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THE USE OF APPLIED TASKS IN THE PROCESS OF MATHEMATICAL TRAINING OF FUTURE ENGINEERS

WYKORZYSTANIE PRZYKŁADOWYCH ZADAŃ W PROCESIE EDUKACJI MATEMATYCZNEJ PRZYSZŁYCH INŻYNIERÓW

ИСПОЛЬЗОВАНИЕ ПРИКЛАДНЫХ ЗАДАЧ В ПРОЦЕССЕ МАТЕМАТИЧЕСКОЙ ПОДГОТОВКИ БУДУЩИХ СТРОИТЕЛЕЙ

Abstracts

Examples of the use of applied problems of professional content in higher mathematics course for students of engineering.have been ordered.

Keywords: *applications, professional orientation, mathematical education, student builders.*

Streszczenie

W artykule opisano znaczenie zastosowań przykładowych problemów zaawansowanej matematyki i zadań w procesie edukacji matematycznej na poziomie studiów wyższych kształcenia przyszłych inżynierów.

Słowa kluczowe*: przykładowe zadania, specjalności zawodowe, edukacja matematyczna, studenci kierunków technicznych.*

Аннотация

Приведены примеры использования прикладных задач профессионального содержания в курсе высшей математики для студентов строительных специальностей.

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Ключевые слова: *прикладная задача, профессиональная направленность, математическая подготовка, студенты-строители.*

Relevance of the research topic. The main task of the national education is the preparation of highly qualified specialists capable of creatively approaching the tasks arising in the professional activity, as well as those capable of education and self-education throughout life. A great potential for the development of these qualities among specialists of various technical specialties, including future builders, has a course in higher mathematics.

First, the professional thinking of future engineers-builders is formed on the basis of mathematical thinking. This is due to the fact that in the process of studying the subject all methods of human mental activity are used: analysis, synthesis, generalization, abstraction, comparison, classification, etc. Therefore, the course of higher mathematics is rightfully leading in the matter of developing the intellect of the individual.

Secondly, mathematical methods are an important means of integrating students' knowledge. As it is known, the abstract nature of mathematical methods makes them universal. Being a "language" of other sciences, mathematics actively penetrates into various fields of knowledge, from technical to humanitarian ones. It is through mathematical methods that students can disclose interdisciplinary connections, and thereby form profound, systematic knowledge.

Third, mathematics is the foundation on which special training of future specialists is being built in higher education. Therefore, it is important to show students how and where mathematics are used in other disciplines on many examples [2].

Dedicated priorities of mathematical preparation can be introduced into the educational process by introducing the application of higher professional skills to higher mathematics.

Analysis of known researches. The scientific basis of the problem of the development of psychological, pedagogical and scientific-methodological provisions for the introduction of applied aspects into the course of higher mathematics consists in the works of such well-known scientists: B.V. Gnedenko, T.V. Krylova, L.D. Kudryavtseva, O. Mordkovich, A.D. Myshkis, B.A. Solonouc, V. Skatecki and others.

The basic principles of selection of applied problems, as well as the requirements for them, are determined in the works of the researchers: G.Ya. Dutka, L.A.Sokolenko, I.M.Shapiro, L.I.Novitskaya.

Separate questions of this problem are also covered in dissertational studies: L.P. Gusak, N.M. Samaruk, A.P. Tomashchuk.

However, despite the above-mentioned studies, further study requires the problem of developing professional tasks for professional content for students of construction specialties.

Main part. We will give examples of tasks for calculating the pressure of a liquid on a vertical plate, and also different ways of solving them.

Let's give a brief theoretical information.

The basic equation of hydrostatics for an incompressible liquid in the state of thermal equilibrium has the form [3]:

$$p = p_0 + (x_0 - x)\rho g, \quad (1)$$

where p, p_0 is the liquid pressure at points M and M_0 with the coordinates x and x_0

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properly; ρ – liquid density; g – acceleration of terrestrial gravity (sometimes instead of these characteristics the specific gravity of the liquid is used $\gamma = \rho g$).

As a rule, the reference point x_0 is co-located with the free surface of the liquid and then p_0 is the external pressure on the free surface, and equation (1) takes the form:

$$p = p_0 + \rho \cdot g \cdot h, \quad (1a)$$

where h is the depth of immersion point M under the free surface level.

Excess liquid pressure $p_{изб}$ is defined as the difference between total pressure and ambient pressure:

$$p_{изб} = p - p_0 = \rho g h, \quad (2)$$

The pressure P of the liquid on a flat horizontal figure with area S, immersed to the depth h, is determined by the formula:

$$P = \rho g h \cdot S. \quad (3)$$

The total pressure force P_n taking into account the pressure of the environment p_0 is come from the following formula:

$$P_n = p_0 \cdot S + \rho g h \cdot S. \quad (4)$$

The determination of the pressure force on a flat vertical figure immersed in a liquid is the problem of integral calculus.

The total force of pressure on the plane figure, which occupies the domain D, has area S and is immersed vertically in the liquid (Figure 1), is determined by means of a double integral:

$$P_n = \iint\limits_D p ds = \iint\limits_D (p_0 + \rho g x) dS = p_0 \iint\limits_D ds + \\ + \rho g \iint\limits_D x ds = p_0 \cdot S + \rho g \iint\limits_D x ds. \quad (5)$$

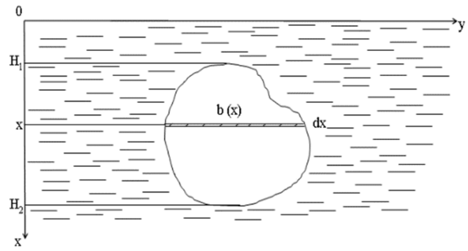

Fig.1. A flat figure immersed vertically in a liquid.

In many problems, the component $p_0 \cdot S$ does not need to take into account and the strength of the liquid pressure is determined by the following formula:

$$P = \rho g \iint\limits_D x ds. \quad (6)$$

If the variable width of the flat figure $b=b(x)$ is known (Figure 1), then the liquid pressure force can be determined with the help of a definite integral by the following formula:

$$P = \rho g \int_{H_1}^{H_2} x b(x) dx. \quad (7)$$

It should be noted that formula (6) can be written differently:

$$P = \rho g \iint\limits_D x ds = \rho g S_y, \quad (8)$$

where S_y is a static moment of the figure relative to the axis OY:

$$S_y = \iint\limits_D x ds. \quad (9)$$

As it is known

$$S_y = x_c \cdot S, \quad (10)$$

where x_c – abscissa of the center of the figure gravity; S – its area.

It is necessary to distinguish the center of the figure gravity from the center of pressure. So, the force of the liquid pressure can be determined by the following formula:

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$$P = \rho g x_c \cdot S. \tag{11}$$

If we denote the pressure in the center of the figure gravity through

p_c ($p_c = \rho g x_c$),

then from formula (11) we obtain a formula for determining the pressure force of a liquid:

$$P = p_c \cdot S, \tag{12}$$

which is used in hydraulic calculations [3]. But from a mathematical point of view, the use of formulas (8) and (11) is more expedient, as in them:

$$S_y = x_c \cdot S$$

is a characteristic of only the geometric properties of the figure and does not depend on the physical properties of the liquid, unlike p_c, which depends not only on the geometric properties of the figure, but also on the physical properties of the liquid.

If the abscissa of the center of the figure gravity is known, then it is advisable to use formula (11) instead of formula (6) or (7) to determine the force of the liquid pressure on a figure immersed vertically, i.e., the integral calculus should not be used.

We consider and compare two methods for solving the same task (No. 1768 [2]).

Task 1. A vertical triangle with a base b and a height h is immersed in a liquid with a vertex downwards so that its base lies on the surface of the liquid (Fig. 2). Find the force of the liquid pressure.

Solution. a) We use a definite integral (formula (7)).

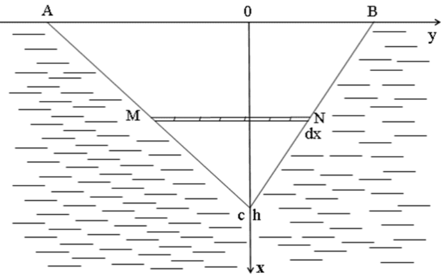

Fig. 2. A vertical triangle submerged in a liquid with a vertex downward

We divide the area of the triangle into elements - stripes of height dx, parallel surfaces of the liquid. The plane of one such element, located at a distance x from the surface, is equal to:

$$ds = b(x)dx.$$

The force of the liquid pressure on this element is:

$$dP = \rho g x \cdot b(x)dx.$$

The variable width $b(x)$ is found from the similarity of the triangles ABC and MNC:

$$\frac{b(x)}{b} = \frac{h-x}{h};\ b(x) = \frac{b}{h}(h-x).$$

Force of pressure on the entire triangle:

$$P = \rho g \int_0^h x b(x) ds = \rho g \int_0^h x \frac{b}{h}(h-x)dx =$$

$$= \rho g \frac{b}{h} \cdot \left(\frac{hx^2}{2} - \frac{x^3}{3}\right)\Big|_0^h = \rho g \frac{bh^2}{6}.$$

b) We use the formula (11). As it is known, for a triangle we have the equalities:

$$x_c = \frac{h}{3};\ S = \frac{1}{2}bh.$$

The force of pressure will be:

$$P = \rho g x_c \cdot S = \rho g \frac{bh^2}{6}.$$

It should be noted that it is much simpler than using the integral calculus (formula 11) to solve the problem of finding the force of the liquid pressure on planar figures immersed vertically if such figures can be divided into a finite number

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of particular regions whose abscissas of centers of gravity are known or easy are calculated.

For such figures, taking into account formulas (8), (9), (11) and the properties of the double integral, the formula for calculating the pressure force takes the following form:

$$P = \rho g \sum_{i=1}^{n} x_{c_i} \cdot S_i , \tag{13}$$

where x_{c_i} – the abscissa of the gravity center of the *i-th* private area; S_i – its area; n – the number of particular areas to which this figure is divided.

Let us illustrate this by solving the following task.

Task 2. The vertical dam has the form of a trapezoid. Find the force of the water pressure on the entire flesh, if it is known that the upper base of the dam a, the lower base b, and the height of the dam h (Figure 3) (No. 1769, [2]).

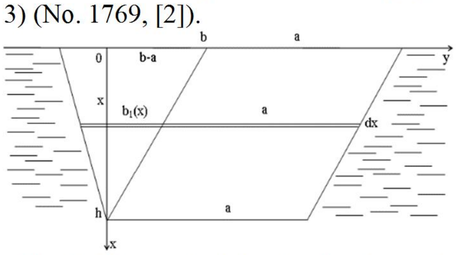

Fig. 3. The vertical dam in the form of a trapezoid, immersed in a liquid.

Solution. We divide the trapezoid into a parallelogram and a triangle.

a) We use a certain integral (formula 7). The variable width of the shape is determined by:

$$b(x) = b_1(x) + a ,$$

where $b_1(x)$ is found from the similarity of triangles as in the previous task:

$$b_1(x) = \frac{b-a}{h}(h-x).$$

Then

$$b(x) = b_1(x) + a = \frac{b-a}{h}(h-x) + a.$$

The force of pressure in this case is equal to:

$$P = \rho g \int_0^h x b(x) dx = \rho g \int_0^h x\left(\frac{b-a}{h}(h-x) + a\right) dx =$$

$$= \rho g \left(\frac{b-a}{h} \cdot \frac{hx^2}{2} - \frac{b-a}{h} \cdot \frac{x^3}{3} + \frac{ax^2}{2}\right)\Big|_0^h =$$

$$= \rho g h^2 \left(\frac{b-a}{2} - \frac{b-a}{3} + \frac{a}{2}\right) = \frac{\rho g h^2}{6}(b + 2a).$$

б) We use the formula (13):

$$P = \rho g \cdot \left(x_{c_1} \cdot S_1 + x_{c_2} \cdot S_2\right),$$

where x_{c_1}, x_{c_2} – abscissas of the gravity centers of a triangle and a parallelogram; S_1, S_2 – their areas. It is known that:

$$x_{c_1} = \frac{h}{3}; \quad x_{c_2} = \frac{2h}{3};$$

$$S_1 = \frac{1}{2}(b-a) \cdot h; \quad S_2 = a \cdot h.$$

The force of pressure is:

$$P = \rho g \left(\frac{h}{3} \cdot \frac{1}{2} \cdot (b-a) \cdot h + \frac{h}{2} \cdot a \cdot h\right) =$$

$$= \rho g h^2 \left(\frac{b}{6} - \frac{a}{6} + \frac{a}{2}\right) = \frac{\rho g h^2}{6}(b + 2a).$$

It is possible to divide the trapezoid into two triangles with the bases a and b and apply the formula (13), where x_{c_1}, x_{c_2} – abscissas of the gravity centers of triangles; S_1, S_2 – their areas. As it is known:

$$x_{c1} = \frac{h}{3}; \quad x_{c2} = \frac{2h}{3}; \quad S_1 = \frac{1}{2} b \cdot h; \quad S_2 = \frac{1}{2} a \cdot h.$$

Force of pressure:

$$P = \rho g \left(\frac{h}{3} \cdot \frac{1}{2} \cdot b \cdot h + \frac{2h}{3} \cdot \frac{1}{2} \cdot a \cdot h\right) = \frac{\rho g h^2}{6}(b + 2a).$$

It should be noted that when applying formulas (8) and (11) to determine the pressure force of a liquid on a vertical flat figure, the Guldin's Theorem

II can be used: *the body volume V, which is circumscribed by a plane figure as it rotates about the axis, lying in the plane of this figure and does not intersect it, is equal to the product of the figure area S by the circumference length, which is circumscribed by the gravity center of this figure*. I.e.:

$$V = S \cdot 2\pi \cdot x_c. \qquad (14)$$

Then

$$x_c = \frac{V}{2\pi \cdot S} \qquad (15)$$

If the formula (10) is taken into account, then the formula (14) is written down as:

$$V = 2\pi \cdot S_y. \qquad (16)$$

Whence

$$S_y = \frac{V}{2\pi}, \qquad (17)$$

i.e., the static moment of a plane figure with respect to an axis lying in the plane of this figure and does not intersect it is equal to the body volume V, which is circumscribed by this figure as it rotates about an axis divided by 2π.

We compare solutions of the same task without applying (case a) and applying the consequences of Guldin's formula (15) (case b) and (17) (case c).

Task 3. Find the pressure force of the liquid acting on a semicircle of R radius that is immersed vertically so that its diameter coincides with the surface of the liquid (Fig. 4).

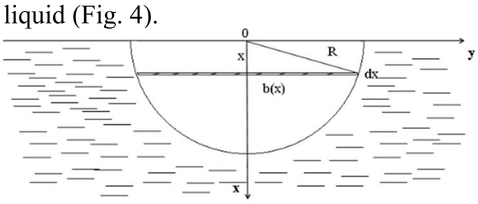

Fig. 4. A semicircle of R radius immersed vertically in a liquid.

Solution. a) As $b(x) = 2\sqrt{R^2 - x^2}$, то

$$P = \rho g\int_0^R xb(x)dx = 2\rho g\int_0^R x\sqrt{R^2 - x^2}\,dx = 2\rho g\int_0^R \sqrt{r^2 - x^2}\,d\left(\frac{x^2}{2}\right) =$$

$$= -\rho g\int_0^R \left(R^2 - x^2\right)^{1/2} d\left(r^2 - x^2\right) = -\rho g \cdot \frac{2}{3} \cdot \left(R^2 - x^2\right)^{3/2} \Big|_0^R = \frac{2}{3}\rho g R^3.$$

It is known that the area of the semicircle is equal to:

$$S = \frac{1}{2}\pi R^2.$$

When the semicircle rotates around the diameter, we obtain a sphere whose volume is equal to:

$$V = \frac{4}{3}\pi R^3.$$

The abscissa of the gravity center:

$$x_c = \frac{V}{2\pi \cdot S} = \frac{\frac{4}{3}\pi R^3}{2\pi \cdot \frac{1}{2}\pi R^2} = \frac{4R}{3\pi}.$$

The pressure is equal to:

$$P = \rho g x_c \cdot S = \rho g \frac{4R}{3\pi} \cdot \frac{1}{2}\pi R^2 = \frac{2}{3}\rho g R^3.$$

c) As the volume of the body rotation is known, then by formula (17) we find:

$$S_y = \frac{V}{2\pi} = \frac{\frac{4}{3}\pi R^3}{2\pi} = \frac{2R^3}{3};$$

$$P = \rho g S_y = \frac{2}{3}\rho g R^3.$$

Task 4. Find the pressure force of the liquid on the plate, which has the shape of a semi-ellipse with axes *2a* and *2b*, immersed vertically so that the major axis coincides with the liquid surface (Fig. 5).

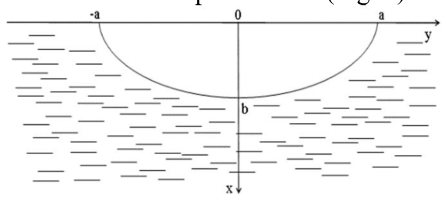

Fig. 5. The plate in the form of a semi-ellipse immersed in a liquid.

Solution. The canonical equation of an ellipse:

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

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When such a plate rotates around the principal axis, we obtain an ellipsoid whose volume:

$$V = \frac{4}{3}\pi b^2 a.$$

Static plate moment:

$$S_y = \frac{V}{2\pi} = \frac{\frac{4}{3}\pi b^2 a}{2\pi} = \frac{2b^2 a}{3}.$$

Liquid pressure:

$$P = \rho g S_y = \frac{2}{3}\rho g b^2 a.$$

To solve this task with the help of a double integral, we need to go over to generalized polar coordinates:

$$\frac{x}{b} = r\cos\varphi;\quad \frac{y}{a} = r\sin\varphi;$$

$$ds = abrdrd\varphi;$$

$$0 \le r \le 1;\quad 0 \le \varphi < \pi/2.$$

The force of the liquid pressure is found from the following formula (6):

$$P = \rho g \iint_D xds = 2\rho g \iint_{D_1} xds =$$

$$= 2\rho g \iint_{D_1} br\cos\varphi abrdrd\varphi =$$

$$= 2\rho g ab^2 \int_0^{\pi/2} \cos\varphi d\varphi \int_0^1 r^2 dr =$$

$$= 2\rho g ab^2 \sin\varphi \Big|_0^{\pi/2} \frac{r^3}{3}\Big|_0^1 = 2\rho g ab^2 \cdot 1 \cdot \frac{1}{3} = \frac{2}{3}\rho g b^2 a.$$

Task 5. Find the liquid pressure on the ABC triangular plate completely immersed vertically in the liquid if the coordinates of the triangle vertices are known in a rectangular Cartesian coordinate system chosen such that the OY axis on the liquid surface and the OX axis is down-directed: $A(x_1; y_1), B(x_2; y_2), C(x_3; y_3)$ (Fig.5).

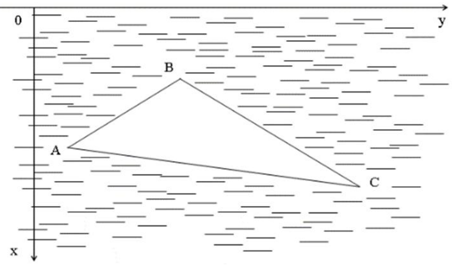

Fig. 6. The triangular plate immersed in liquid.

Solution. Find the lengths of the sides of triangle *a, b, c*, half-perimeter *p*, the area of triangle *S*, the abscissa of the triangle's gravity center x_0 and the pressure force *P*:

$$a = \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2};$$

$$b = \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2};$$

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2};\quad p = \frac{1}{2}\cdot(a + b + c);$$

$$S = \sqrt{p\cdot(p-a)\cdot(p-b)\cdot(p-c)};$$

$$x_0 = \frac{x_1 + x_2 + x_3}{3};$$

$$P = \rho \cdot g \cdot x_0 \cdot S.$$

It should be noted that the above method of calculating the pressure force on a triangular plate can also be applied to polygons, where polygons should be divided into triangles.

We shall give an example of the task of calculating the liquid force pressure on a plate, vertically immersed in a liquid, for which the application of integral calculus is indisputable.

Problem 6. Find the liquid force pressure on a vertical plate bounded by lines: $x = 2y - y^2;\quad x = 0$ (Fig. 8).

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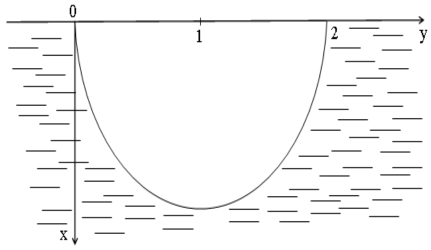

Fig. 7. The vertical plate of parabolic shape, immersed in liquid

Solution. We use the formula (6):

$$P=\rho g\iint\limits_D x\,ds=\rho g\int_0^2 dy\int_0^{2y-y^2}x\,dx=\frac{\rho g}{2}\int_0^2 x^2\Big|_0^{2y-y^2}dy=$$

$$=\frac{\rho g}{2}\int_0^2\left(4y^2-4y^3+y^4\right)dy=\frac{\rho g}{2}\cdot\left(\frac{4y^3}{3}-\frac{4y^4}{4}+\frac{y^5}{5}\right)\Big|_0^2=$$

$$=\frac{\rho g}{2}\cdot\left(\frac{32}{3}-16+\frac{32}{5}\right)=\frac{\rho g}{15}\cdot\left(80-120+48\right)=\frac{8\rho g}{15}.$$

The application of the formulas following from the Guldin's Theorem II, where possible, greatly simplifies the computations in determining the liquid pressure on the vertical plate in comparison with the integral calculus. The integral calculus gives a universal method for solving similar tasks. The most common is the application of the double integral. If the variable width of the plate is easy to determine, then a definite integral is used. However, the use of a double integral to determine the liquid pressure in the rational choice of the order of integration is sometimes more effective than the application of a definite integral. The student should be able not only to solve the task, but also to find the most rational way to solve it.

Conclusions. The quality of the mathematical formation largely depends on how much the mathematical courses in their content reflect the specific character of the future profession of the student. The consolidation of mathematical methods with applied tasks of professional content not only increases the interest in studying the course of higher mathematics, providing the necessary motivation for studying it, but also shapes knowledge, abilities and skills to apply the mathematical apparatus in the future profession activity.

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