627	0.7974	0.7427	0.6880
0.2264	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2339	1.1286	1.0339	0.9486	0.8733	0.8074	0.7527	0.6980
0.2291	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2445	1.1392	1.0445	0.9592	0.8839	0.8180	0.7633	0.7086
0.2318	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2551	1.1498	1.0551	0.9698	0.8945	0.8286	0.7739	0.7192
0.2345	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2657	1.1604	1.0657	0.9804	0.9051	0.8392	0.7845	0.7298
0.2372	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2763	1.1710	1.0763	0.9910	0.9157	0.8498	0.7951	0.7404
0.2399	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2869	1.1816	1.0869	1.0016	0.9263	0.8604	0.8057	0.7510
0.2426	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.2975	1.1922	1.0975	1.0122	0.9369	0.8710	0.8163	0.7616
0.2453	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3081	1.2028	1.1081	1.0228	0.9475	0.8816	0.8269	0.7719
0.2480	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3187	1.2134	1.1187	1.0334	0.9581	0.8922	0.8375	0.7828
0.2507	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3293	1.2240	1.1293	1.0440	0.9687	0.9028	0.8481	0.7934
0.2534	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3399	1.2346	1.1399	1.0546	0.9793	0.9134	0.8587	0.8040
0.2561	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3505	1.2452	1.1505	1.0652	0.9899	0.9240	0.8693	0.8146
0.2588	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3611	1.2558	1.1611	1.0758	1.0005	0.9346	0.8799	0.8252
0.2615	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3717	1.2664	1.1717	1.0864	1.0111	0.9452	0.8905	0.8358
0.2642	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3823	1.2770	1.1823	1.0970	1.0217	0.9558	0.9011	0.8464
0.2669	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.3929	1.2876	1.1929	1.1076	1.0323	0.9664	0.9117	0.8570
0.2696	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4035	1.2982	1.2035	1.1182	1.0429	0.9770	0.9223	0.8676
0.2723	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4141	1.3088	1.2141	1.1288	1.0535	0.9876	0.9329	0.8782
0.2750	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4247	1.3194	1.2247	1.1394	1.0641	0.9982	0.9435	0.8888
0.2777	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4353	1.3300	1.2353	1.1500	1.0747	1.0088	0.9541	0.8994
0.2804	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4459	1.3406	1.2459	1.1606	1.0853	1.0194	0.9647	0.9100
0.2831	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4565	1.3512	1.2565	1.1712	1.0959	1.0300	0.9753	0.9206
0.2858	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4671	1.3618	1.2671	1.1818	1.1065	1.0406	0.9859	0.9312
0.2885	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4777	1.3724	1.2777	1.1924	1.1171	1.0512	0.9965	0.9418
0.2912	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1156	0.1040	1.4883	1.3830	1.2883	1.2030	1.1277	1.0618	1.0071	0.9524
0.2939	0.1400	0.1387	0.1367	0.1337	0.1298	0.1239	0.1									

Furthermore, with increasing volatility of the capacity factor the probability of project continuation decreases faster, more and more unreachable states are observed, and “stop” – decisions occur earlier in time.

### 3.2. Real options model for the investment in a flexibility enhancement measure

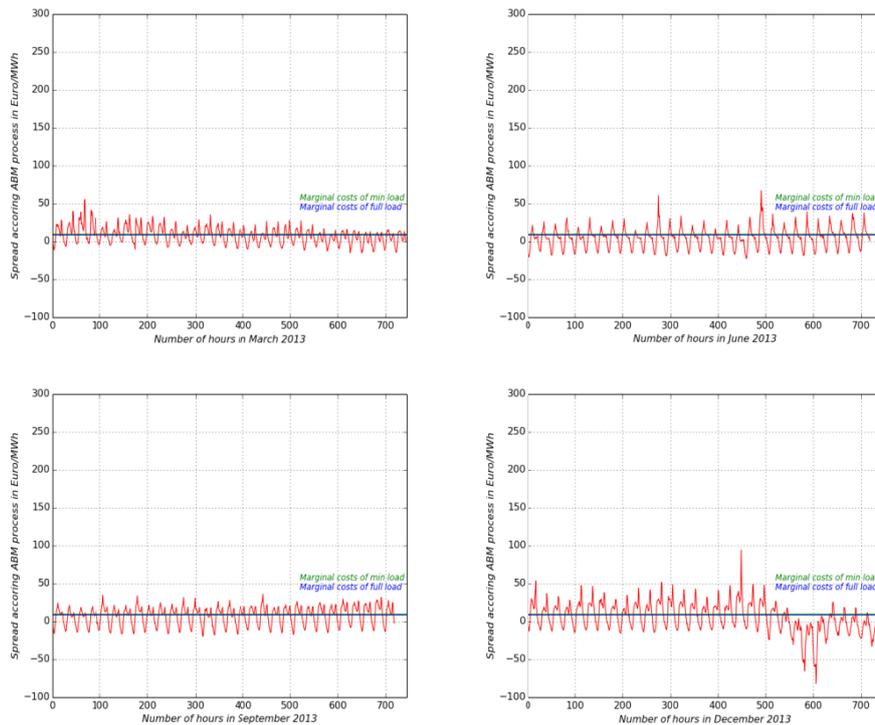
Flexibility or flexible operation of a conventional power plant can be understood in different ways. One of them defines the increase in flexibility by reducing the start-up and shut-down times and costs, by the reduction of the minimum load, or by the increase in part-load operation capabilities or the load gradient. To achieve these effects, the installation of new additional technical components, or upgrading of already existing ones, can be expected. The proposed real options model for the investment in a flexibility measure gives the answer to the questions regarding in which component/s and when the power plant owner should invest.



**Fig. 8.** Electricity price development for March, June, September and December 2013 following an arithmetic Brownian motion process

Source: Glensk and Madlener [2015].

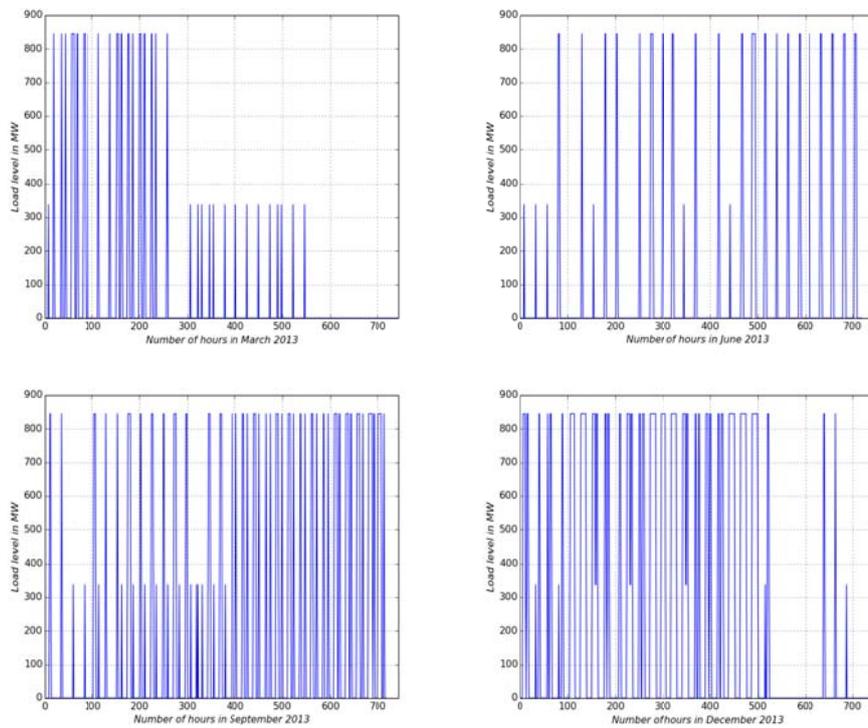
The model's procedure consists of three steps. In the first one, the simplified operation strategy of the power plant is defined. Here only maximum, minimum, and zero operation levels can be determined when applying (1) the electricity price and (2) the spark spread as profitability indicators and sources of uncertainty, respectively. Using an arithmetic Brownian motion process, the development of the electricity price and spark spread for selected months in 2013 as well as the operation strategy are presented in Figures 8-11 (for more information see Glensk and Madlener [2015]).



**Fig. 9.** Spark spread development for March, June, September and December 2013, following an arithmetic Brownian motion process

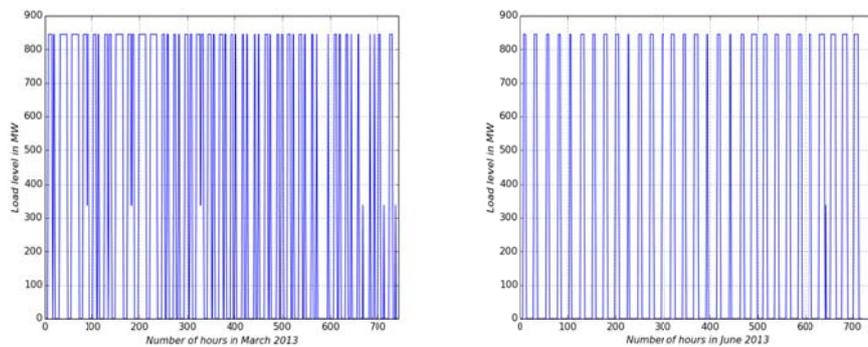
Source: Glensk and Madlener [2015].

The analysis conducted verifies very well the expected future role of conventional power generation, which means an interrupted operation with more shut-downs and start-ups, and delivery of the electricity on demand.



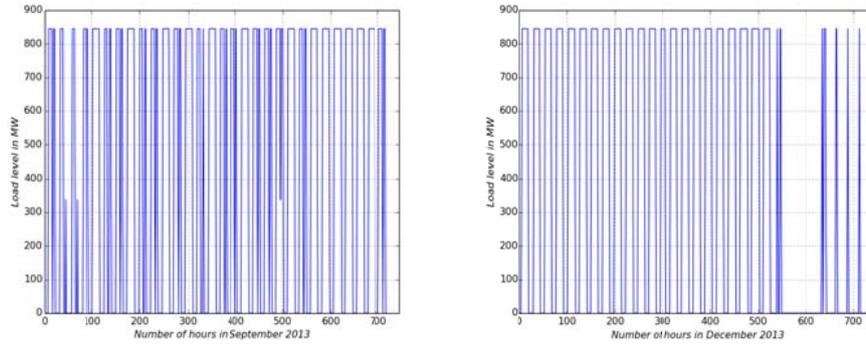
**Fig. 10.** Operation strategy for March, June, September and December in 2013 with the electricity price used as a profitability indicator and source of uncertainty

Source: Glensk and Madlener [2015].



**Fig. 11a.** Operation strategy for March, June, September and December 2013 with the spark spread used as a profitability indicator and source of uncertainty

Source: Glensk and Madlener [2015].



**Fig. 11b.** Operation strategy for March, June, September and December 2013 with the spark spread used as a profitability indicator and source of uncertainty

Source: Glensk and Madlener [2015].

The cash flow values of the power plant can be calculated in the second step of the procedure, using defined above operation strategy. Nevertheless, based on the assumptions made by Deng and Oren [2003], the calculations consider only three possible operation stages (S1 – the power plant is off, S2 – the power plant in at min load operation, and S3 – the power plant in at max load operation), and only three possible actions to be taken (A1 – the power plant runs at full-capacity, A2 – the power plant runs at a low-capacity level, and A3 – the power plant is turned off). The operating cash flow of the power plant for each hour can be presented as follows:

$$CF_t(P_t, A_t, S1) = \begin{cases} \text{for } A_t = A1: -c_{start-up} - c_{ramp-up}(load\ max_t) \\ \text{for } A_t = A2: -c_{start-up} - c_{ramp-up}(load\ min_t) \\ \text{for } A_t = A3: 0 \end{cases} \quad (28)$$

$$CF_t(P_t, A_t, S2) = \begin{cases} \text{for } A_t = A1: -c_{ramp-up}(load\ max_t) \\ \text{for } A_t = A2: P_t(load\ min_t) \\ \text{for } A_t = A3: -c_{shut-down} \end{cases} \quad (29)$$

$$CF_t(P_t, A_t, S3) = \begin{cases} \text{for } A_t = A1: P_t(load\ max_t) \\ \text{for } A_t = A2: P_t(load\ min_t) \\ \text{for } A_t = A3: -c_{shut-down} \end{cases} \quad (30)$$

where:

$$P_t(load_t) = \left( P_{elec\ abm,t} - \frac{P_{gas\ abm,t}}{\eta(load_t)} - \frac{P_{CO_2\ abm,t} \cdot e_{spec}}{\eta(load_t)} - OM_{var} \right) \cdot load_t \quad (31)$$

or:

$$P_t(\text{load}_t) = \left( \text{Spread}_{abm,t} - \frac{P_{CO_2 abm,t} \cdot e_{spec}}{\eta(\text{load}_t)} - OM_{var} \right) \cdot \text{load}_t \quad (32)$$

for the electricity price and spark spread, respectively. Moreover,  $c_{start-up}$ ,  $c_{shut-down}$ ,  $c_{ramp-up}$  define the costs for the start-up, shut-down, and ramp-up;  $\text{Spread}_{abm,t}$ ,  $P_{elec abm,t}$ ,  $P_{gas abm,t}$ , and  $P_{CO_2 abm,t}$  define the development of the spark spread and the electricity, gas und CO<sub>2</sub> prices, according to arithmetic Brownian motion processes over time  $t$ ;  $OM_{var}$  are variable operation and maintenance costs; and  $\eta(\text{load}_t)$  is the load-level dependent net efficiency of the power plant at  $\text{load}_t \in \{\text{load max}_t, \text{load min}_t\}$ .

Finally, in a last step, the option to expand the decision process is considered, defined as follows:

$$\text{if } PV_t < \text{Option to expand} - \text{invest} \quad (33)$$

$$\text{if } PV_t \geq \text{Option to expand} - \text{wait} \quad (34)$$

where:

$$PV_t = \sum_{t=0}^T \frac{CF_t - OM_{fixed,t} - Dep_t}{(1 + WACC)^t} \quad (35)$$

and:

$$\text{Option to expand} = \max(RPV_{t+\Delta t} - retrofit_{investment}, 0) \quad (36)$$

and  $OM_{fixed}$  are fixed operation and maintenance costs,  $Dep_t$  defines depreciation, the WACC (Weighted Average Cost of Capital) is used as discount rate, and  $RPV_{t+\Delta t}$  is the project value after retrofitting considering time ( $\Delta t$ ) needed for the investment in the considered improvement.

In the considered case study, technical enhancement options leading to a decrease in the minimum load level (e.g. variable-pitch guide vanes at the gas turbine compressor or an air preheater) were analyzed. Regarding the electricity price as well as the spark spread as sources of uncertainty, the decision to invest in technical improvement should be undertaken immediately, as Tables 1 and 2 show.

The proposed real options model simply takes advantage from market uncertainties incorporated in the model's structure. It can be a useful and novel approach in the decision-making process regarding investments in flexibility enhancements of conventional power plants.

**Table 1.** Present value of the retrofitted power plant, option value and decision when the electricity price is the source of uncertainty

Turn-off time during the realization (in months)	RPV of power plant (in €)	Option value (in €)	Decision for $PV_{01,2013}^* = -169,414,781$
2	-100,792,111	0.00	invest
3	-99,941,125	0.00	invest
4	-99,383,841	0.00	invest
5	-98,906,139	0.00	invest
6	-98,523,480	0.00	invest
7	-99,087,585	0.00	invest
8	-98,740,655	0.00	invest
9	-97,508,996	0.00	invest
10	-98,113,813	0.00	invest

Source: Glensk and Madlener [2015].

**Table 2.** Present value of the retrofitted power plant, option value and decision when the spark spread is the source of uncertainty

Turn-off time during the realization (in months)	RPV of power plant (in €)	Option value (in €)	Decision for $PV_{01,2013}^* = -77,069,139$
2	-3,748,871	0.00	invest
3	-2,858,149	0.00	invest
4	-1,089,553	0.00	invest
5	220,123	0.00	invest
6	600,109	0.00	invest
7	-122,454	0.00	invest
8	1,057,882	0.00	invest
9	1,462,912	0.00	invest
10	-663,053	0.00	invest

Source: Glensk and Madlener [2015].

## Conclusions and recommendations

Regarding the rapid evolution of the energy sector, power plants operators are often forced to reconsider their investment decisions and optimal operation strategies in light of new risks and uncertainties. Market liberalization, unforeseen policy changes, and the rapidly increasing use of renewable energy sources, challenge the standard toolbox of decision-makers and ask for more sophisticated and powerful tools. Herein this article, we provided an overview of how well-developed financial methods can be applied to real assets such as power

plants, and what new possibilities in the decision-making process their application offers. Moreover, considering the specific character of the power plants as real assets, the technical as well as economic parameters (including e.g. price uncertainties) of the power plant can be straightforwardly incorporated in the analysis.

Furthermore, the results obtained from the application of different MVP models allows to better capture the dynamics in the diversification process of power generation mix (especially regarding new investments), to integrate the investors' aspirations with regard to return and risk (e.g. by using the FSMAD model), as well as rebalancing the generation mix by taking risks and uncertainty more appropriately into account. In the case of ROA, the proposed models can better guide the investment decision-making process in light of the risks and uncertainties inherent in the development in the energy markets. As was shown, both investment and disinvestment problems, and also flexibility enhancement problems, can be supported through different model types and solution methods offered by real options theory.

Finally, a further aspect shown in our article is the graphical visualization of results, which can have positive impact the decision-making process through the straightforward and easy-to-comprehend comparison of graphs and tables, which in our studies were obtained predominantly by the application of MS Excel and Python.

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#### PRZEGLĄD WYBRANYCH METOD OPTYMALIZACJI PORTFELA I PODEJMOWANIA NIEODWRACALNYCH DECYZJI W SEKTORZE ENERGETYCZNYM W WARUNKACH NIEPEWNOŚCI

**Streszczenie:** W niniejszym artykule zostały zaprezentowane przykłady zastosowań wybranych metod optymalizacyjnych portfeli na rynku energetycznym, takich jak klasyczna teoria portfela Markowitza, dynamiczne ujęcie optymalizacji portfela czy wykorzystanie teorii zbiorów rozmytych do tworzenia portfeli. W świetle liberalizacji rynku energetycznego, jak również wzrastającego ryzyka związanego z coraz większym udziałem odnawialnych źródeł energii w procesie generowania energii, autorzy artykułu proponują także zastosowanie teorii opcji realnych w procesie decyzyjnym. Modele opcji realnych pozwalają, między innymi, na określenie optymalnego terminu realizacji inwestycji (czy też dezinwestycji) i mogą służyć jako alternatywna metoda w procesie podejmowania decyzji.

**Słowa kluczowe:** optymalizacja portfela, teoria zbiorów rozmytych, analiza opcji realnych.