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# SEMI-PARAMETRIC RISK MEASURES

**Summary:** Parametric methods of risk analyses have long histories starting from the first Markowitz proposals associated with moments of the distribution of a random variable, means the distribution of return of the risky investment. Real empirical distributions have not the characteristics in accordance with the group of elliptical random variable. The risk assessment should be based on measures being based on other characteristics, and at the same time be characterized by good properties, agreeable with set of axioms formulated with reference to measures of the risk.

**Keywords:** risk measures, quantiles, expectiles, axioms approach.

## Introduction

Researches in the area of the risk analysis are being conducted for the second mid-fifties of the last century. H. Markowitz as first exploited the measure of the statistical description of the distribution of the statistical variable for the risk analysis. Estimators of the distribution of a random variable of rates of return were a next stage of applying the proposed approach. Accepting classical way on statistical analysis we should take into account a lot of such assumptions like the sample size and the class of distribution. A lot in the extensive literature on the subject weak points of the proposed approach portrayed works and examinations, which we will find. There were introduced measures taking into account the different approach of investors, towards the risk of the measure for the description of the type negative approach towards the risk: semi-variance, semi-standard deviation, semi-averages deviation. The conducted research on the decision making in conditions of risk and in conditions is showing next paths of acting unlike classical climbs the uncertainty. Introducing axiomatic properties of measures of the risk eliminated applications of many conventional measures of the risk used in finances. In particular measures using the second moment of the distribution of the rate of return on account of the lack of the monotonicity, proposed through H. Markowitz [1952]. We will move acrobatics of measures of the risk leaving from axiomatic properties of measures of the risk.

## 1. Axiomatic risk measures approach

Answering to the question: how to measure the risk of the portfolio was define an axioms of coherent measures of the risk. Generally, let R denote and monetary risk measure [Artzner et al., 1999].

**Definition 1.** A monetary risk measure is a mapping $R$: $L^p(\Omega, F, P) \to \mathrm{R}$

**Definition 2.** A monetary risk measure $R$ is coherent when is:

1) monotone: $X \geq Y$ implies $R(X) \leq R(Y)$ (MO)
2) subadditive: $R(X + Y) \leq R(X) + R(Y)$ (SA)
3) positively homogenous: $\lambda \geq 0$ implies $R(\lambda X) = \lambda R(X)$ (PH)
4) invariant by translation: $k \in R$ implies $R(X + k) = R(X) - k$ (TI)

**Definition 3.** A monetary risk measure $R$ is can be:

1) positive: $X \geq 0$ implies $R(X) \leq 0$ (PO)
2) invariant in law: $X \sim Y$ implies $R(X) = R(Y)$ (IL)
3) convex: $\forall \lambda \in [0, 1]$, $R(\lambda X + (1 - \lambda Y)) \leq \lambda R(X) + (1 - \lambda)R(Y)$ (CO)

The interpretation of invariant by translation risk is now that:

$$\mathrm{R}(X + \mathrm{R}(X)) = 0$$

and positively homogenous property of risk implies R(0) = 0 (which is also called the grounded property). We can observe, that if R satisfies property invariant by translation and convex then:

$$\mathrm{R}(\mu + \lambda Z) = \lambda \mathrm{R}(Z) - \mu$$

We can find definition of some of this property in literature: invariant by translation [Reich, 1984], positively homogenous [Schmidt, 1989], convex [Deprez, Gerber, 1985]. We now try to look close to quantile risk measures. The quantile was a natural risk measure, when $X$ was a loss. We will define risk measures, that will be large when $-X$ is large.

**Definition 4.** We call Value-at-Risk on the level $\alpha \in (0, 1)$ the quantile on the level $\alpha$:

$$R_\alpha(X) = VaR_\alpha(X) = x_\alpha \text{ or } \mathrm{P}(X \le x_\alpha) = \alpha$$

so it is also possible to notice as:

$$VaR_\alpha(X) = \inf\{x, (X \le x) \ge \alpha\} = F_x^{-1}(\alpha) = Q(\alpha)$$

*Remark:* This measures is increasing function in $\alpha$. We will try to understand the unlying axiomatic, for some random variable *X*. Risk *X* is said to be $VaR_\alpha$ – acceptable, if $VaR_\alpha$(X) ≤ 0 [Jorion, 2007].

*Lemma.* For all $\alpha \in (0, 1)$, if *g* is strictly increasing and left continuous then:

$$VaR_\alpha(g(X)) = F_{g(X)}^{-1}(\alpha) = g\left(F_X^{-1}(\alpha)\right) = g(VaR_\alpha(X))$$

and if *g* is strictly decreasing and right continuous and $F_X$ is a bijective then:

$$VaR_\alpha(g(X)) = F_{g(X)}^{-1}(\alpha) = g\left(F_X^{-1}(1-\alpha)\right) = g(VaR_{1-\alpha}(X))$$

but Value-at-Risk is not sub-additive.

**Example 1.** We can consider two random variable, two risk as Pareto random variable $X \sim Par(1, 1)$ and $Y \sim Par(1, 1)$:

$$P(X > t) = P(Y > t) = \frac{1}{1+t}, t > 0$$

Then:

$$VaR_\alpha(X) = VaR_\alpha(Y) = \frac{1}{1-\alpha} - 1$$

we can write next:

$$P[X + Y \le t] = 1 - \frac{2}{2+t} + 2\frac{ln(1+t)}{(2+t)^2}, t > 0$$

or

$$P[X + Y \le 2VaR_\alpha[X]] = \alpha - \frac{(1-\alpha)^2}{2} ln\left(\frac{1+\alpha}{1-\alpha}\right) < \alpha$$

so, for all $\alpha \in (0, 1)$ we have:

$$VaR_\alpha(X) + VaR_\alpha(Y) < VaR_\alpha(X+Y)$$

**Example 2.** We can consider two random variable, two risk as beta random variable X,Y ~ *B*(92,5%) then:

$$\text{VaR}_{0,9}(\boldsymbol{X}) + \text{VaR}_{0,9}(\boldsymbol{Y}) = 0 + 0 \leq \text{VaR}_{0,9}(X + \boldsymbol{Y}) = 1$$

As a conclusion we observe, that the Value-at-risk is generally not a coherent risk measure. The Value-at Risk is a coherent risk measures for elliptical risk. For all *X*, note that VaR:

$$VaR_\alpha(\boldsymbol{X}) = \inf\{R(X) \text{ such } R \text{ is coherent and } VaR_\alpha(\boldsymbol{X}) \leq R(X)\}$$

Now we notice risk measures, which follow the concept of Value-at-Risk, especially for non-gaussian risk.

**Definition 5.** The Tail-Value-at-Risk on the level $\alpha \in (0, 1)$, notice as $TVaR_\alpha(X)$ is defined as:

$$TVaR_\alpha(X) = \frac{1}{1-\alpha} \int_\alpha^1 VaR_t(X) dt$$

This measure is the average on Value-at-Risk on the level $\alpha \in (0, 1)$.

*Remark:* There exist a distribution function $\widetilde{F}_X$ (transformation of Hardy- Littlewood of $F_X$ [Hardy, Littlewood, 1930]), such that for all $\alpha \in (0, 1)$:

$$\widetilde{F}_X^{-1}(\alpha) = TVaR_\alpha(X)$$

If $\widetilde{X}$ have a distribution function $\widetilde{F}_X$, then:

$$TVaR_\alpha(X) = VaR_\alpha(\widetilde{X})$$

The Tail-Value-at-Risk for *X* is *VaR* transformation of Hardy- Littlewood of *X*. We can observe and notice than:

$$TVaR_0[X] = E[X]$$

and because:

$$TVaR_\alpha[X] = \frac{1}{1-\alpha}\left\{E[X] - \int_0^\alpha VaR_\xi[X]d\xi\right\}$$

We can see, that $TVaR_\alpha(X)$ is increasing function in $\alpha$:

$$TVaR_\alpha[X] \geq TVaR_0(0) = E[X]$$

We can comment, that *Tail-VaR* contains always one loading of security.

**Definition 6.** The Conditional Tail Expectation on the level $\alpha \in (0, 1)$, notice as $CTE_\alpha(X)$ is defined as:

$$CTE_\alpha[X] = E[X|X > VaR_\alpha[X]]$$

Next measures *CVaR* is an average on this risk value, which excess Value-at-Risk (on the level $\alpha \in (0, 1)$), that mean the average of risk crossings over *VaR*.

**Definition 7.** The Conditional-VaR on the level $\alpha \in (0, 1)$, notice as $CVaR_\alpha(X)$ is defined as:

$$\begin{aligned} CVaR_\alpha[X] &= E[X - VaR_\alpha[X]|X > VaR_\alpha[X]] \\ &= e_X(VaR_\alpha[X]) = CTE_\alpha[X] - VaR_\alpha[X] \end{aligned}$$

**Definition 8.** The Expected shortfall on the level $\alpha \in (0, 1)$, notice as $ES_\alpha(X)$ is defined as:

$$ES_\alpha[X] = E[(X - VaR_\alpha[X])_+]$$

Now we try to notice some properties for some probability level $\alpha \in (0, 1)$ [Acerbi, Tasche, 2002]:

**Proposition 1.** The nest equivalences is true:

$$TVaR_\alpha[X] = VaR_\alpha[X] + \frac{1}{1-\alpha} ES_\alpha[X]$$

$$CTE_\alpha[X] = VaR_\alpha[X] + \frac{1}{\overline{F}_X(VaR_\alpha[X])} ES_\alpha[X]$$

**Proposition 2.** Two measures $CTE_\alpha(X)$ and $TVaR_\alpha(X)$ coincident for two risk with continuous distributions:

$$CTE_\alpha(X) = TVaR_\alpha(X)$$

and

$$TVaR_\alpha[X] = VaR_\alpha[X] + \frac{1}{1-\alpha} ES_\alpha[X]$$

$$CTE_\alpha[X] = TVaR_\alpha[X],\ \alpha \in (0,1)$$

## 2. Comparing Risk based on semi-parametric method in risk analyses

We can look on properties on quantile function, which is base for risk measures and base for comparing risk [Trzpiot, 2006].

**Properties 1.** A quantile function, as a function of $X$, is:
1. positive, $X \geq 0$ implies $Q_X(u) \geq 0$, $\forall u \in [0, 1]$
2. monotonne, $X \geq Y$ implies $Q_X(u) \geq Q_Y(u)$, $\forall u \in [0, 1]$
3. positively homogenous, $\lambda \geq 0$ implies $Q_{\lambda X}(u) = \lambda Q_X(u)$, $\forall u \in [0, 1]$
4. invariant by translation, $k \in$ R implies $Q_{X-k}(u) = Q_X(X) - k$, $\forall u \in [0, 1]$
5. invariant in law, $X \sim Y$ implies $QX(u) = QY(u)$, $\forall u \in [0, 1]$

Quantile function is neither convex nor subadditive, so the quantile function as a risk measure might penalize diversification. We can use for comparing risk different relation defined in the set of risks.

**Properties 2.** As first we can use the first stochastic dominance FSD relation which is defined as:
1. $E(g(\boldsymbol{X})) \leq E(g(\boldsymbol{Y}))$ for $g$ no decreasing
2. For all $x \in R$, $P(\boldsymbol{X} \leq x) \geq P(\boldsymbol{Y} \leq x)$
3. For all $x \in R$, $P(\boldsymbol{X} > x) \leq P(\boldsymbol{Y} > x)$
4. For all $x \in [0, 1]$, $VaR_\alpha(\boldsymbol{X}) \leq VaR_\alpha(\boldsymbol{Y})$

Next we can use *VaR* relation or *TVaR* relation, which can be view as equivalent to second stochastic dominance SSD.

**Definition 9.** Risk $Y$, $X$ is less dangers then $Y$, when we base on risk measures $VaR$, if:

$$VaR_\alpha[X] \le VaR_\alpha[Y], \forall \alpha \in (0,1)$$

**Properties 3.** The second is possibility of use the second stochastic dominance SSD relation which is equivalent to:

1. $E(g(\boldsymbol{X})) \le E(g(\boldsymbol{Y}))$ for $g$ no decreasing and convex
2. $E((\boldsymbol{X} - t)^+) \le E((\boldsymbol{Y} - t)^+)$, for all $t \in$ R
3. For all x∈ [0, 1], $\int_0^\alpha \mathrm{VaR}_p(\boldsymbol{X})dp \ge \int_0^\alpha \mathrm{VaR}_p(\boldsymbol{Y})dp$
4. For all x∈ [0, 1], $\int_\alpha^\infty \mathrm{VaR}_p(\boldsymbol{X})dp \le \int_\alpha^\infty \mathrm{VaR}_p(\boldsymbol{Y})dp$
5. For all x∈ [0, 1], $\mathrm{TVaR}_\alpha(\boldsymbol{X}) \le \mathrm{TVaR}_\alpha(\boldsymbol{Y})$

## 3. Estimators for quantiles

We have in the literature few paths to obtain quantile estimators; the usual hypothesis in finance is that risk is normal random variable. We try to point out a few different estimators for random sample.

**Definition 10.** Given a sample $\{X_1, X_2, ..., X_n\}$ the Cornish-Fisher [1937] estimation of the α-quantile is:

$$\hat{q}_n(\alpha) = \hat{\mu} + \hat{z}_n \hat{\sigma}$$

where

$$\hat{\mu} = \frac{1}{n}\sum_{1=1}^{n} X_i \,, \; \hat{\sigma} = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \hat{\mu})^2}$$

$$\hat{z}_n = \Phi^{-1}(\alpha) + \frac{\zeta_1}{6}\left[\Phi^{-1}(\alpha)^2 - 1\right] + \frac{\zeta_2}{24}\left[\Phi^{-1}(\alpha)^3 - 3\Phi^{-1}(\alpha)\right] - \frac{\zeta_1^2}{36}\left[2\Phi^{-1}(\alpha)^3 - 5\Phi^{-1}(\alpha)\right]$$

where $\zeta_1$ is a natural estimator of skewnees and $\zeta_2$ is a natural estimator of the excess kurtosis (obtained by moments method).

For nonparametric estimators' two techniques have been considered to smooth estimation quantiles, further implicit or explicit. We can consider a linear combination of order statistics.

**Definition 11.** The classical empirical quantile estimate is:

$$Q_n(p) = F_n^{-1}\left(\frac{i}{n}\right) = X_{(i)} = X_{([np])}$$

where [.] denote the integer part.

This estimator is simple to obtain, but depends only on one observation. A natural extension will be to use – at – least two observation, if *np* is not an integer.

**Definition 12.** The weighted empirical quantile estimation is then defined as

$$Q_n(p) = (1-\gamma)X_{([np])} + \gamma X_{([np]+1)} \text{ where } \gamma = np - [np]$$

Harrell-Davis [1982] estimator of α-quantile is defined as:

$$Q_n(p) = \sum_{i=1}^{n}\left[\int_{\frac{(i-1)}{n}}^{\frac{i}{n}} \frac{\Gamma(n+1)}{\Gamma((n+1)p)\Gamma((n+1)q)} y^{(n+1)p-1}(1-y)^{(n+1)q-1}\right] X_{i:n}$$

When we want to find a smooth estimator for $F_X$ and then find (numerically) the inverse. The α-quantile is defined as the solution of $F_X \circ q_X(\alpha) = \alpha$ . If $\hat{F}_n$ denotes a continuous estimate of $F$, then a natural estimate for $q_X(\alpha)$ is $\hat{q}_X(\alpha)$, such that $F_X \circ q_X(\alpha) = \alpha$ obtained using, e.g. Gauss-Newton algorithm.

## 4. Generalized quantiles as risk measures

In the statistical and actuarial literature several generalizations of quantiles have been considered, by means of the minimization of a suitable asymmetric loss function. The new results connected with risk properties was proved in [Bellini et al.,2014]. Authors showed that the only M-quantiles, that are coherent risk measures are the expectiles, introduced by W. Newey and J. Powell [1987] as the minimizer of an asymmetric quadratic loss function.

**Definition 13.** The quantiles $q_\alpha$ is the minimizer of a piecewise-linear loss function:

$$q_\alpha(X) = \underset{x \in R}{arg\,min}\left\{\alpha E\left[(X-x)^+\right] + (1-\alpha)E\left[(X-x)^-\right]\right\}$$

where we use the notation $x^+ := \max\{x; 0\}$ and $x^- := \max\{-x; 0\}$.

**Definition 14.** The expectiles $e_\alpha$ (with $\alpha \geq 1/2$ ) of a random variable $X$ may be defined as the minimizer of an asymmetric quadratic loss function:

$$e_\alpha(X) = \underset{x \in R}{\arg\min}\left\{ \alpha E\left[\left((X-x)^+\right)^2\right] + (1-\alpha)E\left[\left((X-x)^-\right)^2\right]\right\}$$

The associated expectile – based risk measure is $R_\alpha(X) = e_\alpha(X) - E(X)$. As proved in [Jones, 1993], expectiles are quantiles, but not associated with $F_X$.

**Example 3.** The case where $X \sim \varepsilon(1)$ and the case $X \sim N(0, 1)$ can be visualized of Fig. 1.

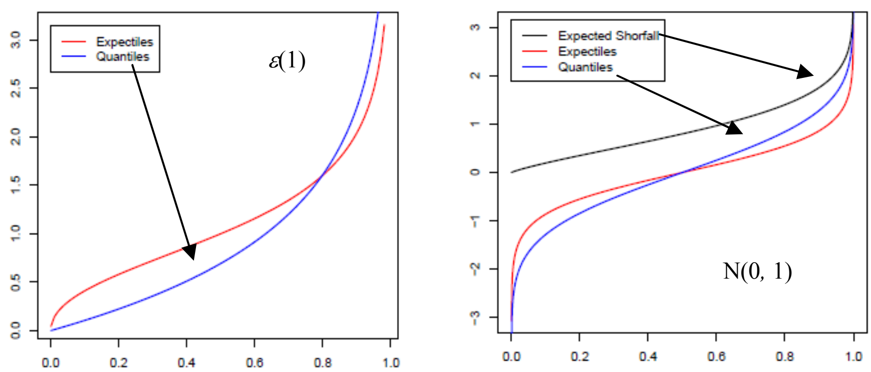


**Fig. 1.** Quantiles, Expected Shortfall and Expectiles, $\varepsilon(1)$ and N(0, 1) risks

We can notice next definition, using notation from J. Breckling and R. Chambers (1988).

**Definition 15.** For $\Phi_1, \Phi_2 : [0;\infty) \to [0; \infty)$ be convex, strictly increasing functions satisfying: $\Phi_1(0) = \Phi_2(0) = 0$ and $\Phi_1(1) = \Phi_2(1) = 1$, we consider the minimization problem:

$$\pi_\alpha(X) = \inf_{x \in R} \pi_\alpha(X, x)$$

where

$$\pi_\alpha(X) = \alpha E\left[\Phi_1\left((X-x)^+\right)\right] + (1-\alpha)E\left[\Phi_2\left((X-x)^-\right)\right]$$

and any minimizer

$$\pi_\alpha^*(X) \in \underset{x \in R}{arg\,min}\, \pi_\alpha(X,x)$$

we call a generalized quantile.

**Example 4.** For $\Phi_1(x) = \Phi_2(x) = x$, generalized quantiles reduce to the usual quantiles ($q_\alpha$).

**Example 5.** For $\Phi_1(x) = \Phi_2(x) = x^2$, generalized quantiles reduce to expectiles. Since $\Phi_1$ and $\Phi_2$ are differentiable the first order condition is given by:

$$\alpha E\left[\Phi_1\left(\left(X - x_\alpha^+\right)^+\right)\right] = (1-\alpha)E\left[\Phi_2\left(\left(X - x_\alpha^+\right)^-\right)\right]$$

Any solution of the equality above is called $\alpha$ - expectile of $X$ ($e_\alpha$).

The new results are the properties of a generalized quantile as a risk measures [Bellini et al., 2014]. Given functions $\Phi_1, \Phi_2$ as above, a probability space ($\Omega$; $F$; $P$) and the space $L^0$ of all random variables $X$ on ($\Omega$;F; P), we recall that the Orlicz heart $M^\Phi$:

$$M^\Phi = \left\{ X \in L^0 : E\left[\Phi\left(\frac{|X|}{a}\right)\right] < \infty, \textit{for every } a > 0 \right\}$$

**Proposition 3.** Let $\Phi_1$, $\Phi_2$ : $[0;\infty) \to [0;\infty)$ be convex, strictly increasing functions (satisfying : $\Phi_1(0) = \Phi_2(0) = 0$ and $\Phi_1(1) = \Phi_2(1) = 1$ ) be convex, strictly increasing and satisfy. Let $X \in M^{\Phi 1} \cap M^{\Phi 2}$, $\alpha \in (0;\ 1)$ and let $\pi_\alpha(X) = \inf_{x \in R} \pi_\alpha(X,x)$.

We denote with $y_\alpha^{*-}$ and $y_\alpha^{*+}$ the (lower and upper) generalized quantiles of Y. Then the following holds:

1) translation equivariance: if $Y = X + h$ with $h \in$ R, then [ $y_\alpha^{*-}$ , $y_\alpha^{*+}$ ] = [ $x_\alpha^{*-} + h$; $x_\alpha^{*+} + h$]
2) positive homogeneity: if $\Phi_1(x) = \Phi_2(x) = x^\beta$, with $\beta \geq 1$, then $Y = \lambda X$ for $\lambda > 0$ ) [ $y_\alpha^{*-}$ , $y_\alpha^{*+}$ ] = [$\lambda x_\alpha^{*-}$ ;$\lambda x_\alpha^{*+}$ ]
3) monotonicity: if $X$ *FSD* $Y$, then $x_\alpha^{*-} \geq y_\alpha^{*-}$ and $x_\alpha^{*+} \geq y_\alpha^{*+}$

4) constancy: if $X = c$, P-a.s., then $x_{\alpha}^{*-} = x_{\alpha}^{*+} = c$
5) internality: if $X \in L^{\infty}$, then $[x_{\alpha}^{*-}; x_{\alpha}^{*+}] \in$ [ess inf $(X)$ ; ess sup $(X)$]
6) monotonicity in α: if $\alpha_1 \le \alpha_2$, with $\alpha_1; \alpha_2 \in (0; 1)$, then $x_{\alpha 1}^{*-} \le x_{\alpha 2}^{*-}$ and $x_{\alpha 1}^{*+} \le x_{\alpha 2}^{*+}$.

In the next proposition [Bellini et al., 2014] we have reasonable properties in the sense of the axiomatic theory of risk measures. In particular, generalized quantiles are positively homogeneous and generalized quantiles are convex. As a consequence, we will show, that the only generalized quantiles, that are coherent risk measures are the expectiles with $\alpha \ge 1/2$.

**Proposition 4.** Let $\Phi_1, \Phi_2 : [0;\infty) \to [0; \infty)$ be strictly convex and differentiable with $\Phi_1(0) = \Phi_2(0) = 0$ and $\Phi_1(1) = \Phi_2(1) = 1$ and $\Phi'_{1+}(0) = \Phi'_{2+}(0) = 0$ .Let $\alpha \in (0; 1)$ and:

$$x_{\alpha}^{*}(X) = \underset{x \in R}{arg\,min} \left\{ \alpha E\left[\Phi_1\left((X-x)^{+}\right)\right] + (1-\alpha)E\left[\Phi_2\left((X-x)^{-}\right)\right]\right\}$$

1) $x_{\alpha}^{*}(X)$ is positively homogeneous, if and only if $\Phi_1(x) = \Phi_2(x) = x^{\beta}$, with $\beta > 1$
2) $x_{\alpha}^{*}(X)$ is convex, if and only if the function $\psi$ : R → R

$$\psi(t) = \begin{cases} -(1-\alpha)\Phi'_2(-t), & t \le 0 \\ \alpha\Phi'_1(t), & t \ge 0 \end{cases}$$

is convex; it is concave if and only if $\psi$ is concave
3) $x_{\alpha}^{*}(X)$ is coherent, if and only if $\Phi_1(x) = \Phi_2(x) = x^2$ and $\alpha \ge 1/2$.

From least proposition we have conclusion, that the only generalized quantiles, that are coherent risk measures (in case $\alpha \ge ½$). The first order condition for expectiles can be written in several equivalent ways:

$$\alpha E\left[(X - e_{\alpha})^{+}\right] = (1-\alpha)E\left[(X - e_{\alpha})^{-}\right]$$

or also

$$e_{\alpha} - E[X] = \frac{2\alpha - 1}{1-\alpha} E\left[(X - e_{\alpha}(X))^{+}\right], \text{ for all } \alpha \in (0; 1)$$

[Newey, Powell 1987]. We remark, that both equations have a unique solution for all $X \in L^1$, so expectiles are well defined on $L^1$, although the loss function can assume the value $+\infty$. From least propositions we know, that expectiles satisfy translation equivariance, positive homogeneity, monotonicity with respect to the FSD order and subadditivity (when $\alpha \geq 1/2$). In the next proposition we collect some further immediate properties.

**Proposition 5.** Let $X$; $Y \in L^1$ and let $e_\alpha(X)$ be the $\alpha$-expectile of $X$. Then:
(a) $X \leq Y$ and $P(X < Y) > 0$ imply, that $e_\alpha(X) < e_\alpha(Y)$ (strong monotonicity)
(b) if $\alpha \leq 1/2$, then $e_\alpha(X + Y) \geq e_\alpha(X) + e_\alpha(Y)$
(c) $e_\alpha(X) = -e_{1-\alpha}(-X)$

## Conclusion

This paper attempts to organize the basic theoretical approaches to the description and measurement of risk using methods semiparametric. It can be notice these methods in the extension to the case of multidimensional or mulivalued case. Application of the proposed methodology can be in investments as well as in insurance or in the description of project risk.

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## SEMI-PARAMETRYCZNE METODY ANALIZY RYZYKA

**Streszczenie:** Parametryczne metody analizy ryzyka mają długą historię, począwszy od pierwszych propozycji H. Markowitza, związanych z momentami rozkładu zmiennej losowej, rozkładu stop zwrotu ryzykownej inwestycji. Rzeczywiste empiryczne rozkłady nie mają charakterystyk zgodnych z grupą rozkładów eliptycznych. Ocena ryzyka powinna opierać się na miarach bazujących na innych charakterystykach, a jednocześnie charakteryzować się dobrymi własnościami, zgodnymi z aksjomatyką sformułowaną w odniesieniu do miar ryzyka.

**Słowa kluczowe:** miary ryzyka, kwantyle, *expectiles*, aksjomaty miar ryzyka.