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PERFORMANCE ANALYSIS OF TWO-STAGE NETWORK PROCESSES WITH FEEDBACK FLOWS AND UNDESIRABLE FACTORS

Network data envelopment analysis (NDEA) is a non-parametric technique to evaluate the relative efficiency of decision-making units (*DMU*s) with network structures. An interesting and important network structure is a two-stage feedback process in which the outputs of the second stage are used as the inputs for the first stage. The existing approach did not consider undesirable products and from experience though we know that in real applications, network structures may consist of desirable and undesirable products outputs in which undesirable products can be used in the systems. The present paper proposes a DEA-based method for evaluating the relative efficiency of such a two-stage-feedback network structure with undesirable factors. Directional distance function along with weak disposability assumption for undesirable outputs has been used to analyse the performance of the network. A real case on ecological system of 31 regions in China is used to illustrate the applicability of the proposed approach.

Keywords: *data envelopment analysis, weak disposability, directional distance function, undesirable output, two-stage, feedback*

1. Introduction

Data envelopment analysis (DEA), introduced by Charnes et al. [2], is a non-parametric method for measuring the relative efficiency of a set of the homogeneous decision-making units (*DMU*s) with multiple incommensurate inputs and outputs. In the last

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three decades, the application of DEA has attracted the attention of many researchers. Theoretical and applied extensions of this technique in different areas show that it has been proven as an excellent and powerful tool in performance evaluation.

One of the most frequently studied subject in the field of DEA is two-stage network structures. The original application of DEA to a two-stage network did not take the internal structure of the network into consideration and the system was considered as a black-box. Ignoring internal structures leads to incorrect evaluations. However, in most situations, the internal structures have a significant impact on the overall performance of the system. So, there is a need to study network structures more precisely and specifically.

Network DEA with internal structure was introduced by Färe and Grosskopf [6]. Their network model allows us to look into the boxes. Afterwards, many studies were conducted in systems with two-stage structures and a lot of models are proposed for measuring the relative efficiency of such processes. Kao and Hwang [8] proposes an efficiency decomposition in a two-stage system. In 2009, Chen et al. [3] provide an additive efficiency decomposition in two-stage DEA. In a new model, Chen et al. [4] use the additive efficiency measure to evaluate the relative efficiency of a two-stage process with shared non-separable inputs. Zha and Liang [20] introduce a product-form cooperative efficiency model for two-stage network processes with freely shared inputs. They suggest a heuristic method to transform their non-linear model into a parametric linear form.

The above-mentioned studies do not consider undesirable products in the process. Fukuyama and Weber [7] suggest a model constructed on the slack-based measure to evaluate the inefficiency scores in two-stage processes with undesirable outputs. Lozano et al. [13] propose a directional distance approach for a two-stage network with undesirable outputs. In the following, Maghbouli et al. [14] utilise the weak disposability assumption to model undesirable outputs and develop non-cooperative and leader-follower models for two-stage network structures with undesirable outputs. Wu et al. [17] use an additive efficiency model to evaluate the overall efficiency of two-stage processes with undesirable outputs. All of these studies consider intermediate measures and final outputs, and feedback flows are not focused on. However, an important class of two-stage network structures is that which consists of feedback flows that are produced from final stage and re-consumed by the first stage. Such variables that simultaneously play input and output roles are called dual-role variables [5, 6].

The problem of recyclable products is studied by Liang et al. [11]. They propose a method for measuring the relative efficiency of two-stage-feedback processes (with only one output for the second stage). Then, they apply their model to evaluate 50 Chinese universities. Unfortunately, by increasing the number of inputs and outputs, the transformation of the model into a linear form is very difficult.

Amirteimoori et al. [1] study the problem of recyclable products. They propose a DEA-based model when there are feedback products from stage 2 to stage 1. As based on the additive efficiency measure and non-cooperative game, Wu et al. [18] propose a model to evaluate the relative efficiency of two-stage processes with shared inputs and undesirable feedback outputs. In the following paper, Wu et al. [19] propose a method for assessing efficiency of two-stage feedback processes with undesirable outputs. Their model has been applied to evaluate the relative efficiency of iron and steel industries of China. A similar method is proposed by Li et al. [12] by which the efficiency of the ecological system in some regions of China is evaluated. The approach is based on game theory of Liang et al. [10] in which stage 1 is considered as a leader. Note that in the two-stage feedback structure of Li et al. [12] there are undesirable dual-role variables and final undesirable output.

Due to the importance and applicability of network systems, the present paper deals with the problem of performance evaluation in two-stage network structures when there are feedback flows in the system. In the system under our consideration, in both stages, undesirable factors play an important role in the process. In order to model the undesirable products, the weak disposability assumption of Shephard [15] is used. After constructing the technology set, directional distance function is used to evaluate performance. The linearity of the model is preserved and our proposed model not only provides an efficiency score to each stage but also evaluates the whole system.

The paper is organized as follows: in the section below, we briefly review two-stage processes with undesirable outputs. In the next section, our two-stage network with feedback flows is introduced. The applicability of the proposed model is illustrated by the real case of China's ecological system. Conclusions appear in the last section.

2. Two-stage process with undesirable factor

Consider a two-stage process (Fig. 1). Suppose there are K DMUs and each DMU_k, $k = 1, ..., K$ consists of two stages. The first stage uses $x_k = (x_{1k}, ..., x_{Nk}) \geq 0$ to produce two types of outputs: the final desirable output $v_k = (v_{1k}, ..., v_{Mk}) \geq 0$ and the undesirable output $w_k = (w_{1k}, ..., w_{Jk}) \geq 0$. The second stage of DMU_k uses its own inputs $z_k = (z_{1k}, ..., z_{Tk}) \geq 0$ and the dedicated inputs $w_k = (w_{1k}, ..., w_{Jk}) \geq 0$ to produce the desirable and final output $y_k = (y_{1k}, ..., y_{Rk}) \geq 0$.

The production technology set in a general form can be written as follows:

$$P = \{(x, y) : x \text{ can produce } y\}$$

in which y is a mixture of desirable and undesirable outputs. The following weak disposability assumption of Shephard [15] is used to incorporate undesirable products in the model:

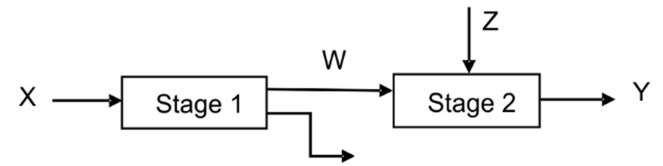

Fig. 1. Two-stage process of DMU_k

Definition 1. Desirable and undesirable outputs are weakly disposable if and only if $(v, w) \in P(x)$ and $0 \le \theta \le 1$, then $(\theta v, \theta w) \in P(x)$, $x \in R_+^N$.

The linear technology set of the foregoing two-stage process under weak disposability assumption is as follows:

$$
\begin{aligned}
& T = \{(x, v, w, z, y) \big| \\
& \text{Stage 1 constraints} \\
& \sum_{k=1}^{K} (\rho^k + \mu^k)\, x_n^k \le x_n, \; n = 1, ..., N \\
& \sum_{k=1}^{K} \rho^k v_m^k \ge v_m, \; m = 1, ..., M \\
& \sum_{k=1}^{K} \rho^k w_j^k = w_j, \; j = 1, ..., J \\
& \text{Stage 2 constraints} \\
& \sum_{k=1}^{K} \rho^k w_j^k = w_j, \; j = 1, ..., J \\
& \sum_{k=1}^{K} (\rho^k + \mu^k)\, z_t^k \le z_t, \; t = 1, ..., T \\
& \sum_{k=1}^{K} (\rho^k + \mu^k)\, y_r^k \ge y_r, \; r = 1, ..., R \\
& \text{Generic constraints} \\
& \sum_{k=1}^{K} (\rho^k + \mu^k) = 1 \\
& \rho^k, \; \mu^k \ge 0, \; k = 1, ..., K\}
\end{aligned}
\tag{1}
$$

ρ^k and μ^k are decision variables and since the right-hand-side of the constraints is free of variables, we can use the following directional distance function model:

$$
\begin{aligned}
& e_o^* = \max\left[\sum_{n=1}^{N}\alpha_n + \sum_{m=1}^{M}\beta_m + \sum_{j=1}^{J}\gamma_j + \sum_{t=1}^{T}\theta_t + \sum_{r=1}^{R}\varphi_r\right] \\
& \text{s.t.} \\
& \text{Stage 1 constraints} \\
& \sum_{k=1}^{K}(\rho^k + \mu^k)\, x_n^k \le x_n^o - \alpha_n d_n^{(x)},\ n = 1,\dots,N \\
& \sum_{k=1}^{K}\rho^k v_m^k \ge v_m^o + \beta_m d_m^{(v)},\ m = 1,\dots,M \\
& \sum_{k=1}^{K}\rho^k w_j^k = w_j^o - \gamma_j d_j^{(w)},\ j = 1,\dots,J \\
& \text{Stage 2 constraints} \\
& \sum_{k=1}^{K}\rho^k w_j^k = w_j^o - \gamma_j d_j^{(w)},\ i = 1,\dots I \\
& \sum_{k=1}^{K}(\rho^k + \mu^k)\, z_t^k \le z_t^o - \theta_t d_t^{(z)},\ t = 1,\dots,T \\
& \sum_{k=1}^{K}(\rho^k + \mu^k)\, y_r^k \ge y_r^o + \varphi_r d_r^{(y)},\ r = 1,\dots,R \\
& \text{Generic constraints} \\
& \sum_{k=1}^{K}(\rho^k + \mu^k) = 1 \\
& \rho^k,\ \mu^k \ge 0,\ k = 1,\dots,K \\
& \alpha_n,\ \beta_m,\ \gamma_j,\ \theta_t,\ \varphi_r \ge 0,\ \text{for all } n,\ m,\ j,\ t,\ r
\end{aligned}
\tag{2}
$$

Model (2) is a directional distance model that leads to the contraction of inputs and undesirable outputs and expansion of desirable outputs in the direction

$$\mathbf{d} = (d^{(x)}, d^{(v)}, d^{(w)}, d^{(z)}, d^{(f)}, d^{(y)}, d^{(h)})$$

The optimal value e_o^* is the overall efficiency of DMU_o. If e_o^* is equal to zero, then the whole system is efficient; otherwise, it is inefficient. Regarding the objective function, we can easily obtain measure of efficiency for both stages. The term

$\left[\sum_{n=1}^{N} \alpha_n + \sum_{m=1}^{M} \beta_m + \sum_{j=1}^{J} \gamma_j \right]$ is the measure of directional efficiency for the first stage, and the term $\left[\sum_{j=1}^{J} \gamma_j + \sum_{t=1}^{T} \theta_t + \sum_{r=1}^{R} \varphi_r \right]$ is the measure of directional efficiency for the second stage. So, we can easily calculate the efficiencies of the stages.

3. The proposed approach

In this section, we consider a two-stage feedback process in which both its intermediate measures and final outputs are the desirable and undesirable product. Figure 2 shows the structure of the network that we consider in this section.

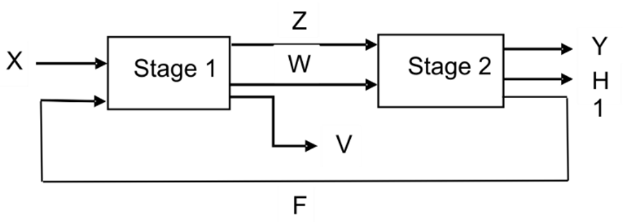

Fig. 2. Two-stage feedback process of DMU_k

Suppose there are K DMU_k, $k = 1, ..., K$. The first stage of a specific DMU_k uses $x_k = (x_{1k}, ..., x_{Nk}) \geq 0$ to produce three types of outputs: the final desirable output $v_k = (v_{1k}, ..., v_{Mk}) \geq 0$, the desirable output $z_k = (z_{1k}, ..., z_{Tk}) \geq 0$, and the undesirable output $w_k = (w_{1k}, ..., w_{Jk}) \geq 0$. The second stage of DMU_k uses the desirable and undesirable outputs $z_k = (z_{1k}, ..., z_{Tk}) \geq 0$ and $w_k = (w_{1k}, ..., w_{Jk}) \geq 0$ from the first stage as input to produce three types of outputs: the final desirable output $h_k = (h_{1k}, ..., h_{Ik}) \geq 0$, the final undesirable output $y_k = (y_{1k}, ..., y_{Rk}) \geq 0$ and the feedback output $f_k = (f_{1k}, ..., f_{Sk}) \geq 0$ which is used as input for the first stage.

As based on the axioms of observing activities, convexity, weak disposability of desirable and undesirable outputs and free disposability for inputs and desirable outputs, the minimal production technology set for the two-stage feedback process with the structure as in Fig. 2 is as follows:

$$
\begin{aligned}
&T = \Big\{(x, v, w, z, f, y, h)\Big| \\
&\text{Stage 1 constraints} \\
&\sum_{k=1}^{K} \lambda^k x_n^k \le x_n,\ n = 1, \dots, N \\
&\sum_{k=1}^{K} \theta^k \lambda^k v_m^k \ge v_m,\ m = 1, \dots, M \\
&\sum_{k=1}^{K} \theta^k \lambda^k w_j^k = w_j,\ j = 1, \dots, J \\
&\sum_{k=1}^{K} \theta^k \lambda^k z_t^k - p_t = z_t,\ t = 1, \dots, T \\
&\sum_{k=1}^{K} \theta^k \lambda^k f_s^k - q_s = f_s,\ s = 1, \dots, S \\
&\text{Stage 2 constraints} \\
&\sum_{k=1}^{K} \theta^k \lambda^k w_j^k = w_j,\ j = 1, \dots, J \\
&\sum_{k=1}^{K} \theta^k \lambda^k z_t^k - p_t = z_t,\ t = 1, \dots, T \\
&\sum_{k=1}^{K} \theta^k \lambda^k f_s^k - q_s = f_s,\ s = 1, \dots, S \\
&\sum_{k=1}^{K} \theta^k \lambda^k y_r^k = y_r,\ r = 1, \dots, R \\
&\sum_{k=1}^{K} \theta^k \lambda^k h_i^k \ge h_i,\ i = 1, \dots, I \\
&\text{Generic constraints} \\
&\sum_{k=1}^{K} \lambda^k = 1, \\
&\lambda^k \ge 0,\ 0 \le \theta^k \le 1,\ k = 1, \dots, K \\
&p_t,\ q_s \ge 0 \ \text{ for all } \ s, t
\end{aligned}
\tag{3}
$$

It can be seen that technology (3) is non-linear. By partitioning the intensity weights λ^k into two parts as $\lambda^k = \rho^k + \mu^k$ (whereby $\mu^k = (1-\theta^k)\,\lambda^k$, $\rho^k = \theta^k \lambda^k$), using the manner of Kuosmanen [9], the above technology is transformed into a linear form as follows:

$$
\begin{aligned}
& T = \{(x, v, w, z, f, y, h) \mid \\
& \text{Stage 1 constraints} \\
& \sum_{k=1}^{K} (\rho^k + \mu^k)\ x_n^k \le x_n,\ n = 1, \dots, N \quad && \text{a)} \\
& \sum_{k=1}^{K} \rho^k v_m^k \ge v_m,\ m = 1, \dots, M && \text{b)} \\
& \sum_{k=1}^{K} \rho^k w_j^k = w_j,\ j = 1, \dots, J && \text{c)} \\
& \sum_{k=1}^{K} \rho^k z_t^k - p_t = z_t,\ t = 1, \dots, T && \text{d)} \\
& \sum_{k=1}^{K} \rho^k f_s^k - q_s = f_s,\ s = 1, \dots, S && \text{e)} \\
& \text{Stage 2 constraints} \\
& \sum_{k=1}^{K} \rho^k w_j^k = w_j,\ j = 1, \dots, J && \text{f)} \\
& \sum_{k=1}^{K} \rho^k z_t^k - p_t = z_t,\ t = 1, \dots, T && \text{g)} \\
& \sum_{k=1}^{K} \rho^k\ f_s^k - q_s = f_s,\ s = 1, \dots, S && \text{h)} \\
& \sum_{k=1}^{K} \rho^k y_r^k = y_r,\ r = 1, \dots, R && \text{i)} \\
& \sum_{k=1}^{K} \rho^k\ h_i^k \ge h_i,\ i = 1, \dots, I && \text{j)} \\
& \text{Generic constraints} \\
& \sum_{k=1}^{K} (\rho^k + \mu^k) = 1 && \text{k)} \\
& \rho^k,\ \mu^k \ge 0,\ k = 1, \dots, K && \text{l)} \\
& p_t,\ q_s \ge 0 \ \text{ for all } s,\ t && \text{m)}
\end{aligned}
\tag{4}
$$

Note that ρ and μ, and slack variables p_t, q_s are the unknown variables for the algebraic representation of production technology set for the two-stage feedback process under variable returns to scale and weak disposability assumption for the desirable and undesirable outputs. Moreover, $z_{tk}(t=1,...,T)$ and $f_{sk}(s=1,...,S)$ play the role of input for stage one and role of desirable output for stage two, simultaneously. Hence, positive slack variables $p_t(t=1,...,T)$ and $q_s(s=1,...,S)$ are considered in the constraints (4d, e, g, h).

By applying the directional distance function aiming to acquire the maximum decrease in inputs and undesirable outputs and the maximum increase in desirable outputs in direction of the suitable vector **d**, the following model is proposed to evaluate the efficiency of *DMU*$_o$:

$$e_o^* = \max\left[\sum_{n=1}^{N}\alpha_n + \sum_{t=1}^{T}\theta_t + \sum_{j=1}^{J}\gamma_j + \sum_{m=1}^{M}\beta_m + \sum_{r=1}^{R}\varphi_r + \sum_{i=1}^{I}\lambda_i + \sum_{s=1}^{S}\zeta_s\right]$$

s.t.

Stage 1 constraints

$$\sum_{k=1}^{K}(\rho^k + \mu^k)\, x_n^k \le x_n^o - \alpha_n d_n^{(x)},\ n=1,...,N \quad \text{a)}$$

$$\sum_{k=1}^{K}\rho^k v_m^k \ge v_m^o + \beta_m d_m^{(v)},\ m=1,...,M \quad \text{b)}$$

$$\sum_{k=1}^{K}\rho^k w_j^k = w_j^o - \gamma_j d_j^{(w)},\ j=1,...,J \quad \text{c)}$$

$$\sum_{k=1}^{K}\rho^k z_t^k - p_t = z_t^o + \theta_t d_t^{(z)},\ t=1,...,T \quad \text{d)} \qquad (5)$$

$$\sum_{k=1}^{K}\rho^k f_s^k - q_s = f_s^o + \zeta_s d_s^{(f)},\ s=1,...,S \quad \text{e)}$$

Stage 2 constraints

$$\sum_{k=1}^{K}\rho^k w_j^k = w_j^o - \gamma_j d_j^{(w)},\ i=1,...,I \quad \text{f)}$$

$$\sum_{k=1}^{K}\rho^k z_t^k - p_t = z_t^o + \theta_t d_t^{(z)},\ t=1,...,T \quad \text{g)}$$

$$\sum_{k=1}^{K}\rho^k f_s^k - q_s = f_s^o + \zeta_s d_s^{(f)},\ s=1,...,S \quad \text{h)}$$

$$\sum_{k=1}^{K}\rho^k y_r^k = y_r^o - \varphi_r d_r^{(y)},\ r=1,...,R \quad \text{i)}$$

$$\sum_{k=1}^{K}\rho^k h_i^k \ge h_i^o + \lambda_i d_i^{(h)},\ i=1,...,I \quad \text{j)}$$

Generic constraints

$$\sum_{k=1}^{K}(\rho^k+\mu^k)=1 \qquad \text{k)}$$

$$\rho^k,\ \mu^k \geq 0,\ k=1,\dots,K \qquad \text{l)}$$

$$p_t,\ q_s \geq 0 \ \text{ for all } s,\ t \qquad \text{m)}$$

$$\alpha_n,\ \theta_t,\ \gamma_j,\ \beta_m,\ \varphi_r,\ \lambda_i,\ \zeta_s \geq 0,\ \text{ for all } n,\ t,\ j,\ m,\ i,\ s \qquad \text{n)}$$

Clearly, the model (5) is a linear programming problem and it is always feasible and bounded. Using the user-defined directional vector

$$\mathbf{d}=(d^{(x)},d^{(v)},d^{(w)},d^{(z)},d^{(f)},d^{(y)},d^{(h)})$$

we can easily evaluate DMU_o and in this sense, the relative efficiencies of the stages are calculated as follows:

$$e_o^{*\text{ stage 1}}=\sum_{n=1}^{N}\alpha_n^*+\sum_{m=1}^{M}\beta_m^*+\sum_{j=1}^{J}\gamma_j^*+\sum_{t=1}^{T}\theta_t^*+\sum_{s=1}^{S}\zeta_s^* \qquad (6)$$

$$e_o^{*\text{ stage 2}}=\sum_{j=1}^{J}\gamma_j^*+\sum_{t=1}^{T}\theta_t^*+\sum_{s=1}^{S}\zeta_s^*+\sum_{r=1}^{R}\varphi_r^*+\sum_{i=1}^{I}\lambda_i^* \qquad (7)$$

Model (5) is divided into three parts; the first two consist of ten constraints related to stage 1 and stage 2, and the third part contains generic constraints. The abatement and expansion factors $\alpha_n,\ \theta_t,\ \gamma_j,\ \beta_m,\ \varphi_r,\ \lambda_i,\ \zeta_s \geq 0$ show the number of units that must be deducted (added) from (to) the number of inputs (outputs) of each unit under evaluation, in order to make them efficient. In generic constraints, $\sum_{k=1}^{K}(\rho^k+\mu^k)=1$ is added to guarantee the convexity assumption.

Definition 2. If the optimal solution of the model (5) is equal to zero, then the unit under evaluation, DMU_o, is said to be efficient; otherwise, it is called inefficient. Inefficient units can be efficient by projecting onto the efficient frontier. The slack variables $s_n^{*(x)}: n=1,\dots,N$, $s_m^{*(v)}: v=1,\dots,M$ and $s_i^{*(h)}: i=1,\dots,I$ are used to project inefficient units to the efficient frontier. The projection point of the inefficient DMU_o is as follows:

$$\begin{cases} \bar{x}_n^o = x_n^o - \alpha_n^* d_n^{(x)} - s_n^{*(x)}, \ n = 1, ..., N \\ \bar{v}_m^o = v_m^o + \beta_m^* d_m^{(v)} + s_m^{*(v)}, \ m = 1, ..., M \\ \bar{w}_j^o = w_j^o - \gamma_j^* d_j^{(w)}, \ j = 1, ..., J \\ \bar{z}_t^o = z_t^o + \theta_t^* d_t^{(z)} + p_t^*, \ t = 1, ..., T \\ \bar{f}_s^o = f_s^o + \zeta_s^* d_s^{(f)} + q_s^*, \ s = 1, ..., S \\ \bar{y}_r^o = y_r^o - \varphi_r^* d_r^{(y)}, \ r = 1, ..., R \\ \bar{h}_i^o = h_i^o + \lambda_i^* d_i^{(h)} + s_i^{*(h)}, \ i = 1, ..., I \end{cases} \tag{8}$$

It is easy to show that the projection point defined in (8) is an efficient point.

4. An illustrated application

To illustrate the real applicability to the proposed approach, we apply it to a real data set consisting the ecosystem of 31 regions in China [12].

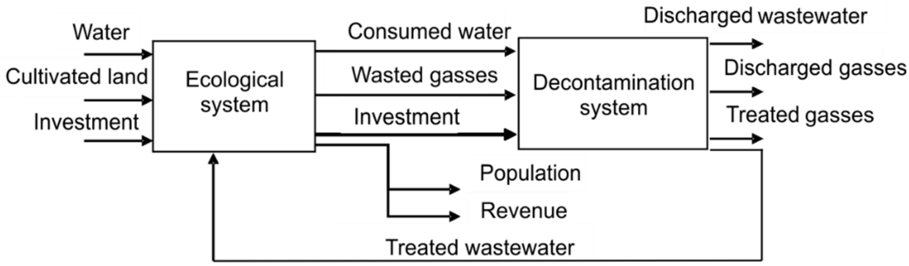

Fig. 3. Two-stage feedback ecosystem

The ecosystem of each 31 regions is divided into two stages: an ecological system (stage 1) and a decontamination system (stage 2). The inputs and outputs of two stages are illustrated as below.

Stage 1 (ecological system) consumes three types of inputs: water (x_1), investment (x_2), cultivated land (x_3) to produce five types of outputs: consumed water (w_1), wasted gasses (w_2) such as nitrogen oxides, sulphur dioxide and soot as undesirable outputs, investment (z) as a desirable output, investment (v_1) and population (v_2) as final desirable outputs.

Stage 2 (decontamination stage) consumes undesirable outputs: consumed water (w_1), wasted gasses (w_2), and desirable output: investment (z) from stage 1 as inputs to produce four types of outputs: discharged waste water (y_1), discharged gasses (y_2) such as nitrogen oxides, sulphur dioxide and soot as final undesirable outputs, treated gasses

(h) as final desirable outputs, and finally, treated wastewater (f) as a feedback output that flows back to stage 1 and is also used as an input for this stage. Data for stage 1 and stage 2 of the ecological system of 31 regions are displayed in Table 1.

Table 1. Data for a real case

Region	x_1	x_2	x_3	v_1	v_2	z	w_1	w_2	h	y_1	y_2	f
Beijing	231.7	8870.8	35.9	17536.8	2069	342.6	19.6	338167	399	14.0175	337768	0.0099
Tianjin	441.1	8853.6	23.1	12736.4	1413	157.6	15.5	642808	292	8.2767	642516	0.0046
Hebei	6317.3	20106	195.3	26008.9	7288	486.1	144	4338188	212	30.5688	4337976	0.0085
Shanxi	4055.8	8311.7	73.4	11784.6	3611	328.2	56	3616588	118	13.4269	3616470	0.0029
Inner Mongolia	7147.2	12174.6	184.4	15435.5	2490	445.1	123.1	3636835	16	10.2404	3636819	0.002
Liaoning	4085.3	24225.6	142.2	24163	4389	683.4	91.1	2821292	126	23.8663	2821166	0.0106
Jilin	5534.6	9694.4	129.8	12585	2750	103.4	67.1	1244090	36	11.9448	1244054	0.0061
Heilong -jiang	11830.1	10400.5	358.9	13473.5	3834	218.1	204.71	1994184	16	16.2568	1994170	0.0021
Shanghai	244	6961.2	116	20047.6	2380	134.1	20	716984	348	21.8889	716636	0.0355
Jiangsu	4763.8	36552.9	552.2	53401.1	7920	657.1	281.9	2914784	964	59.7929	2913820	0.0282
Zhejiang	1920.9	19243.6	198.1	34289.9	5477	375.4	109.7	1688645	675	42.0466	1687970	0.0495
Anhui	5730.2	16587.8	292.6	16881.9	5988	330.2	148.2	1907916	999	25.4155	1906917	0.0174
Fujian	1330.1	13850.7	200.1	19479.3	3748	222.5	90.6	1091097	60	25.5999	1091037	0.0264
Jiangxi	2827.1	12103.1	242.5	12632.8	4504	316.1	107.8	1502155	34	1.9859	1502121	0.026
Shandong	7515.3	33538.2	221.8	49274.1	9685	739.1	143.9	4183047	436	47.8769	4182611	0.0331
Henan	7926.4	21710.1	238.6	29389.8	9406	209.5	134.5	3501630	118	40.3526	3501512	0.0142
Hubei	4664.1	16884.5	299.3	22373.9	5779	285.5	132.6	1612032	173	28.9946	1611859	0.0254
Hunan	3789.4	15898.5	328.8	21963.9	6639	190.3	136.5	1592897	64	30.3812	1592833	0.0402
Guangdong	2830.7	22005.9	451.5	56805.7	10594	260.2	172.8	2430904	1760	83.8009	2429144	0.0542
Guangxi	4217.5	10506.8	303	12844.6	4862	190.5	129.9	1302102	153	24.5457	1301949	0.0121
Hainan	727.5	2755.8	45.3	2810.8	887	44.7	20.7	154135	134	3.7051	154001	0.0052
Chongqing	2235.9	10312	82.9	11222.7	2945	186.9	41.5	1129714	28	13.2288	1129686	0.0142
Sichuan	5947.4	18204	245.9	23694.5	8076	178.3	120.8	1819281	693	28.3427	1818588	0.023
Guizhou	4485.3	5949.1	100.8	6783.3	3484	68.9	46.6	1899123	6	9.1403	1899117	0.0052
Yunnan	6072.1	8047.5	151.8	10177.1	4959	132.4	84.9	1607208	35	15.3499	1607173	0.0111
Tibet	361.6	696.7	29.8	697	308	4	24.3	55088	4	0.4681	55084	0.0002
Shanxi	4050.3	13222.3	88	14273.1	3753	180.6	51.4	2113935	19	12.8663	2113916	0.0086
Gansu	4658.8	5365.8	123.1	5528.8	2578	121.4	80.5	1253438	230	6.2777	1253208	0.0036
Qinghai	542.7	1982.7	27.4	1869.4	573	24.1	17.2	436305	3	2.1987	436302	0.0007
Ningxia	1107.1	1998.3	69.4	2285.6	647	55.7	31.3	1060375	6	3.8937	1060369	0.0011
Xinjiang	4124.6	6572.9	590.1	7250.2	2233	255.1	396.1	2311731	47	9.3769	2311684	0.0041

By considering directional vector $\mathbf{d} = (x_o,\ v_o,\ w_o,\ z_o,\ f_o,\ y_o,\ h_o)$, the efficiencies of ecological system of the 31 regions in China were evaluated by model (5) and the results obtained through the efficiency scores of model (5) and the proposed model by Li et al. [12] are presented in Table 2. The projection points obtained from (8) are shown in Table 3. Columns 2, 3 and 4 from Table 2 show the total system efficiency score and each of its two stages, respectively. Moreover, column 5 from Table 2 shows the efficiency score of the proposed method by Li et al. [12]. The results obtained through model (5) in the directional vector of **d** (in direction of decreasing inputs and undesirable outputs, and also increasing desirable outputs) show that the two-stage feedback process is overall-efficient if and only if both two stages are efficient. Furthermore, as we see, the obtained results show that the number of efficient units obtained through model (5) in the direction of the mentioned directional vector is more than the number of efficient units obtained through the proposed model by Li et al. [12].

Table 2. Results of the model (5) and the proposed model by Li et al. [12]

Region	Model (5)			Supper-efficiency measure	Model by Li et al.
	e_o^*	$e_o^{*\text{stage 1}}$	$e_o^{*\text{stage 2}}$		e_o^*
Beijing	0.0000	0.0000	0.0000	1.7021	1.0000
Tianjin	0.0000	0.0000	0.0000	1.1516	1.0000
Hebei	0.0000	0.0000	0.0000	1.0112	0.8932
Shanxi	0.0000	0.0000	0.0000	1.0835	0.9866
Inner Mongolia	0.0000	0.0000	0.0000	1.0133	0.8554
Liaoning	0.0000	0.0000	0.0000	1.0394	0.7600
Jilin	14.7467	5.3561	13.4359	–	0.8522
Heilongjiang	41.5492	9.6184	40.3678	–	0.8538
Shanghai	0.0000	0.0000	0.0000	1.3494	1.0000
Jiangsu	0.0000	0.0000	0.0000	1.0636	0.9231
Zhejiang	0.0000	0.0000	0.0000	1.108	1.0000
Anhui	0.0000	0.0000	0.0000	1	0.9876
Fujian	10.0903	1.2178	9.4809	–	0.8594
Jiangxi	0.0000	0.0000	0.0000	1.0135	0.9963
Shandong	0.0000	0.0000	0.0000	1.2587	0.9666
Henan	0.0000	0.0000	0.0000	1.0216	0.9350
Hubei	4.4209	1.3168	3.4918	–	0.7946
Hunan	0.0000	0.0000	0.0000	1	0.9690
Guangdong	0.0000	0.0000	0.0000	1.4403	1.0000
Guangxi	4.0715	2.1598	2.9077	–	0.8694
Hainan	0.0000	0.0000	0.0000	1.0197	0.9826
Chongqing	0.0000	0.0000	0.0000	1	0.8943
Sichuan	0.0000	0.0000	0.0000	1.0093	0.9805
Guizhou	0.0000	0.0000	0.0000	1.0086	0.9893
Yunnan	0.0000	0.0000	0.0000	1.0311	0.9825
Tibet	0.0000	0.0000	0.0000	1.7863	1.0000
Shanxi	0.0000	0.0000	0.0000	1	0.8887
Gansu	0.0000	0.0000	0.0000	1.0045	0.9821
Qinghai	0.0000	0.0000	0.0000	1.038	1.0000
Ningxia	13.9640	3.1753	12.3814	–	0.9314
Xinjiang	0.0000	0.0000	0.0000	1.0027	0.8856

As a final point, in this example, the number of regions is 31 with three inputs, three intermediate measures and five final outputs along with one feedback measure. This clearly violates the rule of thumb in DEA. So, the number of efficient regions is increased. This shows that there is a need to run one of the existing ranking techniques on the results. A super-efficiency model is run and the results are given in the fifth column of Table 2.

5. Conclusions

An interesting and frequently studied issue in the field of DEA is performance analysis of two-stage network structures. A lot of extensions and developments of two-stage network structures have been given by different researchers. A new network-structure that has recently attracted considerable attention among production analysts is two-stage network structures with feedback flows. Such a structure frequently occurs in real life situations, such as in ecological systems. This paper develops a DEA model for assessing the relative performance of a two-stage system when there are both undesirable products in the stages and feedback flows from the second stage to the first stage. The proposed approach not only provides an efficiency score to the whole system, but it also gives the efficiencies of the stages. Moreover, it preserves the linear structure of the model.

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