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# AN INTUITIONISTIC FUZZY EXTENSION OF THE CODAS-SORT METHOD

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**Abstract**

Currently, an important issue in multi-criteria decision-making (MCDM) problems are vagueness and lack of precision of decision--making information because of insufficient data and incapability of the decision maker to process the information. Intuitionistic fuzzy sets (IFS) are a solution to eliminate the vagueness and the uncertainty. While fuzzy sets (FS) deal with ambiguity and vagueness problem, IFSs have more advantages. Moreover, the CODAS-SORT method cannot handle the uncertainty and ambiguity of information provided by human judgments. The aim of this study is to develop an IF extension of CODAS-SORT combining this method with the IFS theory. To achieve this, we use the fuzzy weighted Euclidean distance and fuzzy weighted Hamming distance instead of the crisp distances. A case study of a supplier selection assessment is used to clarify the details of our proposed method.

**Keywords:** multicriteria decision aid, sorting methods, CODAS-SORT, intuitionistic fuzzy set.

## 1 Introduction

MCDM helps the decision maker to evaluate several conflicting criteria. In real life, most problems have multiple objectives and need an assessment of several criteria. As a result, MCDM has become a significant problem and a great deal

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of research has gone into helping the decision maker to choose the best decisions. The categorization and classification of MCDM methods are defined in different ways by the authors. According to Roy (1985), the goal of MCDM is to solve one of three types of decision-making problems: (1) identifying a single best alternative or selecting a few best alternatives (choice), (2) ranking the alternatives from the best to the worst (ranking), (3) sorting the alternatives into predefined homogeneous classes (sorting). The study and application of the first two problems has occurred in several areas, while the sorting problems are handled in few studies.

ELECTRE-Tri (Yu, 1992) is the first variant of ELECTRE for the sorting problems. After that, a few studies applied the transformation of ranking methods to deal with sorting problems, e.g., Electre Tri-C (Almeida-Dias, Figueira and Roy, 2010), ELECTRE Tri-nC (Almeida-Dias et al., 2010), ELECTRE-SORT (Ishizaka and Nemery, 2014), ELECTRE Tri-nB (Fernandez et al., 2017). ELECTRE is not the only ranking method that has been adapted to solve the sorting problem. For example, UTADIS was introduced as a sorting variant of the UTA method (Jacquet-Lagreze and Siskos, 1982). The Promethee variants in the sorting environment are the best known. Figueira, Smet and Brans (2004) developed PROMETHEE TRI, which is the first variant of PROMETHEE to solve a sorting problem. PROMSORT is a sorting methodology based on PROMETHEE (Araz and Ozkarahan, 2005). PROMSORT has two important advantages over PROMETHEE TRI. FlowSort (Nemery and Lamboray, 2008) is an variant of Promethee. Then, Ishizaka, Pearman and Nemery (2012) developed a sorting extension of AHP, namely AHPSort, while Nemery et al. (2012) developed GAIASort, an extension of GAIA. TOPSIS-Sort (Sabokbar et al., 2016) supports sorting problems with TOPSIS. MACBETHSort (Ishizaka and Gordon, 2017) is a sorting variant of MACBETH. VIKORSORT (Demir et al., 2018) is a sorting extension of VIKOR and DEASORT (Ishizaka et al., 2018) is a sorting extension of DEA.

Keshavarz Ghorabaee et al. (2016) proposed a different outranking MCDM method which addresses the ranking problem with the calculation of two distances. This advantage gives CODAS more credibility for the decision maker. The Euclidean distance of alternatives from the “negative-ideal” solution is the first measure and the Taxicab distance is the secondary measure. The most desirable alternative is the one farthest from the negative ideal solution. In this method, the Taxicab distance is used as a secondary measure when there are two incomparable alternatives according to the Euclidean distance. According to these cases, the calculation of the assessment score of the alternatives is a combination of the Euclidean and Taxicab distances. The assessment score makes it possible to rank the alternatives from the best to the worst.

For sorting, Ouhibi and Frikha (2019) introduced CODAS-SORT, a variant of CODAS. The assignment rules use two measures. The first measure is the Euclidean distance and the second one is the Taxicab distance. The difference between these two distances define the assignment rules. However, a problem of human judgments is its ambiguity but the CODAS-SORT method cannot deal with this problem. For this reason, we resort to use an IF environment.

According to Zadeh (1975), FST is an extension of classical set theory (Lemaire, 1990). In real-life conditions, the information and data collected are multiple and sometimes contradictory. For this reason, the evaluation criteria are difficult to express. To solve this problem, the concepts from the IFS theory are more appropriate for dealing with vagueness than other generalized FS models (Gautam, Abhishekh and Singh, 2016). Atanassov (1986) introduced an IFS that is an extension of the classical FST; it is characterized by a membership function and a non-membership function.

In this study, an IF extension of the CODAS-SORT method is proposed to handle the sorting problem in an uncertain environment. A case study is added to indicate the reliability of the proposed IF-CODAS-SORT method. The rest of this paper is organized as follows. In Section 2, some basic concepts and definitions of intuitionistic fuzzy sets are presented. In Section 3, an extension of the CODAS-SORT method is proposed to handle IF multi-criteria decision--making. Then the proposed IF-CODAS-SORT method is applied to a case study in Section 4. Finally, conclusions and suggestions for further research are presented.

## 2 Fuzzy sets and intuitionistic fuzzy sets

In this section, the basic definitions for the IFS and some IFS-based MCDM problems are reviewed.

**Definition 1.** Fuzzy sets (Zadeh, 1975)
FST is an extension of classical set theory. However, there is a relaxation of the concept of membership that occurs in the classical theory (Lemaire, 1990). The set $X$ is a universe of discourse, and a fuzzy set $\tilde{a}$ is characterized by a membership function $\mu_{\tilde{a}}(x)$, for $x \in X$, which measures the degree of $x$ belonging to $\tilde{a}$. $\alpha$. $\mu_{\tilde{a}}(x)$ represents the membership of $x$ in $\tilde{a}$.

$$\alpha = \{(x, \mu_{\tilde{\alpha}}(x)) | x \in X\} \tag{1}$$

**Definition 2.** Intuitionistic fuzzy set
IFS introduced by Atanassov (1986) is an extension of the classical FST, which is a suitable way to deal with vagueness.

Assuming that X is a collection of objects $x$ and $\beta \in X$ is a fixed set, the IFS $\beta$ on X is defined as (Atanassov, 1986):

$$\beta = \left\{\left(x, \mu_\beta(x), \upsilon_\beta(x)\right) \middle| x \in X\right\} \tag{2}$$

where $\mu_\beta(x): X \to [0,1]$, $x \in X \to \mu_\beta(x) \in [0,1]$ represents the degree of membership of element $x \in X$ in set $\beta$, and $\upsilon_\beta(x): X \to [0,1]$, $x \in X \to \upsilon_\beta(x) \in$ $\in [0,1]$ is the degree of non-membership of element $x \in X$ in set $\beta$.

$\mu_\beta$ and $\upsilon_\beta(x)$ usually satisfy $0 \le \mu_\beta(x) + \upsilon_\beta(x) \le 1$ for all $x \in X$. Besides the degree of membership and non-membership, an indeterminacy degree, so-called "hesitancy degree" of $x$ to $\beta$, which is different from the numbers $\mu_\beta(x)$ and $\upsilon_\beta(x)$ and which measures the degree of indeterminacy of $x \in X$ to $\beta$ is defined as:

$$\pi_\beta(x) = 1 - \mu_\beta(x) - \upsilon_\beta(x) \tag{3}$$

Accordingly, an intuitionistic fuzzy number $\beta$ can be represented as $\beta = (\mu_\beta, \upsilon_\beta, \pi_\beta)$, which included the degree of membership, of non--membership, and of indeterminacy.

**Definition 3.** Arithmetic operations (Xu and Yager, 2006)
Let $\gamma = \left(\mu_\gamma, \upsilon_\gamma, \pi_\gamma\right)$ and $\beta = (\mu_\beta, \upsilon_\beta, \pi_\beta)$ be two intuitionistic fuzzy numbers; the arithmetic operations on these numbers are defined as follows:

Addition:

$$\begin{gathered} \gamma \oplus \beta = \left(\mu_\gamma, \upsilon_\gamma, \pi_\gamma\right) \oplus \left(\mu_\beta, \upsilon_\beta, \pi_\beta\right) = \\ = \mu_\gamma + \mu_\beta - \mu_\gamma\mu_\beta, \upsilon_\gamma\upsilon_\beta, 1 + \mu_\gamma\mu_\beta - \mu_\gamma - \mu_\beta - \upsilon_\gamma\upsilon_\beta \end{gathered} \tag{4}$$

$$\begin{gathered} \bigoplus_{j=1}^{n} \gamma_j = \bigoplus_{j=1}^{n} \left(\mu_{\gamma_j}, \upsilon_{\gamma_j}, \pi_{\gamma_j}\right) = \\ = 1 - \prod_{j=1}^{n}(1 - \mu_{\gamma_j}), \prod_{j=1}^{n} \upsilon_{\gamma_j}, \prod_{j=1}^{n}(1 - \mu_{\gamma_j}) - \prod_{j=1}^{n} \upsilon_{\gamma_j}) \end{gathered} \tag{5}$$

Multiplication:

$$\begin{gathered} \gamma \otimes \beta = \left(\mu_\gamma, \upsilon_\gamma, \pi_\gamma\right) \otimes \left(\mu_\beta, \upsilon_\beta, \pi_\beta\right) = \\ = (\mu_\gamma\mu_\beta, \upsilon_\gamma + \upsilon_\beta - \upsilon_\gamma\upsilon_\beta, 1 + \upsilon_\gamma\upsilon_\beta - \mu_\gamma\mu_\beta - \upsilon_\gamma - \upsilon_\beta) \end{gathered} \tag{6}$$

$$\begin{gathered} \bigotimes_{j=1}^{n} \gamma_j = \bigotimes_{j=1}^{n} \left(\mu_{\gamma_j}, \upsilon_{\gamma_j}, \pi_{\gamma_j}\right) = \\ = \left(\prod_{j=1}^{n} \mu_{\gamma_j}, \prod_{j=1}^{n}(1 - \upsilon_{\gamma_j}), 1 - \prod_{j=1}^{n} \mu_{\gamma_j}) - \prod_{j=1}^{n}(1 - \upsilon_{\gamma_j})\right) \end{gathered} \tag{7}$$

Scale multiplication:

$$\lambda_\gamma = \left(1-\left(1-\mu_\gamma\right)^\lambda, \left(\upsilon_\gamma\right)^\lambda, \left(1-\mu_\gamma\right)^\lambda - \left(\upsilon_\gamma\right)^\lambda\right) \tag{8}$$

where $\lambda$ is a crisp number.

**Definition 4.** Geometric distance (Szmidt and Kacprzyk, 2000)
The Hamming distance is defined as:

$$D(\gamma,\beta) = \frac{1}{2}\sum_{j=1}^{n}\left(\left|\mu_\gamma(x_j)-\mu_\beta(x_j)\right| + \left|\upsilon_\gamma(x_j)-\upsilon_\beta(x_j)\right| + \left|\pi_\gamma(x_j)-\pi_\beta(x_j)\right|\right) \tag{9}$$

The Euclidean distance is defined as:

$$D(\gamma,\beta) = \sqrt{\frac{1}{2}\sum_{j=1}^{n}\left[\left(\mu_\gamma(x_j)-\mu_\beta(x_j)\right)^2 + \left(\upsilon_\gamma(x_j)-\upsilon_\beta(x_j)\right)^2 + \left(\pi_\gamma(x_j)-\pi_\beta(x_j)\right)^2\right]} \tag{10}$$

## 3 The intuitionistic fuzzy CODAS-SORT method

In this section, we present an IF extension of the CODAS-SORT method to deal with sorting problem. As already declared, CODAS-SORT is a new sorting method based on CODAS. It is easy to apply and simple to deal with for DM. The use of two measures defines the assignment rules. The first measure is the Euclidean distance and the second one is the Taxicab distance. However, we cannot use the Euclidean and Taxicab distances in IF problems, because they are defined in a crisp environment. Because of that, we replaced the Taxicab distance by the Hamming distance. Since the aim of this study is to propose an IF extension of CODAS, instead of crisp distances, we use the fuzzy weighted Euclidean distance and the fuzzy weighted Hamming distance, which were introduced by Li (2007). Suppose that we have *n* alternatives and *m* criteria.

The steps of the IF-CODAS-SORT method are the following:

**Step 1.** Construct the IF decision matrix ($D_X$):

Determining the IF decision-making matrix. Assuming that there are *m* alternatives ($A_1, A_2, \ldots, A_m$) to be evaluated with respect to *n* criteria ($M_1, M_2, \ldots, M_n$):

$$D_X = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{cccc} M_1 & M_2 & & M_n \\ (\mu_{11}^X,\upsilon_{11}^X,\pi_{11}^X) & (\mu_{12}^X,\upsilon_{12}^X,\pi_{12}^X) & \cdots & (\mu_{1n}^X,\upsilon_{1n}^X,\pi_{1n}^X) \\ (\mu_{21}^X,\upsilon_{21}^X,\pi_{21}^X) & (\mu_{22}^X,\upsilon_{22}^X,\pi_{22}^X) & \ldots & (\mu_{2n}^X,\upsilon_{2n}^X,\pi_{2n}^X) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}^X,\upsilon_{m1}^X,\pi_{m1}^X) & (\mu_{m2}^X,\upsilon_{m2}^X,\pi_{m2}^X) & \ldots & (\mu_{mn}^X,\upsilon_{mn}^X,\pi_{mn}^X) \end{array} \tag{11}$$

where $D_X$ is the decision-making matrix, and $\mu_{ij}^X, \upsilon_{ij}^X, \pi_{ij}^X$ are the relative performances of the $i^{th}$ alternative with respect to the $j^{th}$ criterion.

**Step 2.** Construct the IF profile matrix ($D_Y$):

Determining the IF decision-making matrix. Assuming that there are $l$ profiles ($B_1, B_2, \dots, B_l$) to be evaluated with respect to $n$ criteria $M_1, M_2, \dots, M_n$):

$$D_Y = \begin{array}{c|cccc} & M_1 & M_2 & & M_n \\ B_1 & (\mu_{11}^Y, \upsilon_{11}^Y, \pi_{11}^Y) & (\mu_{12}^Y, \upsilon_{12}^Y, \pi_{12}^Y) & \cdots & (\mu_{1n}^Y, \upsilon_{1n}^Y, \pi_{1n}^Y) \\ B_2 & (\mu_{21}^Y, \upsilon_{21}^Y, \pi_{21}^Y) & (\mu_{22}^Y, \upsilon_{22}^Y, \pi_{22}^Y) & \dots & (\mu_{2n}^Y, \upsilon_{2n}^Y, \pi_{2n}^Y) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B_l & (\mu_{l1}^Y, \upsilon_{l1}^Y, \pi_{l1}^Y) & (\mu_{l2}^Y, \upsilon_{l2}^Y, \pi_{l2}^Y) & \dots & (\mu_{ln}^Y, \upsilon_{ln}^Y, \pi_{ln}^Y) \end{array} \tag{12}$$

where $D_Y$ is the profiles matrix, and $\mu_{kj}^X, \upsilon_{kj}^X, \pi_{kj}^X$ are the relative performances of $k^{th}$ profile with respect to $j^{th}$ criterion.

**Step 3.** Determine the IF negative-ideal solution:

$NIS_j$ is the anti-ideal solution of the decision matrix:

$$NIS_j = (u_j^X, v_j^X, \pi_j^X), \text{j} = 1, 2, \dots, \text{n} \tag{13}$$

$$t = arg\min_i(u_{ij}^X) \tag{14}$$

$$t = u_{tj}^X \tag{15}$$

$$u_j^X = u_{tj}^X \tag{16}$$

$$v_j^X = v_{tj}^X \tag{17}$$

$$\pi_j^X = 1 - u_{tj}^X - v_{tj}^X \tag{18}$$

$MIS_j$ is the anti-ideal solution of the profiles matrix:

$$MIS_j = (u_j^Y, v_j^Y, \pi_j^Y), \text{j} = 1, 2, \dots, \text{n} \tag{19}$$

$$t = arg\min_k(u_{kj}^Y) \tag{20}$$

$$t = u_{kj}^Y \tag{21}$$

$$u_j^Y = u_{kj}^Y \tag{22}$$

$$v_j^Y = v_{kj}^Y \tag{23}$$

$$\pi_j^Y = 1 - u_{kj}^Y - v_{kj}^Y \tag{24}$$

**Step 4.** Calculate the IF weighted Euclidean distances of alternatives from the IF negative-ideal solution:

$$E_{a_i} = \sqrt{\tfrac{1}{2}\textstyle\sum_{j=1}^{n}\left(u_{ij}^X - u_j^X\right)^2 + \left(v_{ij}^X - v_j^X\right)^2 + \left(\pi_{ij}^X - \pi_j^X\right)^2} \tag{25}$$

where $E_{a_i}$ denotes the Euclidean distance between the action $a_i$ and the negative-ideal solution $NIS_j$:

$$E_{b_k} = \sqrt{\tfrac{1}{2}\textstyle\sum_{k=1}^{n}\left(u_{kj}^Y - u_j^Y\right)^2 + \left(v_{kj}^Y - v_j^Y\right)^2 + \left(\pi_{kj}^Y - \pi_j^Y\right)^2} \tag{26}$$

where $E_{b_k}$ denotes the Euclidean distance between the limit $b_k$ and the negative--ideal solution $MIS_j$.

**Step 5.** Calculate the IF weighted Hamming distances of alternatives from the IF negative-ideal solution:

$$H_{a_i} = \frac{1}{2}\sum_{j=1}^{n}\left(\left|u_{ij}^{X} - u_j^{X}\right| + \left|v_{ij}^{X} - v_j^{X}\right| + \left|\pi_{ij}^{X} - \pi_j^{X}\right|\right) \tag{27}$$

where $H_{a_i}$ denotes the Hamming distance between the action $a_i$ and the negative--ideal solution $NIS_j$:

$$H_{b_k} = \frac{1}{2}\sum_{j=1}^{n}\left(\left|u_{kj}^{Y} - u_j^{Y}\right| + \left|v_{kj}^{Y} - v_j^{Y}\right| + \left|\pi_{kj}^{Y} - \pi_j^{Y}\right|\right) \tag{28}$$

where $H_{b_k}$ denotes the Hamming distance between the limit $b_k$ and the negative--ideal solution $MIS_j$.

**Step 6.** Determine the relative assessment matrix:

$$R(a_i, b_k) = \left[E_{a_i} - E_{b_k}\right] + (\psi\left[E_{a_i} - E_{b_k}\right] * \left[H_{a_i} - H_{b_k}\right]) \tag{29}$$

where $k \in \{1, 2,\dots, n\}$ and $\psi$ denotes a threshold function to determine the equality of the Euclidean distances of two alternatives, and is defined as follows:

$$\psi(x) = \begin{cases} 1 & \text{if } |x| \geq \tau \\ 0 & \text{if } |x| < \tau \end{cases} \tag{30}$$

In this function, $\tau$ is the threshold parameter that can be set by the DM. It is suggested to fix this parameter at a value between 0.01 and 0.05.

If the difference between the Euclidean distances of two alternatives is less than $\tau$, these two alternatives are also compared by the Hamming distance. In this study, we use $\tau = 0.02$ for the calculations.

**Step 7.** Assign alternatives to categories: To assign an alternative $a_i$ to one of the predefined categories, there are two ways that depend on the type of the available profile provided by the decision maker:

<u>Central profiles:</u>
If central profiles have been defined, the alternative $a_i$ is assigned to the class $C_k$ which has the smallest $|R(a_i, b_k)|$.
If $|R(a_i, b_k)|$ is the smallest then $a_i \in C_k$.

<u>Limiting profiles:</u>
When the difference between the two distances is minimal, the alternative and the center of the category are very near and if the difference is negative or positive, the alternative belongs to the category that has the minimum difference. If limiting profiles have been defined and $|R(a_i, b_k)|$ is the smallest then there are two cases:

– If $R(a_i, b_k) \geq 0$ then alternative $a_i$ is assigned to class $C_k$.
– If $R(a_i, b_k) < 0$ then alternative $a_i$ is assigned to class $C_{k-1}$.

## 4 Case study: Suplier selection

The case problem allows evaluating and assessing seven suppliers ($a_1$, $a_2$, $a_3$, $a_4$, $a_5$, $a_6$, $a_7$). The proposed evaluation framework was applied at a company N, a maker of perfumery, hygiene, health and cosmetic products. An expert evaluates the suppliers with respect to four criteria: Price, Product quality, Delivery and Agility. These seven suppliers are divided into three groups: Worst $C_1$, Moderate $C_2$, and Best $C_3$. After determining the list of alternatives, we evaluate them with regard to each criterion (Table 1).

Table 1: Performance matrix

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $\boldsymbol{a_1}$ | (0.50, 0.10, 0.40) | (0.60, 0.10, 0.30) | (0.40, 0.10,0.50) | (0.70, 0.10, 0.20) |
| $\boldsymbol{a_2}$ | (0.20, 0.40, 0.40) | (0.40, 0.20, 0.40) | (0.50, 0.10, 0.40) | (0.20, 0.30, 0.50) |
| $\boldsymbol{a_3}$ | (0.40, 0.50, 0.10) | (0.50, 0.10, 0.40) | (0.40, 0.20, 0.40) | (0.50, 0.10, 0.40) |
| $\boldsymbol{a_4}$ | (0.50, 0.10, 0.40) | (0.50, 0.10, 0.40) | (0.40, 0.30, 0.30) | (0.20, 0.40, 0.40) |
| $\boldsymbol{a_5}$ | (0.50, 0.20, 0.30) | (0.60, 0.10, 0.30) | (0.50, 0.20, 0.30) | (0.70, 0.20, 0.10) |
| $\boldsymbol{a_6}$ | (0.20, 0.20, 0.60) | (0.40, 0.20, 0.40) | (0.50, 0.10, 0.40) | (0.40, 0.30, 0.30) |
| $\boldsymbol{a_7}$ | (0.40, 0.10, 0.50) | (0.50, 0.00, 0.50) | (0.40, 0.30, 0.30) | (0.50, 0.20, 0.30) |
| $NIS_j$ | (0.20, 0.20, 0.60) | (0.40, 0.20, 0.40) | (0.40, 0.10, 0.50) | (0.20, 0.30, 0.50) |

Thereafter, the decision maker is invited to provide a list of classes. We evaluate the limiting profiles with regard to each criterion (Table 2).

Table 2: Limiting profiles

| | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $\boldsymbol{b_1}$ | (0.40, 0.20, 0.40) | (0.40, 0.20, 0.40) | (0.40, 0.25, 0.35) | (0.40, 0.25, 0.35) |
| $\boldsymbol{b_2}$ | (0.60, 0.30, 0.10) | (0.60, 0.30, 0.10) | (0.55, 0.30, 0.15) | (0.55, 0.30, 0.15) |
| $MIS_j$ | (0.40, 0.20, 0.40) | (0.40, 0.20, 0.40) | (0.40, 0.25, 0.35) | (0.40, 0.25, 0.35) |

Next, we calculate the Euclidean and Hamming distances of the alternatives and limits from the negative-ideal solution (Tables 3 and 4):

Table 3: Euclidian and Hamming distances (actions)

| | Distances | |
|---|---|---|
| Alternatives | $E_{a_i}$ | $H_{a_i}$ |
| $a_1$ | 0.29 | 1 |
| $a_2$ | 0.05 | 0.3 |
| $a_3$ | 0.28 | 1 |
| $a_4$ | 0.125 | 0.7 |
| $a_5$ | 0.36 | 1.2 |
| $a_6$ | 0.05 | 0.3 |
| $a_7$ | 0.17 | 0.9 |

Table 4: Euclidian and Hamming distances (profiles)

| | Distances | |
|---|---|---|
| Profiles | $E_{b_i}$ | $H_{b_i}$ |
| $b_1$ | 0 | 0 |
| $b_2$ | 0.205 | 0.82 |

The construction of the relative evaluation matrix is as follows (Table 5):
First, we set τ = 0.02
Example of calculation:

$$h(a_1, b_1) = (0.29 - 0) + [(0.29 - 0) * (1 - 0)] = 0.58$$

The other relative evaluations are shown in Table 5.

Table 5: Relative evaluation matrix

| | $b_1$ | $b_2$ |
|---|---|---|
| $a_1$ | 0.58 | 0.1 |
| $a_2$ | 0.065 | −0.075 |
| $a_3$ | 0.56 | 0.89 |
| $a_4$ | 0.11 | −0.07 |
| $a_5$ | 0.79 | 0.2 |
| $a_6$ | 0.065 | −0.074 |
| $a_7$ | 0.323 | −0.038 |

The assignment of alternatives to categories is presented in Table 6.

For example:
Since $|R(a_3, b_1)|$ is the smallest, we have $R(a_3, b_1) \geq 0$, and alternative $a_3$ is assigned to class $C_2$.
Since $|R(a_4, b_2)|$ is the smallest, we have $R(a_4, b_2) < 0$, and alternative $a_4$ is assigned to class $C_2$.

Table 6: The final classification of the actions

| Actions | Categories |
|---|---|
| $a_1$ | $C_3$ |
| $a_2$ | $C_2$ |
| $a_3$ | $C_2$ |
| $a_4$ | $C_2$ |
| $a_5$ | $C_3$ |
| $a_6$ | $C_2$ |
| $a_7$ | $C_2$ |

Suppliers $a_1$ and $a_5$ are assigned to the best group, whereas $a_2$, $a_3$, $a_4$, $a_6$ and $a_7$ are assigned to the moderate group.

## Sensitivity analysis

A sensitivity analysis is also performed in this part to demonstrate the stability of the sorting result. First, five values of $\tau$ are generated. Then we solve the problem using each of these cases. The generated values of $\tau$ are shown in Table 7 and the sorting results, in Figure 1.

Table 7: Sorting results with different values of $\tau$

| Actions | 0.01 $C_3$ | 0.02 $C_2$ | 0.03 $C_3$ | 0.04 | 0.05 |
|---|---|---|---|---|---|
| $a_1$ | 3 | 3 | 3 | 3 | 3 |
| $a_2$ | 2 | 2 | 2 | 3 | 2 |
| $a_3$ | 2 | 2 | 1 | 2 | 2 |
| $a_4$ | 2 | 3 | 2 | 2 | 2 |
| $a_5$ | 3 | 3 | 3 | 3 | 3 |
| $a_6$ | 2 | 2 | 2 | 2 | 2 |
| $a_7$ | 2 | 2 | 3 | 1 | 1 |

According to Figure 1 and Table 7, we can notice a good stability in the sorting of actions when the threshold parameter $\tau$ varies from 0.01 to 0.05. However, the modification of the $\tau$ parameter has a minor and neglected impact on the sorting of actions that can undermine the validity of the results. Consequently, we can affirm the performance of the IF-CODAS-SORT method.

As indicated by the conclusions of this analysis, we can claim that our proposed method is proficient to handle MCDM problems.

However, it may be seen from Table 7, that every one of the differences in sorting occurred between the successive classes, which confirms the consistency of the results.

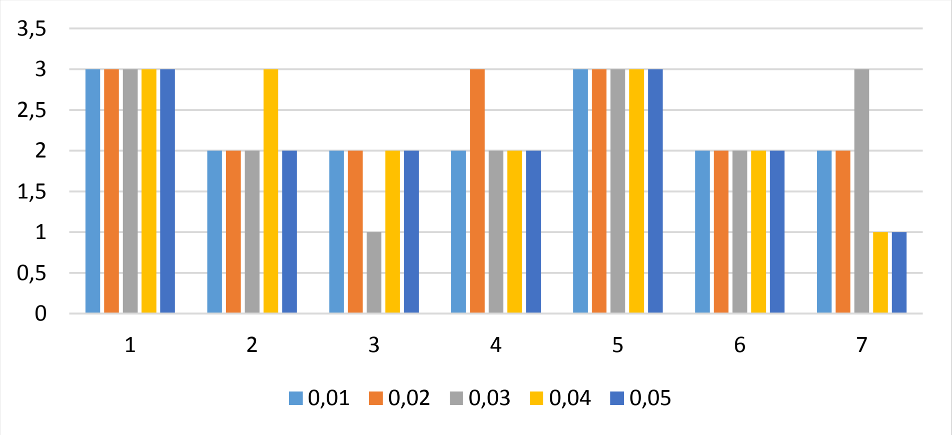


Figure 1: Sorting results with different values of τ

## 5 Application of the IR-CODAS model for risk assessment

The development of an intuitionistic fuzzy CODAS-SORT method is the objective of this study. Indeed, CODAS-SORT deals with the sorting MCDM problem. This method sorts the alternatives into ordered classes based on the central and limiting profiles and using exact values. Since it is difficult for decision makers to precisely express their preferences, we have developed an IF-CODAS-SORT method which uses intuitionistic fuzzy numbers to express uncertain evaluations.

An advantage of our result is that the assignment rules are based on the use of two measures. The first measure is based on the Euclidean distance. The second measure is the Hamming distance. The assignment rules are based on the difference between the two distances.

In the future, we intend to develop an IF-CODAS-SORT approach in the group decision context (IF-GD-CODAS-SORT).

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