Methods of estimating the mean duration of enterprises and the mean additional duration of enterprises in Lodzkie Voivodship

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Abstract: In a duration analysis, the time of enterprises functioning, i.e. from the foundation to their liquidation, is represented by a random variable. The basic characteristic functions used for time description include: density function, distribution function, survival function and hazard function. The first one $f(t)$ defines the probability of enterprise liquidation in the period denoted as t, i.e. in $\langle t^*, t^* + 1 \rangle$ period of time; the second one $F(t)$ expresses the probability of enterprise persistence from the moment of its establishment to the maximum end of the period denoted as t, end of $\langle 0, t^* \rangle$ period of time; the third one $S(t)$ defines the probability of enterprise survival for longer than the end of the period denoted as t, end of $\langle 0, t^* \rangle$ period of time; and the last one $h(t)$ assesses the intensity of the process of enterprise liquidation in the period denoted as t. The article presents theoretical and practical aspects of determining selected distribution parameters of time duration: the mean duration of enterprises and the mean additional duration of enterprises, which are also used for characterizing patterns of companies' survival. The considerations are supported by calculations of the above-mentioned measures for subsequent cohorts of established enterprises (including the liquidated ones) in Lodzkie Voivodship in the years 2001-2015.

Keywords: mean duration of enterprises, mean additional duration of enterprises, Lodzkie Voivodship, Lodz, Piotrkow Trybunalski, Skierniewice

JEL codes: C10, C14, C41

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1. Introduction

A cohort analysis of enterprises' duration is an analysis of one and the same generation of entities and consists in observation of changes occurring in their structure over time. Cohort tables compile the number of entities which experienced the event of liquidation and the number of entities persisting in subsequent time intervals. On the basis of this data included in cohort tables, apart from basic functions, such as density function – probability of enterprise liquidation; distribution function – probability of enterprise persistence; survival function – probability of enterprise survival and hazard function – enterprises liquidation intensity (Machin et al., 2006), one can estimate parameters of the distribution of time duration as well as those of the conditional distribution of time duration of enterprises. The parameters of the enterprises' survival process, which take the form of mean duration or mean additional duration of enterprises, complement the above functions and are useful in characterizing patterns of companies' survival.

The aim of this paper is to present theoretical and practical aspects of determining: mean duration and mean additional duration of enterprises based on annual cohort tables. The theoretical considerations will be illustrated with calculations for the established enterprises (including the liquidated ones) in Lodzkie Voivodship in the years 2001-2015; with a special focus on three cities with the rights of a county (the *'powiat'* status): Lodz, Piotrkow Trybunalski and Skierniewice. The results of the analyses may contribute to implementing a more effective regional policy which would in turn foster the development of entrepreneurship.

2. Data characteristics

In this article, the individual data relating to 115.6 thousand enterprises[1] established in Lodzkie Voivodship in the years 2001-2015, which were liquidated by the end of 2015 were used (see: Table 1). Among the enterprises mentioned above, 41.3 thousand were established and liquidated in Lodz, 3.9 thousand were established-liquidated in Piotrkow Trybunalski and 2.3

[1] The enterprises were selected according to the methodology of survey *SP-3 Report on economic activity of enterprises* (annual survey of enterprises with up to 9 persons employed), conducted by the Central Statistical Office and the Statistical Office in Lodz. The authors omitted the criterion of size of the enterprise (number of employed persons), because the analysis was to concern all enterprises conducting manufacturing, trade or service activity on the free market basis – for profit and for self-employment (regardless of their size) – see: Programme of Statistical Surveys of Official Statistics for 2015 – the Council of Ministers Regulation of 27 August 2014, Journal of Laws 2014, item 1330 with subsequent amendments: 377-379.

thousand started and ended their business activity in Skierniewice. The cities chosen for the analysis: Lodz, Piotrkow Trybunalski and Skierniewice are ones with the rights of a county (the '*powiat'* status) in Lodzkie Voivodship.

The data on the enterprises established and liquidated in Lodzkie Voivodship were obtained from the Statistical Office in Lodz and contained: REGON[2], location, form of ownership, legal form, business activity (according to the Polish Classification of Activities – PKD 2007), respectively the date of the firm's establishment and the date of liquidation of the business and the enterprise's size – measured with the number of employed persons (up to 9, 10-49, 50 and more).

Table 1. Number of the enterprises established-liquidated in Lodzkie Voivodship in the years 2001-2015

| Year of foundation | Enterprises liquidated | | | |
| | Lodzkie Voivodship | of which: | | |
		Lodz	Piotrkow Trybunalski	Skierniewice
Total	**115596**	**41293**	**3948**	**2255**
2001	10282	3696	413	215
2002	8952	3271	344	204
2003	8977	3354	321	178
2004	8505	3151	294	172
2005	10219	3984	320	174
2006	10385	3618	312	205
2007	9827	3504	315	183
2008	9854	3289	354	189
2009	10274	3502	333	191
2010	9947	3340	370	201
2011	7219	2528	220	140
2012	5490	1946	171	89
2013	3729	1352	127	76
2014	1507	586	43	32
2015	429	172	11	6

Source: Mikulec, 2017: 43.

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[2] The REGON register is the only available source of data on established and liquidated enterprises – regardless of the legal form, as by definition, it contains data on all entities of the national economy. Bearing in mind some imperfections of the REGON register, it should be stated that determining the exact size of a defined cohort of enterprises was not possible, hence the obtained data should be treated as a large research sample, see: Markowicz (2012: 103).

3. Mean duration and mean additional duration of enterprises

In a duration analysis it is assumed that the enterprise duration is a particular type of non-negatively determined continuous random variable T, $(T \geq 0)$, which defines the time passing from the moment of the foundation to the moment of a particular event, usually its closing down. It is a reasonable assumption that at the moment of the establishment of a given enterprise it is not possible to answer the question how long it is going to operate on the market. Bearing in mind that the length of enterprise duration may vary – it changes depending on the type of enterprise, in terms of time and location – hence the objective is not the calendar time but the enterprise's own time $(0; \infty)$. Moreover, it is assumed that the entity can be a focus of the analysis only once – Frątczak et al. (1996: 23-25); Markowicz (2012: 34-37); Jackowska (2013: 16-28); Frątczak et al. (2014: 37-40).

Figure 1. Mean duration of enterprises
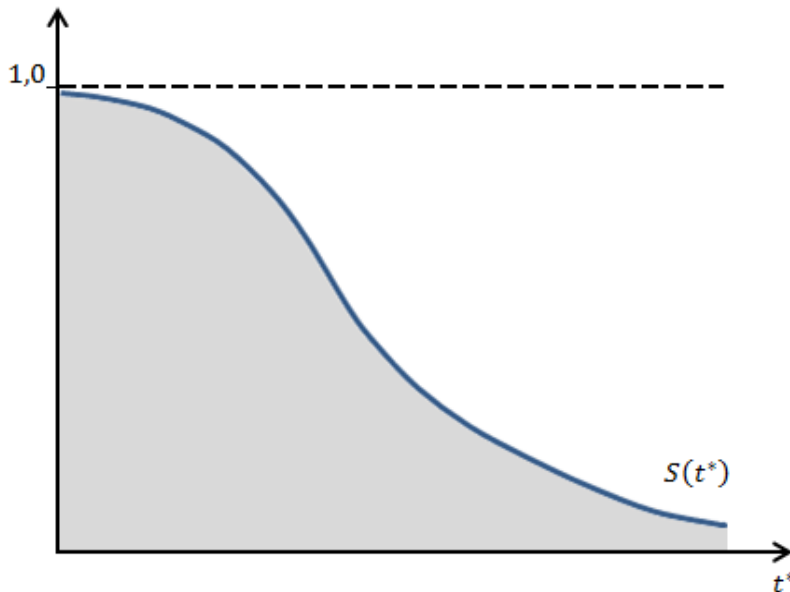
Source: Author's own elaboration based on: Balicki, 2006: 35.

In order to characterize the enterprises' survival process, apart from the basic functions, different distribution parameters of time duration are used, mainly the expected value (average). The expected value of a random variable T – **the mean duration of enterprises** – shall be determined by the probability of density function $f(t^*)$ or survival function $S(t^*)$. In the other

case (see: Figure 1), the expected value is equal to the area below curve $S(t^*)$ in the interval $(0; \infty)$, (Balicki, 2006: 34; Jackowska, 2013: 26):

$$m = E(T) = \int_0^\infty S(t^*)dt^*. \tag{1}$$

On the other hand, the expected value of random variable T of additional duration of enterprise on the basis of survival function $S(t^*)$ in the interval $(t^*; \infty)$ is derived from formula (Balicki, 2006: 67; Jackowska, 2013: 34):

$$m_t = E(T|t^*) = \frac{1}{S(t^*)} \int_{t^*}^\infty S(u)du. \tag{2}$$

In the geometric interpretation (see: Figure 2), **the mean additional duration of enterprises** which survived time t^*, is equal to the quotient of the area under survival curve point t^* and ordinate of function $S(t^*)$ at this point (Balicki, 2006: 39).

Figure 2. Mean additional duration of enterprises
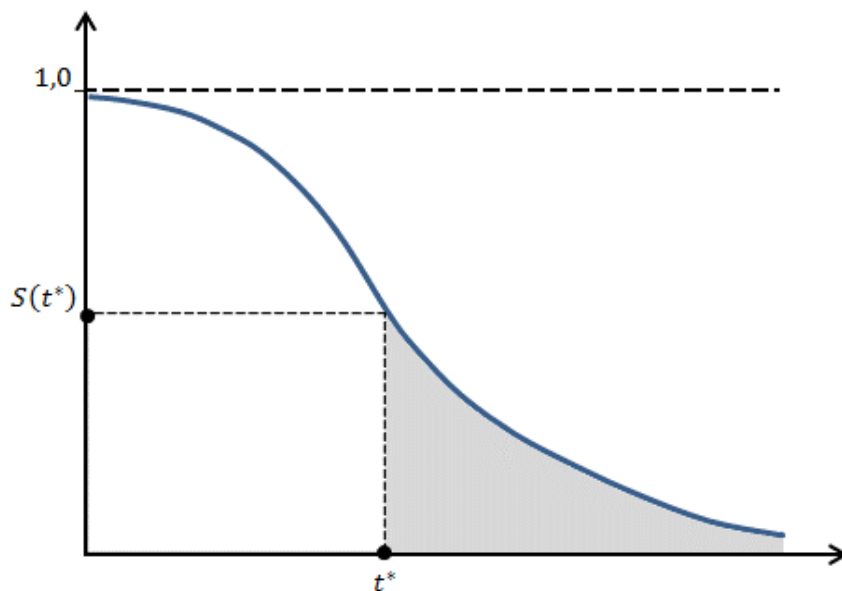
Source: Author's own elaboration based on: Balicki, 2006: 40.

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In general, the exact course of the function of the mean additional duration of enterprises (for any t^*) is not known, as we do not have an analytic form of function $S(t^*)$. Hence, to calculate the expected value we shall use cohort tables of enterprises' duration (non-parametric methods), which contain a discrete estimation of characteristic functions for enterprises' duration. It shall be noticed that in cohort studies, there occur censored[3] entities which were not liquidated during the time of cohort observations. In order to determine the mean duration and the mean additional duration of enterprises we need modified cohort tables[4] – created on the basis of complete data, i.e. without censored data (see: diagram in Mikulec, 2017: 36).

For the purpose of the present analysis, it has been assumed that the cohort were the enterprises founded in a given year, which were liquidated (see: Table 1) and the time of this event.[5] Enterprises' duration from the moment of their establishment till their liquidation was divided into annual intervals $\langle t^*, t^* + 1\rangle$, while the beginning of individual intervals is marked with the letter t^*. As Balicki (2006: 26-27) states, determining the survival model as random variable distribution T, $(T \geq 0)$ results in a representation of any probabilistic function which describes this distribution (or a set of functions). In view of the fact that cohort tables do not provide exact forms of these functions, they are claimed to be based on an **overall survival model**, and formulas presented below allow us to determine assessment (approximations) of particular functions and distribution parameters of time duration.

If, in a cohort table, n_{t+1} is the number of enterprises that survived until the beginning of interval number $t + 1$:

$$n_{t+1} = n_t - z_t, \tag{3}$$

where: $t = 0,1, \ldots, w$ − numbers of intervals in the table, n_t − the number of entities which survived until the beginning of interval number t, that is the time interval $\langle t^*, t^* + 1\rangle$, and n_t for interval number $t = 0$ (n_0) is cohort output size, so the number of enterprises established in a given year, z_t − the number of enterprises liquidated in the interval number t, i.e. time interval

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[3] Balicki (2006: 71-78), Domański and Pruska (2000: 203).
[4] Cohort annual tables of enterprises' duration for entities established-liquidated in Lodzkie Voivodship (in Lodz, Piotrkow Trybunalski and Skierniewice) in the years 2001-2015 – Mikulec (2018) – are available at: https://doi.org/10.13140/RG.2.2.29154.02247.
[5] A single-stated model is examined in this work – registration and deregistration (liquidation) of an enterprise.

$\langle t^*, t^* + 1 \rangle$; then the estimator of the mean duration and the mean additional duration of enterprises can be described with one formula:

$$\hat{e}_t = \frac{1}{n_t} \sum_{j=t}^{w} L_j, \qquad t = 0,1, \dots, w, \tag{4}$$

where $\sum_{j=t}^{w} L_j$ – the sum of duration of all the enterprises in the cohort in the intervals number $j = t, \dots, w$, thus in time intervals $\langle t^*, t^* + 1, \dots, w^* \rangle$.

For grouped empirical data it is frequently assumed that enterprises leaving the cohort – liquidations, spread evenly over each interval. Then, the sum of time L_t survived by the entities in duration interval $\langle t^*, t^* + 1 \rangle$ is:

$$L_t = \frac{x_t(n_t + n_{t+1})}{2}, \qquad t = 0,1, \dots, w, \tag{5}$$

and is equal to the product of average number of entities, which reached the duration defined in the interval number t, thus $(n_t + n_{t+1})/2$, and span x_t of this interval[6].

In the case of annual tables, for each interval number $t = 0,1, \dots, w$ the span of time interval $x_t = 1$, from this we have $L_t = (n_t + n_{t+1})/2$. Estimator $\hat{e}_t$ can be written then as follows:

$$\hat{e}_t = \frac{1}{n_t} \sum_{j=t}^{w} L_j = \frac{1}{n_t} \left(\frac{n_t + n_{t+1}}{2} + \frac{n_{t+1} + n_{t+2}}{2} + \frac{n_{t+2} + n_{\dots}}{2} + \cdots + \frac{n_{\dots} + n_w}{2} + \frac{n_w + n_{w+1}}{2} \right) =$$

$$= \frac{1}{n_t} \left(\frac{n_t}{2} + \frac{n_{t+1}}{2} + \frac{n_{t+1}}{2} + \frac{n_{t+2}}{2} + \frac{n_{t+2}}{2} + \frac{n_{\dots}}{2} + \cdots + \frac{n_{\dots}}{2} + \frac{n_w}{2} + \frac{n_w}{2} + \frac{n_{w+1}}{2} \right) =$$

$$= \frac{1}{n_t} \left(\frac{n_t}{2} + \overbrace{\frac{n_{t+1}}{2} + \frac{n_{t+1}}{2}}^{n_{t+1}} + \overbrace{\frac{n_{t+2}}{2} + \frac{n_{t+2}}{2}}^{n_{t+2}} + \overbrace{\frac{n_{\dots}}{2} + \cdots + \frac{n_{\dots}}{2}}^{n_{\dots} + \cdots + n_{\dots}} + \overbrace{\frac{n_w}{2} + \frac{n_w}{2}}^{n_w} + \frac{n_{w+1}}{2} \right) =$$

$$= \frac{1}{n_t} \left(\frac{n_t}{2} + \sum_{j=t+1}^{w} n_j + \frac{n_{w+1}}{2} \right) = \frac{1}{n_t} \cdot \frac{n_t}{2} + \frac{1}{n_t} \cdot \sum_{j=t+1}^{w} n_j + \frac{1}{n_t} \cdot \frac{n_{w+1}}{2}, \tag{6}$$

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[6] Explanation of determining the sum of time L_t survived by entities and its graphical interpretation is presented in Balicki (2006: 59-60).

since the number of enterprises that survived until the beginning of interval number $w + 1$ is zero ($n_{w+1} = 0$), because all of them were closed down, it can be written as:

$$\hat{e}_t = \frac{1}{2} + \frac{1}{n_t} \cdot \sum_{j=t+1}^{w} n_j, \qquad t = 0,1,\dots,w. \tag{7}$$

Substituting in formula (4) or (7) value $t = 0$, we have the assessment of the mean duration of enterprises, and defining value $t = 1,\dots,w$, we have the estimated mean additional duration of enterprises which survived until the beginning of interval number t, thus until the beginning of time interval $\langle t^*, t^* + 1 \rangle$.

To sum up, in order to determine the mean duration and the mean additional duration of enterprises on the basis of cohort tables we need to make sure that the analysed individual data concern only established-liquidated enterprises. Otherwise, if the database contains censored observations, the calculated value will not be the mean duration of enterprises, but it will be mean observation time of enterprises. It is also possible to calculate the mean duration of enterprises on the basis of individual data and partial results of survival function estimation using the Kaplan-Meier method (Kaplan and Meier, 1958) provided we subtract the censored data (Hozer et al., 2008: 29). The Kaplan-Meier curve by definition takes into account censored observations (Stanisz, 2007: 361).

Moreover, interpreting the value of mean additional duration of enterprises outlined in literature, one should think of the number of entities. For instance, in the paper by Mikulec (2017: 39-40), the mean duration of enterprises was presented as well. However, it was calculated on the basis of data on the established entities, which were liquidated, but regardless of the date of their establishment (a bigger number of the companies than in this article). The number of liquidated entities regardless of the date of their establishment is used in constructing cross tables of enterprise duration.

4. Analysis results

Tables 2-5 present the results of the mean duration and the mean additional duration of enterprises established-liquidated in the years 2001-2014 in Lodzkie Voivodship, in Lodz, Piotrkow Trybunalski, and Skierniewice.[7] It appears that the cohort of enterprises founded in 2014 was the last one for which there was a point in determining the mean duration of companies. The value of the mean duration for the entities established in 2015 in Lodzkie Voivodship, in Lodz, Piotrkow Trybunalski and Skierniewice (observed until the end of the year) is equal to 0.5.

The values in rows of the tables are falling for the next enterprise cohorts in view of the common closing date of observations – 31st December 2015, hence possible comparisons should be made between Lodzkie Voivodship entities and cities with the rights of a county (the *'powiat'* status).

The mean time of enterprises functioning on the market from the moment of their establishment[8] ($t^* = 0$), depending on the cohort, amounts to: from 0.71 to 6.36 years in Lodzkie Voivodship; from 0.70 to 6.34 years in Lodz; from 0.71 to 5.78 years in Piotrkow Trybunalski and from 0.77 to 6.19 years in Skierniewice. In the case of Lodz and Skierniewice, the next generation enterprises were usually characterized with a higher value of the mean duration ($t^* = 0$) than that of the entities in the whole voivodship. It means that the average duration of enterprises in these cities was longer than that of the entities in Lodzkie Voivodship. In the case of Piotrkow Trybunalski, it was only for the cohort of enterprises founded in 2012 that the above-mentioned situations occurred – the average value duration was higher than that of the companies in the voivodship – the shaded values in Tables 3-5, marked [(*)].

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[7] Places in Tables 2-5 marked with "-" are the cases in which defining mean duration or mean additional duration of enterprises was not possible.
[8] Bearing in mind, the numbering of intervals and duration in the cohort tables (as well as at the side of Tables 2-5) begin with 0 – according to enterprise's own time, hence the interval from 0 to 1 is the first year of enterprise operating according to the calendar time and the common meaning. Interpretation of the results is presented according to the calendar time.

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Table 2. Mean duration and mean additional duration of enterprises in Lodzkie Voivodship

Beginning of time interval (year) $\langle t^*, t^* + 1 \rangle$	Year of foundation													
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
0	6.36	5.55	4.84	4.17	3.69	3.56	3.33	3.10	2.77	2.54	2.19	1.79	1.28	0.71
1	5.87[1]	5.04	4.32	3.69	3.34	3.12	2.84	2.48	2.16	1.90	1.57	1.13	0.66	-
2	5.39	4.55[2]	3.89	3.63	3.27	2.84	2.46	2.06	1.75	1.47	1.05	0.62	-	-
3	4.75	3.99	3.62[3]	3.46	3.10	2.71	2.28	1.85	1.45	1.05	0.61	-	-	-
4	4.11	3.61	3.39	3.02	2.71	2.32	1.89	1.50	0.99	0.59	-	-	-	-
5	3.67	3.32	2.89	2.61	2.36	1.94	1.53	1.04	0.58	-	-	-	-	-
6	3.32	2.78	2.48	2.39	1.95	1.56	1.08	0.60	-	-	-	-	-	-
7	2.73	2.40	2.41	1.94	1.53	1.10	0.60	-	-	-	-	-	-	-
8	2.33	2.42**	1.95	1.59	1.07	0.61	-	-	-	-	-	-	-	-
9	2.41**	1.93	1.56	1.10	0.59	-	-	-	-	-	-	-		-
10	1.90	1.54	1.13	0.62	-	-	-	-	-	-	-	-	-	-
11	1.52	1.09	0.62	-	-	-	-	-	-	-	-	-	-	-
12	1.07	0.61	-	-	-	-	-	-	-	-	-	-	-	-
13	0.61	-	-	-	-	-	-	-	-	-	-	-	-	-

Source: Author's own elaboration.

Table 3. Mean duration and mean additional duration of enterprises in Lodz

Beginning of time interval (year) $\langle t^*, t^* + 1 \rangle$	Year of foundation													
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
0	6.34	5.60*	4.89*	4.35*	3.80*	3.62*	3.37*	3.11*	2.77	2.56*	2.21*	1.76	1.22	0.70
1	5.95[1]	5.13	4.45	3.90	3.44	3.17	2.94	2.55	2.26	1.97	1.63	1.13	0.65	-
2	5.58	4.70	4.05	3.74	3.31	2.88	2.54	2.12	1.81	1.52	1.06	0.62	-	-
3	4.98	4.21	3.73	3.47	3.10	2.71	2.34	1.88	1.48	1.08	0.61	-	-	-
4	4.39	3.76	3.36	3.01	2.67	2.34	1.95	1.54	1.04	0.59	-	-	-	-
5	3.93	3.30	2.82	2.57	2.39	1.93	1.56	1.05	0.58	-	-	-	-	-
6	3.33	2.66	2.40	2.40	1.96	1.57	1.09	0.60	-	-	-	-	-	-
7	2.67	2.25	2.45**	1.92	1.52	1.08	0.61	-	-	-	-	-	-	-
8	2.25	2.47**	2.07	1.55	1.06	0.62	-	-	-	-	-	-	-	-
9	2.51**	1.99	1.59	1.10	0.58	-	-	-	-	-	-	-	-	-
10	2.01	1.56	1.10	0.62	-	-	-	-	-	-	-	-	-	-
11	1.52	1.09	0.63	-	-	-	-	-	-	-	-	-	-	-
12	1.08	0.64	-	-	-	-	-	-	-	-	-	-	-	-
13	0.63	-	-	-	-	-	-	-	-	-	-	-	-	-

Source: Author's own elaboration.

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Table 4. Mean duration and mean additional duration of enterprises in Piotrkow Trybunalski

Beginning of time interval (year) $\langle t^*, t^* + 1 \rangle$	Year of foundation													
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
0	5.78	5.06	4.10	3.49	3.33	3.42	3.23	3.05	2.64	2.41	2.17	1.81*	1.23	0.71
1	5.21	4.41	3.63	2.97	3.13	3.07	2.83	2.46	2.02	1.85	1.60	1.13	0.68	-
2	4.55	3.97(2)	3.26	3.18**	3.33**	2.88	2.51	2.06	1.58	1.41	1.07	0.58	-	-
3	3.97	3.35	3.35**	3.26**	3.20	2.70	2.38	1.75	1.38	1.01	0.63	-	-	-
4	3.27	3.38**	3.28	3.09	2.66	2.23	1.91	1.48	0.95	0.59	-	-	-	-
5	3.51**	3.36	3.23	3.10**	2.24	1.92	1.49	1.11	0.66	-	-	-	-	-
6	3.54**	3.18	2.71	2.61	1.86	1.61	1.06	0.62	-	-	-	-	-	-
7	3.22	2.88	2.35	2.12	1.34	1.05	0.56	-	-	-	-	-	-	-
8	2.74	2.16	1.89	1.85	1.00	0.60	-	-	-	-	-	-	-	-
9	2.26	2.07	1.52	1.12	0.63	-	-	-	-	-	-	-	-	-
10	1.88	1.53	1.10	0.64	-	-	-	-	-	-	-	-	-	-
11	1.52	1.06	0.54	-	-	-	-	-	-	-	-	-	-	-
12	0.97	0.58	-	-	-	-	-	-	-	-	-	-	-	-
13	0.58	-	-	-	-	-	-	-	-	-	-	-	-	-

Source: Author's own elaboration.

Table 5. Mean duration and mean additional duration of enterprises in Skierniewice

Beginning of time interval (year) $\langle t^*, t^* + 1 \rangle$	Year of foundation													
	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014
0	6.19	5.20	4.84	4.71*	3.86*	3.58*	3.51*	3.39*	2.83*	2.63*	2.09	1.78	1.19	0.77*
1	6.04	4.97	4.52	4.20	3.51	3.15	2.88	2.82	2.15	2.08	1.54	1.25	0.69	-
2	5.78	4.55	4.41	3.95	3.61**	3.08	2.42	2.35	1.77	1.60	1.01	0.65	-	-
3	5.30	4.08	4.02(3)	3.83	3.28	2.91	2.00	2.11	1.41	1.14	0.65	-	-	-
4	4.88	3.88	3.91	3.49	2.76	2.53	1.72	1.46	0.84	0.63	-	-	-	-
5	4.64	3.62	3.51	2.94	2.47	2.15	1.18	1.06	0.59	-	-	-	-	-
6	4.09	3.43	3.21	2.32	2.00	1.53	0.86	0.63	-	-	-	-	-	-
7	3.52	2.95	2.45	1.74	1.49	1.10	0.63	-	-	-	-	-	-	-
8	2.80	2.26	1.94	1.45	0.96	0.73	-	-	-	-	-	-	-	-
9	2.30	1.76	1.62	1.16	0.67	-	-	-	-	-	-	-	-	-
10	1.99	1.30	1.18	0.64	-	-	-	-	-	-	-	-	-	-
11	1.63	0.91	0.54	-	-	-	-	-	-	-	-	-	-	-
12	1.08	0.50	-	-	-	-	-	-	-	-	-	-	-	-
13	0.54	-	-	-	-	-	-	-	-	-	-	-	-	-

Source: Author's own elaboration.

Reviewing the results of the mean additional duration of enterprises ($t^* = 1, ..., 13$) one should notice that in view of the common closing date of observations – 31st December 2015, the results of particular columns in Tables 2-5 are falling – for the entities of duration 1, 2, 3 etc. Still, there are exceptions, despite the time passing, the mean additional duration of "older" enterprises from a given cohort is rising, not falling – the shaded values in Tables 2-5, marked [**].

Comparing the results of the mean additional duration for all the subgroups of enterprises based in Lodz, Piotrkow Trybunalski and Skierniewice, operating on the market for at least 1 year (from the second row in each of Tables 2-5) with companies from Lodzkie Voivodship, the following can be noticed:

- Lodz: in most cases (65%), <u>higher</u> mean additional duration of the entities in Lodz than in Lodzkie Voivodship has been observed (for example for the enterprise cohort in 2001 which survived until the beginning of the second year of their business activity – value 5.95 years for the entities from Lodz versus 5.87 years for the companies from Lodzkie Voivodship) – the shaded values in Tables 2 and 3, marked [1].

- Piotrkow Trybunalski: in most cases (66%), <u>lower</u> mean additional duration of enterprises in Piotrkow Trybunalski than in Lodzkie Voivodship has been observed (for example for the enterprise cohort in 2002, which survived until the beginning of the third year of their business activity – value 3.97 years for the entities from Piotrkow Trybunalski versus 4.55 years for the companies from Lodzkie Voivodship) – the shaded values in Tables 2 and 4, marked [2].

- Skierniewice: in most cases (68%), <u>higher</u> mean additional duration of entities in Skierniewice than in Lodzkie Voivodship has been observed (for example for the enterprise cohort in 2003, which survived until the beginning of the fourth year of their business activity – value 4.02 years for the entities from Skierniewice versus 3.62 years for the companies from Lodzkie Voivodship) – the shaded values in Tables 2 and 5, marked [3].

5. Conclusion

An estimation of the distribution parameters of time duration and parameters of the conditional distribution of time duration of enterprises in the form of mean duration and mean additional duration made on the basis of cohort tables values is another widening of the conception of tables which contain an estimation of basic functions concerning a duration analysis.

Due to different numbers of entities, on the basis of which the value of their mean duration can be determined, not all the results are compatible. In the article, there have been presented methodical clues and the analysis results for enterprises of Lodzkie Voivodship (including Lodz, Piotrkow Trybunalski and Skierniewice).

The obtained results allow us to define the differences in terms of the mean duration of enterprises or the mean additional duration of enterprises for particular cohorts from Lodz, Piotrkow Trybunalski and Skierniewice, compared with entities based in Lodzkie Voivodship. The situation of particular generations of companies from Lodz and Skierniewice was more favourable compared with companies from the voivodship (higher values of the mean duration), whereas entities from the majority of analysed cohorts from Piotrkow Trybunalski were characterized by lower values of the mean duration.

In further analyses to follow, attempts will be made to determine factors affecting enterprises' (time) duration from particular cohorts in Lodz, Piotrkow Trybunalski and Skierniewice in comparison with the entities in Lodzkie Voivodship, with the use of a model-based approach.

Literature

Balicki, A. (2006). *Analiza przeżycia i tablice wymieralności* (Survival Analysis and Mortality Tables). Warszawa: PWE.

Domański, Cz., Pruska, K. (2000). *Nieklasyczne metody statystyczne* (Non-classical Statistical Methods). Warszawa: PWE.

Frątczak, E., Jóźwiak, J., Paszek, B. (1996). *Zastosowania analizy historii zdarzeń w demografii* (Applications of Event History Analysis in Demography). Warszawa: Oficyna Wydawnicza SGH.

Frątczak, E., Sienkiewicz, U., Babiker, H. (2014). *Analiza historii zdarzeń. Elementy teorii, wybrane przykłady zastosowań* (Event History Analysis. Basic Theory, Selected Examples of Applications). Warszawa: Oficyna Wydawnicza SGH.

Artur MIKULEC

Hozer, J., Markowicz, I., Stolorz, B. (2008). *Zastosowanie metod analizy historii zdarzeń w badaniu czasu funkcjonowania firm* (Application of Methods of Event History Analysis to Research of Time of Firms Working). Szczecin: Wydawnictwo Zapol.

Jackowska, B. (2013). *Modele dalszego trwania życia oraz ich zastosowania w przypadku osób starszych* (Mortality Models and Their Applications in the Case of Elderly People). Gdańsk: Wydawnictwo Uniwersytetu Gdańskiego.

Kaplan, E.L., Meier, P. (1958). Nonparametric estimation from incomplete observations. *Journal of American Statistical Association* 53: 457-481.

Machin, D., Cheung, Y.B., Parmar, M.K.B. (2006). *Survival analysis: a practical approach. Second edition*. Chichester: John Wiley & Sons, Ltd.

Markowicz, I. (2012). *Statystyczna analiza żywotności firm* (Statistical Analysis of Company Lifespan). Szczecin: Wydawnictwo Naukowe Uniwersytetu Szczecińskiego.

Mikulec, A. (2017). Analiza wskaźnikowa trwania przedsiębiorstw w województwie łódzkim (Ratio Analysis of the Enterprises Duration in Lodzkie Voivodship). *Wiadomości Statystyczne* 11(678): 29-55. Available at: http://stat.gov.pl/download/gfx/portalinformacyjny/pl/defaultaktualnosci/5982/7/36/1/ws_11_2017__05_art ur_mikulec__analiza_wskaznikowa_trwania_przedsiebiorstw_w_wojewodztwie_lodzkim.pdf. Accessed 28 February 2018.

Mikulec, A. (2018). Kohortowe roczne tablice trwania przedsiębiorstw powstałych-zlikwidowanych w województwie łódzkim (w tym w Łodzi, Piotrkowie Trybunalskim, Skierniewicach) w latach 2001-2015.pdf (Cohort Annual Tables of Enterprises' Duration for Entities Established-Liquidated in Lodzkie Voivodship (in Lodz, Piotrkow Trybunalski and Skierniewice) in the years 2001-2015.pdf), Available at: https://doi.org/10.13140/RG.2.2.29154.02247. Accessed 11 April 2018.

Program badań statystycznych statystyki publicznej na rok 2015 – Rozporządzenie Rady Ministrów z dnia 27 sierpnia 2014 r., Dz.U. z 2014, poz. 1330 ze zm. (Programme of Statistical Surveys of Official Statistics for 2015 – the Council of Ministers Regulation of 27 August 2014, Journal of Laws 2014, item 1330 with subsequent amendments). Available at: http://bip.stat.gov.pl/dzialalnosc-statystyki-publicznej/program-badan-statystycznych/pbssp-2015/. Accessed 28 February 2018.

Stanisz, A. (2007). *Przystępny kurs statystyki z zastosowaniem STATISTICA PL na przykładach z medycyny. Tom 3. Analizy wielowymiarowe* (An Accessible Course of Statistics with the Use of STATISTICA PL and Examples from Medicine. Volume 3. Multivariate Analyses). Kraków: StatSoft.

METHODS OF ESTIMATING THE MEAN DURATION OF ENTERPRISES AND THE MEAN ADDITIONAL
DURATION OF ENTERPRISES IN LODZKIE VOIVODSHIP

Metody szacowania przeciętnego oraz przeciętnego dalszego czasu
trwania przedsiębiorstw w województwie łódzkim

Streszczenie

Czas funkcjonowania przedsiębiorstw – od momentu ich powstania do likwidacji – jest w analizie trwania zmienną losową. Do podstawowych funkcji charakterystycznych, służących do jego opisu (czasu) należą: funkcja gęstości, dystrybuanta, funkcja przetrwania oraz funkcja hazardu. Pierwsza z nich, $f(t)$ określa prawdopodobieństwo likwidacji przedsiębiorstwa w okresie numer t, a więc w przedziale czasu $\langle t^*, t^* + 1 \rangle$; druga $F(t)$ wyraża prawdopodobieństwo dotrwania przedsiębiorstwa od momentu jego powstania co najwyżej do końca okresu numer t, przedziału czasu $\langle 0, t^* \rangle$; trzecia $S(t)$ określa prawdopodobieństwo przetrwania przedsiębiorstwa dłużej niż do końca okresu numer t, przedziału czasu $\langle 0, t^* \rangle$; a ostatnia $h(t)$ ocenia intensywność procesu likwidacji przedsiębiorstwa w okresie numer t. W artykule przedstawiono teoretyczne i praktyczne aspekty wyznaczania wybranych parametrów rozkładu czasu trwania: średniego czasu trwania oraz średniego dalszego czasu trwania przedsiębiorstw, które również służą do charakteryzowania wzorców przeżycia przedsiębiorstw. Rozważania teoretyczne poparte zostały obliczeniami wyżej wymienionych miar dla kolejnych kohort przedsiębiorstw powstałych (w tym zlikwidowanych) w województwie łódzkim w latach 2001-2015.

Słowa kluczowe: średni czas trwania przedsiębiorstw, średni dalszy czas trwania przedsiębiorstw, województwo łódzkie, Łódź, Piotrków Trybunalski, Skierniewice.