University of Economics in Katowice

Volume 11 2013

Journal of

Economics & Management

*Eloy Hontoria, Victoria de-la-Fuente Aragon, Lorenzo Ros-McDonnell, Marija Bogataj*

# MONTE CARLO SIMULATION APPLIED TO THE LOGISTICS OF CERAMICS

## Introduction

The current tough competition on the market is forcing companies to improve their processes in order to make them more efficient. Improved transport-related logistics supports clients by improving satisfaction in terms of timing and cost. In the case of ceramic companies, this study is oriented toward the processes connected to the *grouping* of merchandises and customers orders. The lack of appropriate tools to support optimal logistics of such items at very volatile demand and supply has led to development of different techniques to facilitate both calculation and the implementation of these tasks. Our plan is to improve *An approximate algorithm for optimal logistics of heavy and variable size items* described by Ros-McDonnell et al. (2010). With the objective of picking up all orders from all clients on a certain date, we will run a Monte Carlo simulation aiming to find a new, more well-organized vehicle assignment solution, with lower costs than those calculated by mathematical model initially proposed by the cited authors, where the constraints of the loading capacity and time window for an individual vehicle could be slightly relaxed up to p%, if probability density function of this relaxation is supposed to be uniform on the interval (0, ε) and the maximal number of available trucks in a procedure is not fixed by logistic operator in advance, but could be stochastic variable.

Section 2 of the paper presents a brief review of the literature related to the capacitated vehicle routing problem (CVRP), and lays out the group of constraints associated with the description of the CVRP and the discussion on the solutions contributed by mathematical and simulation models. Sections 3 and 4 describe the real problem of a logistic operator, the objectives pursued, and the constraints that must be obey as well as the calculation method used to simulate the new solutions. Section 5 describes a real situation logistic problem, the solutions obtained and the analysis of those results by comparing them with the original solution of Ros McDonnell et al. The new solution will be validated.

## 1. The capacitated vehicle routing problem

The Vehicle Routing Problem (VRP) is addressing the optimal allocation of vehicles on the routes where we have several clients. These vehicles serve a group of clients. The problem has been thoroughly studied in several papers (Laporte, 1992; Laporte et al., 2000; Toth & Vigo, 2002; Bachelet & Yon, 2007). Many of the requirements and the operative constraints, such as route specifications and length, number of vehicles in the fleet and their loading capacities, composition of orders, type of demands, number of warehouses, etc., may be imposed on the practical applications of the VRP problem (Ralphs et al., 2003).

At day by day operations of logistic operators, normally some variables are changing fast during the day and new solutions must be given to solve the new context rapidly. Some drivers also take a risk to load a very small percentage over the declared loading capacity. Related with this volatile characteristics, we can find for example travel time (important to consider the rush hour in urban logistic) or loading the trucks. Important constrains appears according to capacity constraint modelled at Capacitated Vehicle Routing, especially in ceramic industry.

In the basic CVRP the demands are deterministic, the service involves deliveries or collections but not both, the vehicles are based at a single depot, the capacity restrictions and windows for the vehicles are deterministic or stochastic variables, the objective is to minimize the total cost needed to serve all the clients. Generally, travel cost between each pair of customer locations is the same in both directions, therefore the resulting cost matrix is symmetric, whereas in some applications, like urban areas distribution in with one-way directions imposed on the roads, the cost matrix is asymmetric (Toth & Vigo, 2002).

When researchers are looking for solutions in this context, they promote optimization techniques based on mathematical models such as Branch & Bound, which imply simplifications of all possible constraints. Nevertheless, these approaches have important difficulties in their practical application, mainly when changes in the definition of the problem appear, such as the additional constraints to the problem or their relaxation. At this stage, the problem could cease to be a robust one, and not have a solution through the previous optimization techniques. In this situation, simulation models permits descriptions of complex systems without too many assumptions, obtaining near-optimal solutions and guaranteeing their feasibility (Bachelet & Yon, 2007).

Due to the variability of assignments, capacities and schedules, the development of exact mathematical models has been traditionally used in such studies. However, it is very difficult to find the optimal solutions, due to the complexity of the real world problems and the mathematical model that represents them. Use of heuristics offers different easier approaches. These techniques facilitate the search for solutions, reducing the calculation time and simplifying the real problems. The first attempt to give a solution for the real world problem of the logistic operator in a ceramics industry was given by Ros-McDonnell et al. (2010). These authors developed an approximate algorithm which was based on the Branch & Bound technique (Fischetti et al., 1994) and consisted on several priority rules:

- Rule 1: Assignment of the means of road transport in decreasing order of load associated to each of routes.
- Rule 2: Assignment of the vehicles to a route if its capacity is greater or equal to the load picked up on the route.
- Rule 3: Division of orders if the load to be collected on a route is greater than the capacity of the assigned vehicle.

– Rule 4: Division of the orders if the load to be collected for one supplier is only greater than the capacity of the allocated vehicle.
– Rule 5: The division of orders if the load to be collected on a route is less than the capacity of the assigned vehicle, but fails to meet the constraint of the maximum.
– Rule 6: The assignment of vehicles for the collection of possible little orders (routes with a load less than 3.000 kg) and the remainder of orders through the combination of loads not yet assigned to the different routes.

The procedure is running on 5 stages:

Stage 1: The grouping of suppliers by route.
Stage 2: Calculation of daily loads.
Stage 3: Assigning the means of transport.
Stage 4: Calculation of the efficiencies in the assignment of the means of road transport.
Stage 5: Calculation of cost.

Constrains of max net load were not relaxed and time windows have been strictly 8 hours. This algorithm was robust enough to obtain solutions according to constraints, but the disadvantage of this method is the high level of complexity for the companies' procedure, without qualified employees to deal with this algorithms (see: WWW1). Here we will present a new procedure based on Monte Carlo simulation, starting from the solution of the less complex algorithm given by Ros-McDonnell et al.

## 2. Problem description

The case studied is applied at a logistics operator (LO) receiving daily orders from distributors. These orders relate to ceramics products which must be picked up from several manufacturers who are located on different routes. At the end of the day all orders received must be grouped by routes in the optimal sequences, to be picked up next early morning, but first is necessary to obtain a quick assignment of available trucks for collection the cargo. The trucks required to collect the ceramics products of several manufacturers dispersed on different routes must be hired the evening before the operation. **This strategy warranties logistics operator a better use of the logistical resources, and minimizes the associated costs.** However, there are a number of constrains that LO must bear in mind when it comes to providing these services:

- Capacity of each transport vehicle at road (with relaxation added).
- Possible pick-up routes.
- Maximal number of suppliers that can be visited per vehicle per route (the value determined by LO as deterministic (Ros-McDonnell et al) or stochastic parameter.

- Cost of each vehicle.
- Time windows (with relaxations).

The variables and constrains are given at Tables 1, 2, and 3.

Table 1 shows the capacity of each type of vehicle.

Table 1

Capacities of vehicles

| VEHICLES | T1 | T2 | T3 | T4 | T5 | T6 | T7 |
|---|---|---|---|---|---|---|---|
| CAPACITY (kg) | 25.000 | 16.000 | 8.000 | 5.500 | 4.500 | 4.000 | 1.100 |

In the case studied six possible routes are available:

$$R_k = [A, B, C, D, E, F]$$

Based on its experience, the LO has determined a maximum number of suppliers which can be visited by each type of vehicles.

Table 2

The maximal number of suppliers (Si) that can be visited per vehicle (Tj) according to route

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| S1($_{T1)}$ | 12 | 10 | 8 | 6 | 6 | 9 |
| S2($_{T2)}$ | 12 | 10 | 8 | 6 | 6 | 9 |
| S3($_{T3)}$ | 15 | 14 | 12 | 6 | 6 | 14 |
| S4($_{T4)}$ | 15 | 14 | 12 | 6 | 6 | 14 |
| S5($_{T5)}$ | 17 | 18 | 13 | 6 | 6 | 15 |
| S6($_{T6)}$ | 17 | 18 | 13 | 6 | 6 | 15 |
| S7($_{T7)}$ | 18 | 18 | 14 | 6 | 6 | 15 |

The daily (in the time window of 8 hour + relaxation) cost per vehicle is given at the Table 3.

Table 3

Daily cost per vehicle (in €)

| VEHICLE | T1 | T2 | T3 | T4 | T5 | T6 | T7 |
|---|---|---|---|---|---|---|---|
| EUR/DAY | 300 | 220 | 170 | 160 | 160 | 160 | 125 |

There are multiple combinations of these variables and constraints even for feasible solutions without relaxation, which needs to be known by LO before previous working day is finished and assignment of vehicles is done. For these reasons, it is necessary to use a fast procedure to find the best solution.

## 3. Monte Carlo method

Monte Carlo method is a computational algorithm that relies on repeated random sampling to compute their results. Because of their reliance on repeated computation of random numbers, these methods are most suited when it is infeasible or impossible to obtain an exact result with a deterministic algorithm. The idea is not to get an exact solution after an infinite amount of calculation time, but to have a good approximation quickly (Jackel, 2002).

The method applies to problems with no probabilistic content as well as to those with inherent probabilistic structure (Fishman, 1996) like in our relaxations and the constraints in number of vehicles hired for the individual road.

Monte Carlo Simulation (MCS) have been several times base on Fernandez de Cordoba (1998) who developed a heuristic algorithm based in MC methods to the rural postman problem and Juan et al. (2009) who applied MCS to solve the capacitated vehicle routing problem. MCS can also be complemented with others methods like Markov Chains to solve discrete and combinatorial optimization problems (Vrugt et al., 2011) especially in case of given time window like here if service time would be stochastic variable. The aim is to find a variety of solutions that can be used under the additional constraints or relaxations.

This improved version, in which all the objectives are met in terms of order collection, aims to find as good or better solutions (based on the cost minimization) than solutions given by the approximate algorithm of Ros-McDonnell et al. Random numbers are first generated by the computer simulating the maximum quantity of vehicles of each type needed to collect the orders, which is in previous paper determined in advance by LO as the fixed value, based on the previous experiences of LO. It can be now uniformly distributed around previously fixed value. The output of each simulation will show to the LO the best set of the fleet of vehicles, its load capacity and the associate cost.

The variety of solutions are easily available for the LO in a short time running the MCS using Excel with an extremely low computational cost. These solutions can be used under the additional constraints or relaxations previously explained. A subsequent validation process of the solutions found will be developed using Google Maps web application.

The demand is deterministic because the information about it, is known by the LO at the end of the previous working day. But here additionally the load is allowed to be 0% to ε% over the limits in capacity of vehicles and is uniformly distributed.

The procedure for searching solutions is as follows:

1. Calculation of the load capacity to be hired on a daily basis. This capacity of the fleet must be greater or equal than the total load to be picked up on any particular day. Upper limit is uniformly distributed up to a given value.
2. Calculation of the possible solutions, by means of a Monte Carlo simulation, for the fleet to be used to collect the daily orders. Random variables will model the fleet of vehicles which has to pick up all the orders. Up to ε% uniformly distributed overload and up to δ% of time window is allowed but not always accepted by drivers (uniformly distributed).
3. Calculation of costs at each solution found in the previous step. The solutions granting lower costs than those of the approximate algorithm must be selected.
4. The final choice is the cheapest solution of fleet and route allocations for each vehicle, along several alternative routes under additional relaxations in constrains of time window or capacity of individual vehicle.

A short example with four simulations is shown in Table 4, where LO has to collect orders in amount of 85.611 kg with a maximum cost of 1.550 €. Solutions representing a fleet of vehicles with a lower capacity and/or higher cost will not be accepted. The constraints of capacity and time window are relaxed as described above.

Table 4

Simulation example with combination of fleets in columns 4-10, capacities in column 2 and costs in column 3

| SIMULATION | | TOTAL KG TO BE COLLECTED = 85,611 | | | | COST < 1,550 € | | | |
|---|---|---|---|---|---|---|---|---|---|
| OUTPUTS | KG | COST (€) | T1 | T2 | T3 | T4 | T5 | T6 | T7 |
| VALID SOLUTION | 91.100 | 1.455 | 2 | 1 | 3 | 0 | 0 | 0 | 1 |
| INVALID SOLUTION | 56.900 | 1.850 | 0 | 1 | 1 | 2 | 3 | 1 | 4 |
| VALID SOLUTION | 93.200 | 1.490 | 3 | 0 | 2 | 0 | 0 | 0 | 2 |
| INVALID SOLUTION | 114.000 | 2.350 | 0 | 4 | 3 | 0 | 4 | 2 | 0 |

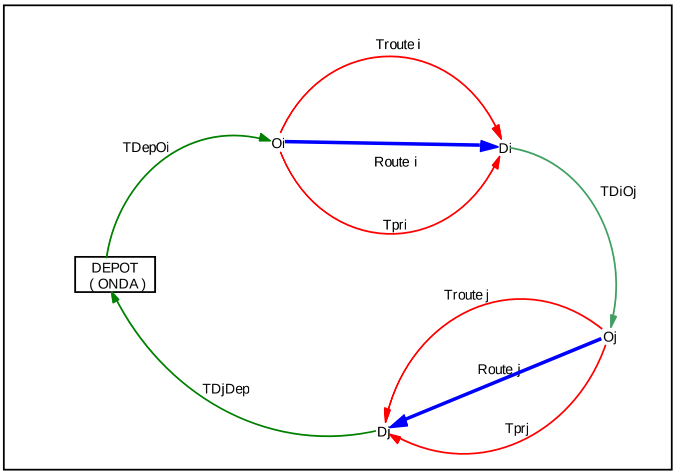
The valid solution presented in the first line of Table 4 is the simulation output of 2 vehicles “T1”, 1 vehicle “T2”, 3* T3 and 1*T7. According to Table 4, this means a fleet with a maximum capacity of 91.100 kg, which is higher than 85.611 kg to be collected. The cost to hire this fleet is 1.455 € which is lower than the cost given by the approximate algorithm, which equals 1.550 €.

## 4. Monte Carlo simulation solution

The vehicle leaves from one single depot and must travel one or several routes, with the limitations of routes. As an example, Figure 1 shows the itinerary for a vehicle that is making two routes and collect suppliers orders in those routes in a working day.

Service time is defined as the time needed for the vehicle to leave the route, enter the supplier's facilities, pick up the order and return to the route. We assumed that it is equal to 30 minutes for each supplier. Service time is added to the time needed for the vehicle to travel the whole route, which has been calculated by using Google Maps web application. The total time needed by a vehicle to collect all the orders in the routes that have been assigned to it, is calculated by the following formula:

$$Ttotal = TDep\ Oi + Trutai + Tpri + TDiOj + Trutaj + Tprj + TDjDep$$




Terms:

| | |
|---|---|
| TDep $O_i$ | time elapsed from the depot to the start of the route $i$ |
| $Troute_i$ | Travelling time needed to complete the route $i$ . |
| $Tpr_i$ | Time needed to collect all orders from all suppliers on the route $i$ (calculated service time at each supplier's facilities is 30 minutes). |
| $TDiO_j$ | Time consumed from the completion of the route $i$ to the start of the route $j$ . |
| $Troute_j$ | travelling time of the route $j$ |
| $Tpr_j$ | Time needed to collect all orders from all suppliers on the rout $j$ (30 minutes). |
| $TD_jDep$ | Return time from the end of the last route to the depot. |

Figure 1. Diagram for the calculation of vehicle total travel and service times

The optimal logistics on January 19th 2006 when the quantity collected (in kg) at each road was as presented at the Table 5 is as follows.

Table 5

Loads per route (in kg)

| ROUTE A | ROUTE B | ROUTE C | ROUTE D | ROUTE E | ROUTE F | TOTAL |
|---|---|---|---|---|---|---|
| 40.906 | 24.402 | 4.577 | 12.517 | 1.885 | 23.898 | 108.185 |

Table 6

Number of suppliers per route

| **route** | **ROUTE A** | **ROUTE B** | **ROUTE C** | **ROUTE D** | **ROUTE E** | **ROUTE F** | **TOTAL** |
|---|---|---|---|---|---|---|---|
| **number** | **14** | **22** | **4** | **2** | **1** | **18** | **61** |

Starting from approximate algorithm (Ros-McDonnell et al., 2010) the assignments of the following fleet has been determined: 3xT1, 2xT2, 1xT4, 1xT5 and 1xT6. The cost of total service has been equal to 1.820 € of. The number of operations performed by the simulation can be fixed in advance. We have made the constraints to 400.000 iterations. Using Monte Carlo Simulation by excel under given constrains, we obtained the following solutions, which are given at Figure 2. The solution at lower cost than 1.550 € and the appropriate total capacities (higher than 580.611 kg + ε) are valid solutions, marked by square. If we choose ε = 0, the optimal solution corresponds to the fleet: (3*T1, 2*T2, 2*T7) with a load capacity of 109.200 kg.

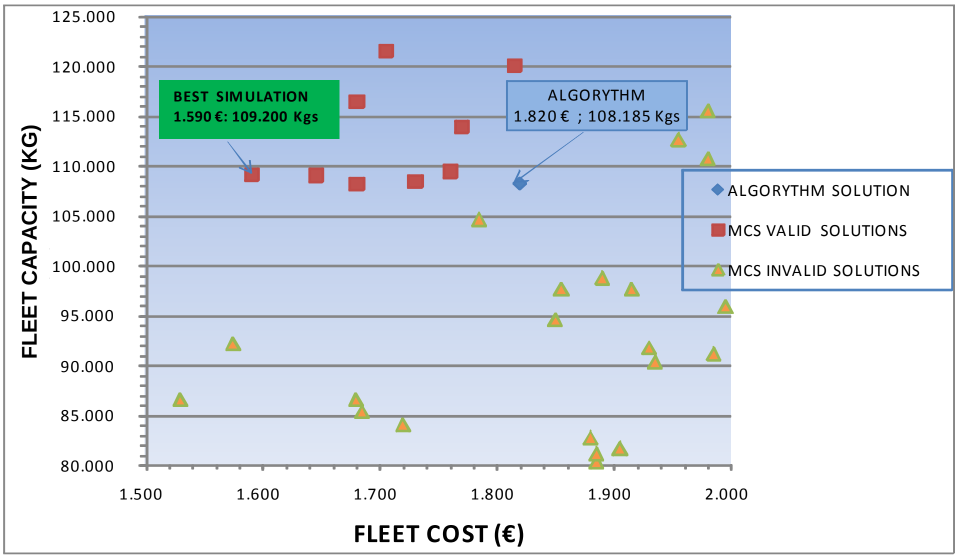



Figure 2. Solutions obtained with Monte Carlo Simulation method (MCS)

Once the assignment of routes to vehicles is completed, the solution must be validated to verify that the fleet can in fact carry out the collection of all the merchandise in prescribed time window of 8 hour which could be exceeded for less than a certain percentage (3%). Using Google Map the admissible solutions can be given as presented at Figure 3.

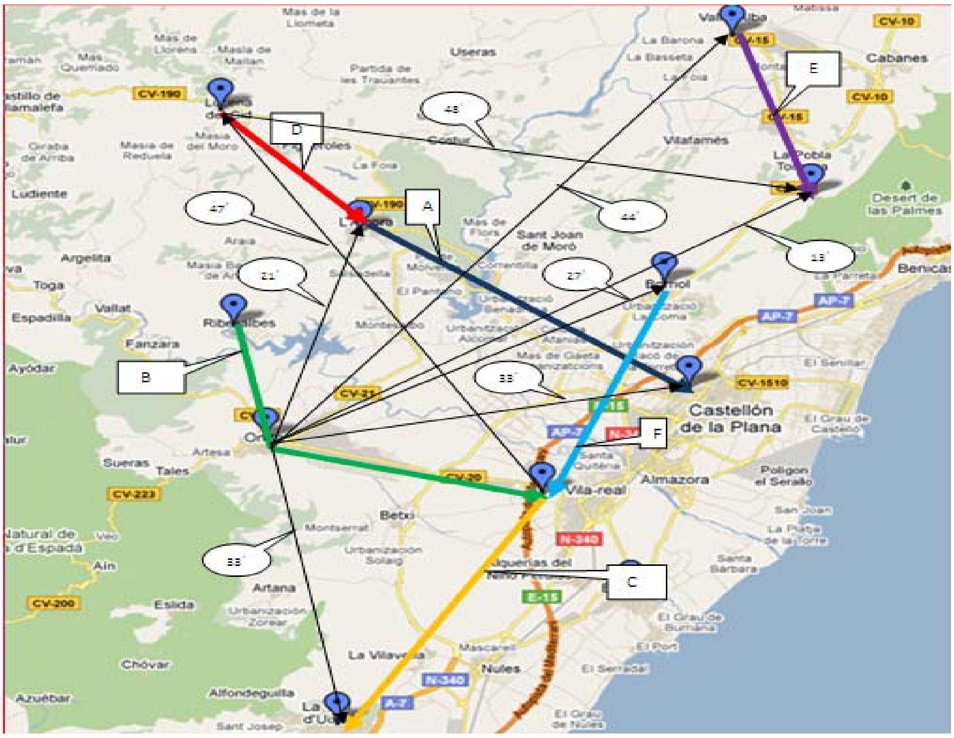



Figure 3. Route´s Map and travelling times at Google Maps web application

Figure 3 represents total travel times given by Google Maps which are used to verify if each type of vehicles meets time windows under 8 hours (Table 7). Fleet (3*T1, 2*T2, 2*T7) exceeds time window only at one vehicle for 0,25%.

Table 7

The optimal solution

| VEHICLE | ROUTE | SUPPLIERS VISITED | TIME | |
|---|---|---|---|---|
| 1 T1 | A | 10 | 6,07 | OK |
| 2 T1 | A-B | 9 | 5,98 | OK |
| 3 T1 | F-B | 10 | 6,85 | OK |

contd. table 7

| | | | | |
|---|---|---|---|---|
| 1 T2 | F-C-B | 10 | 8,02 | OK |
| 2 T2 | E-D-F | 6 | 6,42 | OK |
| 1 T7 | F-C | 7 | 5,8 | OK |
| 2 T7 | A | 8 | 6,1 | OK |

This optimal solution meets all the constraints (Table 1 and 2) and requirements (enough load capacity, costs less than algorithm of Ros McDonnells et al and time window has been exceeded for 0,25% only at only one truck T2).

## Conclusions

In real everyday operations CVRP solutions having limited and slightly relaxed time window and some other constrains, are needed to be achieved quickly, and with relevant economic consequences for a company. Orders being collected on one particular day will determine the capacity of the total fleet for the following day. This situation, together with the lack of an appropriate method, may result in solutions that are quick and effective, but not efficient enough. The Monte Carlo-based calculation method improves the results obtained by the approximate algorithm of Ros McDonnell et al. (2010) allowing higher volatility of parameters. The improvements obtained are reflected in a reduction of costs close to 20%, and in the increase of efficiency of the load of vehicles in the fleet. The advantages given by the method (as opposed to other existing methods) are in easy and faster way to obtain the better results when very small relaxation of constrains are allowed.. All the obtained solutions are operative in Spanish ceramics industry and they have been verified and validated by Google Maps time & distance tool.

## References

Bachelet B., Yon L. (2007): *Model Enhancement: Improving Theoretical Optimization with Simulation*. Simul Model Pract Th 15, pp. 703-715.

Fernandez de Cordoba P., Garcia Rafi L.M., Sanchis J.M. (1998): *A Heuristic Algorithm Based on Monte Carlo Methods for the Rural Postman Problem*. “Computers and Operation Research” 25(12), pp. 1097-1106.

Fischetti M., Toth P., Vigo D. (1994): *A Branch-and-bound Algorithm for the Capacitated Vehicle Routing Problem on Directed Graphs*. Oper. Res., 42, pp. 846-859.

Fishman G.S. (1996): *Monte Carlo. Concepts, Algorithms, and Applications*. Springer, New York.

Jackel P. (2002): *Monte Carlo Methods in Finance*. John Wiley & Sons, New Jersey.

Juan A.A., Adelantado F., Faulin J., Grasman S.E., Montoya-Torres J.R (2009): *Solving the Capacitated Vehicle Routing Problem with Maximum Travelling Distance and Service Time Requirements: An Approach Based on Monte Carlo Simulation. Proceedings – Winter Simulation Conference*, art. no. 5429663, pp. 2467-2475.

Laporte G. (1992): *The Vehicle Routing Problem: An Overview of Exact and Approximate Algorithms*. Eur J Oper Res 59, pp. 345-358.

Laporte G., Gendreau M., Potvin J.Y., Semet F. (2000): *Classical and Modern Heuristics for the Vehicle Routing Problem*. Int Tra Oper Res 7, pp. 285-300.

Toth P., Vigo D. (2002): *Models, Relaxations and Exact Approaches for the Capacitated Vehicle Routing Problem*. Discrete Appl Math 123, pp. 487-512.

Ralphs T.K., Kopman L., Pulleyblank W.R., Trotter L.E. (2003): *On the Capacitated Vehicle Routing Problem*. Math Program 94, pp. 343-359.

Ros-McDonnell L., de-la-Fuente Aragon M.V., Bogataj M. (2010): *An Approximate Algorithm for Optimal Logistics of Heavy and Variable Size Items.* Cent Eur J Oper Res doi: 10.1007/s10100-010-0155-6.

Vrugt J.A.ab, Ter Braak C.J.F. (2011): DREAM(D): *An Adaptive Markov Chain Monte Carlo Simulation Algorithm to Solve Discrete, Noncontinuous, and Combinatorial Posterior Parameter Estimation Problems*. "Hydrology and Earth System Sciences", Vol. 15, Iss. 12, pp. 3701-3713.

(WWW1): http://rd.springer.com/article/10/1007/s10100-010-0155-6