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A NEW PROCEDURE OF CRITERIA WEIGHT DETERMINATION WITHIN THE ARAS METHOD

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Abstract

In most of multicriteria aggregation methods, we need to elicit parameters that are generally determined directly by the decision-maker (DM). Direct assigning of parameters and criteria weights presents a crucial and difficult step in the decision-making process. However, this kind of information is too subjective and may affect the reliability of the results. To overcome this issue, we suggest a weighting method based on mathematical programming to incorporate the DM's preferences indirectly within the ARAS method.

Keywords: MCDA, preference disaggregation, ARAS, criteria weights.

1 Introduction

Multiple criteria decision analysis (MCDA) is a general framework for supporting complex decision-making situations with multiple and often conflicting objectives. Commonly, the multicriteria methods require setting criteria weights in order to be implemented. Therefore, the problem of criteria weight determination has gained the interest of many researchers during the past decades. There are two ways of weight elicitation: 'a priori weights' that are determined directly by the experts and 'a posteriori weights' obtained from the data. This paper adopts the 'a posteriori approach'. Hence, we focus on reducing the subjectivity and the unreliability of weight values when they are directly determined by the DM

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without excluding him from the decision making process. Thus, we propose a new procedure of preference disaggregation in order to elicit criteria weights in the ARAS method. This approach is based on preference relations provided by the decision maker, as well as on comparisons between differences of criteria weights. Our weight elicitation method is based on solving a linear program which takes into account the DM's preferences.

Our paper consists of six sections. Section 2 will give a brief survey of the state of the art of selected weighting methods; selected preference disaggregation approaches will be described. In Section 3, the different steps of the ARAS method will be presented. In section 4, we will develop a criteria determination approach based on the ARAS method. In section 5, a case study will be presented to discuss the feasibility of the proposed model. In section 6, we present conclusions and perspectives for future research.

2 A review of the literature

Chiang (2009) noted that “one of the most difficult tasks in multiple criteria decision analysis (MCDA) is determining the weights of individual criteria so that all alternatives can be compared based on the aggregate performance of all criteria”. For this reason, many methods have been developed to objectively determine the values of criteria weight. For instance, Figueira and Roy (2001) proposed a version of the Simos method which takes into account a new kind of information supplied by the DM and changed some computing rules. In addition, a new software package based on the revised Simos' procedure has been implemented. In addition, Chiang (2009) proposed a measure of the relative distance, which involved the calculation of the relative position of an alternative between the anti-ideal and the ideal for ranking to seek the shortest absolute distance between an alternative and the ideal one. The author showed that the relative distance produces consistent rankings for any set of weights, regardless of how they are determined. Thus, this method is suitable for cases where no prior information can be used for determining the weights. Furthermore, Rezaei (2009) proposed a new method called BWM (Best-Worst Method). First, the DM gives the best and the worst criterion. Then, pairwise comparisons are conducted between each of these two criteria (best and worst) and the remaining ones. After that, a maximin problem is formulated and solved to determine the weights of different criteria. In the same context, Roszkowska (2013) presented a comparative overview on several rank ordering weight methods that convert the ordinal ranking of a number of criteria into numerical weights. Also, Siskos and Tsotsolas (2015) proposed a set of complementary robustness analysis rules and measures integrated in a robust Simos method for the elicitation of the criteria

weights. The goal was to aid the DM and the analysts to gain insight on the whole set of weighting solutions, to select a single set of criteria weights and to apply robust rules based on multiple sets of acceptable weights.

Approaches to preference disaggregation

In the aggregation paradigm, the aggregation model is known a priori, whereas the global preference is unknown. On the other hand, the philosophy of the disaggregation involves the inference of preference models from the given global preferences.

The development of preference disaggregation methods was initiated in 1978. In the disaggregation-aggregation approach, iterative interactive procedures are used to be aggregated later to a value system (Siskos, 1980; Jacquet-Lagrèze and Siskos, 1982, 2001; Siskos and Yannacopoulos, 1985; Siskos et al., 1993). The first developed preference disaggregation method was the UTA method proposed by Jacquet-Lagrèze and Siskos (1982). The purpose of this method is to infer additive value functions from a given ranking through linear programming. Besides, Mousseau and Slowinski (1998) developed a global inference approach to determine ELECTRE III's parameters. In the same way, Lourenço and Costa (2004) developed a disaggregation approach for the determination of weight coefficients as well as a category of reference profiles of ELECTRE III. Furthermore, Dias and Mousseau (2006) developed a mathematical program to determine the veto thresholds of the ELECTRE III method. Nevertheless, Corrente et al. (2014) opted for the Robust Ordinal Regression (ROR) to determine the different values of ELECTRE parameters. On the other hand, Frikha et al. (2018) determined the ELECTRE I parameters based on the outranking relations given by the DM. In addition, Mousseau et al. (2001) solved a linear program to infer criteria weights in the ELECTRE III method. They used a pure maxmin and a standard additive objective function. In the same context, Kadzinski et al. (2017) developed a disaggregation approach to elicit the parameters of the ELECTRE III-C method. Indeed, Frikha et al. (2010) determined the relative importance of the criteria of the PROMETHEE method based on some preference relations and other information provided by the DM. Also, Frikha et al. (2011a) developed an interactive disaggregation approach to infer the indifference thresholds of the PROMETHEE II method based on some preference relations. Later, Frikha et al. (2011b) proposed an approach to elicit both preference and indifference thresholds of the PROMETHEE method. Moreover, Frikha et al. (2017) developed a mathematical programming model to determine the relative importance of the criteria as well as the preference and the indifference thresholds in the PROMETHEE method. Disaggregation methods in multi-criteria decision analysis use linear programming, in particular goal

programming, in eliciting preference aggregation models (Siskos, 1983). For instance, Charnes et al. (1955) proposed a linear model by disaggregating pairwise comparisons and given measures. Greco et al. (2010) used robust ordinal regression to describe an interactive multiobjective optimization methodology called NEMO. Likewise, Kadziński et al. (2013) used ROR to establish the rank of the alternatives. Furthermore, Corazza et al. (2015) determined the parameter values of the MULTicriteria RAnking MEthod (MURAME), while Valkenhoef and Tervonen (2016) considered the elicitation of incomplete preference information for the additive utility model in terms of linear constraints on the weights using holistic pairwise comparisons given by the DM. Likewise, De Almeida et al. (2016) used partial holistic information to determine criteria weights based on Multi-Attribute Value Theory (MAVT). Furthermore, Kadziński et al. (2017) developed a set of interactive evolutionary multiple objective optimization (MOO) methods, called NEMO-GROUP.

In this paper, we propose a new approach to elicit criteria weights of the ARAS method.

3 The ARAS method

The ARAS (*Additive Ratio ASsessment*) method proposed by Zavadskas and Turskis (2010) as a ranking method. Its purpose is to select the best alternative among others. It has been applied in several fields such as technology, construction, investments, etc., to validate the selection of a decision alternative.

The steps of the ARAS method are:

Step 1

The first stage of ARAS is to create the decision-making preference matrix consisting of m alternatives and n criteria.

Let

x_{ij} be the performance value of the alternative i according to the criterion j ;

m be the number of alternatives and

n be the number of criteria.

$$X = \begin{bmatrix} x_{01} & \dots & x_{0j} & \dots & x_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{i1} & \dots & x_{ij} & \dots & x_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mj} & \dots & x_{mn} \end{bmatrix} \quad i=0, \dots, m ; j=1, \dots, n$$

Step 2

The second stage in most of MCDM methods is the normalization of the decision matrix. The purpose of any normalization technique is to unify incommensurable criteria measures so that all the performances can be compared. In the literature, two normalization ways are suggested:

The criteria whose preferable values are maxima, are normalized as follows:

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^m x_{ij}} \quad (1)$$

The criteria whose preferable values are minima, are normalized as follows:

$$x_{ij} = \frac{1}{x_{ij}^*}; \quad (2)$$

$$\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^m x_{ij}} \quad (3)$$

where

\bar{x}_{ij} are the normalized values of the normalized decision matrix \bar{X} and

x_{ij}^* is the optimal value of the criterion j .

x_{0j} is the initial value of the minimized criterion j .

If the optimal value of criterion j is unknown, then $x_{0j} = \max x_{ij}$, if $\max x_{ij}$ is preferable and

$x_{0j} = \min x_{ij}^*$, if $\min x_{ij}^*$ is preferable.

Thus, the general form of the normalized decision matrix \bar{X} is:

$$\bar{X} = \begin{bmatrix} \bar{x}_{01} & \dots & \bar{x}_{0j} & \dots & \bar{x}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{i1} & \dots & \bar{x}_{ij} & \dots & \bar{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \bar{x}_{m1} & \dots & \bar{x}_{mj} & \dots & \bar{x}_{mn} \end{bmatrix} \quad i = 0, \dots, m; j = 1, \dots, n$$

Step 3

The third stage consists in creating the weighted-normalized matrix \hat{X} .

The weighted-normalized values of all the criteria are calculated as follows:

$$\hat{x}_{ij} = \bar{x}_{ij} w_j; \quad i = 0, \dots, m; j = 1, \dots, n \quad (4)$$

where

\bar{x}_{ij} is the normalized evaluation value of the alternative i according to the criterion j ;

w_j is the weight of the criterion j and

$\sum_{j=1}^n w_j = 1$ (criteria weights must be normalized)

The weighted normalized matrix is:

$$\hat{X} = \begin{bmatrix} \hat{x}_{01} & \dots & \hat{x}_{0j} & \dots & \hat{x}_{0n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{x}_{i1} & \dots & \hat{x}_{ij} & \dots & \hat{x}_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{x}_{m1} & \dots & \hat{x}_{mj} & \dots & \hat{x}_{mn} \end{bmatrix} \quad i=0, \dots, m ; j=1, \dots, n$$

Step 4

The objective of this step is to determine the values of the optimality function, denoted by S_i , such that

$$S_i = \sum_{j=1}^n \hat{x}_{ij} ; i = 0, \dots, m \tag{5}$$

Step 5

In ARAS, the value K_i of the utility function determines the relative efficiency of a feasible alternative a_i . It can be calculated as follows:

$$K_i = \frac{S_i}{S_0} ; i = 0, \dots, m \tag{6}$$

where S_0 is the optimal value (i.e., the maximum value of S_i) and the calculated values K_i are in the interval $[0,1]$.

Step 6

The last step of the ARAS method consists in ranking, in an increasing order, the values K_i of the utility function. As a result, we obtain the rank of all the alternatives and therefore also the best one.

Thus, we choose to change the normalization formula of ARAS to a more convenient one (normalization by the minimum-maximum) because the linear normalization technique is not symmetric. Actually, the normalized values of the alternative are lower for the benefit criteria and higher for the cost criteria (Vafaei et al., 2015).

The minimum-maximum normalization technique can be described as follows:

In the case of maximization criteria, we replace the formula ($\bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=0}^m x_{ij}}$) by

$$\bar{x}_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})} \tag{7}$$

In the case of minimization criteria, we use:

$$\bar{x}_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})} \tag{8}$$

Thus, we propose a new procedure of preference disaggregation to elicit the criteria weights of ARAS.

4 The proposed model for the determination of ARAS criteria weights

In most procedures, ranking is a necessary first step for eliciting accurate weights. Usually, criteria weights are obtained from the rank order of each criterion. Thus, ARAS has a serious flaw: the criteria weights are too subjective since they are provided directly by the DM. Therefore, we propose a mathematical programming model that aims to determine the criteria weights objectively, but without excluding the DM. For that purpose, the decision maker is asked to provide pairwise comparisons of alternatives and criteria weights. The provided information is integrated into the following program.

Program 1

$$\max \sum_{i=1}^p g_i \quad (9)$$

Subject to

$$\sum_{j=1}^n w_j \bar{x}_{Bj} - \sum_{j=1}^n w_j \bar{x}_{Qj} - g_i \geq 0 \quad \forall B, Q \in A; \forall i=1, \dots, p \quad (10)$$

$$w_k - w_l \geq w_r - w_v, \quad k, l, r, v \in [1, \dots, n] \quad (11)$$

$$w_k \geq w_l, \quad k, l \in [1, \dots, n] \quad (12)$$

$$g_i \geq \frac{1}{2^{(p-1)}} \quad \forall i=1, \dots, p \quad (13)$$

$$w_j \geq e \quad \forall j=1, \dots, n \quad (14)$$

$$\sum_{j=1}^n w_j = 1 \quad (15)$$

Let:

A : be the set of alternatives;

p : be the number of relations between pairwise preferences among alternative preferences provided by the decision-maker;

w_j be the weight of the j^{th} criterion;

e be a threshold.

Within ARAS, alternative B is preferable over alternative Q ($B > Q$) if $K_B \geq K_Q$. The degree of preference of B over Q (g_i) is the difference between the two utility degrees with respect to all the criteria, that is, $K_B - K_Q = g_i$ for every preference relation i provided by the DM.

In order to ensure strict preference and to avoid the relationship of indifference between two alternatives, we have to maximize the sum of slack variables g_i given in Equation (9).

In addition, in ARAS, all alternatives are ranked according to the decreasing order of the values of their utility degrees. As we said before, alternative B is preferable to Q is equivalent to: the utility degree of B is greater than that of Q .

$$\text{Then, } K_B \geq K_Q \quad (16)$$

$$\text{Consequently, } \frac{S_B}{S_0} \geq \frac{S_Q}{S_0} \tag{17}$$

where S_0 is the best value.

$$\sum_{j=1}^n \widehat{x}_{Bj} \geq \sum_{j=1}^n \widehat{x}_{Qj} \tag{18}$$

where \widehat{x}_{Bj} and \widehat{x}_{Qj} are the normalized-weighted values of all the criteria

$$\sum_{j=1}^n w_j \bar{x}_{Bj} \geq \sum_{j=1}^n w_j \bar{x}_{Qj} \tag{19}$$

where \bar{x}_{Bj} and \bar{x}_{Qj} are the normalized values of the decision matrix.

Then, the preference relations expressed by the DM are modeled in the mathematical program as $\sum_{j=1}^n w_j \bar{x}_{Bj} - \sum_{j=1}^n w_j \bar{x}_{Qj} - g_i \geq 0 \forall B, Q \in A; \forall i=1, \dots, p$ Equation (10).

In addition to the preference relations, the DM should provide two other pieces of information. The first one concerns the comparisons of the differences of adjacent weights written as:

$w_k - w_l \geq w_r - w_v$ Equation (11). Therefore, the gap between the importance of criteria k and l is more important than that between r and v .

The second piece of information concerns a partial pre-order on criteria weights. The DM is asked to supply pairwise comparisons of criteria weights in the form $w_k \geq w_l \forall k \in [1, \dots, n]; \forall l \in [1, \dots, n]$ Equation (12). The number of partial pre-order constraints must not exceed $(n-1)$.

In order to guarantee the preference between the pairs of preferences provided by the DM and to avoid the situation of indifference, we impose the condition that all slack variables (g_i) are strictly positive. Consequently, we have to set a minimum threshold for each g_i according to each preference relation. It is evident that the threshold value is strongly dependent on the number of preference relationships, hence it can be equal to $\frac{1}{2^{(p-1)}}$. Thus, we introduce the constraint $g_i \geq \frac{1}{2^{(p-1)}} \forall i=1, \dots, p$ Equation (13).

The constraint (14) is related to a threshold of the weight values. Indeed, in the constraints of the weight determination, we should take into account the condition that all criteria weights should be strictly positive ($w_j > 0$) in order to prevent any criterion from being null and therefore ignored. Since mathematical programming deals with weak inequalities and not with strict inequalities, we should set a small positive threshold e associated with each importance coefficient w_j . Depending on the value of e , the criterion may be meaningless. The value of e is dependent on the number of criteria. Then, we should add the constraint $w_j \geq e \forall j= 1, \dots, n$ to the mathematical program.

Moreover, we should take into account that all criteria weights are normalized. This means that the sum of all the weights is equal to 1. For example, if we have n criteria, then $\sum_{j=1}^n w_j = 1$ Equation (15).

Our approach is iterative and interactive. In the iterative process of determining ARAS criteria weights, the DM is free to add or to remove information whenever needed. The additional information consists in adding or even removing one or more preference relations. Each additional information and each preference relation will be modeled in the mathematical program as constraints. In real-world decision problems, the decision-makers have difficulty in providing reliable information due to time constraints and their cognitive limitations. Therefore, the preferences of the decision makers are not necessarily stable: they can evolve over time and can even contain conflicting and inconsistent information. The role of an interactive tool is to help the DM to understand his preferences and their representation in a specific aggregation method. Inconsistencies occur when the DM's preferences cannot be obtained from the aggregation method used.

5 An illustrative example

Rainwater source control is usually considered as an alternative solution of water evacuation by sanitation networks. The alternatives (infiltration and retention basin, porous pavements with tank structure, infiltration wells, draining trenches, berms, storage roofs and buried pools) are subject to pollution and floods caused by rainwater in urban areas. Therefore, water managers face many obstacles related to the diversity of management techniques of a source of rainwater. Decision support tools are therefore required to guide the water managers in the choice of the best alternative. Therefore, multiple criteria methods are needed to develop such decision support (Martin and Legret, 2005).

A storm water Best Management Practice (BMP) is a practice that is suitable for reducing the volume of overflow and treating pollutants in storm water runoff. Therefore, the alternatives represent the eight types of Best Management Practice (BMP).

- **A1:** Wet pond (retention basin): “A retention basin or wet pond is a storm water control structure with a permanent pool of water into which storm runoff is directed. Runoff from each storm is retained, allowing suspended sediment particles and associated pollutants to settle out. Water in the basin infiltrates or is displaced by runoff from a subsequent storm” (Kathryn et al., 2011).
- **A2:** Dry pond (detention basin): “A detention basin or dry pond is a structure into which storm water runoff is directed, held for a period of time (detained), and slowly released to a surface water body. A dry pond is not designed to permanently contain water. It can help to improve water quality by allowing suspended solids to settle over a period of time. The temporary storage of storm runoff water also decreases downstream peak flow rates which can reduce potential flooding” (Kathryn et al., 2011).

- **A3:** Buried pool: “Hidden basins but remaining accessible, intended to store underground rainwater” (Iowa Drainage Law Manual).
- **A4:** Berm: “A horizontal strip or shelf built on or cut into an embankment to break the continuity of a long slope, usually to reduce erosion or increase the size of the embankment” (Iowa Drainage Law Manual).
- **A5:** Porous pavement with tank structure: “Porous, permeable or pervious pavement includes several methods and materials that allow water and air to move through the pavement and into the underlying soil. Some examples of permeable pavement include specially designed and constructed concrete, asphalt, paving stones or bricks. Permeable pavement sometimes includes an underlying reservoir for additional water storage” (Kathryn et al., 2011).
- **A6:** Draining trenches (storm sewer): “A natural or artificial waterway where a stream of water flows periodically or continuously or forms a connecting link between bodies of water. Also a conduit such as a pipe conveys water” (Iowa Drainage Law Manual).
- **A7:** Storage roofs: “waterproofing coating installed on the roofs of buildings protected by grave land designed to temporarily retain rainwater” (Iowa Drainage Law Manual).
- **A8:** Infiltration wells: “an infiltration basin is a shallow impoundment designed to infiltrate storm water runoff into the soil. Infiltration basins do not release water except by infiltration, evaporation, or emergency overflow” (Kathryn et al., 2011).

These alternatives are evaluated according to eight criteria which are:

- **C1:** pollution retention (to be maximized)
- **C2:** probability of dysfunction (to be minimized)
- **C3:** need for and frequency of maintenance operations (to be minimized)
- **C4:** impact on groundwater quality (to be minimized)
- **C5:** level of approval (to be maximized)
- **C6:** contribution to development policies (to be maximized)
- **C7:** equity stake (to be maximized)
- **C8:** maintenance costs (to be minimized)

The criteria: pollution retention (C1), need for and frequency of maintenance operations (C2), impact on groundwater quality (C4), level of approval (C5) and contribution to development policies (C6) have been evaluated on the basis of the analysis of the results of a satisfaction survey on the use of alternative techniques in rain water sanitation. They are evaluated on a scale of 1 to 5 or 1 to 3.

The criterion probability of dysfunction (C2) is evaluated in %, according to a bibliographic study on different alternative techniques.

The criteria equity stake (C7) and maintenance costs (C8) are valued numerically, in € and € / year, respectively.

The DM provides the following decision matrix (Table 1).

Table 1: Decision matrix

| Criteria Alternatives | C1 max | C2 min | C3 min | C4 min | C5 max | C6 max | C7 max | C8 min |
|--------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| A1 | 4 | 20 | 3 | 2 | 5 | 3 | 38 | 32 |
| A2 | 4 | 20 | 3 | 2 | 5 | 3 | 54 | 32 |
| A3 | 4 | 20 | 2 | 2 | 1 | 1 | 370 | 32 |
| A4 | 4 | 40 | 3 | 2 | 3,5 | 3 | 13 | 30 |
| A5 | 4 | 60 | 2 | 2 | 2,5 | 2 | 54 | 4,5 |
| A6 | 4 | 60 | 2 | 2 | 2,5 | 2 | 39 | 1,2 |
| A7 | 1 | 40 | 2 | 5 | 1 | 2 | 0 | 2 |
| A8 | 4 | 60 | 2 | 1 | 1 | 1 | 4 | 2 |

The normalization of the decision matrix is based on equations 7 and 8.

We get the normalized values and hence the normalized decision matrix (Table 2).

Table 2: Normalized decision matrix

| Criteria Alternatives | C1 max | C2 min | C3 min | C4 min | C5 max | C6 max | C7 max | C8 min |
|--------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| A1 | 1 | 1 | 0 | 0,75 | 1 | 1 | 0,103 | 0 |
| A2 | 1 | 1 | 0 | 0,75 | 1 | 1 | 0,146 | 0 |
| A3 | 1 | 1 | 1 | 0,75 | 0 | 0 | 1 | 0 |
| A4 | 1 | 0,5 | 0 | 0,75 | 0,625 | 1 | 0,035 | 0,065 |
| A5 | 1 | 0 | 1 | 0,75 | 0,375 | 0,5 | 0,146 | 0,893 |
| A6 | 1 | 0 | 1 | 0,75 | 0,375 | 0,5 | 0,105 | 1 |
| A7 | 0 | 0,5 | 1 | 0 | 0 | 0,5 | 0 | 0,974 |
| A8 | 1 | 0 | 1 | 1 | 0 | 0 | 0,011 | 0,974 |

Thus, the manager of the civil engineering department gave the following pairwise preference relations among the alternatives:

$$A_8 > A_4;$$

$$A_3 > A_7;$$

$$A_6 > A_8;$$

$$A_2 > A_5;$$

$$A_1 > A_3;$$

He also gave some comparisons between differences of criteria weights:

$$w_5 - w_6 \geq w_1 - w_4$$

$$w_3 - w_2 \geq w_7 - w_8$$

The gap between criteria 5 and 6 is more important than that between criteria 1 and 4 (Equation 7).

Moreover, some pairwise comparisons among criteria weights are given:

$$w_1 \geq w_5$$

$$w_4 \geq w_3$$

The information provided is incorporated into the following mathematical program (program 2).

Program 2

$$\begin{aligned} &\max \sum_{i=1}^5 g_i \\ &\sum_{j=1}^8 w_j \bar{x}_{A_8j} - \sum_{j=1}^8 w_j \bar{x}_{A_4j} - g_1 \geq 0 \\ &\sum_{j=1}^8 w_j \bar{x}_{A_3j} - \sum_{j=1}^8 w_j \bar{x}_{A_7j} - g_2 \geq 0 \\ &\sum_{j=1}^8 w_j \bar{x}_{A_6j} - \sum_{j=1}^8 w_j \bar{x}_{A_8j} - g_3 \geq 0 \\ &\sum_{j=1}^8 w_j \bar{x}_{A_2j} - \sum_{j=1}^8 w_j \bar{x}_{A_5j} - g_4 \geq 0 \\ &\sum_{j=1}^8 w_j \bar{x}_{A_1j} - \sum_{j=1}^8 w_j \bar{x}_{A_3j} - g_5 \geq 0 \\ &w_5 - w_6 \geq w_1 - w_4 \\ &w_3 - w_2 \geq w_7 - w_8 \\ &w_1 \geq w_5 \\ &w_4 \geq w_3 \\ &g_i \geq 0.0625 \quad \forall i=1, \dots, 5 \\ &w_j \geq 0.05 \quad \forall j=1, \dots, 8 \\ &\sum_{j=1}^8 w_j = 1 \end{aligned}$$

We choose to solve the proposed model using the LINGO commercial software package. As a result, by solving this mathematical program, we obtain the following criteria weights:

$$\begin{aligned} w_1 &= 0.123 \\ w_2 &= 0.255 \\ w_3 &= 0.065 \\ w_4 &= 0.079 \\ w_5 &= 0.123 \\ w_6 &= 0.05 \\ w_7 &= 0.05 \\ w_8 &= 0.256 \end{aligned}$$

Sensitivity analysis is crucial at this stage. It investigates how the uncertainty in the output of a mathematical model can be divided into different sources of uncertainty in its inputs. It is also known as the what-if analysis. In this sensitivity analysis, we will study the effect of different normalization forms on criteria weights. These forms are as follows.

The minimum-maximum normalization technique:

$$\text{In the case of benefit criteria: } \bar{x}_{ij} = \frac{x_{ij} - \min(x_{ij})}{\max(x_{ij}) - \min(x_{ij})}$$

$$\text{In the case of cost criteria: } \bar{x}_{ij} = \frac{\max(x_{ij}) - x_{ij}}{\max(x_{ij}) - \min(x_{ij})}$$

The normalization technique by the maximum:

$$\text{In the case of benefit criteria: } \bar{x}_{ij} = \frac{x_{ij}}{\max x_{ij}}$$

$$\text{In the case of cost criteria: } \bar{x}_{ij} = 1 - \frac{x_{ij}}{\max x_{ij}}$$

The linear normalization technique:

$$\text{In the case of benefit criteria: } \bar{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}}$$

$$\text{In the case of cost criteria: } \bar{x}_{ij} = \frac{\frac{1}{x_{ij}}}{\sum_{i=1}^m \frac{1}{x_{ij}}}$$

The vector normalization technique:

$$\text{In the case of benefit criteria: } \bar{x}_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

$$\text{In the case of cost criteria: } \bar{x}_{ij} = 1 - \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}}$$

Once the decision making matrix is normalized, we solve the mathematical model using the LINGO software package to get the criteria weights (Table 3).

Table 3: Weights obtained using each normalization form

| Normalization form | min-max | max | linear | vector |
|--------------------|---------|-------|--------|--------|
| w_1 | 0,123 | 0,151 | 0.148 | 0.259 |
| w_2 | 0,255 | 0,205 | 0.05 | 0.05 |
| w_3 | 0,065 | 0,05 | 0.05 | 0.05 |
| w_4 | 0,079 | 0,067 | 0.356 | 0.05 |
| w_5 | 0,123 | 0,151 | 0.148 | 0.259 |
| w_6 | 0,05 | 0,05 | 0.05 | 0.05 |
| w_7 | 0,05 | 0,084 | 0 | 0.073 |
| w_8 | 0,256 | 0,242 | 0.198 | 0.208 |

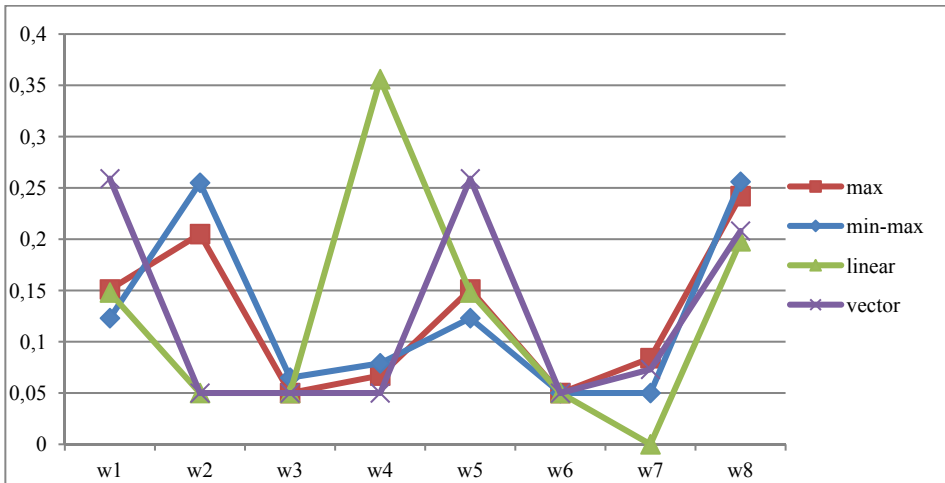


Figure 1: The curves of each normalization technique

As can be seen, the curves vary in different ways. However, there is a significant disparity in the variation of each curve. The fluctuation differs from one curve to another. As a consequence, we can conclude that this approach is sensitive to a change in the normalization technique.

The next step consists in building the weighted-normalized decision matrix in which we calculate the values of the optimality function (S_t), and the utility degree (K_t) to obtain a ranking of all the alternatives (Table 4).

Table 4: The weighted normalized decision matrix and solution

| Criteria \ Alternatives | C1 max | C2 min | C3 min | C4 min | C5 max | C6 max | C7 max | C8 min | S_t | K_t | Rank |
|-------------------------|--------|--------|--------|--------|--------|--------|---------|--------|--------|-------|------|
| A1 | 0,123 | 0,255 | 0 | 0,059 | 0,123 | 0,05 | 0,005 | 0 | 0,615 | 0,997 | 2 |
| A2 | 0,123 | 0,255 | 0 | 0,059 | 0,123 | 0,05 | 0,007 | 0 | 0,617* | 1 | 1 |
| A3 | 0,123 | 0,255 | 0,065 | 0,059 | 0 | 0 | 0,05 | 0 | 0,552 | 0,894 | 5 |
| A4 | 0,123 | 0,1275 | 0 | 0,059 | 0,077 | 0,05 | 0,002 | 0,017 | 0,455 | 0,737 | 8 |
| A5 | 0,123 | 0 | 0,065 | 0,059 | 0,046 | 0,025 | 0,007 | 0,229 | 0,554 | 0,897 | 4 |
| A6 | 0,123 | 0 | 0,065 | 0,059 | 0,046 | 0,025 | 0,005 | 0,256 | 0,579 | 0,939 | 3 |
| A7 | 0 | 0,1275 | 0,065 | 0 | 0 | 0,025 | 0 | 0,249 | 0,467 | 0,756 | 7 |
| A8 | 0,123 | 0 | 0,065 | 0,079 | 0 | 0 | 0,00055 | 0,249 | 0,517 | 0,837 | 6 |

* $S_0 = 0.617$ (the greater value).

The final ranking of the alternatives is: $A2 \succ A1 \succ A6 \succ A5 \succ A3 \succ A8 \succ A7 \succ A4$.

This means that A2 (dry pond / detention basin) is the best alternative for retaining excess rainwater since it reduces peak rate of runoff and alleviates flooding. It is also regarded as cost effective. A dry pond can be designed to improve water quality. A detention basin has the advantage that the space

surrounding the pond can be landscaped to enhance the beauty of the place and provide a habitat for the inhabitants.

On the other hand, berms are considered to be the worst alternative for retaining excess rainwater because they require a lot of space. Unless fill is available nearby, the cost of transporting it to the site may be prohibitive.

The obtained results are different from those found in the paper Martin and Legret (2005). The authors used the ELECTRE III method to classify the different BMPs according to three strategies (planning, urban development and environment protection) in France. The following figure shows the resulting outranking relations.

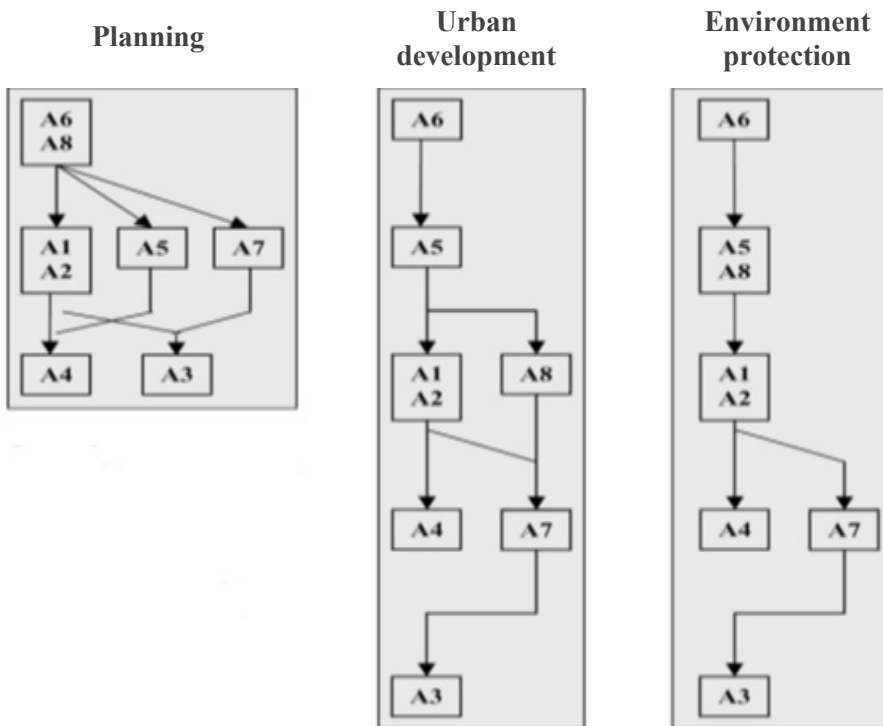
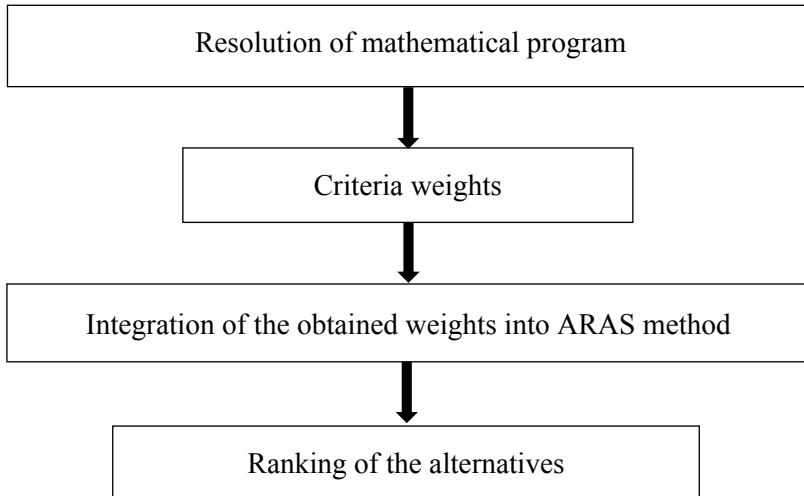


Figure 2: The outranking graphs (Martin and Legret, 2005)

The difference in the results is due to the fact that multi-criteria methods do not give the same output. In fact, the choice of a multi-criteria method is itself considered to be a multi-criteria problem. Indeed in MCDM, there is no optimal solution, rather a satisfying one (unlike in the exact methods). For instance, ELECTRE and ARAS cannot give the same result. Also, the preference relations obtained from the DM do not contradict the final rankings founded in Martin and Legret (2005), apart from two constraints ($A_2 > A_5$ and $A_3 > A_7$) which give the proposed method more consistency and reliability.

In the final analysis, the proposed model can be summarized by the following algorithm.



6 Conclusion and perspectives

In this paper, we have proposed an approach to criteria weight determination for the ARAS method. In most multicriteria aggregation problems, the DM determines directly the weight values using his own intuition. However, this information is too subjective which makes the results unreliable. To overcome this flaw, we suggested a weighting method that involves the DM indirectly in the decision-making process. The DM was asked to provide pairwise preferences among alternatives and criteria weights. On the basis of his preferences, we formulated a mathematical program using the ARAS method and solved it with the LINGO software package. Having obtained the weight values, we ranked the alternatives from the best to the worst. Finally, a case study in rainwater management in urban areas was given in order to implement the model. The main contribution of this paper is that the DM is not directly involved in the elicitation of weights, which reduces the subjectivity of the results. The proposed method can be applied to several real-world case studies. However, the proposed mathematical program is valid only for the ARAS method. It does not accept any threshold, either. In future research, we will consider eliciting criteria weights in a hierarchical structure of criteria.

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