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The application of selected methods of multivariate statistical analysis to study objective quality of life in Polish and Belarusian regions

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Abstract. The concept of life quality has been studied by specialists from a variety of scientific fields: economics, social geography, sociology, psychology, medicine, political sciences, and others. This contributes to the complementariness of the notion and broadens its interdisciplinary perspective, but on the other hand, it leads to a lack of unanimity in terms of the definition and measurement of the quality of life. Meanwhile, all developed countries in the world regard enhancing life quality as a priority of state policy. With the further advancement of our civilisation, quality of life will become a major issue in economic development. Therefore, monitoring this aspect of economic life, at both country and regional level, seems to be of particular significance. The paper aims to assess the suitability of selected methods of multivariate statistical analysis for the construction of a synthetic measure of objective quality of life. The study employs two methods of constructing synthetic measures of objective life quality: the linear ordering method – TOPSIS, and factor analysis. The results obtained by means of multivariate statistical analysis methods made it possible to create ratings of Polish and Belarusian regions in terms of objective quality of life and to further divide the regions into typological groups.

Keywords: objective quality of life, TOPSIS, factor analysis

JEL: C38; I31; R13

1. Introduction

As advanced integration processes are being implemented, globalisation is becoming a significant factor of economic and social change in modern society. Not only are socio-economic inequalities between the populations of different countries persisting, they are actually increasing. These inequalities, accompanied by a declining standard and quality of life (QoL), undermine economic growth, have an adverse effect on social stability, and exclude large groups of individuals from participating in the political, economic and social life of their countries. Therefore, in all developed countries in the world, improving life quality is considered a priority of state policy. It is becoming increasingly evident that with the further advancement of civilisation, quality of life is bound to be a major factor of economic development. The standard and quality of life will fully reflect the efficiency of state structures and the social policy of governments.

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Internal disparities exist among regions both in Poland and Belarus (voivodships and oblasts, respectively) as far as QoL is concerned. This is confirmed not only by statistical analyses of indicators relating to QoL (National Statistical Committee of the Republic of Belarus [Belstat], 2018; Central Statistical Office [GUS], 2017), but also by research conducted by numerous scientists (Bąk & Szczecińska, 2016; Lialikava et al., 2017; Lialikava & Kalinina, 2016; Nowak, 2018; Winiarczyk-Raźniak & Raźniak, 2011).

Polish strategic documents, directly or indirectly, refer to the category 'quality of life'. The improvement of Polish citizens' life quality is the main strategic goal of the long-term national development strategy (Ministry of Administration and Digitization, 2013, p. 42). The strategy provides for increased expenditure in the following areas: education, health, infrastructure, research and development, and culture. Also the medium-term national development strategy (Ministry of Regional Development [MRD], 2012, p. 20) is concerned with improving QoL. Its main aim is to strengthen and exploit economic, social and institutional potentials to ensure faster and sustainable growth of the economy, and the improvement of the quality of life of the population.

As regards Belarus, the national strategy for sustainable socio-economic development (Ministry of Economy of the Republic of Belarus [MINEC], 2015) confirms the urgency of the task of enhancing people's life quality. The strategy highlights the following aspects of quality of life: accessibility of high-quality education and health services, ensuring high-quality housing, a wide access to cultural goods and high standards of personal and environmental security. Monitoring the QoL in regions, analysing interregional variations, and seeking factors that could contribute to reducing socio-economic inequalities thus seem crucial here. This is acknowledged by state authorities, institutions which gather and analyse data, as well as economic researchers.

The purpose of this paper is to assess the suitability of selected methods of multivariate statistical analysis (MSA) for the construction of a synthetic measure of objective life quality, in particular by means of the TOPSIS method and factor analysis. Sixteen Polish voivodships and seven Belarusian regions are considered in the study. Due to the variety of differences between the countries, including the level of economic development, political systems, cultural backgrounds, and considering the fact that the present paper is a pilot study, the two countries were analysed separately. Year 2016 was selected as the period of interest because of the availability of statistical data.

Section 2 discusses the theoretical aspects of the 'quality of life' category. Subsequently, a description of the multivariate statistical analysis methods used in the

research is provided. Section 4 contains a presentation of the diagnostic variables used to construct synthetic measures of objective QoL. Sections 5 and 6 are devoted to a discussion of the results. Section 7 offers a comparative analysis of the outcomes of the study. The paper closes with conclusions on the findings.

2. The concept of quality of life

The term 'quality of life' was first used in 1958 by a British economist A. C. Pigou (see Pigou, 1920, p. 32), but it did not gain much popularity at the time. The first scientific approach to the problem appeared in the theories proposed by Bell (1976), Galbraith (1958), and Toffler (1980). In the 1960s, a quantitative approach to the concept in question prevailed. With time, however, a qualitative perspective became more prominent. In the 1970s, the so-called 'binary' concept of life quality was developed. The level of life quality, understood as physical, emotional, material and social well-being, began to be considered not only with regard to objective facts, but also individual, subjective notions and perceptions. The 1980s saw an increased interest in quality of life as referred to an individual. Apart from investigating the socio-economic aspects, researchers also began to analyse non-material factors, like the welfare of a person or life satisfaction (Lialikava et al., 2017).

Despite the fact that the phrase 'quality of life' has functioned in the theory of economics and in economic practice for many years, the debate on its precise definition is still going on. In the debate, one can distinguish two opposing positions (Borys, 2015, pp. 2–3):

- the belief that quality of life cannot be universally defined, because there are too many ways of interpreting the notion and too many dimensions which would have to be taken into account should a uniform definition be adopted;
- attempts at creating a universal definition of life quality in spite of the numerous difficulties resulting from a number of factors, including the complexity of the problem, its interdisciplinary character or an overlapping of the scientific and colloquial understanding of the phrase 'quality of life'.

The term 'quality of life' is used interchangeably with the following phrases: well-being, living conditions, level of living, living standards, way of life or lifestyle. The differences or similarities between these expressions have not been clearly identified, which often leads to theoretical and practical contradictions (Borys, 2015, p. 2).

Table 1 presents selected approaches to defining QoL.

Table 1. Selected approaches to defining quality of life

Author	Definition
T. Słaby	All aspects of human life associated with the existence of a person, being someone, and experiencing various emotional states caused by, e.g. having a family, colleagues, friends (Słaby, 1990, p. 8).
T. Borys	The image of life perceived on the basis of a specific system of values (axiological system). This image (as a collective attribute of an individual or a group) can be described in a subjective or objective manner, from a one-dimensional or a multidimensional perspective etc., depending on the tools used. The tools applied to describing quality of life create its different typologies (Borys, 2015, p. 4).
E. Skrzypek	A combination of objective conditions: economic circumstances, leisure time, housing conditions, natural environment, health, social environment, and subjective conditions, which are perceived in a unique way by every individual and are reflected in their well-being (Skrzypek, 2001, p. 8).
R. Kolman	The degree to which the spiritual and material needs of individuals and society as a whole are satisfied, the degree to which the expectations of contractual normality in everyday activities of individuals and the society are met (Kolman, 2000, p. 2).
WHO	An individual's perception of their position in life in the context of culture and systems of values in which they live and in relation to their goals, expectations, standards and concerns.[1]

Source: authors' work.

The empirical research presented in the paper concerned objective QoL and included such areas as the quality of the population, material living conditions, social sphere, environment, and cultural sphere.

3. Research methodology

TOPSIS is a linear ordering method. It involves calculating the distance of each multi-attribute object from the pattern and anti-pattern, followed by a linear ordering of the objects. In taxonomic studies, the first linear ordering method using a pattern was presented by a Polish statistician Zdzisław Hellwig in 1968 (Hellwig, 1968). Hellwig's article initiated intensive research in this field, carried on by other Polish scientists, including Bartosiewicz (1976), Borys (1978), Cieślak (1974), Pluta (1976), Strahl (1978) and Walesiak (1993). In terms of the decision theory, the first linear ordering method with a pattern and anti-pattern was proposed by C.L. Hwang and K. Yoon in 1981, and was named TOPSIS (Hwang & Yoon, 1981).

The study's objective was achieved in the following 6 stages:

Stage 1. Selection of diagnostic variables on the basis of substantive and statistical factors. The diagnostic variables which were initially chosen for analysis (see

[1] See: WHOQOL: Measuring Quality of Life, https://www.who.int/healthinfo/survey/whoqol-qualityoflife/en/ (access: 12.08.2020)

Table 2.) were universally acknowledged, substantively valuable, measurable and confirmed by accessible statistics. In statistical terms, the level of variation was examined (a 10% value of the classical coefficient of variation was assumed as critical), as was the level of correlation (in order to eliminate excessively correlated variables, the inverse correlation matrix by Malina and Zeliaś (1997) was applied).

Stage 2. Division of diagnostic variables into stimulants (a higher value of such a variable means a higher level of the studied phenomenon) and destimulants (a higher value of such a variable means a lower level of the studied phenomenon).[2]

Stage 3. Normalisation of the values of diagnostic variables. The zero unitarisation procedure was adopted,[3] as represented by the equations below (Kukuła, 2000):

- for stimulants

$$z_{ik} = \frac{x_{ik} - \min_i\{x_{ik}\}}{\max_i\{x_{ik}\} - \min_i\{x_{ik}\}}, \tag{1}$$

- for destimulants

$$z_{ik} = \frac{\max_i\{x_{ik}\} - x_{ik}}{\max_i\{x_{ik}\} - \min_i\{x_{ik}\}}, \tag{2}$$

where
i – number of region ($i = 1, 2, \ldots, n$),
k – number of diagnostic variable ($k = 1, 2, \ldots, m$).

Stage 4. Calculation of the Euclidean distance of each region from the pattern $z_k^+ = [1,\ 1, \ldots,\ 1]$ and from the anti-pattern $z_k^- = [0,\ 0, \ldots,\ 0]$, according to the following equations:

- distance from the pattern

$$d_i^+ = \sqrt{\sum_{k=1}^{m} (z_{ik} - z_k^+)^2}, \tag{3}$$

- distance from the anti-pattern

[2] The concepts of stimulants and destimulants were introduced into the literature by Hellwig (1968).
[3] In Hwang and Yoon's original work, the quotient transformation was used to normalise the variables (Hwang & Yoon, 1981, pp. 131–132).

$$d_i^+ = \sqrt{\sum_{k=1}^{m} (z_{ik} - z_k^-)^2}\,, \tag{4}$$

where $(i = 1, 2, \ldots, n)$.

Stage 5. Calculation of the value of the synthetic measure for each region, by means of the following formula (see Hwang & Yoon, 1981, p. 132):

$$q_i = \frac{d_i^-}{d_i^- + d_i^+}. \tag{5}$$

The values of the synthetic measure fall within the range [0,1]. The measure takes the value of 1 for the pattern and 0 for the anti-pattern. The closer the value of the measure to 1, the less the given region diverges from the pattern.

Stage 6. Ordering of the studied regions and their division into typological groups.

The process of ordering was conducted on the basis of the value of the synthetic measure calculated in the previous stage. The boundaries of the intervals were established through arithmetic means and standard deviation of the synthetic measure:

- group I (very high and high objective QoL): $q_i \geq \bar{q} + s_q$,
- group II (medium-higher objective QoL): $\bar{q} \leq q_i < \bar{q} + s_q$,
- group III (medium-lower objective QoL): $\bar{q} - s_q \leq q_i < \bar{q}$,
- group IV (low and very low objective QoL): $q_i < \bar{q} - s_q$.

Factor analysis is a collection of techniques and procedures used to reduce a large number of studied variables to a far smaller group of mutually independent factors or principal components. It consists of the classical factor analysis and the principal component analysis. The former, whose main ideas were developed by Spearman (1904) and Thurstone (1931), is primarily used to investigate the internal relationships between variables. The latter, whose theoretical foundations were devised by Hotelling (1933) and Pearson (1901), is applied for analysing interdependencies within sets of variables or for studying the structures of sets of observations.

In the case of factor analysis, the following algorithm was used:

Stage 1. Selection of diagnostic variables on the basis of substantive and statistical reasons. Here, the applied substantive criteria were analogous to those used in the TOPSIS method and therefore, the set of diagnostic variables was also analogous (see Table 2.). The level of correlation of diagnostic variables was studied in statistical terms – the use of factor analysis is only justified if at least some of the variables are correlated.

Stage 2. Division of diagnostic variables into stimulants and destimulants.

Stage 3. Normalisation of the values of diagnostic variables – the zero unitarisation procedure was applied, according to Equations (1) and (2).

Stage 4. Estimation of the factor analysis model using the principal components method (Härdle & Simar, 2015, pp. 367–375; Timm, 2002, pp. 502–506).

Stage 5. Establishing the number of principal factors.

In the paper, the Kaiser criterion was adopted, which involves the elimination of those principal factors whose singular values are lower than 1 (Jolliffe, 2002, pp. 114–115).

Stage 6. Calculation of the value of the synthetic measure for each region, by means of the equation below:

$$R = \left(\sum_{i=1}^{m} \lambda_i\right)^{-1} (\lambda_1 F_1 + \cdots + \lambda_m F_m) \times 100, \tag{6}$$

where

F_i – values of the first m factors

λ_i – singular values of the covariance matrix.

Stage 7. Ordering of the studied regions and their division into typological groups on the basis of the values of synthetic measure. The division into groups was conducted through k-means clustering (Härdle & Simar, 2015, pp. 385–406; Timm, 2002, pp. 522–523).

4. Diagnostic variables

The lack of a single, widely accepted definition of QoL results in the fact that there is no unambiguous method of measuring this category. International organisations, e.g. the European Union, the United Nations, the World Bank, the OECD, as well as individual countries, including Poland[4] and Belarus,[5] have been involved in developing criteria for assessing the standard and quality of people's lives. Scientists and practitioners make attempts at constructing synthetic measures of quality of life. The

[4] Research into QoL in Poland is conducted by Statistics Poland, whose published reports (every two years) contain indicators on the following areas of QoL: material situation, work, health, education, free time and social relations, personal safety, state quality and basic rights, quality of the natural environment in the place of residence as well as the subjective well-being. In addition, the Social Monitoring Board has been conducting research within the framework of the project 'Social Diagnosis' since 2000. Data relating to households and attitudes, state of mind and behaviours of their members are obtained.

[5] Research into QoL in Belarus is conducted by the National Statistical Committee of the Republic of Belarus. The published reports contain socio-economic indicators measuring the quality and standard of living of the inhabitants of Belarusian cities and regions.

most widely known ones include: the Human Development Index, the Physical Quality of Life Index, Gross National Happiness, the Happy Planet Index, The Economist Intelligence Unit's Quality-of-Life Index, the Gallup-Healthways Global Well-being Index, and the Legatum Prosperity Index. Also this paper undertakes to develop synthetic measures of quality of life on a regional level.

Table 2 presents a set of diagnostic values used in the study. The set is the result of a compromise between knowledge and experience in measuring objective QoL and the accessibility of comparable data for the two groups of regions investigated by the authors. The variables were classified according to five categories: quality of the population, material living conditions, social sphere, environment and cultural sphere. The statistical data were obtained from the official databases of statistical offices in Poland and Belarus. The study covers the year 2016.

Table 2. Diagnostic variables of objective QoL

Symbol	Diagnostic variable	Type of variable[a]
	Quality of population	
*X*1	Net migration rate	S
*X*2	Birth rate per 1,000 population	S
*X*3	Life expectancy	S
*X*4	Infant mortality rate	D
*X*5	Pre-working age population per 1,000 persons of working age	S
*X*6	Post-working age population per 1,000 persons of working age	D
*X*7	Percentage of population with tertiary education employed in the economy (Polish regions) Percentage of population with tertiary education employed in organisations (Belarusian regions)	S
*X*8	Number of deaths per 1,000 population	D
*X*9	Number of marriages per 1,000 population	S
*X*10	Number of divorces per 1,000 population	S
	Material living conditions	
*X*11	Average monthly salary	S
*X*12	Average usable floor area of residential premises per person	S
*X*13	Number of passenger cars per 1,000 population	S
*X*14	GDP per capita	S
*X*15	Retail sale of goods per person	S
*X*16	Percentage of households below poverty line	D
	Social sphere	
*X*17	Unemployment rate	D
*X*18	Employment rate	S
*X*19	Number of doctors per 10,000 population	S
*X*20	Number of nurses and midwives per 10,000 population	S
*X*21	Number of injured in accidents at work per 1,000 employed	D
*X*22	Number of offences per 1,000 population	D

a S – stimulant, D – destimulant.

Table 2. Diagnostic variables of objective QoL (cont.)

Symbol	Diagnostic variable	Type of variable[a]
	Environment	
*X*23	Emission of particulate pollutants by plants of significant impact on air quality	D
*X*24	Industrial and municipal waste water requiring treatment discharged into waters or into the ground	D
	Cultural sphere	
*X*25	Audience in theatres and music institutions per 1,000 population	S
*X*26	Number of museum admissions per 10,000 population	S

a S – stimulant, D – destimulant.
Source: authors' work.

5. Results obtained by means of TOPSIS

The diagnostic variables from Table 2 were verified statistically in order to eliminate data which were insufficiently varied or excessively correlated. Table 3 contains diagnostic variables used to construct the synthetic measures of objective QoL in the Polish and Belarusian regions. Each of the specified areas of objective QoL was represented by at least one variable.

Table 3. Diagnostic variables used for constructing synthetic measures of objective QoL

Polish regions	Belarusian regions
Quality of population	
*X*1, *X*4, *X*10	*X*1, *X*4, *X*7
Material living conditions	
*X*14, *X*16	*X*14
Social sphere	
*X*17, *X*21, *X*22	*X*17, *X*19, *X*21
Environment	
*X*23	*X*23
Cultural sphere	
*X*25	*X*25, *X*26

Source: authors' work.

The values of the variables were normalised according to Equations (1) and (2). Next, the values of the synthetic measures of objective QoL were calculated and, on this basis, linear ordering of the regions was performed, followed by their division into typological groups. The division into groups was conducted with the help of the

mean and standard deviation of the synthetic measures. The results are presented in Tables 4 and 5.

The variation of the value of the synthetic measure was approximately 36%. In the case of seven voivodships, the value of the synthetic measure was higher than the average (i.e. 0.49).

It was found that in 2016, Mazowieckie offered the best objective QoL in Poland, whereas in Warmińsko-Mazurskie, the QoL was the poorest of all the voivodships. In Mazowieckie Voivodship, five out of ten diagnostic variables reached the best values (these were $X1$, $X4$, $X14$, $X16$, $X21$), while the majority of diagnostic variables in Warmińsko-Mazurskie assumed the lowest values. The only exception was variable $X23$, in which Warmińsko-Mazurskie ranked the highest of all the voivodships. Mazowieckie, Małopolskie and Pomorskie were those three voivodships in which QoL was very high or high. The objective QoL of inhabitants of Dolnośląskie, Łódzkie, Podlaskie, and Wielkopolskie was medium-high. Eight voivodships: Podkarpackie, Lubelskie, Opolskie, Świętokrzyskie, Śląskie, Lubuskie, and Kujawsko-Pomorskie offered medium-low objective QoL. Only one voivodship, Warmińsko-Mazurskie, offered low or very low QoL.

Table 4. Ordering and classification of Polish voivodships in terms of objective QoL in 2016 – the TOPSIS method

Voivodship	Value of synthetic measure	Rating position	Group
Mazowieckie	0.904	1	1
Małopolskie	0.751	2	1
Pomorskie	0.689	3	1
Dolnośląskie	0.606	4	2
Łódzkie	0.584	5	2
Podlaskie	0.509	6	2
Wielkopolskie	0.491	7	2
Podkarpackie	0.474	8	3
Lubelskie	0.429	9	3
Opolskie	0.416	10	3
Świętokrzyskie	0.411	11	3
Śląskie	0.359	12	3
Lubuskie	0.358	13	3
Kujawsko-Pomorskie	0.349	14	3
Zachodniopomorskie	0.347	15	3
Warmińsko-Mazurskie	0.178	16	4

Source: authors' calculation.

The City of Minsk clearly stood out from the rest of the Belarusian regions. It was at the top of the rating and was the only region with very high or high objective QoL. For the City of Minsk, eight out of ten diagnostic variables took the highest values (except for $X4$ and $X23$). In the other six regions, the objective QoL was medium-low,

which shows the vastness of the gap between the capital city and the rest of the country. The variation in the value of the synthetic measure, assessed by means of the classical coefficient of variation, reached almost 56%, whereas after the exclusion of the City of Minsk region, it was only 20%.

Table 5. Ordering and classification of Belarusian regions in terms of objective QoL in 2016 – the TOPSIS method

Region	Value of synthetic measure	Rating position	Group
City of Minsk	0.8862	1	1
Minsk Region	0.3687	2	3
Grodno Region	0.3603	3	3
Gomel Region	0.3214	4	3
Vitebsk Region	0.2961	5	3
Brest Region	0.2406	6	3
Mogilev Region	0.1947	7	3

Source: authors' calculation.

6. Results obtained by means of factor analysis

The factor analysis procedure began with the analysis of the correlation matrix of the diagnostic variables in order to find whether at least some of the variables are correlated. After assessing that the condition was fulfilled, the variables were normalised according to Equations (1) and (2). Next, estimation of the model was performed and the principal factors were identified. Table 6 presents the obtained results.

In the case of Polish voivodships, the first principal factor explains 34% of the total variation, the second – approximately 20%, and the third – 15.5%. Therefore, the first three principal factors explain together 70.3% of the total variation. As regards the Belarusian regions, the first principal factor explains about 60% of the total variation, whereas the second one – approximately 20%, which accounts for nearly 80% of the total variation.

Table 6. Results of the factor analysis

Factor	Polish regions		Belarusian regions	
	percentage of total variation	cumulative percentage of total variation	percentage of total variation	cumulative percentage of total variation
F_1	34.21	34.21	59.39	59.39
F_2	20.56	54.77	19.69	79.08
F_3	15.54	70.32	–	–

Source: authors' calculation.

The synthetic measure for assessing objective QoL in the Polish and Belarusian regions was constructed on the basis of the following formulae:

$$R_{PL} = 34.21F_1 + 20.56F_2 + 15.54F_3, \tag{7}$$

$$R_{BLR} = 59.39F_1 + 19.96F_2, \tag{8}$$

where F_1, F_2, F_3 are the estimated values of the first three principal factors, while the accompanying coefficients represent the percentages of the total variation given in Table 6.

Table 7 provides the values of the synthetic measure and the rating and division of Polish voivodships into typological groups. The division into typological groups was conducted according to the k-means clustering method.

Table 7. Ordering and classification of Polish voivodships in terms of objective QoL in 2016 – results of factor and cluster analysis

Voivodship	Rating position	Value of synthetic measure	Group
Mazowieckie	1	98.15	1
Małopolskie	2	72.01	1
Pomorskie	3	52.58	1
Wielkopolskie	4	33.00	1
Dolnośląskie	5	7.33	2
Podkarpackie	6	3.22	2
Podlaskie	7	–2.93	2
Lubelskie	8	–15.98	2
Śląskie	9	–18.39	2
Lubuskie	10	–19.61	2
Kujawsko-Pomorskie	11	–22.37	2
Zachodniopomorskie	12	–29.66	3
Opolskie	13	–32.24	3
Łódzkie	14	–34.54	3
Świętokrzyskie	15	–38.77	3
Warmińsko-Mazurskie	16	–51.80	3

Source: authors' calculation.

Mazowieckie, Małopolskie, Pomorskie and Wielkopolskie were the leaders of the rating. They formed the first group of voivodships, characterised by very high or high objective QoL. The second group consisted of seven voivodships: Dolnośląskie,

Podkarpackie, Podlaskie, Lubelskie, Śląskie, Lubuskie and Kujawsko-Pomorskie. The remaining five regions were classified in the third, and last, typological group.

The results of the ordering and classification of the Belarusian regions are shown in Table 8.

Table 8. Ordering and classification of Belarusian regions in terms of objective QoL in 2016 – results of factor and cluster analysis

Region	Rating position	Value of synthetic measure	Group
City of Minsk	1	131.07	1
Minsk Region	2	25.95	2
Grodno Region	3	–11.56	3
Brest Region	4	–24.05	3
Mogilev Region	5	–38.81	3
Gomel Region	6	–39.22	3
Vitebsk Region	7	–43.37	3

Source: authors' calculation.

The City of Minsk was the leader of the Belarusian regions and alone formed the first typological group. The other regions were classified in the second (Minsk Region) or the third group (Grodno Region, Brest Region, Mogilev Region, Gomel Region and Vitebsk Region).

7. Comparison of research results

Tables 9 and 10 provide the results of the ordering of the Polish and Belarusian regions in terms of objective QoL, done by means of the multivariate statistical analysis methods.

The ratings of Polish regions are characterised by high correlation, as reflected by the values of Spearman's rank correlation coefficients, at 0.77. In both ratings, the first, second, third and sixteenth places are occupied by the same voivodships (Mazowieckie, Małopolskie, Pomorskie, and Warmińsko-Mazurskie, respectively). The largest difference between the two ratings was obeserved for Łódzkie Voivodship. The low rank of the region in the rating constructed through factor analysis may result from the fact that Łódzkie was rated near the bottom in terms of four diagnostic variables which were strongly correlated with the estimated values of the first principal factor (high values of factor loadings).

Table 9. Ordering of Polish voivodships in terms of objective QoL – comparison of results obtained by means of applied MSA methods

Voivodship	TOPSIS	Factor analysis	Difference in rating
Dolnośląskie	4	5	1
Kujawsko-Pomorskie	14	11	3
Lubelskie	9	8	1
Lubuskie	13	10	3
Łódzkie	5	14	9
Małopolskie	2	2	0
Mazowieckie	1	1	0
Opolskie	10	13	3
Podkarpackie	8	6	2
Podlaskie	6	7	1
Pomorskie	3	3	0
Śląskie	12	9	3
Świętokrzyskie	11	15	4
Warmińsko-Mazurskie	16	16	0
Wielkopolskie	7	4	3
Zachodniopomorskie	15	12	3

Source: authors' calculation.

Table 10. Ordering of Belarusian regions in terms of objective QoL – comparison of results obtained by means of the applied MSA methods

Region	TOPSIS	Factor analysis	Difference in rating
City of Minsk	1	1	0
Brest Region	6	4	2
Grodno Region	3	3	0
Gomel Region	4	6	2
Minsk Region	2	2	0
Mogilev Region	7	5	2
Vitebsk Region	5	7	2

Source: authors' calculation.

The level of correlation among the ratings of the Belarusian regions is similar to that among the Polish voivodships. The value of the Spearman's rank amounts to 0.71. In both ratings of the Belarusian regions, the first three places are the same: the City of Minsk, Minsk Region and Grodno Region. The rankings of the other regions vary.

Tables 11 and 12 represent the division of the Polish and Belarusian regions into typological groups created through the use of k-means clustering and the method based on the mean and standard deviation of the synthetic measure determined by TOPSIS.

Table 11. Division of Polish voivodships into typological groups in terms of objective QoL in 2016 – comparison of results obtained by means of the applied MSA methods

TOPSIS / Cluster analysis	Group I	Group II	Group III	Group IV
Group I	Mazowieckie, Małopolskie, Pomorskie	Wielkopolskie		
Group II		Dolnośląskie, Podlaskie	Podkarpackie, Lubelskie, Śląskie, Lubuskie, Kujawsko-Pomorskie	
Group III		Łódzkie	Opolskie, Świętokrzyskie, Zachodniopomorskie	Warmińsko-Mazurskie

Source: authors' calculation.

The cluster analysis revealed three clusters of Polish voivodships and three clusters of Belarusian regions. The classification based on the results of the TOPSIS procedure led to the division of Polish voivodships into four typological groups, whereas in the case of the Belarusian regions, it was two groups. While the results concerning the Belarusian regions are similar, the groupings of the Polish voivodships manifest considerable differences.

Table 12. Division of Belarusian regions into typological groups in terms of objective QoL in 2016 – comparison of results obtained by means of the applied MSA methods

TOPSIS / Cluster analysis	Group I	Group II	Group III	Group IV
Group I	City of Minsk			
Group II			Minsk Region	
Group III			Grodno Region, Gomel Region, Vitebsk Region, Brest Region, Mogilev Region	

Source: authors' calculation.

Research into quality of life at regional level has been conducted by various specialists (e.g. Bąk & Szczecińska, 2016; Lialikava et al., 2017; Lialikava & Kalinina, 2016; Nowak, 2018; Winiarczyk-Raźniak & Raźniak, 2011). However, it is difficult, and in many cases even impossible, to compare the results of these studies, since they pertain to different periods or examine different research objects.

8. Conclusions

The paper presents the results of research into objective QoL in Polish and Belarusian regions, obtained through the application of selected methods of multivariate statistical analysis. Two very different methods – TOPSIS and factor analysis – were used in order to construct synthetic measures of objective QoL and to create ratings and typological groups of the studied regions. The methods differed already at the stage of selecting diagnostic variables. In TOPSIS, the set of diagnostic variables had to be narrowed down by eliminating too strongly correlated variables, whereas in the factor analysis procedure, quite the opposite – the variables had to be correlated. The TOPSIS method involved studying the distance of the regions from the pattern and anti-pattern, and the obtained values of the synthetic measure fell within the range [0, 1]. Factor analysis involved searching for hidden factors in a set of diagnostic variables, and the obtained values of the synthetic measure were infinite. Due to the above-mentioned differences between the applied methods, discrepancies occurred in the ratings and typological groups. The choice of the method affects the outcome of the study, so it should be determined by the purpose of the research and by what is expected of the selected method, as well as on the statistical properties of the analysed set of diagnostic variables.

Evaluating QoL at the regional level is a particularly significant task in the context of socio-economic analyses. The results of the present study can serve as a tool for planning or monitoring the utilisation of financial resources granted to local government units. They can also be used to assess the efficiency of the already implemented socio-economic policies.

Appendix

Table 1A. Basic descriptive statistics of diagnostic variables in the 'quality of population' area – Polish regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*1 (S)	–2.07 Warmińsko-Mazurskie	2.41 Mazowieckie	–0.38	326.78
*X*2 (S)	–2.98 Łódzkie	2.04 Pomorskie	–0.40	355.61
*X*3 (S)	76.58 Łódzkie	79.26 Podkarpackie	77.95	0.91
*X*4 (D)	3.27 Mazowieckie	5.86 Lubuskie	4.18	16.60
*X*5 (S)	25.21 Opolskie	31.59 Pomorskie	28.66	5.82
*X*6 (D)	29.20 Warmińsko-Mazurskie	37.00 Łódzkie	32.48	6.36
*X*7 (S)	26.70 Łódzkie	42.91 Mazowieckie	32.15	11.48
*X*8 (D)	8.98 Podkarpackie	12.14 Łódzkie	10.09	7.85
*X*9 (S)	4.65 Warmińsko-Mazurskie	5.36 Pomorskie	5.01	4.57
*X*10 (S)	1.20 Podkarpackie	1.90 Warmińsko-Mazurskie, Lubuskie	1.62	12.20

Source: authors' calculation.

Table 2A. Basic descriptive statistics of diagnostic variables in the 'material living conditions' area – Polish regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*11 (S)	3619.16 Warmińsko-Mazurskie	5240.86 Mazowieckie	3993.79	9.94
*X*12 (S)	24.2 Warmińsko-Mazurskie	29.9 Mazowieckie	27.11	5.58
*X*13 (S)	485.19 Podlaskie	626.59 Wielkopolskie	555.82	7.30
*X*14 (S)	33371 Lubelskie	77359 Mazowieckie	43765.31	24.81
*X*15 (S)	8597 Opolskie	40383 Mazowieckie	16295.69	56.77
*X*16 (D)	8.5 Mazowieckie	21.3 Podkarpackie	13.32	31.34

Source: authors' calculation.

Table 3A. Basic descriptive statistics of diagnostic variables in the 'social sphere' area – Polish regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*17 (D)	4.90 Wielkopolskie	14.20 Warmińsko-Mazurskie	9.09	26.36
*X*18 (S)	49.00 Warmińsko-Mazurskie	56.60 Mazowieckie	52.19	3.98
*X*19 (S)	35.97 Wielkopolskie	71.32 Mazowieckie	52.59	18.87
*X*20 (S)	50.32 Wielkopolskie	76.45 Śląskie	66.95	11.71
*X*21 (D)	4.84 Mazowieckie	9.17 Warmińsko-Mazurskie	7.38	16.50
*X*22 (D)	11.02 Podkarpackie	25.73 Dolnośląskie	18.87	19.21

Source: authors' calculation.

Table 4A. Basic descriptive statistics of diagnostic variables in the 'environment' and 'cultural sphere' areas – Polish regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*23 (D)	0.03 Warmińsko-Mazurskie	0.74 Śląskie	0.14	112.47
*X*24 (D)	2.00 Podlaskie	30.10 Śląskie	7.60	89.22
*X*25 (S)	100 Podkarpackie	716 Dolnośląskie	332.50	54.76
*X*26 (S)	2412 Opolskie	29363 Małopolskie	7674.49	90.86

Source: authors' calculation.

Table 5A. Basic descriptive statistics of diagnostic variables in the 'quality of population' area – Belarusian regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*1 (S)	–1.75 Grodno Region	4.96 City of Minsk	0.37	793.15
*X*2 (S)	–3.52 Vitebsk Region	2.65 City of Minsk	–0.47	–381.11
*X*3 (S)	73.1 Minsk Region	76.5 City of Minsk	74.00	1.50
*X*4 (D)	2.8 Vitebsk Region, Gomel Region	3.8 Minsk Region	3.14	10.46
*X*5 (S)	267 City of Minsk	344 Brest Region	310.43	8.15
*X*6 (D)	369 City of Minsk	486 Vitebsk Region	450.29	7.90
*X*7 (S)	21.9 Minsk Region	40.7 City of Minsk	26.09	23.07
*X*8 (D)	8.7 CIty of Minsk	14.6 Vitebsk Region	12.93	14.15
*X*9 (S)	6.3 Vitebsk Region, Gomel Region	7.7 City of Minsk	6.69	6.61
*X*10 (S)	3.0 Grodno Region, Brest Region	3.8 City of Minsk	3.40	8.02

Source: authors' calculation.

Table 6A. Basic descriptive statistics of diagnostic variables in the 'material living conditions' area – Belarusian regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*11 (S)	241.2 Gomel Region	445.0 City of Minsk	285.51	23.49
*X*12 (S)	22.5 City of Minsk	29.7 Minsk Region	27.14	8.24
*X*13 (S)	267 Gomel Region	352 Grodno Region	310.14	8.74
*X*14 (S)	6295.1 Viciebsk Region	12960.0 City of Minsk	8001.43	28.18
*X*15 (S)	3251.7 Mogilev Region	11285.4 City of Minsk	5274.77	48.13
*X*16 (D)	1.4 City of Minsk	8.1 Brest Region	5.81	39.07

Source: authors' calculation.

Table 7A. Basic descriptive statistics of diagnostic variables in the 'social sphere' area – Belarusian regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*17 (D)	0.5 City of Minsk	1.0 Vitebsk Region, Gomel Region	0.84	19.91
*X*18 (S)	63.1 Gomel Region	71.5 City of Minsk	66.31	4.28
*X*19 (S)	32.8 Minsk Region	58.7 City of Minsk	42.76	20.07
*X*20 (S)	120.6 Minsk Region	137.2 Grodno Region	132.69	3.97
*X*21 (D)	0.30 City of Minsk	0.52 Grodno Region	0.44	16.49
*X*22 (D)	826 Brest Region	1203 Minsk region	974.57	11.68

Source: authors' calculation.

Table 8A. Basic descriptive statistics of diagnostic variables in the 'environment' and 'cultural sphere' areas – Belarusian regions in 2016

Symbol	Min	Max	Mean	Coefficient of variation
*X*23 (D)	1.45 Mogilev Region	60.33 City of Minsk	10.38	196.52
*X*24 (D)	0.09 City of Minsk	0.22 Minsk Region	0.14	29.26
*X*25 (S)	413.8 Minsk Region	3976.7 City of Minsk	1135.89	106.48
*X*26 (S)	434 Mogilev Region	827 City of Minsk	655.14	20.12

Source: authors' calculation.

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