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ANALYSIS OF THE MULTIPLE ATTRIBUTE DECISION MAKING PROBLEM WITH INCOMPLETE INFORMATION ABOUT PREFERENCES AMONG THE CRITERIA

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Abstract

The paper presents an overview of methods used in solving Multiple Attribute Decision Making (MADM) problems in the case of incomplete information about preferences among criteria, which are defined by explicit attributes of the problems. The paper presents the following methods: dominance, maxmin, maxmax, based on game theory, ELECTRE IV and parametric approach associated with Linear Partial Information and AHP. The presented methods focus on the problem of evaluation of investment projects in a hard coal mine.

Keywords: incomplete inter-criteria information, ELECTRE IV, Linear Partial Information, AHP.

1 Introduction

In the paper we discuss Multiple Criteria Decision Making (MCDM) problems with a finite set of decision variants, that is, Multiple Attribute Decision Making (MADM) problems. MADM problems are MCDM problems with clearly defined attributes of decision variants. The criteria are defined by the attributes and the set of decision variants is mostly complete. The problems considered in the paper are discrete MCDM problems. Many methods and approaches to solve such problems have been developed. An overview of such procedures can be found in the following papers: Roy (1985); Figueira, Greco and Ergott (eds.) (2005); Tzeng, Chiang and Li (2011); Trzaskalik (ed.) (2014a, 2014b).

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In the paper Hwang and Yoon (1981) the main features of MADM problems are defined and compared with Multiple Objective Decision Making (MODM) problems.

Table 1: Characteristics of MADM vs MODM

	MADM	MODM
Criteria defined by	attributes	objectives
Objective	ill defined (implicit)	clearly defined (explicit)
Attribute	explicit	implicit
Constraint	inactive (incorporated into attributes)	active, clearly defined
Decision variant	predefined, usually a finite number	infinite number of variants
Decision problem	discrete	continuous
Interaction with DM	occasionally	mostly
Application	choice, selection, classification, evaluation	design

Source: Hwang and Yoon (1981, p. 4).

The present paper deals with methods of decision support when information about preferences among criteria is not available. Such analytical situations occur when the decision maker does not want to or cannot determine the relation among the criteria importance which usually happens at the beginning of the process of solving the problem. The goal of the paper is to present methods that do not require information about preferences among the criteria. The main objective of the present paper is to present selected methods of multiple attribute decision-making support synthetically when no information about preferences among the criteria is available. The methods focus on the actual problem of evaluation of investment projects in a hard coal mine.

The methods presented cover a selected spectrum of preferences modeling. The ELECTRE IV method (Roy and Bouyssou, 1993) is one of methods consisting in constructing a relational system of preferences of the decision maker based on the outranking relation.

The most common alternative to this approach is the AHP method (Saaty, 1980). This method allows to transform a verbal assessment into a numeric one using pairwise comparison, and therefore determines an ordering of the decision variants.

Simple methods: dominance, maxmin, maxmax (Hwang and Yoon, 1981) and methods based on game theory (Madani and Lund, 2011) usually do not require explicit aggregation of assessments of decision variants. The relations between variants result from comparison of assessments associated with each criterion.

The use of the idea of Linear Partial Information (Kofler, 1993) to solve MCDM problems (Michalska, 2011, 2012; Michalska and Pospiech, 2010, 2011;

Pospiech, 2014) involves taking into account: partial information about preferences, analysis of marginal distributions of criteria weights and determining the order of decision variants by using Wald's maxmin criterion.

The diversity of the presented approaches makes it difficult to present them clearly and comprehensively, or to compare them. Therefore the discussion in this paper focuses on the most important aspect of our topic, which is – to put it simply – the lack of requiring full information about relations between the criteria or full information about their weights.

In our paper we do not discuss the interactive approach where information about preferences in iterations is given. We assume that information about preferences among the criteria is not available: the decision maker cannot or does not want to give it. However, the situation analyzed in this paper can occur at the beginning of the interactive procedure.

2 Basic methods: Dominance, Maxmin, Maxmax and methods based on game theory

The dominance method consists in reducing the number of decision variants by removing dominated variants. The application of the Dominance method by the decision-maker shows a passive attitude or may be a preliminary part of analysis that allows to reduce a set of decision variants.

The Maxmin method requires standardization of decision variants. In this method, for each decision variant, the worst estimate for the variant is determined in terms of the criteria analyzed, and then the best one is selected among the estimates determined. The selected estimate indicates the best variant.

When using the Maxmax method the procedure is similar as in the Maxmin method. For each decision variant the best estimate is determined according to the criteria. The highest estimate indicates the best decision variant. This method assumes an optimistic approach of the decision-maker to the decision problem and the selected variant allows to reach at least one objective at the highest level possible.

The Maxmin approach was the basis for defining multi-criteria problem as two-person zero-sum game (Kofler, 1967). Later models were developed in the form of n-person games in which the player is associated with a criterion, the strategy with a decision variant, and payoffs of each player with the variant's estimate according to a given criterion. The game defined in this way may be considered as played once (Wolny, 2007) or in many moves until a stable solution is achieved (Madani and Lund, 2011). In the former case, using the general theory of equilibrium selection and risk dominance (Harsanyi and Selten, 1990), an equilibrium (in Nash's sense) is indicated that represents accordingly the best decision variant. In the latter case different equilibriums are considered (starting with Nash's equilibrium, through general meta-rationality, symmetric meta-rationality, sequential stability, limited moves stability to non-myoptic stability), and analysis of stable solutions points out the solutions to the original multiple attribute problem.

3 The ELECTRE IV method

This method belongs to the family of ELECTRE methods (ELimination Et Choix Traduisant la REalité − ELimination and Choice Expressing Reality), introduced by the so-called French school in multi-criteria decision analysis (Roy, 1990; Roy and Bouyssou, 1993), and is characterized by modeling of the decision-maker's preferences by means of constructing a relative system of his preferences based on outranking. The feature distinguishing the ELECTRE IV method is the lack of requirement of weights for the criteria analyzed: it is only assumed that none of the criteria is more important than half of them. All the methods from the ELECTRE group are based on pairwise comparison of decision variants. For each criterion the threshold values are usually defined. The thresholds are as follows: q − indifference, p − preference and veto. In the ELECTRE IV method the comparison of two variants consists in verifying whether at least one type of relation occurs: quasi-dominance, canonical dominance, pseudo-dominance, sub-dominance and veto-dominance. All the aforementioned types of dominance represent weakening premises for the occurrence of outranking – if quasi-dominance occurs, all the other ones also appear, if canonical dominance occurs, all the other ones appear except for quasi-dominance etc. On the basis of these relations two partial preorders are set using the distillation procedure. The combination of two such preorders generates the final preorder (Vallée and Zielniewicz, 1994).

4 Linear Partial Information in the AHP method and in additive methods

The analytical hierarchy process (the AHP method) has a wide range of applications. It was introduced by Saaty (1977, 1980) and has been developed since then. AHP allows to estimate decision variants according to the criteria by determining the relative weights that reflect the usability of variants for each criterion. The relative weights are determined based on the transformation of the so-called comparison matrices, which, in turn, are generated using pairwise comparison of decision variants and criteria.

One matrix is generated for each criterion and, additionally, the criteria comparison matrix. These matrices are used in constructing a partial series of decision variants according to each criterion and criteria ranking. The values of relative weights inform about the decision-maker's preferences: the higher the weight the better the variant or criterion.

Relative weights resulting from comparison decision variants generate matrix W; furthermore, relative weights of criteria form vector w. The final ranking is obtained using vector weights $w^{*} = W \cdot w$. However, it should be noted that the values of matrix W may be treated as standardized estimates of decision variants (similarly to vector values w as standardized criteria weights). Therefore, the aggregation of estimates is performed according to all criteria by using a weighted sum, which is a feature of additive methods.

In the problem analyzed in this paper, the components of vector w are unknown; in the AHP procedure the decision-maker does not want or cannot present his preferences in relation to criteria or he reveals them partially, e.g. in the form of linear bounds such as: 'the first criterion is at least as important as the second criterion' ($w_1 \geq w_2$), 'the second criterion is at least as important as the third and fourth ones together' ($w_2 \geq w_3+w_4$). In such a situation the application of the idea of Linear Partial Information (LPI) (Kofler, 1993) is proposed to solve the multi-criteria problem (Michalska and Pospiech, 2011). The idea of this approach consists in:

1) determining the extremal distribution of the criteria weights – the space of feasible values of weights is a simplex and each vertex of the simplex defines an extremal distribution of weights,
2) solving the problem for these distributions (a ranking of variants is established for each distribution) using the AHP method (in general, any MCDM method which allows to order variants and requires weights of criteria can be used),
3) determining the final ranking of variants on the basis of the rankings of variants and using Wald's criterion.

5 Lack of preferences and equivalence of criteria

Let us analyze the problem of the evaluation of investment projects in a hard coal mine involving longwalls. Four criteria were set for this problem (deposit size, total costs, methane hazard, rockburst hazard). The data are presented in Table 2. In the case of minimized criteria, negative estimates were adopted to obtain the same direction of optimization.

The data presented in Table 2 were subject to multi-criteria analysis as shown in the papers Sojda and Wolny (2014); Wolny (2014). It may be noted that variant a_1 dominates a_3; a_2 dominates a_3 and a_8; a_4 dominates a_5; a_6 dominates a_3, a_7 and a_9; a_8 dominates a_3. Nevertheless, the dominance method does not order the set of the variants analyzed.

Table 2: Decision variant estimates according to the criteria analyzed

Investment project – longwall	f_1 – output volume, resources estimated [thousand tons]	f_2 – total cost [PLN thousand]	f_3 – methane hazards (category of hazards)	f_4 – rockburst hazards (category of hazards)
a_1	411	-55 252	-2	-1
a_2	469	-58 251	-1	-1
a_3	297	-82 739	-3	-1
a_4	1581	-89 022	-2	-2
a_5	1092	-99 118	-2	-2
a_6	966	-78 119	-2	-1
a_7	650	-84 084	-4	-1
a_8	414	-68 300	-1	-1
a_9	737	-85 071	-4	-1

Source: Data from a mining company.

In the paper Wolny (2015) the use of the ELECTRE IV method is presented using the threshold values from Table 3. (The criterion f_4 does not differentiate much among the decision variants taking into account the threshold values: all the variants can be treated equivalently with respect to this criterion. However, this criterion was not removed from the analysis performed in the paper for two reasons. First, the differences in the assessments of variants for the indifferent variants are important for the formation of quasi- and canonic dominance relations in the ELECTRE IV method. Second, this criterion is important for the decision maker).

Table 3: Threshold values: indifference, preference and veto

	Indifference threshold	Preference threshold	Veto threshold
Criterion	$q_k[f_k(a_i)]$	$p_k[f_k(a_i)]$	$v_k[f_k(a_i)]$
f_1	10	50	1000
f_2	100	1000	50000
f_3	0	1	3
f_4	2	3	4

Source: Data obtained from the decision-maker.

For the purpose of this paper, for all methods requiring standardization of decision variants estimates, the relative weights used which resulted from the application of the AHP method as well as information about preferences related to each criterion expressed by threshold values are included in Table 3. This means that the final unification of assessments of decision variants (Table 4) are approximations of preferences expressed by thresholds.

Table 4: Standardized estimates of decision variants

Investment project – longwall	**f_1 – output amount, resources estimated [thousand tons]**	**f_2 – total costs [PLN thousand]**	**f_3 – methane hazards (category of hazards)**	**f_4 – rockburst hazards (category of hazards)**
a_1	0.019	0.164	0.126	0.125
a_2	0.019	0.158	0.238	0.125
a_3	0.017	0.107	0.007	0.125
a_4	0.730	0.094	0.126	0.063
a_5	0.124	0.018	0.126	0.063
a_6	0.028	0.116	0.126	0.125
a_7	0.022	0.104	0.006	0.125
a_8	0.019	0.137	0.238	0.125
a_9	0.023	0.102	0.006	0.125

When calculating the extreme distribution of weights (in the method using the LPI idea), it was additionally assumed that the weight of each criterion constitutes at least 20% of weight of the other criteria – in this way the significance of the analyzed criteria was defined. The following constrains should be taken account: $w_k = 0,2 \cdot \sum_{\substack{i=1 \\ i \neq k}}^{4} w_i, k = 1,2,3,4$ and $\sum_{i=1}^{4} w_i = 1$, where w_k is the weight of kth criterion. Consequently, we obtain the following extreme distributions of weights (w_1, w_2, w_3, w_4): (0.500, 0.167, 0.167, 0,167), (0.167, 0.500, 0.167, 0.167), (0.167, 0,167, 0.500, 0.167), (0.167, 0.167, 0.167, 0.500). These weights generate orderings presented in Table 5.

Table 5: Rankings resulting from extreme distributions of weights

	Extreme distributions of weights and the corresponding rankings				**MAX (pessimistic place in order)**	**Ranking**
	(0.500,0.167, 0.167,0.167)	(0.167,0.500, 0.167,0.167)	(0.167,0.167, 0.500,0.167)	(0.167,0.167, 0.167,0.500)		
a_1	5	4	4	4	5	**4**
a_2	2	2	2	2	2	**2**
a_3	9	6	7	6	9	**7**
a_4	1	1	1	1	1	**1**
a_5	3	9	6	9	9	**7**
a_6	6	5	5	5	6	**5**
a_7	8	7	8	7	8	**6**
a_8	4	3	3	3	4	**3**
a_9	7	8	9	8	9	**7**

The orderings (rankings) obtained by different methods, as compared with the solution obtained by including different weight values of the criteria are presented in Table 6.

Table 6: Final rankings of decision variants for compared multi-criteria methods

Variant	Minmax method	Maxmax method	AHP + LPI	ELECTRE IV	Equivalent criteria	
					AHP	ELECTRE III
a_1	5	3	4	3	4	3
a_2	3	2	2	2	2	1
a_3	7	5	7	9	7	7
a_4	1	1	1	4	1	2
a_5	6	4	7	7	6	5
a_6	2	4	5	1	5	2
a_7	8	5	6	8	8	6
a_8	4	2	3	5	3	4
a_9	8	5	7	6	9	6

The rankings obtained differ, but they are a result of the transformation of the same set of information. It should be noted that all the methods analyzed, except for the methods from the ELECTRE family, are consistent in terms of optimum (according to these methods, the best variant is a_4). Differentiation is an obvious consequence of different approaches and notions that characterize the methods analyzed. The idea of using LPI in the AHP method (due to the method of standardization of estimates used here, a simple additive method is identical with it in this example) is based on the use of the Minmax method for the rankings generated by the extreme distributions of weights – from this point of view this approach is compared with a simple application of the Minmax method and the AHP method with equivalent weights of criteria. The ELECTRE IV method, in turn, uses a completely different approach, therefore the ranking obtained with it is compared with the ranking generated by the ELECTRE III method with equivalent criteria.

The values of correlation coefficients of Spearman ranks between the achieved rankings were adopted in order to examine the similarity of rankings. The data are presented in Table 7.

Table 7: Values of correlation coefficients of Spearman ranks between the examined orderings together with their critical value of significance level

	Minmax method	Maxmax method	AHP + LPI	ELECTRE IV	Equivalent criteria	
					AHP	ELECTRE III
Minmax method		**.8196** **p = .007**	**.8204** **p = .007**	**.7811** **p = .013**	**.8876** **p = .001**	**.8938** **p = .001**
Maxmax method	**.8196** **p = .007**		**.9519** **p = .000**	.5746 p = .106	**.9678** **p = .000**	**.7809** **p = .013**
AHP + LPI	**.8204** **p = .007**	**.9519** **p = .000**		.6375 p = .065	**.9363** **p = .000**	**.8230** **p = .006**
ELECTRE IV	**.7811** **p = .013**	.5746 p = .106	.6375 p = .065		.6333 p = .067	**.9252** **p = .000**
AHP	**.8876** **p = .001**	**.9678** **p = .000**	**.9363** **p = .000**	.6333 p = .067		**.8391** **p = .005**
ELECTRE III	**.8938** **p = .001**	**.7809** **p = .013**	**.8230** **p = .006**	**.9252** **p = .000**	**.8391** **p = .005**	

The results obtained indicate a strong correlation between the rankings. It should be taken into account that similar rankings are obtained when all the criteria are assigned equal weights and no information about preferences among the criteria is available (this applies to the ELECTRE methods as well as to other methods). The strong correlation of rankings obtained using the Maxmax method and the AHP method with the inclusion of the LPI idea as well as with equivalent criteria is also interesting – it may be explained by the method of standardization of estimates of decision variants based on the AHP method (the values of all estimates are non-negative and sum up to one).

6 Summary

To summarize the analysis performed, it may be stated that the egalitarian approach to criteria, consisting in assigning identical weights to them, is also a kind of approximation of decision-maker's preferences. It is consistent but not the same as in the case of the methods developed strictly to support the decision-maker in multiple attribute decisions with imperfect information about preferences among the criteria.

The methods connected with game theory were described, using Wald's criterion – from a simple Minmax method to using its idea in the AHP method without inter-criteria information or the notion of linear partial information. Next to the method related to AHP, the ELECTRE IV method was presented which does not require determination of the weights of the criteria analyzed. Further in the paper, using an example, rankings generated by various methods were compared in order to answer the question: can the solution of the problem without information about preferences among the criteria be identified with the solution of the problem with equivalent criteria?

The main conclusion of the paper follows from the analyses which show that the rankings of variants generated by the various methods are similar but not identical. However, a strong correlation of the orderings, lack of perfect information about the preferences among the criteria on the one hand, and lack of premises questioning the egalitarian approach to the criteria on the other hand, indicate a possibility of equivalent understanding of criteria in this type of problems. This approach to the criteria is not identical with treating them equivalently, but it implies similar results as in the case of equivalent criteria.

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