Fedia Daami Remadi[*]
Hela Moalla Frikha[**]

THE MULTICRITERIA GROUP DECISION MAKING FLOWSORT METHOD UNDER UNCERTAINTY

Abstract

Crisp values are insufficient to model real-life situations and imprecise ideas are frequently represented in multicriteria decision aid analysis. In fact, it is difficult to treat the evaluation criteria precisely and to fix exact preferences rating. The triangular intuitionistic fuzzy numbers succeeded to treat this kind of ambiguity in a great deal of research than other forms of fuzzy representation functions. The field of sorting issues is an active research topic in the multiple criteria decision aid (MCDA). This study extended one of the sorting methods, FLOWSORT, for solving multiple criteria group decision-making problems. This extension described the preferences rating of alternatives as linguistic terms which can be easily expressed in triangular intuitionistic fuzzy numbers. To validate our extension, an illustrative example as well as an empirical comparison with other multi-criteria decision making methods is presented.

Keywords: multicriteria group decision making, sorting problematic, intuitionistic fuzzy set, FlowSort method.

---

[*] Research Laboratory "Optimisation, Logistique et Informatique Décisionnelle" (OLID), University of Sfax, Higher Institute of Industrial Management of Sfax, Sfax, Tunisia, e-mail: fedia.daami008@gmail.com, ORCID: 0000-0003-3635-9819.

[**] Research Laboratory "Optimisation, Logistique et Informatique Décisionnelle" (OLID), University of Sfax, Higher Institute of Industrial Management of Sfax, Sfax, Tunisia, e-mail: hela_frikha_moalla@yahoo.fr, ORCID: 0000-0002-0233-697X.

1   Introduction

Multi-criteria decision making (MCDM) is considered an essential part of modern decision science and operational research. It is the process of finding the best compromise among the feasible alternatives. It provides a wide variety of methodologies and techniques that enable the systematic treatment of decision problems under multiple criteria. The MCDA methods can be applied to four different kinds of analyses that can be performed in order to provide significant support to decision-makers (Remadi and Frikha, 2019). These are: (1) the choice of the best alternative, (2) the ranking of the set of the alternatives from the best to the worst, (3) the description of the features of the alternatives and (4) the classification of the alternatives into predefined homogenous groups.

In this paper, we study the ordinal classification problem, also called the sorting problem. It consists in orienting a decision problem to an assignment of alternatives to one of the predefined, ordered and homogenous categories or classes. Each class is a set of alternatives with similar properties or even values for the same properties, when compared to the alternatives of from the other classes. Many methods have been proposed during the previous decades. Among these, we can mention the well-known sorting methods, the ELECTRE-TRI (Shen, Xu and Xu, 2016), the THESEUS (Fernandez and Navarro, 2011), etc. Relying on the PROMETHEE (Brans, Mareschal and Vincke, 1984) methodology, several authors proposed the PROMETHEE-TRI (Figueira, Smet and Brans, 2004), the PROMSORT (Araz and Ozkarahan, 2007) and the FlowSort (Nemery and Lamboray, 2007). In fact, the PROMETHEE is one of the best known MCDM methods, since it is easy to use, simple to process and uses fewer parameters than the other MCDM methods such as ELECTRE (Govindan and Jepsen, 2016). Figueira, Smet and Brans (2004) were pioneers in the PROMETHEE-TRI method, extending it to the sorting context, but it used incompletely ordered categories. In 2007, Araz and Ozkarahan (2007) proposed the PROMSORT method which used completely ordered categories, but the assignment of the alternatives was not independent.

Developed by Nemery and Lamboray in 2007, FlowSort (Nemery and Lamboray, 2007) was proposed for assigning actions to completely ordered categories defined by limiting profiles or central profiles. It solves the drawbacks of PROMETHEE-TRI (Figueira, Smet and Brans, 2004) and PROMSORT (Araz and Ozkarahan, 2007) and treats the problematic sorting issue for independent assignments and completely ordered categories. The evaluation of alternatives and preference parameters of FlowSort are defined as crisp values. But, in a real-world situation, decisional problems are multidimensional and ambiguous in nature.

So, it is difficult to express the evaluation criteria precisely. Many extensions of FlowSort have been developed to solve these problems. Indeed, Janssen and Nemery (2012) proposed an extension of FlowSort to the case of input data imprecision. Moreover, Campos, Mareschal and Almeida (2015) extended FlowSort to introduce a fuzzy sorting method called Fuzzy FlowSort (F-FlowSort). For a simplified FlowSort version, Assche and De Smet (2016) found the parameters of a sorting model using classification examples in the context of traditional sorting and interval sorting. Moreover, Pelissari et al. (2019) suggested a new multicriteria method, SMAA-Fuzzy-FlowSort, for sorting problems under uncertainty through applying the Stochastic Acceptability Analysis to Fuzzy FlowSort.

As stated above, the fuzzy set (FS) theory (Zadeh, 1965) has been successfully applied in a good number of studies. However, this theory is not flawless as it uses only the membership degree of an element to a fuzzy set which is between zero and one. Actually, it is necessary to define the non--membership degree of an element to a fuzzy set, because it is not necessarily equal to 1 minus the degree of membership. To overcome this limitation, the intuitionistic fuzzy set theory concept seems more suitable to deal with uncertainty than other generalized fuzzy sets forms (Zhang, Jin and Liu, 2013). Furthermore, compared to the traditional fuzzy sets, the IFS can describe the fuzzy nature of the real world more comprehensively (Wang, Han and Zhang, 2012). In fact, it provides more flexibility to treat real-life problems under an uncertain environment, because when the area of applications changes, the intuitionistic fuzzy sets are easy to modify (Zhang, Jin and Liu, 2013).

Due to the complexity of the socio-economic environment, single decision--makers are unable to express their opinions or preferences on multiple criteria. In fact, multiple criteria group decision making (MCGDM) problems constitute an important research area that has drawn the attention of many researchers. In addition, the intuitionistic fuzzy set theory was applied to solve real-life complex Multicriteria Group Decision Making problems. Park, Cho and Kwun (2011), for instance, extended the group decision-making VIKOR method to an interval-valued intuitionistic fuzzy environment, in which the information about attribute weights was partially known. In addition, Chen (2015) developed an extended TOPSIS (Chen and Hwang, 1992) method which included the comparison approach to address multiple criteria group decision-making medical problems in the interval--valued intuitionistic fuzzy set framework. In the context of sorting problem, Shen, Xu and Xu (2016) provided a new outranking sorting method for solving Multi--Criteria Group Decision Making (MCGDM) problems using Intuitionistic Fuzzy Sets (IFS). Furthermore, Lolli et al. (2015) introduced a group decision support system, named FlowSort-GDSS, for sorting failure modes into priority classes.

Thus, the first aim of our research, which is also at the heart of its originality, was to develop an extension of the FlowSort method to deal with the imprecision issue, using the IFS theory to solve MCGDM problems. It consists in aggregating the individual sorting results in a collective one and calculating the personal and the group satisfaction degrees. Shen, Xu and Xu (2016) defined the personal satisfaction degree as the mean average of the comparison of the group sorting results and the individual sorting results and the group satisfaction degree as the weighted average of the personal satisfaction degrees. If satisfaction is low, it will be necessary to recollect the input data.

In addition, human judgments including preferences are difficult to define as numerical values. Also, the linguistic terms can simplify the process of an alternative rating by decision makers (DMs). Several operations on fuzzy numbers have been used to convert linguistic terms into IF numbers in the literature; the easiest to use are Triangular intuitionistic fuzzy numbers (Gautam, Singh and Singh, 2016). And here comes our second main original contribution, which lies in our choice to describe our decisional matrix through linguistic terms which are then, converted into triangular intuitionistic fuzzy values.

The remaining of this paper is organized as follows: in the second section, we present the FlowSort method using crisp evaluations. We introduce the IFS theory notations and definitions in the third section. The fourth section is devoted to develop developing an extension of the FlowSort method based on the IFS theory to solve the Multicriteria group decision making problem. Section five includes a numerical example and a comparison of the achieved results with those of other MCDA methods. The final section provides conclusions and suggests further research issues.

2   The FlowSort method

The FlowSort method is an ordinal classification method based on the ranking methodology of the PROMETHEE method. We first summarized the PROMETHEE algorithm which is based on the principle of pairwise comparisons of the alternatives. It aggregates the preference information of a DM through valued preference relations (Brans, Mareschal and Vincke 1984; Brans and Mareschal, 2005). Let $A = \{a_1, a_2, \dots, a_n\}$ be a set of alternatives and $G = \{g_1, g_2, \dots, g_m\}$ be a set of criteria. A w_k weight, $k = 1, \dots, m$, for each criterion should be well-known by the DM.

The preference function $P^k(a_i, a_j)$ represents the preference intensity of a_i over a_j according to criterion g_k, for $i = 1, \dots, n, j = 1, \dots, n$ and $k = 1, \dots, m$: $P^K(a_i, a_j) = P[d^K(a_i, a_j)]$, where $d^K(a_i, a_j) = g_k(a_i) - g_k(a_j)$ for a criterion to maximize and $d^K(a_i, a_j) = g_k(a_j) - g_k(a_i)$ for a criterion to minimize.

Six different types of preference functions were defined by Brans and Mareschal (2005).

Therefore, we need to calculate the outgoing flow $\phi_i^+(a_i) = \frac{1}{N-1}\sum_{x\in A}(a_i, x)$ and the incoming flow $\phi_i^-(a_i) = \frac{1}{N-1}\sum_{x\in A}(x, a_i)$, for each alternative a_i. Three relations can be defined as follows:

- The preference (P):

  If $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) \le \phi_i^-(a_j)$; or $\phi_i^+(a_i) = \phi_i^+(a_j)$ and $\phi_i^-(a_i) < \phi_i^-(a_j)$ or $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) = \phi_i^-(a_j)$; $a_i P\, a_j$,

- The indifference (IND):

  If $\phi_i^+(a_i) = \phi_i^+(a_j)$ and $\phi_i^-(a_i) = \phi_i^-(a_j)$; $a_i \text{IND}\, a_j$,

- The incomparability (INC):

  If $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) < \phi_i^-(a_j)$; or $\phi_i^+(a_i) > \phi_i^+(a_j)$ and $\phi_i^-(a_i) < \phi_i^-(a_j)$; $a_i \text{INC} a_j$.

PROMETHEE II proposed the net flow $\phi = \phi_i^+(a_i) - \phi_i^-(a_i)$ to overcome the incomparability of alternatives. Two rules can be defined as follows:

- the preference (P): $a_i P\, a_j$ iff $\phi(a_i) > \phi(a_j)$;
- the indifference (IND): $a_i \text{ IND } a_j$ iff $\phi(a_i) = \phi(a_j)$.

The FlowSort was proposed to assign a set of n alternatives A to k ordered categories $C_1, C_2, \dots, C_k$ evaluated according to m criteria G. Each category is defined by a set of limiting profiles $R = \{r_1, r_2, \dots, r_{k+1}\}$ or by a set of k central profiles (centroids) for k ordered categories $\tilde{R} = \{\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_k\}$ defined by the DM. So, to avoid conflicts, we note that each category can be defined by a set of reference profiles $R^* = \{r_1^*, r_2^*, \dots\}$ founded by Nemery and Lamboray (2007). For each alternative a_i for all $i \in \{1, 2, ..., n\}$, let us define a set $R_i^* = R^* \cup \{a_i\}$, where a_i is the action to be assigned.

The assignment of alternatives is deduced from their relative position with respect to the reference profiles, in terms of positive, negative and net flows. It depends on the simultaneous comparison of the alternative with all the reference profiles (Nemery and Lamboray, 2007). The positive, negative and net flows are computed as follows by using equation (1):

$$\phi_{R_i^*}^+(x) = \frac{1}{|R_i^*|-1}\sum_{y\in R_i^*}\pi(x, y), \tag{1}$$

$$\phi_{R_i^*}^-(x) = \frac{1}{|R_i^*|-1}\sum_{y\in R_i^*}\pi(y, x), \tag{2}$$

$$\phi_{R_i^*}(x) = \phi_{R_i^*}^+(x) - \phi_{R_i^*}^-(x), \tag{3}$$

where $|R_i^*|$ is the number of elements in the set R_i^*.

Three different assignment rules based on the positive, negative and the net flows are defined as follows:

$$C_{\phi^+}(a_i) = C_K \ \text{if} \ \phi^+_{R_i^*}(r_k) > \phi^+_{R_i^*}(a_i) \geq \phi^+_{R_i^*}(r_{k+1}), \tag{4}$$

$$C_{\phi^-}(a_i) = C_k \ \text{if} \ \phi^-_{R_i^*}(r_k) \leq \phi^-_{R_i^*}(a_i) < \phi^-_{R_i^*}(r_{k+1}), \tag{5}$$

$$C_{\phi}(a_i) = C_K \ \text{if} \ \phi_{R_i^*}(r_k) > \phi_{R_i^*}(a_i) \geq \phi_{R_i^*}(r_{k+1}). \tag{6}$$

3 Intuitionistic fuzzy set theory

To deal with uncertainty and vagueness, fuzzy set theory (Zadeh, 1965) was used as an efficient tool, and has had a great success in innumerable fields. Let X denotes a universe of discourse. A fuzzy set A in X is defined as a set of ordered pairs:

$$A = \{<x, \mu_A(x)>/x \in X\}, \mu_A(x) \in [0, 1] \text{ is the degree}$$
$$\text{of belongingness of x in A.} \tag{7}$$

The intuitionistic fuzzy set theory (Atanassov, 1986) is a generalization of the fuzzy set theory (Zadeh, 1965). It solves the problem that a non-membership degree is not always equal to $1 - \mu_A(x)$ in real life. The IFS theory is characterized by assigning a membership degree and a non-membership degree to each element. Let a set X be fixed, an intuitionistic fuzzy set (IFS) A in X is defined as follows:

$$A = \{<x, \mu_A(x), \nu_A(x)>/x \in X\}, \mu_A(x), \nu_A(x) \in [0, 1], \tag{8}$$

where $\mu_A(x)$ and $\nu_A(x)$ are defined, respectively, as the degree of membership and the degree of non-membership of the element $x \in X$, $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The fuzzy set (Zadeh, 1965) is defined by $A = \{<x, \mu_A(x)>/x \in X\}$ and can be defined as an IFS by $A = \{<x, \mu_A(x), 1 - \mu_A(x)>/x \in X\}$. For each IFS A in X, the degree of hesitancy of x to A is $\theta_A(x) = 1 - \mu_A(x) - \nu_A(x)$. If $\theta_A(x) = 0$, then A is reduced to a fuzzy set.

The IFS is able to describe the data which may involve uncertain information. An ill-known quantity may therefore be expressed with an intuitionistic fuzzy number (IFN). Several functions such as trapezoidal (Banerjee, 2012) triangular (Li, Nan and Zhang, 2012), interval number (Sengupta and Pal, 2009), among others, can be used to explain the intuitionistic fuzzy numbers. The simplest one is to present the membership and the non-membership functions by the triangular fuzzy numbers (TIFNs).

The TIFN (Li, Nan and Zhang, 2012) is represented by the two sets of triplets $A_{(TIFN)}= \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$, where a_2 is the mean value of the intuitionistic fuzzy numbers $\mu_A(x)$ and $\nu_A(x)$, a_1 and a_3 are, respectively, the left and the right boundaries of $\mu_A(x)$, a'_1 and a'_3 are, respectively, the left and the right boundaries of $\nu_A(x)$, and $a'_1 \leq a_1 \leq a_2 \leq a_3 \leq a'_3$. The TIFN membership and non-membership are given as follows:

$$\mu_{A_{(TIFN)}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, for\ a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2}, for\ a_2 \leq x \leq a_3, \\ \quad 0\ otherwise \end{cases} \tag{9}$$

$$\nu_{A_{(TIFN)}}(x) = \begin{cases} \frac{a_2-x}{a_2-a'_1}, for\ a'_1 \leq x \leq a_2 \\ \frac{x-a_2}{a'_3-a_2}, for\ a_2 \leq x \leq a'_3. \\ \quad 1\ otherwise \end{cases} \tag{10}$$

In many real-life situations, the information cannot be evaluated exactly in numerical values but rather in linguistic variables. The linguistic terms are words and sentences of a natural language. The Linguistic Intuitionistic Fuzzy Number (LIFN) is a special intuitionistic fuzzy number which can describe the vagueness existing in real-life decision-making more easily (Liu and Qin, 2017). The linguistic variables have some special transformations forms to IFNs. These may include the trapezoidal, triangular, and rectangular forms. The most popular kind of IFNs are triangular numbers. We opted for the Gautam, Singh and Singh (2016) transformations because of their simplicity and ease of operation. They express the linguistic variables as positive TIFNs as shown in Tables 1 and 2.

Table 1: Linguistic variables for the rating

Very Poor (VP)	<0, 0, 1; 0, 0, 2>
Poor (P)	<0, 1, 3; 0, 1 ,4>
Medium Poor (MP)	<1, 3, 5; 0.5, 3 ,5.5>
Fair (F)	<3, 5, 7; 2, 5, 8>
Medium Good (MG)	<5, 7, 9; 4.5, 7, 9.5>
Good (G)	<7, 9, 10; 6, 9, 10>
Very Good (VG)	<9, 10, 10; 8, 10, 10>

Source: Gautam, Singh and Singh (2016).

Table 2: Linguistic variables for the weight importance of each criterion

Very Low (VL)	<0, 0, 0.1; 0, 0, 0.2>
Low (L)	<0, 0.1 ,0.3; 0, 0.1, 0.4>
Medium Low (ML)	<0.1, 0.3, 0.5; 0.05, 0.3, 0.5, 0.5>
Medium (M)	<0.3, 0.5, 0.7; 0.2, 0.5, 0.8>
Medium High (MH)	<0.5 ,0.7, 0.9; 0.45, 0.7, 0.95>
High (H)	<0.7, 0.9, 1; 0.6, 0.9, 1>
Very High (VH)	<0.9, 1, 1; 0.8, 1, 1>

Source: Gautam, Singh and Singh (2016).

Let us consider $A_{(TIFN)} = \{(a_1, a_2, a_3); (a'_1, a_2, a'_3)\}$ and $B_{(TIFN)} = \{(b_1, b_2, b_3);$ $(b'_1, b_2, b'_3)\}$. The operations on triangular intuitionistic fuzzy numbers are the following:

$$A_{(TIFN)} + B_{(TIFN)} = \{(a_1 + b_1, a_2 + b_2, a_3 + b_3); (a'_1 + b'_1, a_2 + b_2, a'_3 + b'_3)\}, \quad (11)$$

$$A_{(TIFN)} - B_{(TIFN)} = \{(a_1 - b_3, a_2 - b_2, a_3 - b_1); (a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1)\}, \quad (12)$$

$$A_{(TIFN)} * B_{(TIFN)} = \{(a_1 * b_1, a_2 * b_2, a_3 * b_3); (a'_1 * b'_1, a_2 * b_2, a'_3 * b'_3)\}. \quad (13)$$

Let k be a scalar number:

If $k > 0$ then $k * A_{(TIFN)} = \{(k * a_1, k * a_2, k * a_3); (k * a'_1, k * a_2, k * a'_3)\}, \quad (14)$

If $k < 0$ then $k * A_{(TIFN)} = \{(k * a_3, k * a_2, k * a_1); (k * a'_3, k * a_2, k * a'_1)\}. \quad (15)$

Gani and Abbas (2014) defined the defuzzification of a triangular intuitionistic number to ordinal number as follows:

$$A = \frac{(a_1 + 2a_2 + a_3) + (a'_1 + 2a_2 + a'_3)}{8}. \quad (16)$$

4   IFS-FlowSort for multicriteria group decision making

Our research aim is to develop an IFS FlowSort method where an ill-known quantity is expressed with an intuitionistic fuzzy number. Our proposed extension adopts linguistic values as input data to simplify the collection of data. Next, we have to transform the linguistic preference rating and the linguistic weights to triangular intuitionistic fuzzy numbers (TIFNs). In addition, our extension solves the multicriteria group decision making problems (MCGDM). It consists in aggregating the individual sorting results in a collective one and calculating the personal and the group satisfaction degrees. If there is a low satisfaction, it will be necessary to recollect the input data.

As presented in Figure 1, IFS-FlowSort for an MCGDM algorithm can be divided into four phases: (*i*) the construction of the linguistic evaluation matrix, (*ii*) the implementation of IFS-FlowSort (Remadi and Frikha, 2019) of each individual decision maker separately, (*iii*) the aggregation of the individual sorting results in a collective one, (*iv*) the satisfaction evaluation.
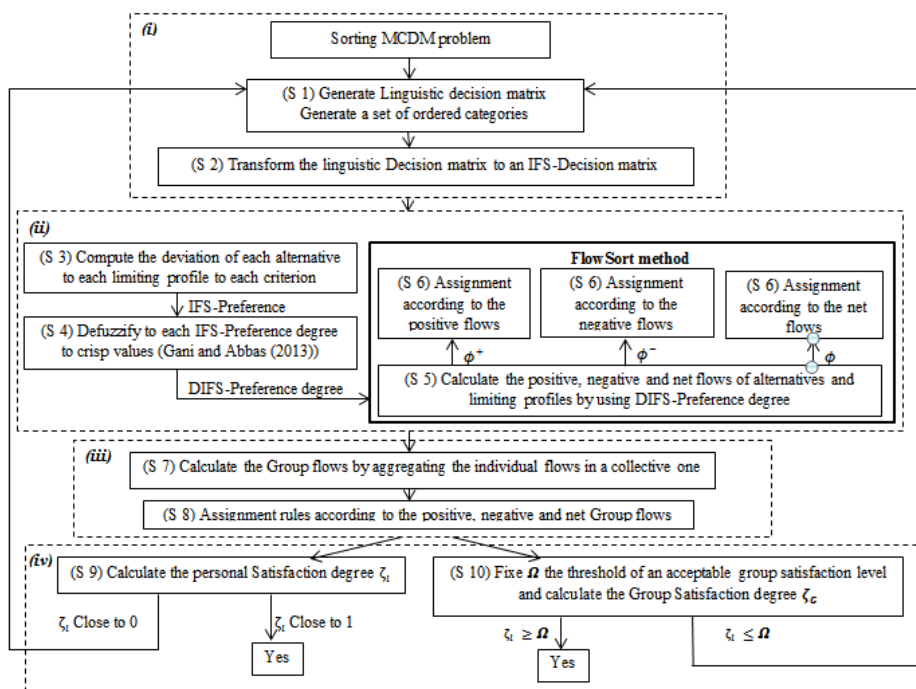
Figure 1: The procedure of the IFS-FlowSort method for an MCGDM problem

According to the definitions in Sections 2 and 3, and as presented in Figure 1, the implementation of the IFS-FlowSort method for MCGDM is as follows:

In the first phase, we have to create a linguistic evaluation matrix. To solve the sorting of the MCGDM problem, it is necessary to describe:

$A = \{a_1, a_2, \ldots, a_n\}$ a set of n alternatives.

$G = \{g_1, g_2, \ldots, g_m\}$ a set of m criteria evaluated by $W_g = \{w_{g1}, w_{g2}, \ldots, w_{gm}\}$ criteria weights.

$C = \{c_1, c_2, \ldots, c_t\}$ a set of t classes.

$D = \{d_1, d_2, \ldots, d_y\}$ a set of y decision makers (DM) evaluated by $\lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_y\}$ DM weights. The DM weights are assumed to be crisp numbers.

$X_{(l)} = (x_{ij(l)})_{n*m}$ is the linguistic performance rating for the alternative a_i ($i = 1, 2, \ldots, n$) on criterion c_j ($j = 1, 2, \ldots, m$) according to the DM d_l ($l = 1, 2, \ldots, y$).

The parameter values such as the criteria weights, the DM weights and the preference and the indifference degrees are assumed to be unique for all the DMs.

Then, the DMs are also invited to determine the set of ordered categories $C_1 \rhd C_2 \rhd \ldots \rhd C_t$, where $C_h \rhd C_l$ for $h < l$, denote that the category C_h is preferred to the category C_l. Each category is defined by one central profile or two reference

profiles. Let $R = \{r_1, r_2, \ldots, r_{t+1}\}$ be the set of limiting profiles, where r_h and r_{h+1} are the upper and the lower bounds of C_h, respectively. There are t central profiles (centroids) for t ordered categories $\tilde{R} = \{\tilde{r}_1, \tilde{r}_2, \ldots, \tilde{r}_t\}$ defined by the DM. When there is no distinction between the set of limiting profiles and the set of centroids, there are exist the reference profiles $R^* = \{r_1^*, r_2^*, \ldots\}$. Let us define the set $R_i^* = = R^* \cup \{a_i\}$ where a_i is the action to be assigned (Step 1).

The second phase is to transform the linguistic performance ratings decisional matrix $X_{(l)}$, the linguistic criterion weights and the linguistic DM weights to triangular intuitionistic fuzzy numbers. Table 1 and Table 2 show the linguistic scales and the corresponding IFNs according to Gautam, Singh and Singh (2016):

$$x_{ij(l)} = \{(x_{ij(l)}^1, x_{ij(l)}^2, x_{ij(l)}^3); (x_{ij(l)}'^1, x_{ij(l)}^2, x_{ij(l)}'^3)\}, i = 1, 2, \ldots, n,$$

$$j = 1, 2, \ldots, m, l = 1, 2, \ldots, y. \tag{17}$$

where $x_{ij(l)}^2$ is the mean value of the intuitionistic fuzzy numbers $\mu(x_{ij(l)})$ and $\nu(x_{ij(l)})$, a_1 and a_3 are, respectively, the left and the right boundaries of $\mu(x_{ij(l)})$, a'_1 and a'_3 are, respectively, the left and the right boundary of $\nu(x_{ij(l)})$, and $x_{ij(l)}'^1 \leq x_{ij(l)}^1 \leq x_{ij(l)}^2 \leq x_{ij(l)}'^3 \leq x_{ij(l)}^3$ (Step 2).

After that, we should construct and exploit the individual Intuitionistic Fuzzy FlowSort procedure:

The preference degrees $\pi(A, B)$ of each alternative A over an alternative B are computed using the arithmetic operation on triangular intuitionistic fuzzy numbers for all the alternatives A, B of R_i^* (Step 3).

$$\pi(A, B) = \sum w_j * P_j(A, B), \tag{18}$$

$$\pi(A, B) = \sum w_j * P_j(f_j(A) - f_j(B)),$$

where $f_j(A) = (a_1, a_2, a_3; a'_1, a_2, a'_3)$, $f_j(B) = (b_1, b_2, b_3; b'_1, b_2, b'_3)$ and $w_j = (w_1, w_2, w_3; w'_1, w_2, w'_3)$ are triangular intuitionistic fuzzy numbers.

$$\pi(A, B) = \sum w_j * P_j((a_1, a_2, a_3; a'_1, a_2, a'_3) - (b_1, b_2, b_3; b'_1, b_2, b'_3)),$$

$$\pi(A, B) = \sum w_j * P_j(a_1 - b_3, a_2 - b_2, a_3 - b_1; a'_1 - b'_3, a_2 - b_2, a'_3 - b'_1),$$

$$\pi(A, B) = \sum w_j * P_j(\alpha_1, \alpha_2, \alpha_3; \alpha'_1, \alpha_2, \alpha'_3),$$

where $\alpha_1 = a_1 - b_3, \alpha_2 = a_2 - b_2, \alpha_3 = a_3 - b_1, \alpha'_1 = a'_1 - b'_3, \alpha'_3 = a'_3 - b'_1$.

$$\pi(A, B) = \sum w_j * (\alpha_1^{P_j}, \alpha_2^{P_j}, \alpha_3^{P_j}; \alpha_1'^{P_j}, \alpha_2'^{P_j}, \alpha_3'^{P_j}),$$

$$\pi(A, B) = \sum (w_{1j}\alpha_1^{P_j}, w_{2j}\alpha_2^{P_j}, w_{3j}\alpha_3^{P_j}; w'_{1j}\alpha_1'^{P_j}, w_{2j}\alpha_2'^{P_j}, w'_{3j}\alpha_3'^{P_j}),$$

$$\pi(A, B) = (\sum w_{1j}\alpha_1^{P_j}, \sum w_{2j}\alpha_2^{P_j}, \sum w_{3j}\alpha_3^{P_j}; \sum w'_{1j}\alpha_1'^{P_j}, \sum w_{2j}\alpha_2'^{P_j}, \sum w'_{3j}\alpha_3'^{P_j}). \tag{19}$$

Then, each preference degree (A, B) should be defuzzified to transform the intuitionistic fuzzy number into a real number. We suggest the use of Gani and Abbas (2014) operator given in (12), since it is easier to use and, therefore, the use of IFS-FlowSort will be simple (Step 4):

$$\pi_d(A, B) = \frac{\left(\sum w_{1j}\alpha_1^{P_j} + 2*\sum w_{2j}\alpha_2^{P_j} + \sum w_{3j}\alpha_3^{P_j}\right) + \left(\sum w'_{1j}\alpha_1'^{P_j} + 2*\sum w_{2j}\alpha_2'^{P_j} + \sum w'_{3j}\alpha_3'^{P_j}\right)}{8}. \tag{20}$$

The positive, negative and net flows of each alternative A of R_i^* are computed according to the defuzzified outranking degree (A, B) (Step 5):

$$\phi_{R_i^*}^+ (A) = \frac{1}{|R_i^*|-1} \sum_{B \in R_i^*} \pi_d (A, B), \tag{21}$$

$$\phi_{R_i^*}^- (A) = \frac{1}{|R_i^*|-1} \sum_{B \in R_i^*} \pi_d (B, A), \tag{22}$$

$$\phi_{R_i^*} (A) = \phi_{R_i^*}^+ (A) - \phi_{R_i^*}^- (A). \tag{23}$$

As in FlowSort, three different assignment rules based on the positive, negative and net flows are defined as follows (Step 6):

$$C_{\phi^+} (a_i) = C_t \text{ if } \phi_{R_i^*}^+ (r_t) > \phi_{R_i^*}^+ (a_i) \geq \phi_{R_i^*}^+ (r_{t+1}), \tag{24}$$

$$C_{\phi^-} (a_i) = C_t \text{ if } \phi_{R_i^*}^- (r_t) \leq \phi_{R_i^*}^- (a_i) < \phi_{R_i^*}^- (r_{t+1}), \tag{25}$$

$$C_{\phi} (a_i) = C_t \text{ if } \phi_{R_i^*} (r_t) > \phi_{R_i^*} (a_i) \geq \phi_{R_i^*} (r_{t+1}). \tag{26}$$

The third phase is the implementation of the group decision making IFS FlowSort procedure:

We calculate the positive, negative and net flows of the group of DMs by aggregating the individual flows in collective ones (Step 7):

$$\phi_{R_i^*}^{+G} = \sum_{l=1}^y \lambda_l * \phi_{R_i^*}^+ (A), \tag{27}$$

$$\phi_{R_i^*}^{-G} = \sum_{l=1}^y \lambda_l * \phi_{R_i^*}^- (A), \tag{28}$$

$$\phi_{R_i^*}^G = \sum_{l=1}^y \lambda_l * \phi_{R_i^*} (A). \tag{29}$$

Afterwards, we assign alternatives according to the group flows values (Step 8):

$$C_{\phi^{+G}} (a_i) = C_t \text{ if } \phi_{R_i^*}^+ (r_t) > \phi_{R_i^*}^{+G} (a_i) \geq \phi_{R_i^*}^+ (r_{t+1}), \tag{30}$$

$$C_{\phi^{-G}} (a_i) = C_t \text{ if } \phi_{R_i^*}^- (r_t) \leq \phi_{R_i^*}^{-G} (a_i) < \phi_{R_i^*}^- (r_{t+1}), \tag{31}$$

$$C_{\phi^G} (a_i) = C_t \text{ if } \phi_{R_i^*} (r_t) > \phi_{R_i^*}^G (a_i) \geq \phi_{R_i^*} (r_{t+1}). \tag{32}$$

In the last phase, we have to calculate the personal and the group satisfaction degrees.

The personal satisfaction degree is the mean average of the comparison of the group sorting results and the individual sorting results (Step 9):

$$\zeta_l = \frac{\sum_{i=1}^n \Psi_l(A_i)}{n} \tag{33}$$

where $\Psi_l(A_i) = \begin{cases} 1 \text{ if } s_l(A_i) = S(A_i) \\ 0 \qquad otherwise \end{cases}$,

where $s_l(A_i)$ is the alternative A_i sorting result of the lth person, $S(A_i)$ is its group sorting result. If ζ_l is close to 1, it means that the personal satisfaction is high, while if ζ_l is close to 0, there is a low personal satisfaction, and consequently it is necessary to recollect data.

The group satisfaction degree is the weighted average of the personal satisfaction degrees (Step 10):

$$\zeta_G = \sum_{l=1}^y \lambda_l \zeta_l = \sum_{l=1}^y \lambda_l \frac{\sum_{i=1}^n \Psi_l(A_i)}{n}, \tag{34}$$

DMs are invited to fix $\boldsymbol{\Omega} \in [0, 1]$ as a threshold of an acceptable group satisfaction level. If $\zeta_G \geq \boldsymbol{\Omega}$, it means that there is a group agreement; else, it is also necessary to recollect data.

5 A numerical example

At this step of our research, we tested the applicability of the proposed IFS-FlowSort method for MCGDM through its application to the example of Gautam, Singh and Singh (2016). In fact, we considered an MCDM sorting problem concerning the assignment of alternatives applied to IFS-TOPSIS (Chen, 2015) to illustrate the implementation of our proposed approach.

In this decision problem, a software company desires to hire a system analyst. After a preliminary screening, four candidates $\{A_1, A_2, A_3, A_4\}$ remain for further assignment to three categories: C_1 is to be selected, C_2 is to be discussed and C_3 is to be rejected. The four potential alternatives can be evaluated by three DMs according to five criteria: g_1 (Emotional steadiness), g_2 (Oral communication skill), g_3 (Personality), g_4 (Past experience), g_5 (Self-confidence). The weights of the five criteria and the performance rating (shown in Table 3) are described using the linguistic term set $w_j = \{H, VH, H, VH, MH\}$. As already mentioned, the weights of the DMs are assumed to be a crisp number $\lambda_l = \{0.3; 0.5; 0.2\}$. We suppose that the indifference threshold $q_j = 0$ and the preference threshold $p_j = 7$ for $j = 1, \ldots, 5$. The limiting profiles of the criteria are given in Table 4.

Table 3: The candidates' ratings according to the three DMs

Criteria	Alternatives	Decision Makers		
		DM_1	DM_2	DM_3
g_1	A_1	MG	G	MG
	A_2	G	G	MG
	A_3	VG	G	F
	A_4	F	F	F
g_2	A_1	G	MG	F
	A_2	VG	VG	VG
	A_3	MG	G	VG
	A_4	MP	P	P
g_3	A_1	F	G	MG
	A_2	VG	VG	VG
	A_3	G	MG	VG
	A_4	MG	MP	P
g_4	A_1	VG	G	F
	A_2	VG	VG	VG
	A_3	G	VG	MG
	A_4	F	F	F
g_5	A_1	F	F	F
	A_2	VG	MG	G
	A_3	G	G	MG
	A_4	F	MP	P

Table 4: The limiting profiles

	g_1	g_2	g_3	g_4	g_5
IR_1	10	10	10	10	10
IR_2	6	6	6	6	6
IR_3	4	4	4	4	4
IR_4	0	0	0	0	0

To construct the intuitionistic fuzzy decision matrix, we transformed the linguistic performance rating (shown in Table 3) into triangular intuitionistic fuzzy number by employing equation (19) (Table 5).

The transformed intuitionistic fuzzy set weight of each criterion is the following: w_j = {<0.7, 0.9, 1; 0.6, 0.9, 1>, <0.9, 1, 1; 0.8, 1, 1>, <0.7, 0.9, 1; 0.6, 0.9, 1>, <0.9, 1, 1; 0.8, 1, 1>, <0.5, 0.7, 0.9; 0.45, 0.7, 0.95>}.

Table 5: The IFS Decision matrix

Criteria	Alternatives	Decision Makers		
		DM_1	DM_2	DM_3
g_1	A_1	<5, 7, 9; 4.5, 7, 9.5>	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_2	<7, 9, 10; 6, 9, 10>	<7, 9,10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_3	<9, 10, 10; 8, 10, 10>	<7, 9, 10; 6, 9, 10>	<3, 5, 7; 2, 5, 8>
	A_4	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>
g_2	A_1	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>	<3, 5, 7; 2, 5, 8>
	A_2	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>
	A_3	<5, 7, 9; 4.5, 7, 9.5>	<7, 9, 10; 6, 9,10>	<9, 10, 10; 8, 10, 10>
	A_4	<1, 3, 5; 0.5, 3, 5.5>	<0, 1, 3; 0, 1, 4>	<0, 1, 3; 0, 1, 4>
g_3	A_1	<3, 5, 7; 2, 5, 8>	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_2	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10, 10>
	A_3	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>	<9, 10, 10; 8, 10, 10>
	A_4	<5, 7, 9; 4.5, 7, 9.5>	<1, 3, 5; 0.5, 3, 5.5>	<0, 1, 3; 0, 1, 4>
g_4	A_1	<9, 10, 10; 8, 10,10>	<7, 9, 10; 6, 9, 10>	<3, 5, 7; 2, 5,8>
	A_2	<9, 10, 10; 8, 10,10>	<9, 10, 10; 8, 10, 10>	<9, 10, 10; 8, 10,10>
	A_3	<7, 9, 10; 6, 9, 10>	<9, 10, 10; 8, 10, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_4	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>
g_5	A_1	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>	<3, 5, 7; 2, 5, 8>
	A_2	<9, 10, 10; 8, 10, 10>	<5, 7, 9; 4.5, 7, 9.5>	<7, 9, 10; 6, 9, 10>
	A_3	<7, 9, 10; 6, 9, 10>	<7, 9, 10; 6, 9, 10>	<5, 7, 9; 4.5, 7, 9.5>
	A_4	<3, 5, 7; 2, 5, 8>	<1, 3, 5; 0.5, 3, 5.5>	<0, 1, 3; 0, 1,4>

We applied individual procedures to each DM evaluation. First, we computed the deviation of each pair of alternatives according to each criterion using the arithmetic IFS operations to obtain the intuitionistic fuzzy preference degrees as mentioned in Step 3. Then, we defuzzified the IF-preference degrees to crisp numbers using equation (22). Finally, we calculated the positive, negative and net flows values of each DM (see Tables 6-9).

The individual results show that, according to DM1 and DM2, the candidates A_1, A_2 and A_3 are assigned to C_1 (to be selected), but candidate A_4 is assigned to C_3 (to be rejected). As for DM3, A_1 is assigned to C_2 (to be discussed).

Table 6: The positive, negative and net flows of DM1

ϕ_{DM1}		IR_1	IR_2	IR_3	IR_4	A_i
R_1	$\phi_{R_1}^+$	2.51	1.51	1.08	0.88	1.52
	$\phi_{R_1}^-$	0.41	0.73	1.333	2.62	0.6
	ϕ_{R_1}	2.09	0.78	−0.24	−1.73	0.89
R_2	$\phi_{R_2}^+$	2.50	1.72	1.39	0.97	2.09
	$\phi_{R_2}^-$	0.089	0.59	1.29	2.61	0.093
	ϕ_{R_2}	2.41	1.13	0.10	−1.64	1.99
R_3	$\phi_{R_3}^+$	2.50	1.58	1.23	0.96	1.77
	$\phi_{R_3}^-$	0.26	0.623	1.3	2.615	0.29
	ϕ_{R_3}	2.25	0.96	−0.06	−1.66	1.48
R_4	$\phi_{R_4}^+$	2.50	1.35	0.8	0.72	0.87
	$\phi_{R_4}^-$	0.74	0.95	1.43	2.615	1.24
	ϕ_{R_4}	1.76	0.397	−0.63	−1.896	−0.37

Table 7: The positive, negative and net flows of DM2

ϕ_{DM2}		IR_1	IR_2	IR_3	IR_4	A_i
R_1	$\phi_{R_1}^+$	2.50	1.52	1.13	0.88	1.53
	$\phi_{R_1}^-$	0.37	0.68	1.31	2.615	0.47
	ϕ_{R_1}	2.13	0.84	−0.18	−1.73	1.05
R_2	$\phi_{R_2}^+$	2.50	1.66	1.33	0.96	1.96
	$\phi_{R_2}^-$	0.154	0.60	1.29	2.615	0.17
	ϕ_{R_2}	2.35	1,06	0.035	−1.65	1.79
R_3	$\phi_{R_3}^+$	2.50	1.60	1.25	0.97	1.82
	$\phi_{R_3}^-$	0.23	0.62	1.29	2.615	0.267
	ϕ_{R_3}	2.27	0.98	−0.04	−1.645	1.56
R_4	$\phi_{R_4}^+$	2.50	1.33	0.72	0.62	0.67
	$\phi_{R_4}^-$	0.86	1.16	1.59	2.61	1.73
	ϕ_{R_4}	1.64	0.17	−0.87	−1.99	−1.06

Table 8: The positive, negative and net flows of DM3

ϕ_{DM3}		IR_1	IR_2	IR_3	IR_4	A_i
R_1	$\phi_{R_1}^+$	2.50	1.36	0.84	0.79	0.99
	$\phi_{R_1}^-$	0.67	0.87	1.37	2.615	1.02
	ϕ_{R_1}	1.83	0.49	−0.53	−1.82	−0.03
R_2	$\phi_{R_2}^+$	2.50	1.66	1.32	0.96	1.95
	$\phi_{R_2}^-$	0.16	0.61	1.29	2.615	0.18
	ϕ_{R_2}	2.34	1.05	0.03	−1.65	1.77
R_3	$\phi_{R_3}^+$	2.50	1.54	1.15	0.91	1.60
	$\phi_{R_3}^-$	0.35	0.68	1.315	2.615	0.46
	ϕ_{R_3}	2.15	0.86	−0.17	−1.70	1.14
R_4	$\phi_{R_4}^+$	2.50	1.33	0.74	0.54	0.61
	$\phi_{R_4}^-$	0.83	1.14	1.58	2.615	1.67
	ϕ_{R_4}	1.67	0.19	−0.85	−2.07	−1.06

We aggregate the individual results into a collective one. The group positive, negative and net flows are presented in Table 9. The group results show that candidates A_1, A_2 and A_3 are assigned to C_1 (to be selected) and candidate A_4 is assigned to C_3 (to be rejected).

Table 9: The group of positive, negative and net flows

	ϕ_G	IR_1	IR_2	IR_3	IR_4	A_i
	$\phi_{R_1}^+$	2.50	1.48	1.06	0.86	1.41
R_1	$\phi_{R_1}^-$	0.44	0.73	1.33	2.615	0.61
	ϕ_{R_1}	2.06	0.75	−0.27	−1.75	0.8
	$\phi_{R_2}^+$	2.50	1.68	1.34	0.96	2.00
R_2	$\phi_{R_2}^-$	0.13	0.60	1.29	2.615	0.15
	ϕ_{R_2}	2.36	1.08	0.05	−1.65	1.85
	$\phi_{R_3}^+$	2.50	1.58	1.22	0.95	1.77
R_3	$\phi_{R_3}^-$	0.26	0.63	1.29	2.615	0.32
	ϕ_{R_3}	2.24	0.95	−0.08	−1.66	1.45
	$\phi_{R_4}^+$	2.50	1.34	0.75	0.63	0.72
R_4	$\phi_{R_4}^-$	0.82	1.09	1.54	2.615	1.57
	ϕ_{R_4}	1.68	0.24	−0.79	−1.98	−0.85

By calculating the personal satisfaction degrees ($\zeta_1(A_i) = 1$, $\zeta_2(A_i) = 1$ and $\zeta_3(A_i) = 0.75$), we can conclude that there is a full satisfaction for the DM1 and DM2 and a high satisfaction for DM3. After fixing the threshold of an acceptable group satisfaction level to $\boldsymbol{\Omega} = 0.9$, the group satisfaction degree ($\zeta_G = 0.95$) shows an agreement among the group of DMs.

In order to compare results, the same input data were used and applied to the FlowSort, the F-FlowSort, the PROMETHEE (Brans, Mareschal and Vincke, 1984), the TOPSIS (Chen and Hwang, 1992) and the IFS-TOPSIS (Chen, 2015) methods. As can be seen in Table 10, assignments are closely similar except for the fourth alternative, when considering the assignment based on the positive and negative flows for F-FlowSort. So, the alternative 4 can be unambiguously assigned to category 3. IFS-FlowSort can successfully correct this ambiguous assignment by using the perfect information given by the IFS values. In addition, we have found identical results when applying FlowSort. Also, some relationship can be noticed when comparing the results given by the ranking methods. In fact, the results given by PROMETHEE (Brans, Mareschal and Vincke, 1984), TOPSIS (Chen and Hwang, 1992) and IFS-TOPSIS (Chen, 2015), and by IFS-FlowSort for MCGDM are almost the same. As it can be seen in Figure 2, if we can group alternatives into three ordered categories from the best to the worst; the 1st, 2nd and 3rd alternatives are always the most preferred, so

it can be logically assigned to the first category, there is no alternative that can be middle preferred and the 4th alternative is always the worst one. However, this observation cannot be generalized, since many studies are wanted in this area.

Table 10: Comparison with other sorting methods

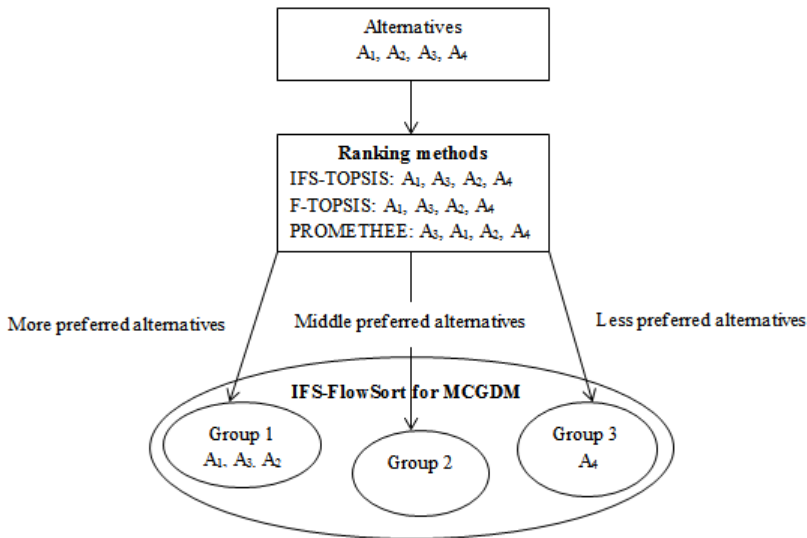| Scenarios	FlowSort			F-FlowSort			IFS-FlowSort-GDM		
	K_{ϕ^+}	K_{ϕ^-}	K_Φ	K_{ϕ^+}	K_{ϕ^-}	K_Φ	K_{ϕ^+}	K_{ϕ^-}	K_Φ
A_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_2	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_3	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1	K_1
A_4	K_3	K_3	K_3	K_3	K_2	K_2	K_3	K_3	K_3

Figure 2: Comparison with ranking methods

6 Conclusion

The ordinal classification MCDM problem is one of the most important issues in management science and operational research. It is the process of structuring and sorting decision problems into ordered predefined categories when multiple conflicting criteria are deployed. The FlowSort method succeeded in solving this issue in a great deal of research. It classified items into ordered categories using the limiting and the centroid profiles based on exact values. However, the process of decision making is often prone to uncertainty and imprecision as it implies human judgement and cognitive thinking. So, the use of crisp values becomes inefficient to solve MCDM problems. The concept of intuitionistic

fuzzy sets (IFS) achieves a great success to deal with the fuzziness of MCDM problems. For that reason, we introduced it and modeled IFS FlowSort. However, it is sometimes difficult for DMs to describe their opinions as intuitionistic fuzzy information. Thus, in this paper we presented preference ratings as linguistic terms and suggested transforming them into triangular intuitionistic fuzzy numbers. In addition, this study focused on a group decision making problem where a group of individuals collectively shares the responsibility for sorting a set of alternatives. In fact, we integrated the MCGDM problem by proposing the FlowSort method. To illustrate this extension, a practical example was presented and validated through a comparison with other MCDM methods As a result, we can conclude that our extension seems coherent in a sorting context and in the uncertainty logic. The proposed FlowSort method is simple to process and easy to use, especially for decision-makers who are familiar with PROMETHEE. As a future research perspective, we can modify the suggested method to solve MCGDM problems based on the input aggregation procedure.

References

Araz C., Ozkarahan I. (2007), *Supplier Evaluation and Management System for Strategic Sourcing Based on a New Multicriteria Sorting Procedure*, International Journal of Production Economics, 106, 585-606.

Assche D.V., De Smet Y. (2016), *FlowSort Parameters Elicitation Based on Categorization Examples*, Int. J. Multicriteria Decision Making, 6, 191-210.

Atanassov K.T. (1986), *Intuitionistic Fuzzy Sets*, Fuzzy Sets and Systems, 20, 87-96.

Banerjee S. (2012), *Arithmetic Operations on Generalized Trapezoidal Fuzzy Number and Its Applications*, Turkish Journal of Fuzzy Systems, 3, 16-44.

Brans J.P., Mareschal P., Vincke P. (1984), *PROMETHEE: A New Family of Outranking Methods in Multicriteria Analysis*, Operational Research, 3, 477-490.

Brans J.-P., Mareschal B. (2005), *Promethee Methods* [in:] J. Figueira, S. Greco, M. Ehrogott (eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, 163-186.

Campos A.C., Mareschal B., Almeida A. (2015), *Fuzzy FlowSort: An Integration of the FlowSort Method and Fuzzy Set Theory for Decision Making on the Basis of Inaccurate Quantitative Data*, Information Sciences, 293, 115-123.

Chen S.J., Hwang C.L. (1992), *Fuzzy Multiple Attribute Decision Making: Methods and Applications*, Springer-Verlag, Berlin.

Chen T.Y. (2015), *The Inclusion-based TOPSIS Method with Interval-valued Intuitionistic Fuzzy Sets for Multiple Criteria Group Decision Making*, Applied Soft Computing, 26, 57-73.

Fernandez E., Navarro J. (2011), *A New Approach to Multi-criteria Sorting Based on Fuzzy Outranking Relations: The THESEUS Method*, European Journal of Operational Research, 213, 405-413.

Figueira J., de Smet Y., Brans J.P. (2004), *MCDA Methods for Sorting and Clustering Problems: Promethee TRI and Promethee CLUSTER*, Université Libre de Bruxelles, Service de Mathématiques de la Gestion Working Paper.

Gani A., Abbas S. (2014), *A New Average Method for Solving Intuitionistic Fuzzy Transportation Problem*, International Journal of Pure and Applied Mathematics, 93, 491-499.

Gautam S.S., Singh A., Singh S.R. (2016), *TOPSIS for Multi Criteria Decision Making in Intuitionistic Fuzzy Environment*, International Journal of Computer Applications, 156(8), 0975-8887.

Govindan K., Jepsen M.B. (2016), *Supplier Risk Assessment Based on Trapezoidal Intuitionistic Fuzzy Numbers and ELECTRE TRI-C: A Case Illustration Involving Service Suppliers*, Journal of the Operational Research Society, 67, 339-376.

Hwang C.L., Yoon K. (1981), *Multiple Attributes Decision Making Methods and Applications*, Springer, Berlin, Heidelberg.

Janssen P., Nemery P. (2012), *An Extension of the FlowSort Sorting Method to Deal with Imprecision*, Quarterly Journal of Operations Research, 11, 171-193.

Li D.F., Nan J.X., Zhang M.J. (2012), *A Ranking Method of Triangular Intuitionistic Fuzzy Numbers and Application to Decision Making*, International Journal of Computational Intelligence Systems, 3, 522-530.

Liu P., Qin X. (2017), *Maclaurin Symmetric Mean Operators of Linguistic Intuitionistic Fuzzy Numbers and Their Application to Multiple-attribute Decision-Making*, Journal of Experimental & Theoretical Artificial Intelligence, 29, 1173-1202.

Lolli F., Ishizaka A., Gamberini R., Rimini B., Messori M. (2015), *FlowSort-GDSS – A Novel Group Multi-Criteria Decision Support System for Sorting Problems with Application to FMEA*, Expert Systems with Applications, 42(17-18), 6342-6349.

Nemery P., Campos A.C.S.M., Mareschal B., de Almeida A.T. (2015), *Addendum on: Fuzzy FlowSort: An Integration of the FlowSort Method and Fuzzy Set Theory for Decision Making on the Basis of Inaccurate Quantitative Data*, Information Sciences, 315, 54-55.

Nemery P., Lamboray C. (2007), *FlowSort: A Flow-based Sorting Method with Limiting or Central Profiles*, TOP (J. Span. Soc. Stat. Oper. Res.), 16, 90-113.

Park J.H., Cho H.J., Kwun Y.C. (2011), *Extension of the VIKOR Method for Group Decision Making with Interval-valued Intuitionistic Fuzzy Information*, Fuzzy Optimization and Decision Making, 10, 233-253.

Pelissari R., Oliveira M.C., Amor S.B., Abackerli A.J. (2019), *A New FlowSort-based Method to Deal with Information Imperfections in Sorting Decision-making Problems*, European Journal of Operational Research, 276(1), 235-246.

Remadi F.D., Frikha H.M. (2019), *The FlowSort for Multi Criteria Decision Making in Intuitionistic Fuzzy Environment*, Control, Decision and Information Technologies (CoDIT), Paris.

Roy B. (1985), *Méthodologie Multicritère d'aide à la décision*, Economica, Paris.

Sengupta A., Pal T.K. (2009), *On Comparing Interval Numbers: A Study on Existing Ideas*, Stud. Fuzziness Soft Comput., 238, 25-37.

Shen F., Xu J., Xu Z. (2016), *An Outranking Sorting Method for Multi-criteria Group Decision Making Using Intuitionistic Fuzzy Sets*, Information Sciences, 338-353.

Wang J.Q., Han Z.Q., Zhang H. (2012), *Multi-Criteria Group Decision-Making Method Based on Intuitionistic Interval Fuzzy Information*, Group Decision and Negotiation, 23, 715-733.

Yu W. (1992), *ELECTRE TRI. Aspects méthodologiques et guide d'utilisation*, Document du LAMSADE, Université Paris-Dauphin.

Zadeh L.A. (1965), *Fuzzy Sets*, Information and Control, 8, 338-353.

Zhang X., Jin F., Liu P. (2013), *A Grey Relational Projection Method for Multi-attribute Decision Making Based on Intuitionistic Trapezoidal Fuzzy Number*, Applied Mathematical Modelling, 37, 3467-3477.

Zopounidis C., Doumpos M. (2002), *Multicriteria Classification and Sorting Methods: A Literature Review*, European Journal of Operational Research, 138, 229-246.