M I S C E L L A N E A

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SOME REMARKS ON COINCIDENCE OF AN ECONOMETRIC MODEL

Abstract

In this paper concept of coincidence of variable and methods for checking coincidence of model and variables are presented. Particularly Hellwig's hypothesis and methods for constructing model with difference compensators are described. It makes possible keeping non coincidentional variables in model.

Keywords: matrix, variables, vector, coefficient, econometric model, coincidence
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Problem of coincidence of an econometric model is very important. Usually lack of variable coincidence means lack of possibility for correct interpretation of structural parameters estimation. Many econometricians were working with this problem. This article describes the most important achievements concerning coincidence of variable and model.

In the paper an econometric model is considered

$$Y = \alpha_1 Z_1 + \alpha_2 Z_2 + \ldots + \alpha_k Z_k + e \qquad (1)$$

Its variables are standardized.
Given is a matrix of observations and all variables of the model (1)

$$Q = \begin{bmatrix} Z & y \end{bmatrix} \qquad (2)$$

where matrix *Z* is of order $n \times k$ (its rank equals *k*) and *y* is an *n* – dimensional vertical vector.
Because variables of the model (1) are standardized hence:

$$\frac{1}{n} Z^T Z = R(k) \quad \frac{1}{n} Z^T y = R_0(k) \qquad (3)$$

The matrix $R(k)$ and the vector $R_0(k)$ are matrix and vector of correlation coefficients between variables in pair respectively (Z_i, Z_j) and (Z_i, Y) *i*, *j* = 1, 2 … *k*. In other words coefficients $r_{ij} = r(Z_i, Z_j)$ are elements of the matrix $R(k)$ and $r_i = r(Y, Z_i)$ are components of the vector $R_0(k)$.
Henceforth we shall talk about a pair of correlation $(R(k), R_0(k))$. It is a regular pair if (see (Hellwig, 1976))

$$0 < r_1 \le r_2 \ldots \le r_k \tag{4}$$

The pair of correlation exists if and only if $r^2 \in [0,1]$ where (Hauke & Pomianowska, 1987)

$$r^2(k) = R_0^T(k) R^{-1}(k) R_0(k) \tag{5}$$

This coefficient is measuring "quality" of model (1) or the correlation pair $(R(k), R_0(k))$.

Estimation of a_i of parameters α_i of the model (1) obtained by LSM are components of the vector $A(k)$ satisfying the system:

$$R(k)A(k) = R_0(k) \tag{6}$$

The problem of coincidence of the model (1) (or of the correlation pair $(R(k), R_0(k))$) has been given by Z. Hellwig in his paper (Hellwig, 1976).

Definition 1

The explanatory variable Z_i $(1 \le i \le k)$ of the model (1) has the property of coincidence if

$$\operatorname{sign} a_i = \operatorname{sign} r_i \tag{7}$$

Where a_i and r_i are components of vectors respectively $A(k)$ and $R_0(k)$.

Definition 2

If the relation (7) is satisfied for all $I = 1, 2 \ldots k$ then the model (1) (or the correlation pair $(R(k), R_0(k))$) has the property of coincidence.

Of course, if the pair $(R(k), R_0(k))$ is regular then (7) leads to

$$\operatorname{sign} a_i = +1 \tag{8}$$

In other words the model (1) (or the correlation pair $(R(k), R_0(k))$) has the property of coincidence if and only if all components of the vector $A(k)$ are positive. M. Kolupa has proved that the explanatory variable Z_i $(1 \le i \le k)$ of the model (1) has the property of coincidence if and only if

$$r_i > \rho_i R_{ii}^{-1} R_{0i} \tag{9}$$

Where ρ_i is the i – th row of matrix $R(k)$ without its i – th element, matrix R_{ii} is obtained by dropping the i – th row and i – th column of matrix $R(k)$ and the vector R_{0i} is created by dropping the i – th component of the vector $R_0(k)$ (Kolupa, 1980).

In order to utilize the inequality (8) we use a bordered matrix

$$U_i = \begin{bmatrix} R_{ii} & R_{0i} \\ \rho_i & r_i \end{bmatrix} \tag{10}$$

The matrix U_i is transformed into matrix U_i^* by elementary transformations as follows:

α - the matrix R_{ii} is transformed to an upper triangular one with its diagonal elements equal 1

β - the vector ρ_i is transformed to a zero vector

Hence

$$U_i \approx U_i^* = \begin{bmatrix} R_{ii}^* & R_{0i}^* \\ 0 & d_i \end{bmatrix} \tag{11}$$

where

$$d_i = r_i - \rho_i R_{ii}^{-1} R_{0i} \tag{12}$$

If $d_i > 0$ ($d_i < 0$) then explanatory variable Z_i has the property of coincidence (has not this property). The theory of bordered matrices and their applications is given in monograph (Kolupa & Szczepańska-Gruźlewska, 1991).

The property of coincidence of the model (1) allows to give a proper economical interpretation of estimators of the model's parameters. It explains the interest taken for the coincidence property. Z. Hellwig has defined an universal matrix in paper (Hellwig, 1976) as follows.

Definition 3

Matrix $G(k) = \lfloor g_{ij} \rfloor_{k \times k}$ where

$$g_{ij} = \begin{cases} 1 & for \;\; i = j \\ r_i r_j & for \;\; i \neq j \end{cases} \tag{13}$$

is called an universal one.

The properties of this matrix are given in (Kolupa, 1980). In 1976 Z. Hellwig has given the following hypothesis:

If

$$I(k) < R(k) < G(k) \tag{14}$$

where $I(k)$, $R(k)$, $G(k)$ are matrices of order $k \times k$ respectively unit, correlation and universal, then an econometric model described by a regular correlation pair $(R(k), R_0(k))$ has the property of coincidence.

Many interesting results concerned with Hellwig's hypothesis have been obtained and they are presented in (Kolupa, 1980). Hellwig's hypothesis was proved by M. Kolupa (see (Kolupa, 1996)).

Because the property of coincidence is very important for practice it is the reason for treating it. This property is reached by elimination from the model (1) of non – coincidentional variables. It is not good procedure as can be seen from the following example.

Let us consider a model given pair of correlations:

$$R(3)=\begin{bmatrix}1 & 0 & 0{,}1\\ 0 & 1 & 0\\ 0{,}1 & 0 & 1\end{bmatrix} \quad R_0(3)=\begin{bmatrix}0{,}01\\ 0{,}2\\ 0{,}3\end{bmatrix} \tag{15}$$

This model is non – coincidential, because:

$$\operatorname{sign} a_1=-1 \quad \operatorname{sign} a_2=+1 \quad \operatorname{sign} a_3=+1 \tag{16}$$

The explanatory variable Z_i is eliminated from the model (15). We obtained a model described by a new pair:

$$R_1(2)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad R_{01}(2)=\begin{bmatrix}0{,}2\\ 0{,}3\end{bmatrix} \tag{17}$$

which has property of coincidence. The same result can be obtained by elimination from the model (15) of variable Z_3 which has the property of coincidence in the model given by the pair (14).

Z. Hellwig has introduced a new concept how to guarantee the property of coincidence of the model (1) without elimination of explanatory variables which have not the property of coincidence (Hellwig, 1987).

Without loss of generality we suppose the non – coincidental variables in the model (1) are Z_1, Z_2 … Z_f and the coincidental ones are Z_{f+1} … Z_k. Among variables Z_{f+1} … Z_k we pick out one for example Z_t $\left(f+1 \le t \le k\right)$ and put:

$$v_i = Z_t - Z_i \quad \text{for} \quad \mathrm{i}=1,2\ldots f \tag{18}$$

$$v_j = Z_j \quad \text{for} \quad \mathrm{j}\neq \mathrm{t},\ \mathrm{j}=f+1,\ldots t \tag{19}$$

We obtain a new model described by the pair $\left(\overline{R}(k),\overline{R}_0(k)\right)$. Its explanatory variables are ones given in (18) and (19). The variables given by (18) are called Hellwig's difference compensators. For this reason the model described by the pair $\left(\overline{R}(k),\overline{R}_0(k)\right)$ is called a model with compensators (Kolupa & Śleszyński, 1989). Its quality is given by the coefficient $\overline{r}^2(k)$ where

$$\overline{r}^2(k)=\overline{R}_0^T(k)\overline{R}^{-1}(k)\overline{R}_0(k) \tag{20}$$

Let b_i $I = 1, 2 \ldots k$ denote components of the vector $B(k)$ satisfying the system

$$\overline{R}(k)B(k)=\overline{R}_0(k) \tag{21}$$

Hence (Kolupa & Marcinkowska-Lewandowska & Radzio, 1991), (Kolupa & Radzio, 1991) (Radzio, 1991)

$$b_i=-d_{it}a_i \quad i=1,2\ldots f \tag{22}$$

$$b_j=a_j \quad j\neq t,\ j=f+1,\ldots k \tag{23}$$

$$b_t=a_1+\ldots+a_f+a_t \tag{24}$$

where

$$d_{it} = r(Z_t, Z_i) \quad i = 1,2\ldots t \tag{25}$$

From (22) (23) and (24) it can be seen that the model described by the pair $\left(\overline{R}(k), \overline{R}_0(k)\right)$ has the property of coincidence if and only if

$$a_1 + a_2 + \ldots + a_f + a_t > 0 \tag{26}$$

Coefficients $r^2(k)$ and $\overline{r}^2(k)$ calculated for the correlation pairs respectively $\left(R(k), R_0(k)\right)$ and $\left(\overline{R}(k), \overline{R}_0(k)\right)$ are equal:

$$r^2(k) = \overline{r}^2(k) \tag{27}$$

At and, we shall give some remarks about literature concerned with the problem of coincidence of the model (1). The first paper was Hellwig's work (Hellwig, 1976) in 1976. It caused a great interest among many Polish econometricians. Today there exists over 50 works concerned with the problem of coincidence of the model (1). We have quoted only some of them which were directly involved in our problem.

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