

Elżbieta Kasperska
Andrzej Kasperski
Elwira Mateja-Losa
Silesian University of Technology

SENSITIVITY ANALYSIS AND OPTIMIZATION ON SOME MODELS OF ARCHETYPES USING VENSIM – EXPERIMENTAL ISSUE

Introduction

The main content of the paper are the results of sensitivity analysis and optimization of systems archetypes. Let's first concentrate on sensitivity analysis. The System Dynamical (SD) models contain usually many parameters. It is interesting to examine the effect on their variation on simulation output. We select some parameters and assign maximum and minimum values along with a random distribution over which to vary them to see their impact on model behaviour.

Vensim has a method of setting up such sensitivity simulation. Monte Carlo multivariate sensitivity works by sampling a set of numbers from within bounded domains. To perform one multivariate test, the distribution for each parameter specified is sampled, and the resulting values used in a simulation. When the number of simulation is set, for example, at 200, this process will be repeated 200 times.

In order to do sensitivity simulation you need to define what kind of probability distribution values for each parameter will be drawn from. The simplest distribution is the Random Uniform Distribution, in which any number between the minimum and maximum values is equally likely to occur. The Random Uniform Distribution is suitable for most sensitivity testing and is selected by default. Another commonly-used distribution is the Normal Distribution (or Bell Curve) in which value near the mean or more likely to occur than values far from the mean. Results of sensitivity testing can be displayed in different formats. Time graphs display behaviour of a variable over a period of time. The variables spread of values, at any period in time, are displayed either in terms of confidence bounds, or a separate values which combine to form individual simulation trace.

Results of experiments for sensitivity analysis for some system archetypes

Authors have executed many experiments type sensitivity analysis on chosen archetypes. Let's present their results in graph form.

Archetype "Eroding Goal"

In archetype "Eroding Goal" there are two interesting parameters: T_1 , T_2 (see: mathematical model). Authors have performed three types of investigation. First we used so called univariate type, that means "change one at time". In experiment 1 the maximum and minimum values are chosen to bound parameter T_1 and in 2 to parameter T_2 appropriately. The results of such simulation experiments are presented of Figures 1, 2, 3, 4 in form of confidence bounds for variables x_1 , x_2 . On the contrary in experiment 3 we used so called multivariate type, that means "change all together". Now, the parameters T_1 and T_2 were changing their values simultaneously (from the maximum and minimum appropriately). The results of such simulation experiments are presented on Figures 5 and 6 in form of confidence bounds for variables x_1 , x_2 .

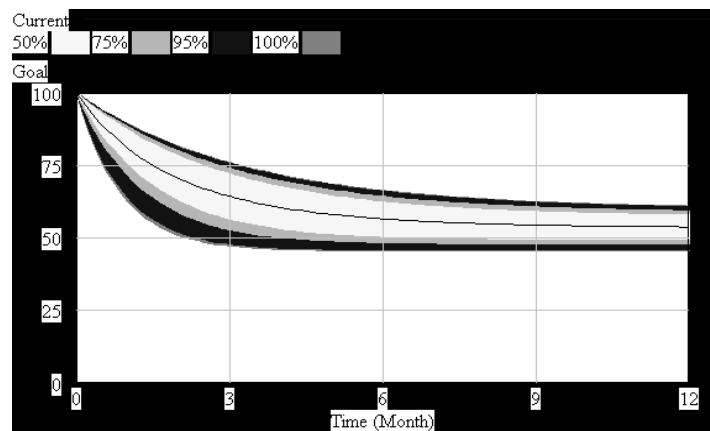


Fig. 1. Confidence bounds for variable "Goal" (x_1) for interval (1,5) for parameter T_1
Source: Own results.

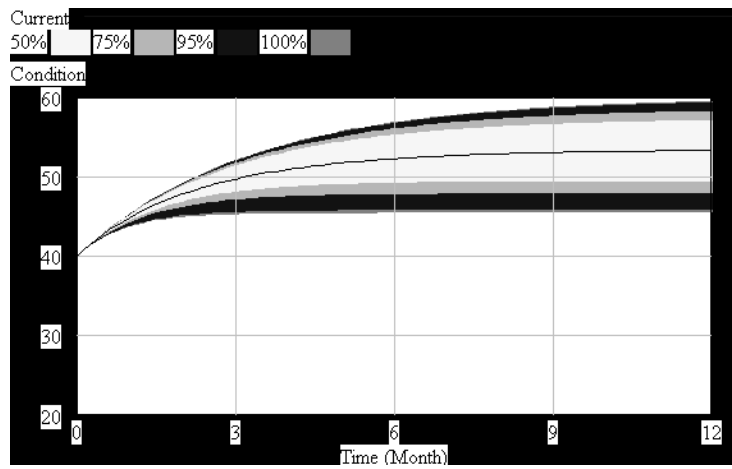


Fig. 2. Confidence bounds for variable “Condition” (x_1) for the interval (1,5) for parameter T_1

Source: Own results.

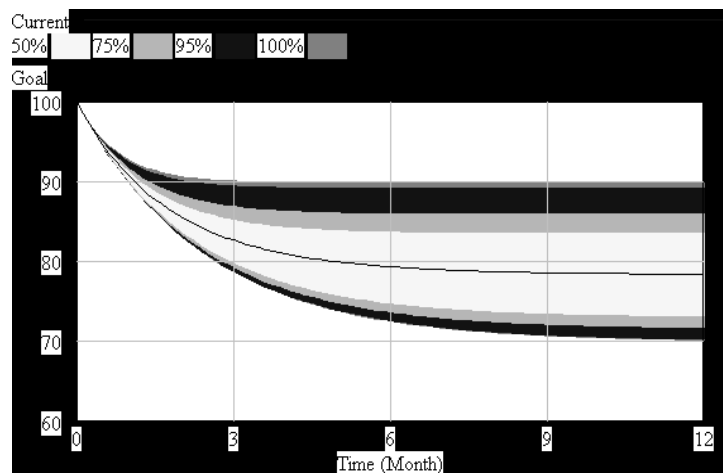


Fig. 3. Confidence bounds for variable “Goal” (x_1) for the interval (5,10) for parameter T_2

Source: Own results.

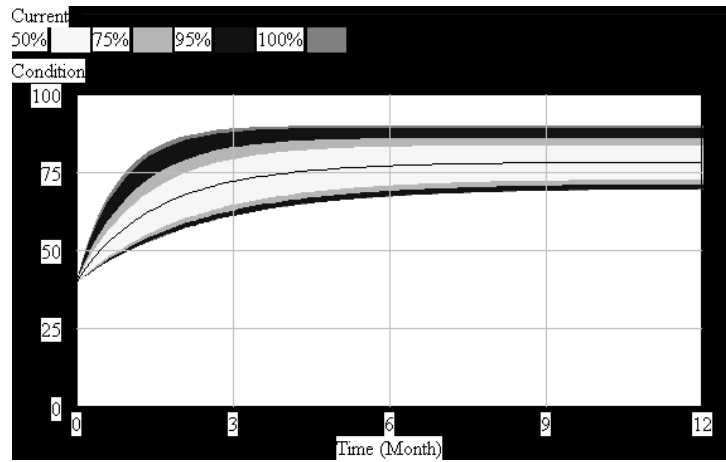


Fig. 4. Confidence bounds for variable “Condition” (x_2) for the interval (5,10) for parameter T_2

Source: Own results.

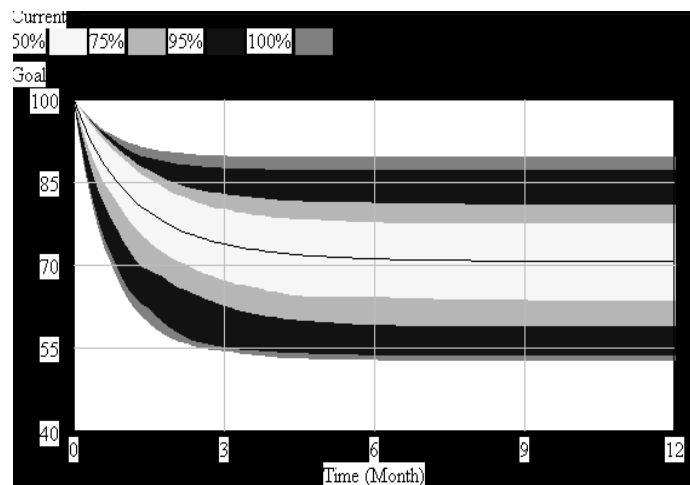


Fig. 5. Confidence bounds for variable “Goal” (x_1) for the interval (1,5) for parameter T_1 and (5,10) for parameter T_2

Source: Own results.

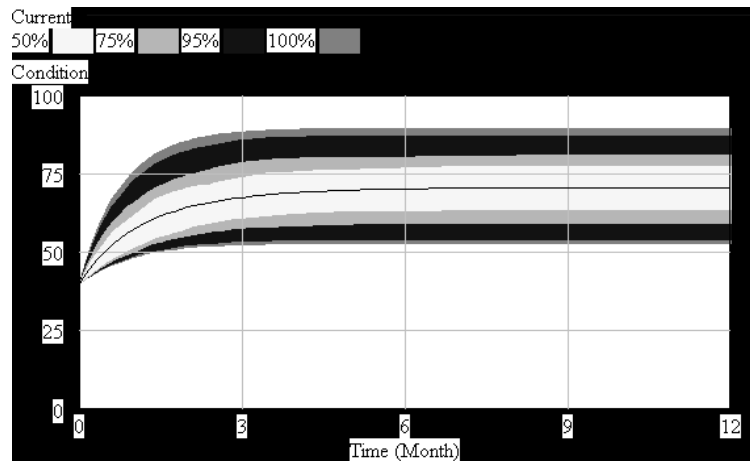


Fig. 6. Confidence bounds for variable “Condition” (x_2) for the interval (1,5) for parameter T_1 and (5,10) for parameter T_2

Source: Own results.

Time for conclusion of that results

Sensitivity analysis can be the entrance for optimization process. Such process can be done by Vensim too. Let the aim of optimization will be maximization of value of variable “Goal”. In Table 1 the comparison of values of objective function for different intervals for parameter T_1 (also with parameter T_2) is given. Moreover in Table 2 we can see the influence of changing the intervals for initial values of levels “Goal” and “Condition” for searching the values of objective function. Moreover in Table 3 the comparison of influence together: T_1 , T_2 , x_{10} for objective function is located. In Table 4 the comparison of influence together: T_1 , T_2 , x_{10} , x_{20} for objective function is located. Possibilities of such effective search of values of objective function, are practically unlimited. It should be stress that like the objective function we can choose “Condition” and the optimization process will be the minimization of such objective function, that time.

Table 1

Effective search of values of objective function – comparison of some results for different intervals for parameters T_1, T_2

| Archetype “Eroding Goal” | | |
|---|------------------|---|
| intervals for sensitive parameters for model of archetype | | values of objective function for optimization: MAX “Goal” |
| $T_1 \in (1,5)$ | | $OF = 54.5454$ |
| $T_1 \in (1,7)$ | | $OF = 54.5454$ |
| $T_1 \in (2,5)$ | | $OF = 49.974$ |
| $T_1 \in (1,5)$ | $T_2 \in (5,10)$ | $OF = 54.5454$ |
| $T_1 \in (1,5)$ | $T_2 \in (1,5)$ | $OF = 50.000$ |
| $T_1 \in (1,7)$ | $T_2 \in (5,10)$ | $OF = 54.5454$ |
| $T_1 \in (2,5)$ | $T_2 \in (5,10)$ | $OF = 49.974$ |
| $T_1 \in (2,5)$ | $T_2 \in (1,5)$ | $OF = 42.8512$ |

Table 2

Effective search of values of objective function – comparison of some results for different intervals for initial values of levels

| Archetype “Eroding Goal” | | |
|--|----------------------|--|
| intervals for initial values of “Goal” (x_{10}) and “Condition” (x_{20}) | | values of objective function (OF) for optimization: MAX “Goal” |
| $x_{10} \in (90,100)$ | | $OF = 39.0184$ |
| $x_{10} \in (75,100)$ | | $OF = 39.0184$ |
| $x_{10} \in (100,150)$ | | $OF = 71.5337$ |
| $x_{10} \in (90,100)$ | $x_{20} \in (30,40)$ | $OF = 39.0184$ |
| $x_{10} \in (100,150)$ | $x_{20} \in (20,40)$ | $OF = 71.5337$ |
| $x_{10} \in (50,150)$ | $x_{20} \in (30,40)$ | $OF = 71.5337$ |
| $x_{10} \in (90,150)$ | $x_{20} \in (20,40)$ | $OF = 39.0184$ |
| $x_{10} \in (75,100)$ | $x_{20} \in (20,40)$ | $OF = 39.0184$ |

Table 3

Effective search of values of objective function – comparison of some results for different intervals for parameters: T_1, T_2 and initial values of level “Goal”

| Archetype “Eroding Goal” | | | |
|--|------------------|------------------------|---|
| intervals for sensitive parameters: T_1, T_2 and for initial values of “Goal” (x_{10}) | | | values of objective function for optimization: MAX “Goal” |
| $T_1 \in (1,5)$ | $T_2 \in (5,10)$ | $x_{10} \in (90,100)$ | $OF = 54.5454$ |
| $T_1 \in (2,5)$ | $T_2 \in (5,10)$ | $x_{10} \in (90,100)$ | $OF = 49.974$ |
| $T_1 \in (1,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (90,100)$ | $OF = 50.000$ |
| $T_1 \in (1,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (100,150)$ | $OF = 91.667$ |
| $T_1 \in (2,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (100,150)$ | $OF = 78.5605$ |
| $T_1 \in (2,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (90,100)$ | $OF = 42.8512$ |

Sometimes the results are intuitually quite obvious and anticipated, but such system like archetype “Eroding Goal” is partially simple. In the case of more complicated systems (with more feedbacks), the possibilities of previously searching sensitive parameters in mathematical models of systems is very valuable, and of course choosing the scopes of intervals of that parameters has the influence for searching objectives function.

Table 4

Effective search of values of objective function – comparison of some results for different intervals for parameters: T_1 , T_2 and initial values of level “Goal” and “Conditions”

| Archetype “Eroding Goal” | | | | |
|---|------------------|------------------------|----------------------|---|
| intervals for sensitive parameters: T_1 , T_2 and for initial values of “Goal” (x_{10}) | | | | values of objective function for optimization: MAX “Goal” |
| $T_1 \in (1,5)$ | $T_2 \in (5,10)$ | $x_{10} \in (90,100)$ | $x_{20} \in (30,40)$ | $OF = 54.5454$ |
| $T_1 \in (2,5)$ | $T_2 \in (5,10)$ | $x_{10} \in (90,100)$ | $x_{20} \in (30,40)$ | $OF = 49.974$ |
| $T_1 \in (1,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (90,100)$ | $x_{20} \in (30,40)$ | $OF = 50.000$ |
| $T_1 \in (1,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (100,150)$ | $x_{20} \in (30,40)$ | $OF = 91.667$ |
| $T_1 \in (2,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (100,150)$ | $x_{20} \in (40,50)$ | $OF = 78.5605$ |
| $T_1 \in (2,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (100,150)$ | $x_{20} \in (40,60)$ | $OF = 48.5605$ |
| $T_1 \in (2,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (90,100)$ | $x_{20} \in (40,50)$ | $OF = 42.8125$ |
| $T_1 \in (2,5)$ | $T_2 \in (1,5)$ | $x_{10} \in (90,100)$ | $x_{20} \in (40,60)$ | $OF = 42.8125$ |

Archetype named “Fixes that Fail”

In archetype “Fixes that Fail” there are three “proportionally” parameters: a , b , c . There is possibility of many experiments type “sensitivity analysis” with many combination on bounds for values. We have perform four. First we used univariate types for parameter a , than for parameter b , and parameter c . Second we used multivariate type for a and b appropriately and then for a , b , c simulatiously. The results of such simulation experiments are presented on Figures 7, 8, 9, 10, 11, 12, 13, 14 in form of confidence bounds for variables x_1 . Similar form can be obtain for variable x_2 .

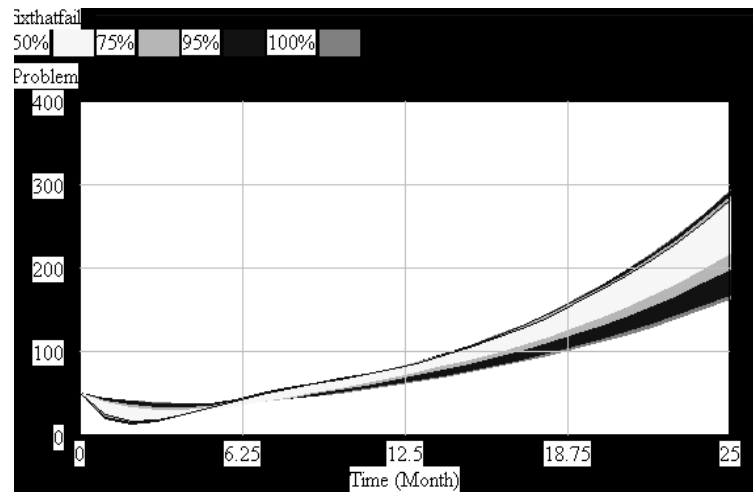


Fig. 7. Confidence bounds for variable “Problem” (x_1) for the interval (0.1,0.6) for parameter “a”

Source: Own results.

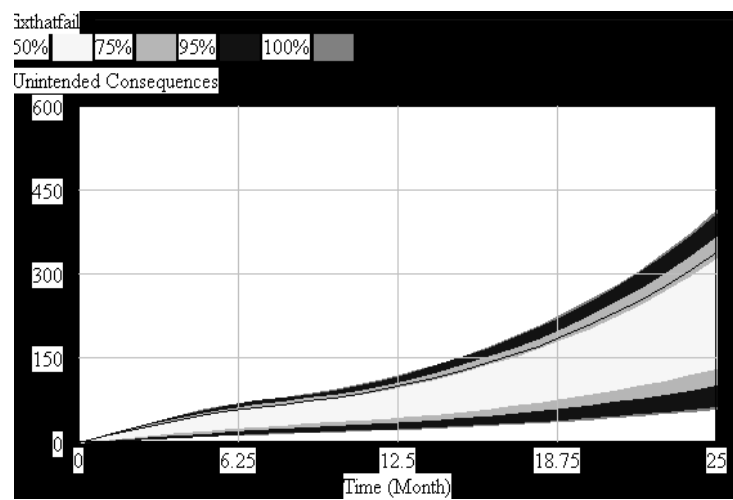


Fig. 8. Confidence bounds for variable “Unintended Consequences” for the interval (0.1,0.6) for parameter “a”

Source: Own results

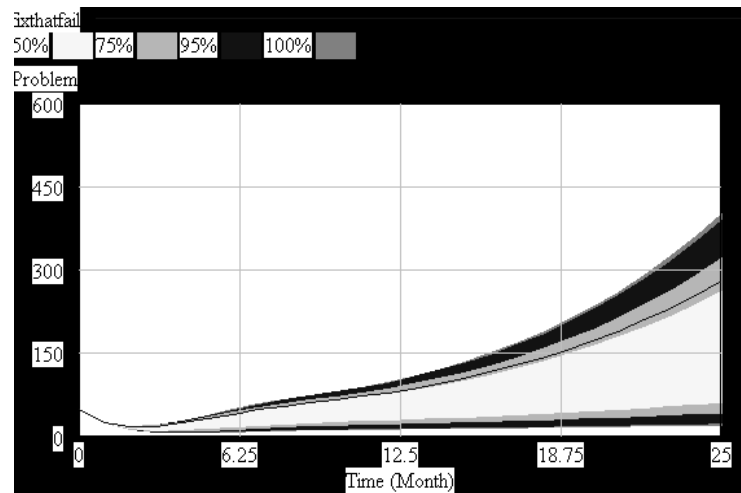


Fig. 9. Confidence bounds for variable "Problem" (x_1) for the interval (0.1,0.6) for parameter "b"

Source: Own results.

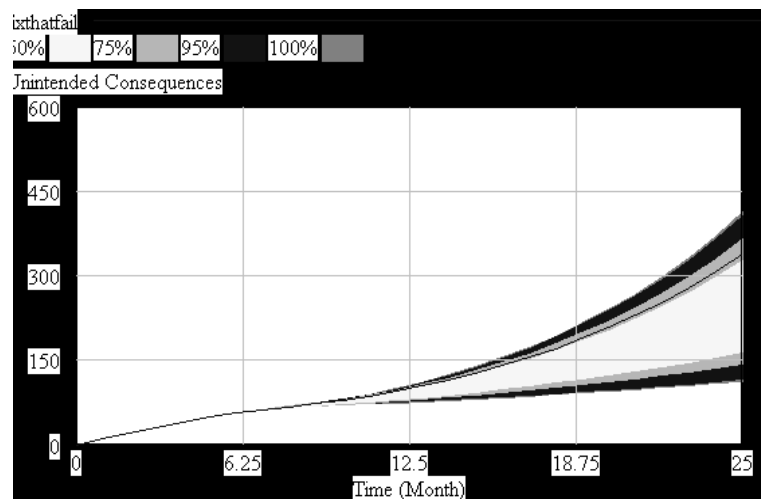


Fig. 10. Confidence bounds for variable "Unintended Consequences" for the interval (0.1,0.6) for parameter "b"

Source: Own results.

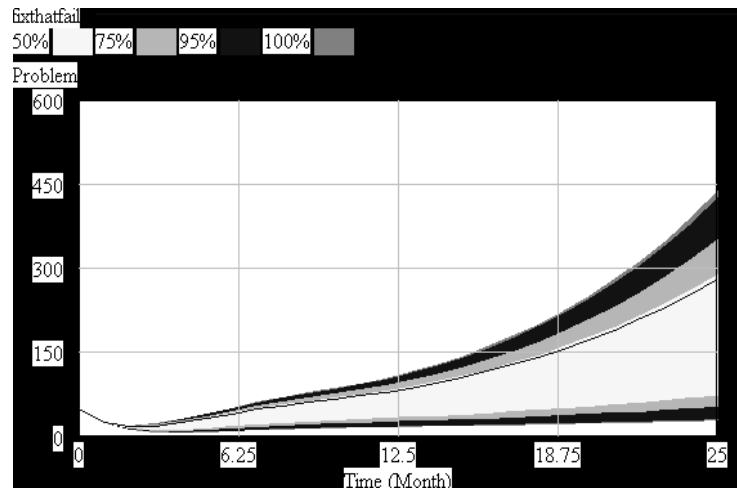


Fig. 11. Confidence bounds for variable “Problem” (x_1) for the interval $(0.1, 0.5)$ for parameter “c”

Source: Own results.

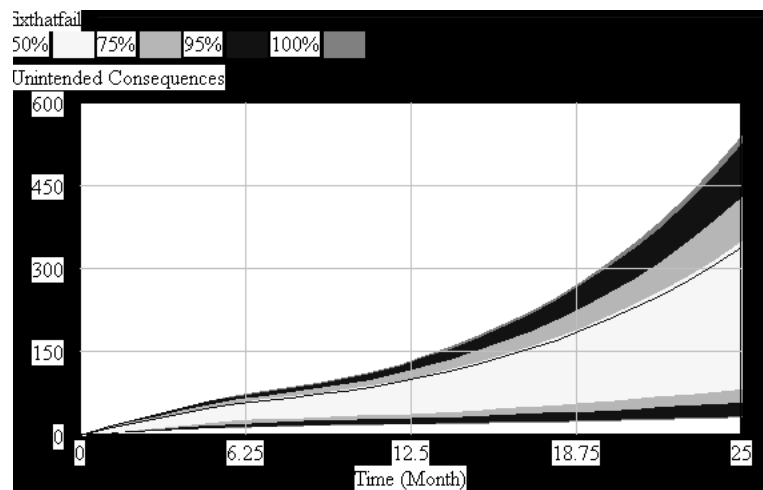


Fig. 12. Confidence bounds for variable “Unintended Consequences” for the interval $(0.1, 0.5)$ for parameter “c”

Source: Own results.

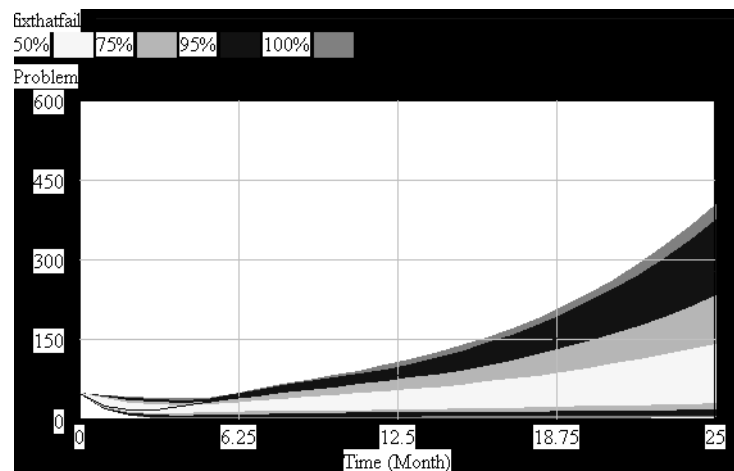


Fig. 13. Confidence bounds for variable “Problem” (x_1) for the intervals: (0.1,0.6) for parameter “a”, (0.1,0.6) for parameter “b”, (0.1,0.5) for parameter “c”

Source: Own results.

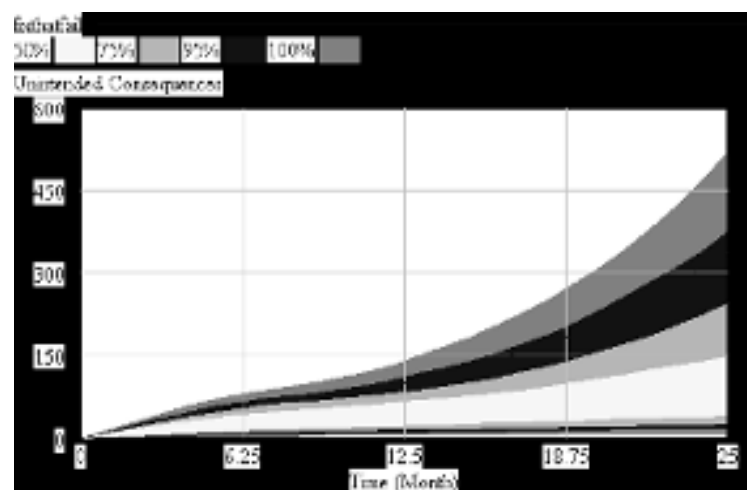


Fig. 14. Confidence bounds for variable “Unintended Consequences” for intervals: (0.1,0.6) for parameter “a”, (0.1,0.6) for parameter “b”, (0.1,0.5) for parameter “c”

Source: Own results.

We can do similar optimization experiments for archetype “Fixes that Fail” like for archetype “Eroding Goal”. In that case the objective function “Problem” can be choose, and different intervals for sensitive parameters: a , b , c , will show their influence for objective function.

Archetype “Success to the Successful”

In archetype “Success to the Successful” there are two “proportionally” parameters: a , b . There is possibility of many experiments type “sensitivity analysis”.

The three of them were chosen. First we used univariate types for parameter a and b appropriately. And then we have used multivariate type for both parameters: a and b .

The results of such simulation experiments are presented on Figures 15, 16, 17, 18, 19, 20 in form of confidence bounds for variables x_1 , x_2 .

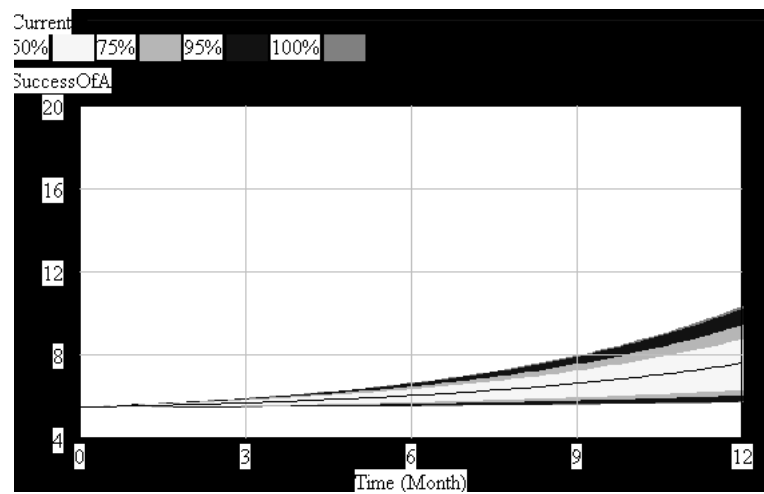


Fig. 15. Confidence bounds for variable “SuccessOfA” for the interval (0.01,0.1) for parameter “a”

Source: Own results.

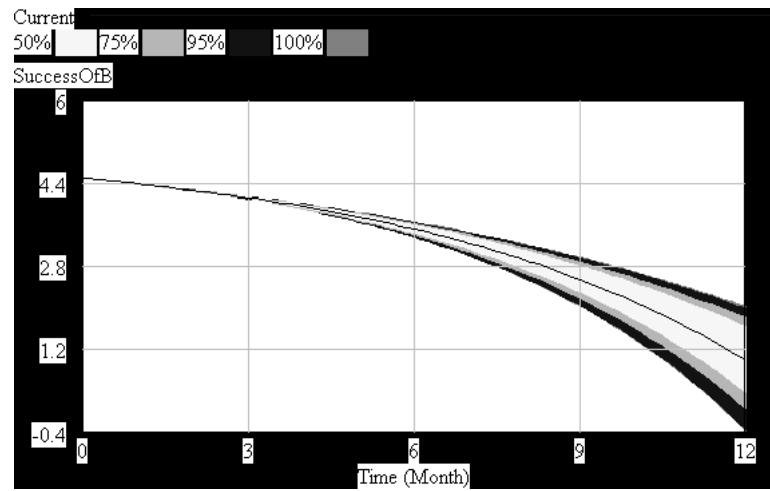


Fig. 16. Confidence bounds for variable "SuccessOfB" for the interval (0.01,0.1) for parameter "a"

Source: Own results.

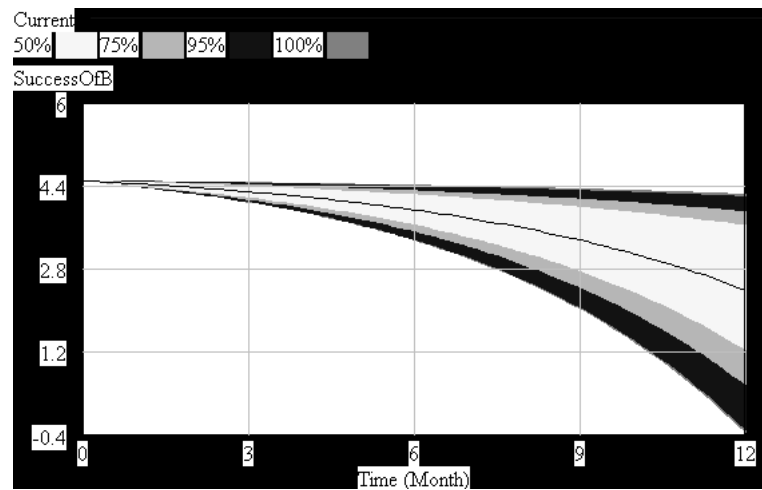


Fig. 17. Confidence bounds for variable "SuccessOfA" for the interval (0.01,0.1) for parameter "b"

Source: Own results.

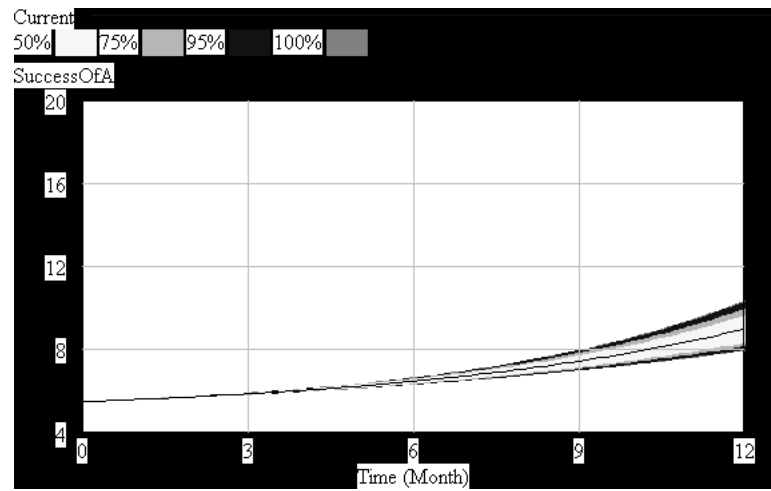


Fig. 18. Confidence bounds for variable “SuccessOfB” for the interval $(0.01, 0.1)$ for parameter “ b ”

Source: Own results.

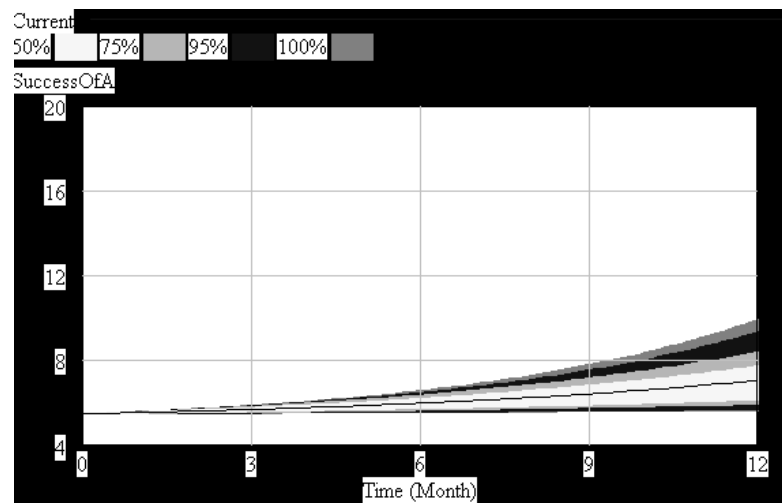


Fig. 19. Confidence bounds for variable “SuccessOfA” for the intervals: $(0.01, 0.1)$ for parameter “ a ” and $(0.01, 0.1)$ for parameter “ b ”

Source: Own results.

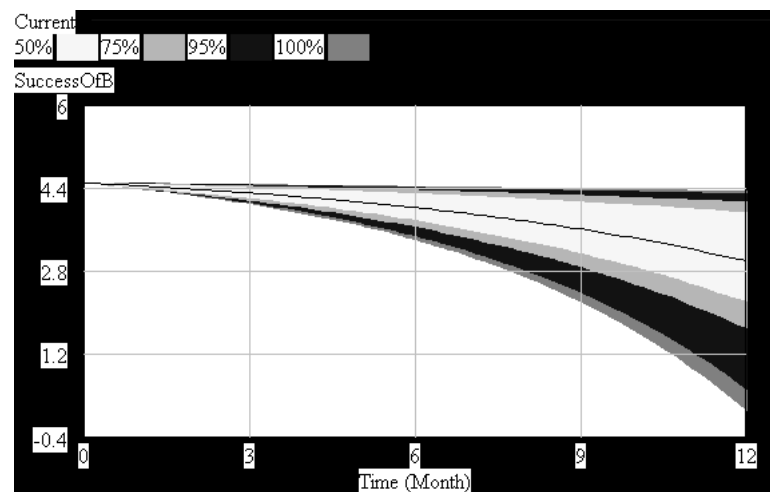


Fig. 20. Confidence bounds for variable “SuccessOfB” for the intervals: (0.01,0.1) for parameter “a” and (0.01,0.1) for parameter “b”

Source: Own results.

Archetype “Accidental Adversaries”

In this archetype there are many parameters: $a, b, c, d, e, f, t_1, t_2$. Authors have performed many experiments type “sensitivity analysis”. Some of them we described below.

First we used so called univariate type, that means “change one at time”. In experiment 1 the maximum and minimum values are chosen to bound parameter “a” and in experiment 2 to parameter “b” appropriately.

The results of such simulation experiments are presented on Figures 21, 22, 23, 24 in form of confidence bounds for variables x_1, x_2 . On the contrary in experiment 3 we used so called multivariate type, that means “change all together”. Now, the parameters a, b, c, d, e, f , changing their values simultaneously. The results of such simulation experiments are presented on Figures 25, 26 in form of confidence bounds for variables x_1, x_2 . Interesting experiments number 4 was performed. That time the maximum and minimum values were chosen to bound parameters: t_1, t_2 . The influence to dynamics of variables x_1, x_2 is presented on Figures 27, 28.

Possibilities of sensitivity analysis are practically unlimited and this is entrance for optimization experiments. Like the objective function we choose multicriterial function type:

$$OF = w_1 Asuccess + w_2 Bsuccess$$

And we performed such types of optimization:

- A. Maximum of *Asuccess* (that means $w_1 = 1, w_2 = 0$).
- B. Maximum of *Bsuccess* (that means $w_1 = 0, w_2 = 1$).
- C. Maximum of sum of *Asuccess* and *Bsuccess* ($w_1 = 1, w_2 = 1$).
- D. Maximum of sum of *Asuccess* and *Bsuccess* (*Bsuccess* is a kind of penalty) ($w_1 = 1, w_2 = -1$).
- E. Minimum of *Asuccess* (that means $w_1 = 1, w_2 = 0$).
- F. Minimum of *Bsuccess* (that means $w_1 = 0, w_2 = 1$).
- G. Minimum of sum of *Asuccess* and *Bsuccess* ($w_1 = 1, w_2 = 1$).

The results of such types of experiments are presented on Figures 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41.

There are interesting conclusions.

The dynamics of variable x_1, x_2 is a result of “cooperation” of loops in structure of archetype. For example on Figures 33, 34 we can see the results of dominating outer loops **R1** and **R2** (see: the structure of archetype), which in consequences gives the exponential growth of variables: x_1, x_2 . Contrary on Figures 38, 39 we can see the influence of balancing loops **B1, B2** together with the acting of “obstructions” (delaying) factors in structure. The results of such acting is a damping oscillations characteristics of variables x_1, x_2 .

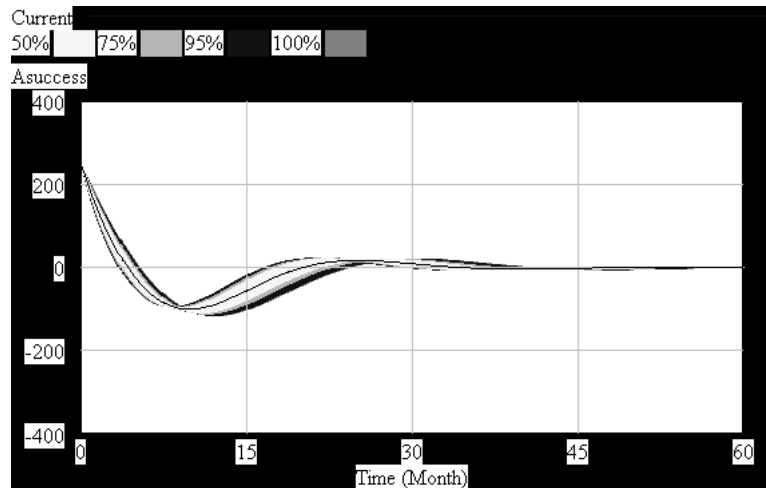


Fig. 21. Confidence bounds for variable “Asuccess” for the interval: (0.2,0.4) for parameter “a”

Source: Own results.

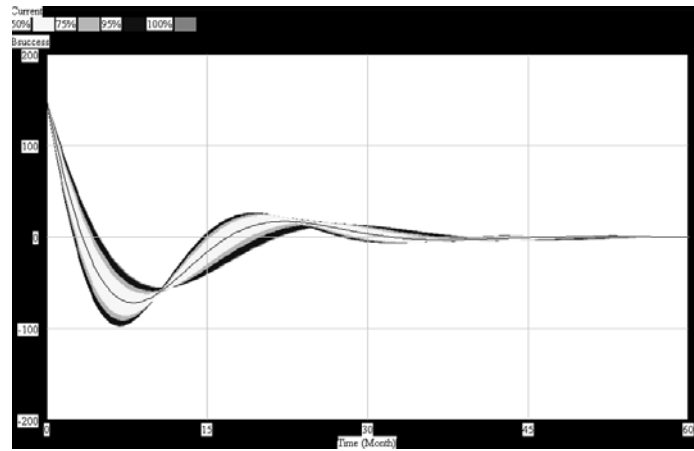


Fig. 22. Confidence bounds for variable “Bsuccess” for the interval: (0.2,0.4) for parameter “a”

Source: Own results.

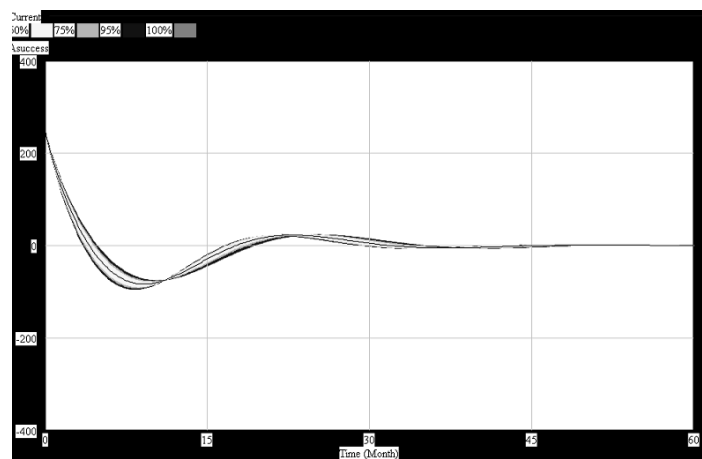


Fig. 23. Confidence bounds for variable “Asuccess” for the interval: (0.2,0.4) for parameter “b”

Source: Own results.

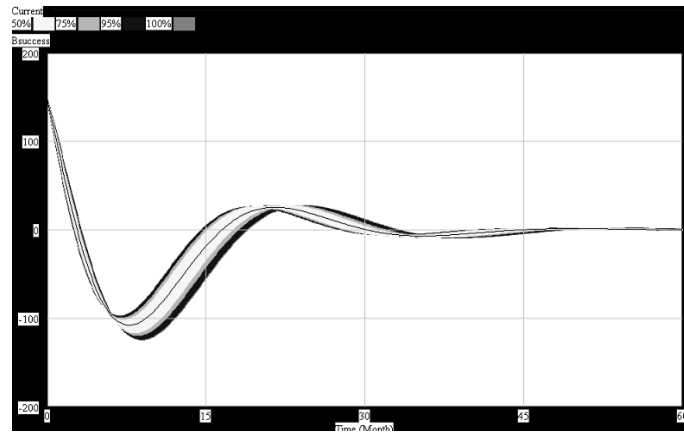


Fig. 24. Confidence bounds for variable “Bsuccess” for the interval: (0.2,0.4) for parameter “b”

Source: Own results.

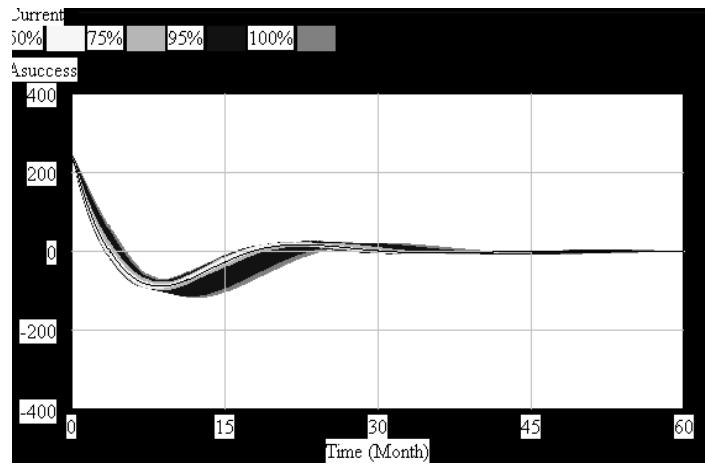


Fig. 25. Confidence bounds for variable “Asuccess” for the intervals for parameters:

$$a \in (0.2, 0.4), b \in (0.2, 0.4), c \in (0.1, 0.2), d \in (0.1, 0.2), g \in (0.4, 0.6), h \in (0.4, 0.6)$$

Source: Own results.

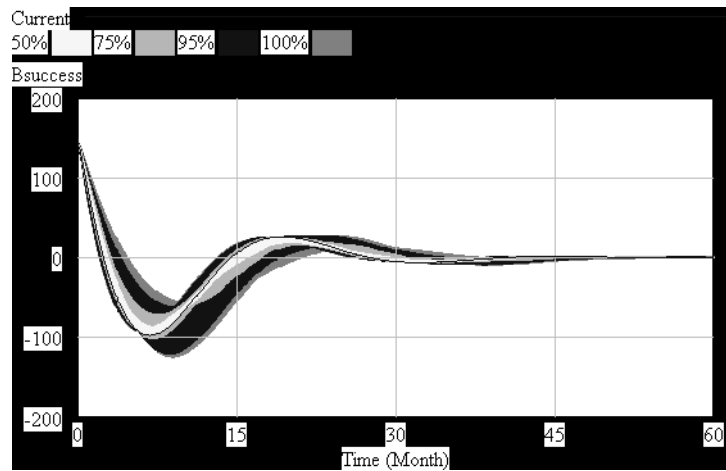


Fig. 26. Confidence bounds for variable "Bsuccess" for the intervals for parameters: $a \in (0.2, 0.4)$, $b \in (0.2, 0.4)$, $c \in (0.1, 0.2)$, $d \in (0.1, 0.2)$, $g \in (0.4, 0.6)$, $h \in (0.4, 0.6)$

Source: Own results.

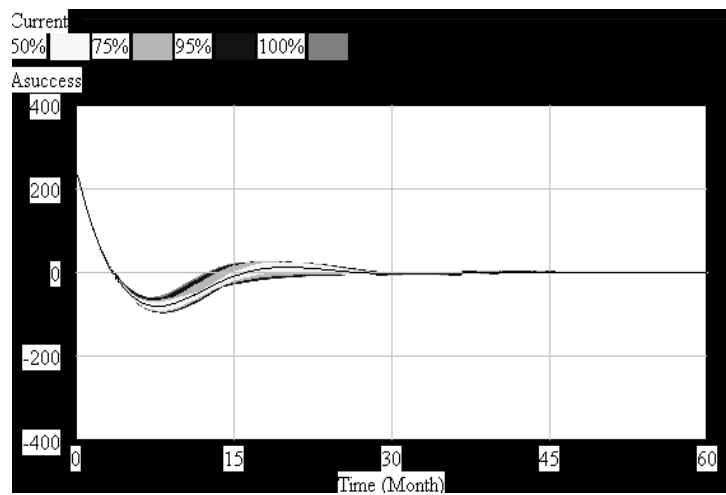


Fig. 27. Confidence bounds for variable "Asuccess" for the intervals: (1,5) for parameter t_1 , and (5,10) for parameter t_2

Source: Own results.

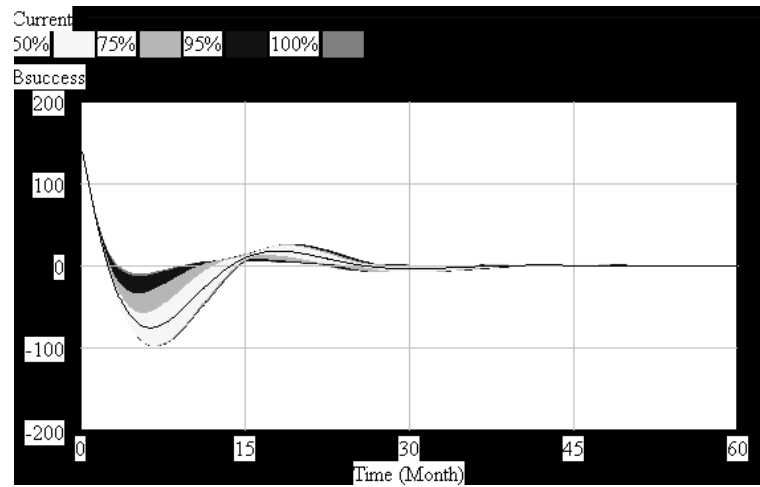


Fig. 28. Confidence bounds for variable “Bsuccess” for the intervals: (1,5) for parameter t_1 , and (5,10) for parameter t_2

Source: Own results.

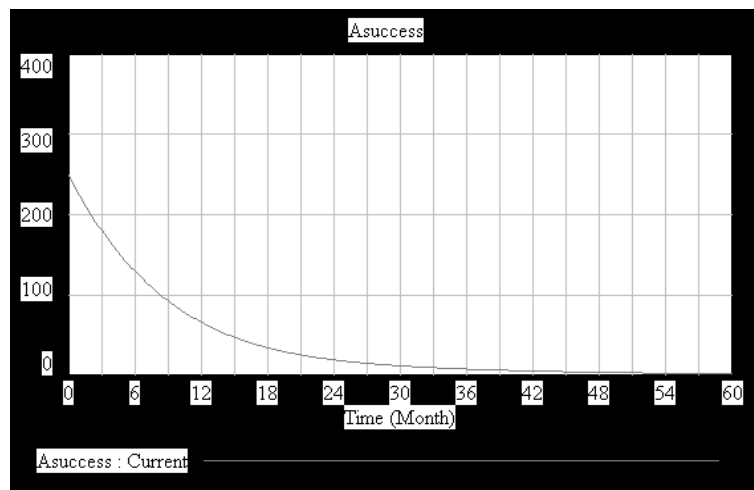


Fig. 29. The dynamics of variable $ASuccess$ for objective function: $OF = ASuccess$ for “Accidental Adversaries” archetype

Source: Own results.

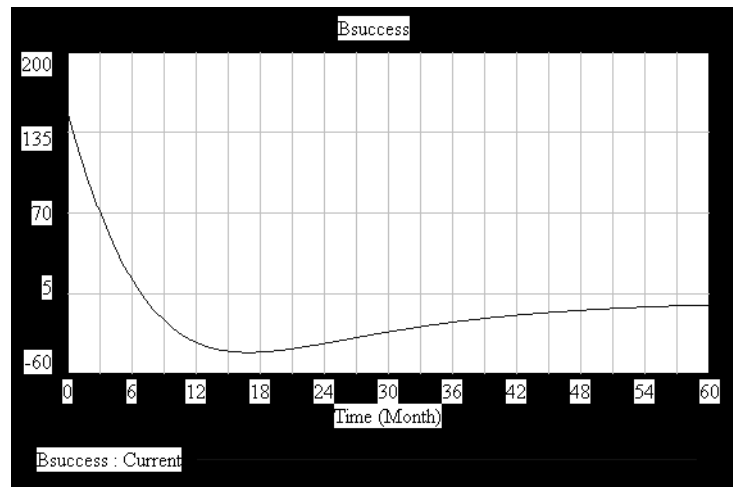


Fig. 30. The dynamics of variable B_{Success} for objective function: $OF = A_{\text{Success}}$ for “Accidental Adversaries” archetype

Source: Own results.

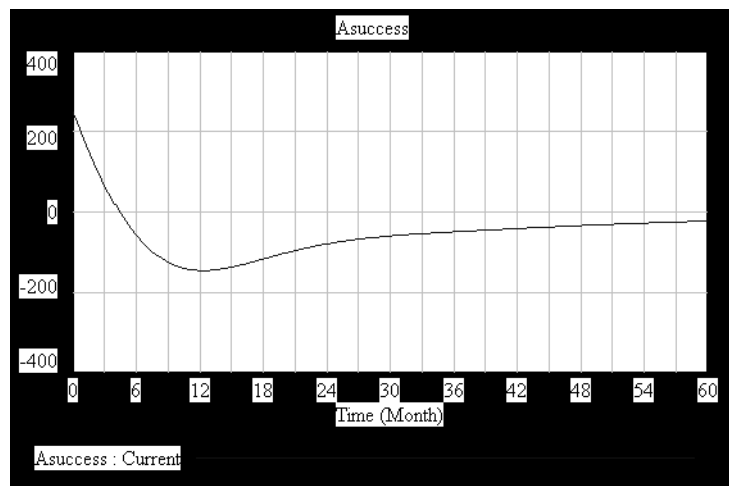


Fig. 31. The dynamics of variable A_{Success} for objective function: $OF = B_{\text{Success}}$ for “Accidental Adversaries” archetype

Source: Own results.

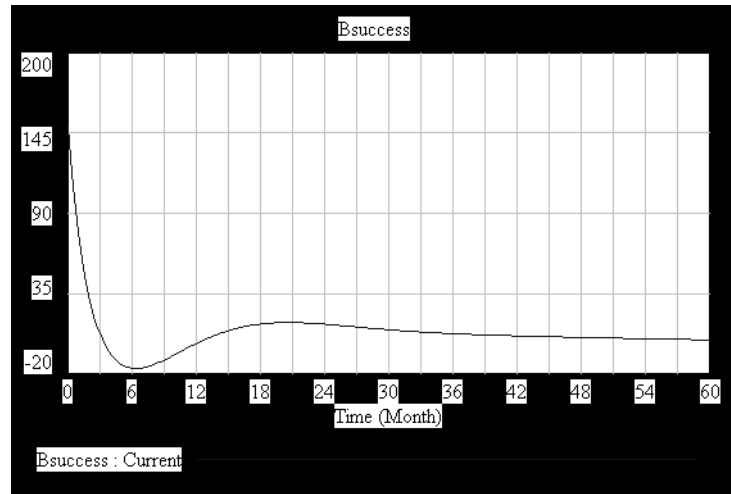


Fig. 32. The dynamics of variable B_{Success} for objective function: $OF = B_{\text{Success}}$ for “Accidental Adversaries” archetype

Source: Own results.

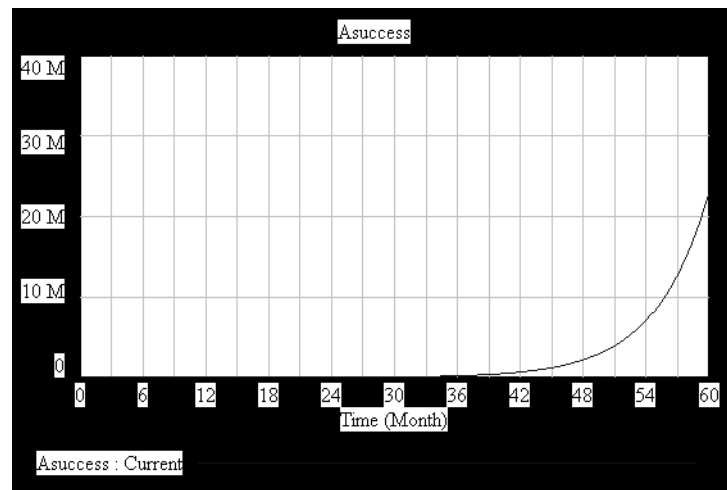


Fig. 33. The dynamics of variable A_{Success} for objective function: $OF = A_{\text{Success}} + B_{\text{Success}}$ for “Accidental Adversaries” archetype

Source: Own results.

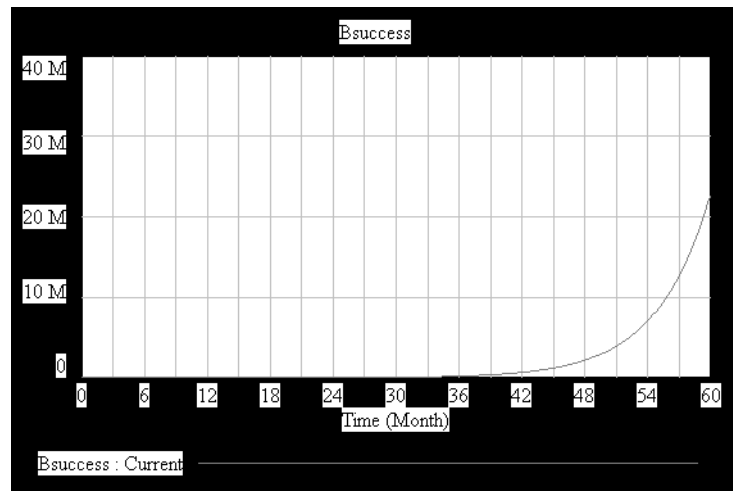


Fig. 34. The dynamics of variable $B_{Success}$ for objective function: $OF = A_{success} + B_{success}$ for “Accidental Adversaries” archetype
Source: Own results.

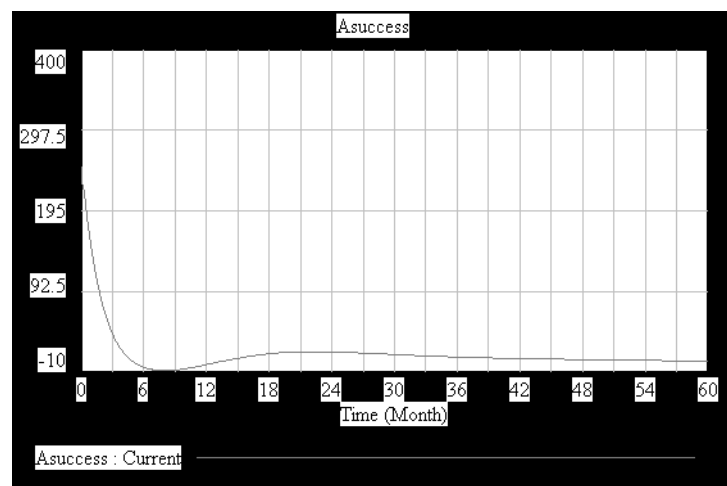


Fig. 35. The dynamics of variable $A_{Success}$ for objective function: $OF = A_{success} - B_{success}$ for “Accidental Adversaries” archetype
Source: Own results.

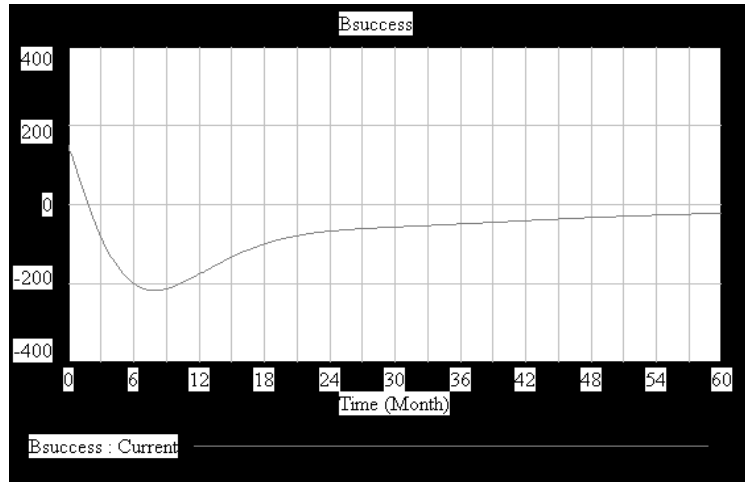


Fig. 36. The dynamics of variable B_{Success} for objective function: $OF = A_{\text{Success}} - B_{\text{Success}}$ for “Accidental Adversaries” archetype

Source: Own results.

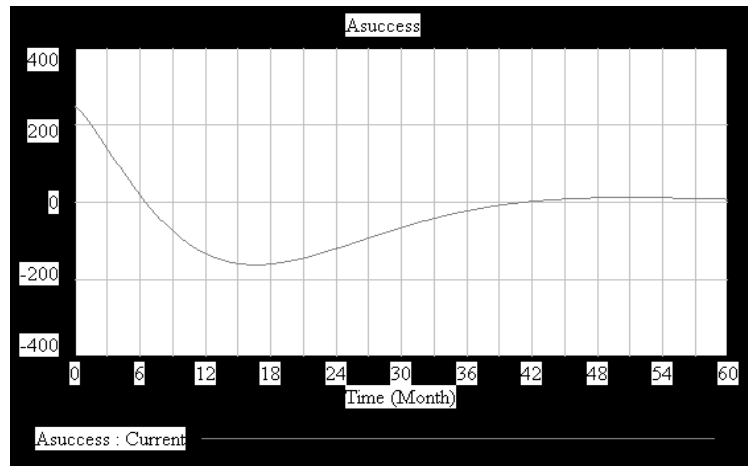


Fig. 37. The dynamics of variable A_{Success} for objective function: $OF = A_{\text{Success}}$ (minimization) for “Accidental Adversaries” archetype

Source: Own results.

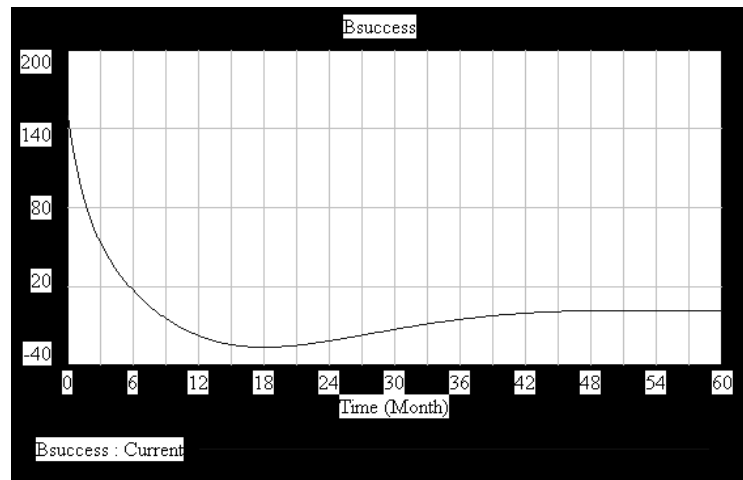


Fig. 38. The dynamics of variable $B_{Success}$ for objective function: $OF = A_{success}$ (minimization) for "Accidental Adversaries" archetype

Source: Own results.

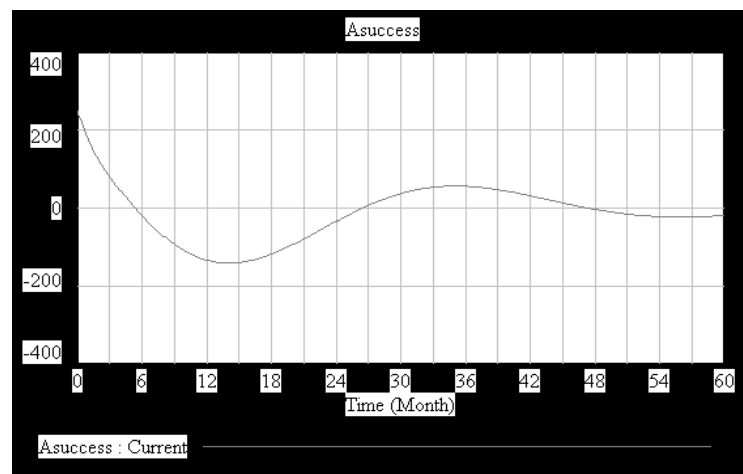


Fig. 39. The dynamics of variable $A_{Success}$ for objective function: $OF = B_{success}$ (minimization) for "Accidental Adversaries" archetype

Source: Own results.

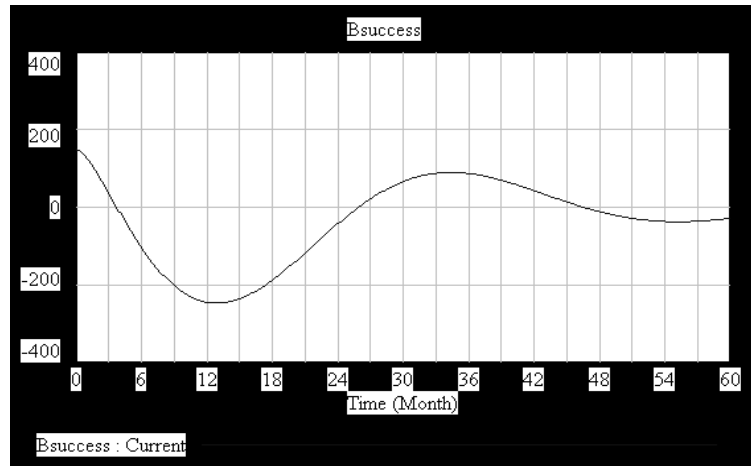


Fig. 40. The dynamics of variable B_{Success} for objective function: $OF = B_{\text{Success}}$ (minimization) for “Accidental Adversaries” archetype

Source: Own results.

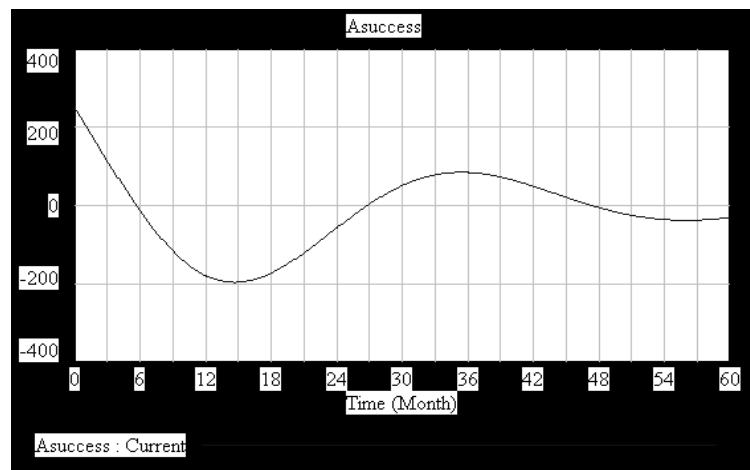


Fig. 41. The dynamics of variable A_{Success} for objective function: $OF = A_{\text{Success}} + B_{\text{Success}}$ (minimization) for “Accidental Adversaries” archetype

Source: Own results.

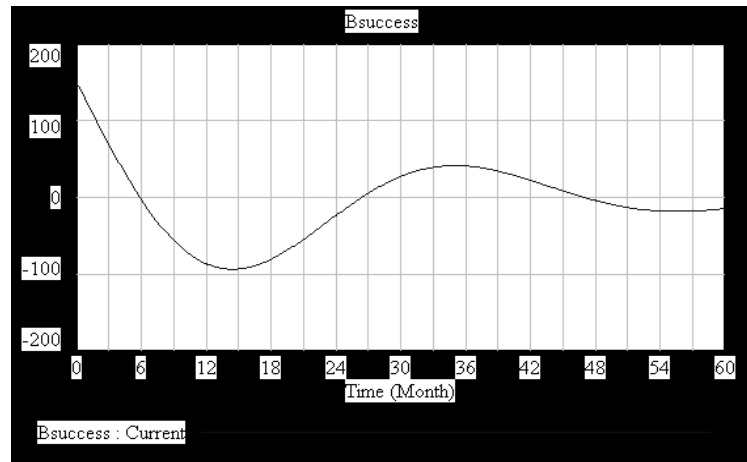


Fig. 42. The dynamics of variable $B_{Success}$ for objective function: $OF = A_{success} + B_{success}$ (minimization) for “Accidental Adversaries” archetype

Source: Own results.

In Table 5 the comparison of results of optimization, for different type of objective function, is located. We can see the difference in searching optimum values of parameters and of course for objective function too.

Table 5

Comparison of results of optimization for different types of objective function, for Accidental Adversaries archetype

| Type of optimization | Results on figures | Optimum values for parameters | Optimum value for objective function |
|-------------------------------------|----------------------------------|--|--------------------------------------|
| MAX $A_{success}$ | <i>Fig. 29</i> <i>Fig. 30</i> | $a = b = 0.1$ $c = d = 0$ $g = 0.6$ $h = 0.1$ | 2387.6 |
| MAX $B_{success}$ | <i>Fig. 31</i> <i>Fig. 32</i> | $a = 0.1$ $b = 0.5$ $c = d = 0$ $g = h = 0.6$ | 508.983 |
| MAX ($A_{success} + B_{success}$) | <i>Fig. 33</i> <i>Fig. 34</i> | $a = b = 0.1$ $c = d = 0.3$ $g = h = 0.1$ | 237364 |

Table 5 continued

| | | | |
|--|----------------------------------|---|---------|
| MAX (<i>Asuccess</i> - <i>Bsuccess</i>) | <i>Fig. 35</i> <i>Fig. 36</i> | $a = 0.5$ $b = 0.1$ $c = d = 0$ $g = h = 0.6$ | 5183.12 |
| MIN <i>Asuccess</i> | <i>Fig. 37</i> <i>Fig. 38</i> | $a = 0.1$ $b = 0.5$ $c = 0.089919$ $d = 0.3$ $g = 0.1$ $h = 0.6$ | 2032.69 |
| MIN <i>Bsuccess</i> | <i>Fig. 39</i> <i>Fig. 40</i> | $a = 0.5$ $b = 0.1$ $c = d = 0.3$ $g = h = 0.6$ | 2283,72 |
| MIN (<i>Asuccess</i> + <i>Bsuccess</i>) | <i>Fig. 41</i> <i>Fig. 42</i> | $a = 0.179673$ $b = 0.5$ $c = d = 0.3$ $g = h = 0.6$ | 1552.48 |

Comparison of optimum values of parameters, for different types of optimization, allows for some remarks:

1. The exponential growth in experiments type: MAX (*Asuccess*+*Bsuccess*), is a result of domination of reinforcing loops (see: the structure of model, Figure 7, and see the value of parameters: $c = d = 0.3$).
2. When *Bsuccess* is a kind of “penalty” for objective function (see: MAX (*Asuccess*-*Bsuccess*), then algorithm choose small fixing for B ($b = 0.1$) big fixing for A ($a = 0.5$), switching out the “activities” ($c = d = 0$) and switching on “obstacles” ($g = h = 0.6$).
3. When we consider minimization of sum *Asuccess* and *Bsuccess* then big “obstacles”, are switching on ($g = h = 0.6$) and interesting value of “fixing” parameters are chosen ($a = 0.179673$, $b = 0.5$).

The precise watching of changing dominance loops for both: *Asuccess*, *Bsuccess* is practically no possible. Because of this such experiments are very interesting from methodological point of view. The structure of “Accidental Advisory” archetype contains two delays, and because of this the precise solving such set of different equations with delayed arguments [Bo10; Ha77; HaSt88], is

very complicated. Authors very slowly read the literature of this subject, so now it is not possible to say something about stability of such structure like for this archetype. This is interesting subject for future study.

Final remarks and conclusion

The aim of this paper was a presentation of some new results of authors investigation in the area of sensitivity analysis and optimization for some system archetypes. First we presented mathematical models of chosen archetype, like:

- Eroding Goal,
- Fixes that Fail,
- Success to the Successful,
- Accidental Adversaries,
- Limit to Growth.

Then we executed many experiments type sensitivity analysis and optimization, using simulation language Vensim.

Specially we presented the comparisons of value of objective functions for different intervals of values of sensitive parameters and intervals for initial values of levels. On the base of results, especially on the base of exact mathematical solution from some models of archetype, we can discussed the problem of stability for these archetypes. This problem is very interesting from methodological point of view. Authors plan to undertake the searching of stability in next papers.

Now, some conclusions are as follow:

1. The more complicated system (more feedbacks), the most interesting is the role of parameters and initial values of levels, especially for process of searching the optimization value of objective function.
2. The chosen intervals of that parameters can be narrowing (for investigating the stability (or chaos), or widen for process of optimization.
3. Different objective function (with weight for multicriterial function) can model different priorities, with penalty factors for constrains.

References

- [Bo10] Bodnar M., Piotrowska M.J., O równaniach różniczkowych z opóźnieniem – teoria i zastosowania, „Matematyka Stosowana” 2010, No 52.
- [Ha77] Hale J.K., Theory of Functional Differential Equations, Springer, New York 1977.
- [HaSt88] Hale J.K., Sternberg M., On Set of Chaos in Differentia Delay Equations, “J. Comp. Phys” 1988, No 77 (1).
- [Ve02] Ventana S.E., Vensim User’s Guide Version 5, Ventana Simulation Environment, 2002.

ANALIZA WRAŻLIWOŚCI I OPTYMALIZACJA NA PEWNYCH MODELACH ARCHETYPÓW Z UŻYCIEM VENSIMA – UJĘCIE EKSPERYMENTALNE

Streszczenie

Artykuł jest kontynuacją artykułu pt. „Analiza wrażliwości i optymalizacja na pewnych modelach archetypów z użyciem Vensima – ujęcie teoretyczne” tych samych autorów. Na bazie możliwości języka symulacyjnego Vensim przeprowadzono wiele eksperymentów i zaprezentowano w postaci „przedziałów ufności”, które są bardzo ładną wizualizacją trajektorii zmiennych modeli.