M A T H E M A T I C A L E C O N O M I C S

No. 7(14) 2011

EXTRACTION OF CYCLICAL FLUCTUATIONS – TWO METHODS ILLUSTRATED BY THE EXAMPLE OF A DEMOGRAPHIC VARIABLE

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Abstract. The purpose of the study is to identify cyclical fluctuations by means of two methods used to determine the cyclical nature of various phenomena. The first approach is based on the modified Harvard method. The other approach uses the Hodrick-Prescott filter and the Baxter-King filter. Cyclical fluctuations are observed not only in series of economic variables, but also in the case of demographic variables. The cyclicity of demographic data has been investigated by the author previously, and so time series for the number of births in Poland and Sweden were used for the study. The conducted analysis of the time series indicated that the number of births is subject to cyclical fluctuations and enabled their principal morphological properties to be established, i.e., the periods in which turning points occur, the fluctuation cycle periods, their lengths and amplitudes.

Keywords: cyclical fluctuations, turning points, cycle, modified Harvard method, Hodrick--Prescott filter, Baxter-King filter, number of births

JEL Classification: C22, J11.

1. Identification of a cyclic component in a time series

In analysing economic and social phenomena, use is made of time series, frequently very long ones. An analysis of changes over time calls for the determination of regularities characterising a given time series; hence the need for decomposing a time series, i.e. identifying its components. The literature on the subject distinguishes between a systematic component and a random component (random fluctuations). A systematic component may make itself visible in a series in various forms: as a development trend or a periodical component (as seasonal or cyclical fluctuations) (cf. (Cieślak (Ed.), 2005, pp. 64-66)). When analysing components, the first step is to examine the time series plot. Graphic analysis is not always sufficient to

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unambiguously determine the form of the systematic component, and some forms are impossible to observe in a series. This is why use is made of formalised methods of decomposing time series. The standard approaches, which are widely described in the literature and implemented in econometric applications, refer to the methods of estimating the cyclic component (cyclical fluctuations).

The procedure for extracting the cyclic component from a time series was first detailed by economists studying business cycles. Initially, the series was cleaned using the Harvard economic situation analysis method (Persons, 1919), encompassing first of all the determination of the linear function of the trend (by estimating the parameters by means of the classical least-squares method), next the calculation of coefficients of seasonal fluctuations, and finally the determination of cyclical fluctuations, together with random fluctuations, as follows:

$$\frac{y_t - S_{wt} \cdot T_{wt}}{T_{wt}} \cdot 100\% \tag{1}$$

or using a later variant:

$$\frac{y_t - S_{wt} \cdot T_{wt}}{S_{wt} \cdot T_{wt}} \cdot 100\%, \tag{2}$$

where:

y_t – the original absolute values of variable Y in period t;
T_{wt} – the estimated trend function value in period t;
S_{wt} – the estimated values of seasonality indicators in period t.

In the Harvard method, the extracted trend was estimated for the entire time series of a variable of interest. Later empirical research indicated that the cycle length varied. This is why the procedure has changed over time, in the 1990s studies of economic fluctuations applied moving average models to determine trends. The method used for the purpose was called PAT (Phase Average Trend). Trends were calculated on the basis of successive changes in the average values of individual business cycle phases. The method led to a slightly undulating trend line, as each time the trend was adjusted to its previously extracted segments, i.e. cycle phases. The PAT method, which is universally used in the OECD countries, has two main parts. In the first part tentative turning points in the studied time series, from which seasonal fluctuations were previously removed, are identified. In the second part the trend is calculated using averages of business cycle phases (cf. (Nilsson 1991; *OECD...*, 1987).

Nowadays, filtration is used in order to extract a time series component of interest. To decompose a series to the trend-cycle form the following filters are universally applied: the Hodrick-Prescott filter, the Baxter-King filter, the Christiano-Fitzgerald filter, as well as their modifications. The first two filters are described in Section 2 of this paper. Very long time series with high frequency data call for such sophisticated component identification methods. Unfortunately, a majority of computer applications containing the filtration functionality are commercial and therefore the freeware GRETL program has been used for this study.

Cyclical fluctuations are observed not only in series of economic variables, also demographic variables have a cyclical nature. The cyclicality of demographic variables has already been studied by the author of this paper (e.g. cf. (Krupowicz, 2009; Krupowicz, 2011a; Krupowicz, 2011b)). This study focuses on time series of the number of births in Poland and Sweden. The Poland series had 64 observations, while the Sweden series – 259 observations. The very long time series of the number of births in Sweden has been divided into three shorter series, covering the following periods: 1749--1849; 1801-1914 and 1901-2009, and so the series lengths exceed 100 observations. The paper presents two methods used to extract cyclical fluctuations. The first is based on the Harvard method, which has been modified for the study and which has previously been used by the author to identify the cyclical character of demographic variables (cf. (Krupowicz, 2011a)).The other method uses the Hodrick-Prescott filter and the Baxter--King filter. The purpose of the study is to identify cyclical fluctuations by means of these two methods.

2. Procedure for extracting cyclical fluctuations

The first method involves a modified procedure based on the Harvard method, which has successfully been applied in studying business cycles. It has already been used by the author in her previous studies (cf. (Krupowicz 2009; Krupowicz, 2011a)).The conduct is divided into four stages.

Stage 1 consists in extracting the development trend in the analysed time series. The criterion for selecting an appropriate trend function form is the function graph for the series in question indicating the observed regularity.[1] During Stage 2 the trend is removed from the time series. A trend can be eliminated in two ways.[2] The first consists in determining absolute deviations, in accordance with formula (3), and the other one in determining relative deviations, in accordance with formula (4):

$$y_t^{'} = y_t - f(t), \tag{3}$$

$$y_t^{'} = \frac{y_t}{f(t)}, \tag{4}$$

where:

y_t – the variable value in period t;

$f(t)$ – the variable trend function value in period t.

Stage 3 consists in eliminating random fluctuations, i.e., smoothing the obtained deviations by calculating the centred moving average (the smoothed value is assigned to the middle observation from the section fragment from which the average was obtained). Depending on the length of the studied time series and the amplitude of random fluctuations, the smoothing constant can be changed.[3] Where the time series of a variable does not show considerable random fluctuations, Stage 3 may be omitted. During the last stage, on the basis of the obtained smoothed values of deviations of the studied variable from the trend function, the morphological properties of cyclical fluctuations, i.e. the properties of the cyclical nature of the variable changes, are established (see the description in Section 3 of this paper).

The other method is based on time series filtration. For the purpose of this study, the Hodrick-Prescott and the Baxter-King filters were applied. The use of filters in studies of economic activity is extensively described in Skrzypczyński (2010); Wośko (2009).

The **Hodrick-Prescott (HP) filter** assumes the presence of a trend independent of the cyclic component (Hodrick, Prescott, 1997). The filter does not remove seasonal or random fluctuations, and so in the case of series in which such fluctuations occur the extraction of the trend-cycle component is

[1] Because of cyclical fluctuations and numerous random fluctuations that usually occur, no high determination coefficient values should be expected.

[2] Generally, it does not make any difference which of the two methods is applied. In this study the second way of eliminating the trend from the time series has been applied, i.e. relative deviations in accordance with formula (3) have been determined.

[3] It should be borne in mind that a bigger smoothing constant reduces the time series more than a smaller smoothing constant.

recommended.[4] Assuming that the time series y_t (t = 1, 2, ..., T) has been deseasoned and the random component is negligible, decomposition into unobservable components has the additive form:

$$y_t = g_t + c_t, \tag{5}$$

where:

g_t – the development trend;

c_t – the cyclic component.

The development trend g and the cyclic component c are estimated by minimising the function Φ:

$$\Phi = \sum_{t=1}^{T} (y_t - g_t)^2 + \lambda \sum_{t=3}^{T} (\Delta^2 g_t)^2, \tag{6}$$

where:

Δ – the difference operator ($\Delta^k y_t = y_t - y_{t-k}$);

λ – the smoothing parameter, which reflects the “smoothness” of the trend.

The first sum in formula (6) indicates estimation accuracy (the rest around the trend), while the other represents the trend smoothness. The smoothing parameter λ shows the weight of the two components in the total sum. Where λ is close to 0, the match of the trend with the observed time series becomes increasingly stronger, particularly when $\lambda = 0$, then $y_t = g_t$. An increase in the value of λ makes the trend smoother, in particular when λ approaches infinity the trend resulting from the application of the HP filter is the same as the deterministic linear trend adjusted with the least-squares method to variable y_t. The authors of this method – R.J. Hodrick and E. Prescott – proposed $\lambda = 100$ for annual data, $\lambda = 1600$ for quarterly data, and $\lambda = 14400$ for monthly data. The literature also describes other ways of establishing the smoothing parameter value.[5]

The cyclic component is calculated by subtracting the HP filter trend from the source series. It should be emphasized that the application of the tool only makes sense in the case of non-stationary (integrated) variables, as

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[4] That is, the calculation of a component cleared of seasonal and random fluctuations, e.g. obtained by using the TRAMO/SEATS procedure. The STATISTICA application enables the so-called Henderson’s curve to be produced.

[5] The smoothing parameter may be determined by using, for instance, the Ravn & Uhlig rule: $\lambda = (0.25 \cdot k)^4 \cdot 1600$, where k corresponds to the number of observations per year. For annual data $k = 1$, and so $\lambda = 6.25$, for monthly data $k = 12$ and $\lambda = 129600$ (Ravn, Uhlig, 2001).

the objective is to estimate and to remove the stochastic trend from input data. The HP filter was used as implemented in the GRETL application.

The **Baxter-King (BK) filter** is a bandpass filter, which means that as a result of filtration it removes both short- and long-term fluctuations (a trend), i.e. it lets through fluctuations from the band defined by the researcher. In the case of the BK filter, one must also remember the influence of seasonal fluctuations on the obtained cyclic component estimator. Thus, the use of the BK filter requires an earlier removal of seasonal fluctuations from the input time series.

The time series $y_t (t = 1, 2, ..., T)$ is assumed to be a (non)stationary variable, with seasonal fluctuations removed. The objective is to estimate the cyclic component variable y_t, i.e. to estimate the component y_t^c, which is only effective in the frequency band from $\underline{\omega}$ to $\overline{\omega}$.The estimator of the component y_t^c, resulting from the use of the BK filter, is given as (cf. (Baxter, King, 1999)):

$$\hat{y}_t^c = \hat{B}_J(L) y_t, \tag{7}$$

where:

$$\hat{B}_J(L) = \sum_{j=-J}^{J} \hat{B}_j L^j \text{ for } t = 1 + J, 2 + J, ..., T\text{-}J, \tag{8}$$

$\hat{B}_j$ – weights satisfying the symmetry requirement, i.e., $\hat{B}_j = \hat{B}_{-j}$.

The weight set $\hat{B}_j$ for the defined parameter J constitutes the solution to the problem of looking for the function Q minimum:

$$Q = \frac{1}{2\pi} \int_{-\pi}^{\pi} \left| B\left(e^{-i\omega}\right) - \hat{B}_J\left(e^{-i\omega}\right) \right|^2 d\omega, \tag{9}$$

where:

$B\left(e^{-i\omega}\right)$ – the strengthening of the “ideal” bandpass filter;

$\hat{B}_J\left(e^{-i\omega}\right)$ – the strengthening of the approximated BK bandpass filter.

The limiting factor for problem (9) is the requirement of summability of weights to zero, i.e. $\sum_{j=-J}^{J} \hat{B}_j = 0$. Thus, the weights $\hat{B}_j$ are given as:

$$\hat{B}_j = B_j + \Theta, \tag{10}$$

where:

B_j – weights corresponding to the weights of the “ideal” bandpass filter;
Θ – a parameter ensuring the summability of weights to zero, defined as:

$$\Theta = \frac{-\sum_{j=-J}^{J} B_j}{2J+1} . \qquad (11)$$

The parameter J is referred to as a truncation lag. The determination of its value is of key importance to estimating the cyclic component of a time series by means of the BK filter. The authors of this method – M. Baxter and R.G. King – suggested that $J = 12$ should be adopted in the case of quarterly data, which means that the component obtained by means of the BK filter is shorter by 12 quarterly observations at the beginning and at the end of the observation sample as compared to the input time series. For annual data the filter authors recommended $J = 3$, and for monthly data $J = 36$, which is consistent with the value for quarterly data that they proposed (cf. (Baxter, King, 1999)). The bigger the value of parameter J, the closer is the filter to the “ideal” filter although it shortens the length of the time series obtained by filtration. Consequently, considering the BK filter structure (a symmetrical moving average), the obtained cyclic component estimator is a time series composed of $T – 2J$ of observations.

The BK filter can be used both to extract cyclic components based on stationary variables and non-stationary variables, at most second-order integrated ones. The BK filter was used as implemented in the GRETL application.

3. Morphological properties of cyclical fluctuations

Morphological properties of cyclical fluctuations should be understood as the properties of extracted fluctuations. The determination of the properties of cyclicity consists in defining peaks and troughs, establishing and measuring the amplitudes of upturn and downturn phases, the lengths of upturn phases, downturn phases and the cycle, and in measuring the intensity of upturn and downturn phases.

The principal morphological property characterising cyclical fluctuations is the cycle length, i.e. its duration. If cycle phases have been identified, their lengths are established. The lengths of the cycle and its individual phases is connected with the notion of a turning point. There are single-name and different-name turning points. The former indicate phase changes identical in terms of direction (e.g. two peaks or troughs), while the latter indicate phase changes different in terms of direction (e.g. a peak and a trough). A peak occurs at a point at which the variable reaches its maximum value, and a trough at a point at which the variable reaches its minimum value. It is worth noting that cyclic components of economic time series that reflect deviations of a variable from the trend are positive at peaks and negative at troughs.

The cycle length is defined as the time between two neighbouring single-name turning points (peaks or troughs). Thus, the cycle length is the number of observations between two successive troughs or peaks, taking account of the periods to which such troughs or peaks belong.[6] The phase length is defined as the time between two neighbouring different-name turning points. In other words, the phase length is the number of observations between turning points, taking account of the periods to which such turning points belong. An upturn phase occurs then between a trough and a peak, and a downturn phase between a peak and a trough.

An amplitude of a given cycle should be understood as the maximum – in terms of a module – value of a deviation of its turning points from the adopted reference line. An amplitude of a cycle phase is usually understood as the absolute value of the difference between values corresponding to its turning points. It is worth mentioning that cyclical fluctuations occur if repeated upturn and downturn phases are observed. The intensity of upturn and downturn phases is determined by means of changeability measures, e.g. standard deviation. The intensity is understood as high if the changeability measure assumes increasingly bigger values. However, there are no arbitrarily established values indicating a high or low intensity of a phase.

4. Results of the conducted study

The time series of the variables were subjected to the presented procedures corresponding to the analysed methods. It must be emphasized that the time series regarded annual data. In the case of the first method, at the very beginning development trends in each series were identified. These were linear trend functions or quadratic polynomial functions. Next, absolute deviations of actual values from the identified development trend for the studied variable were calculated. Because of observed numerous random fluctuations, the third phase of the procedure was carried out, i.e. the obtained values of absolute deviations were smoothed. To this end a five-element moving average for the relative deviations of the studied time series was computed.[7]

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[6] A cycle can therefore be identified using troughs or peaks.

[7] The selected smoothing constant ensured an effective smoothing of the series characteri-

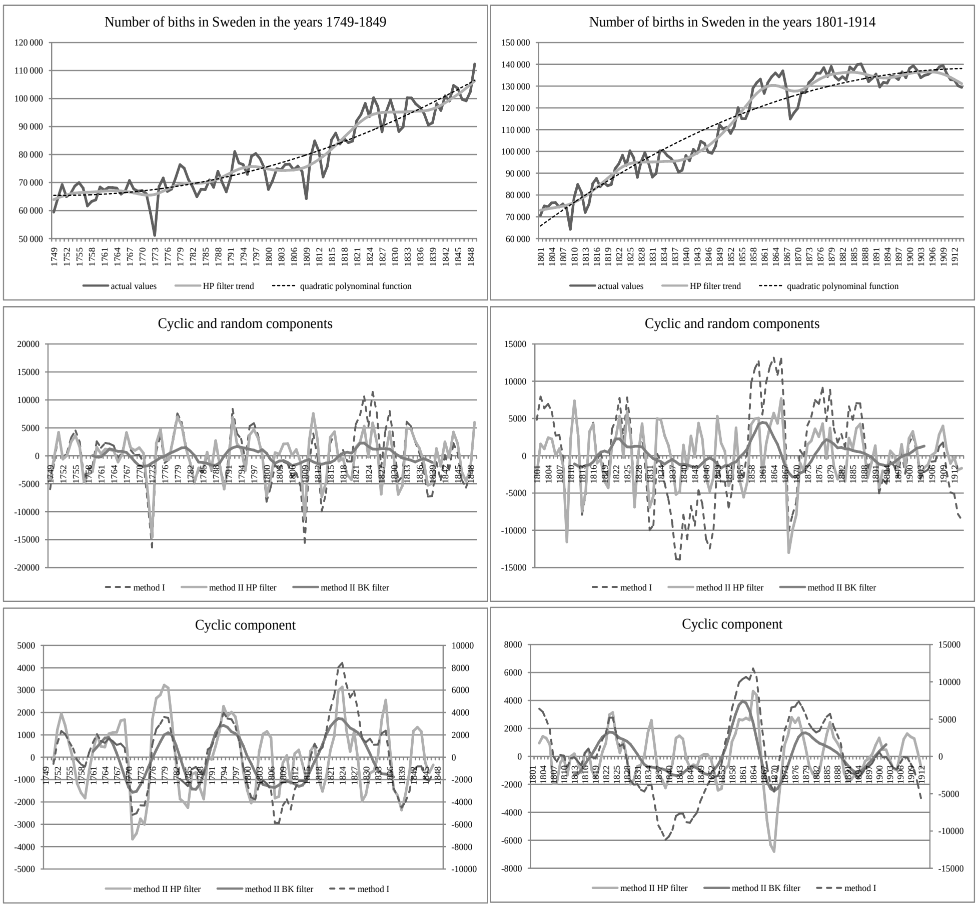

Fig. 1. Decomposition of the time series of the number of births in Sweden between 1749-1849 (left) and 1801-1914 (right)

Source: author's own calculations based on data from the Human Mortality Database.

The second method involved the use of the HP and BK filters. In the case of the HP filter the smoothing parameter λ amounted to 100 (as recommended by the authors of the method). In the case of the BK filter, it was assumed that the cyclical fluctuations of the number of births fall within the range of 18 to 35 years. Various possible values of parameter J were considered, disregarding the authors' suggestion, as the parameter value (a truncation lag) of 10 years was accepted for the Sweden series and

zed by numerous random fluctuations, and at the same time it did not cause any significant loss of information, i.e. a material shortening of the lengths of the time series.

7 years for the Poland series. Because the series were not smoothed earlier to remove random fluctuations, at the end of the procedure the random component was eliminated, using a five-element centred moving average.[8] Figure 1 and 2 present the results of the considered methods of extracting cyclical fluctuations. A visual assessment of the plots indicates that the time series contain cyclical fluctuations, but these have various cycle lengths and amplitudes.

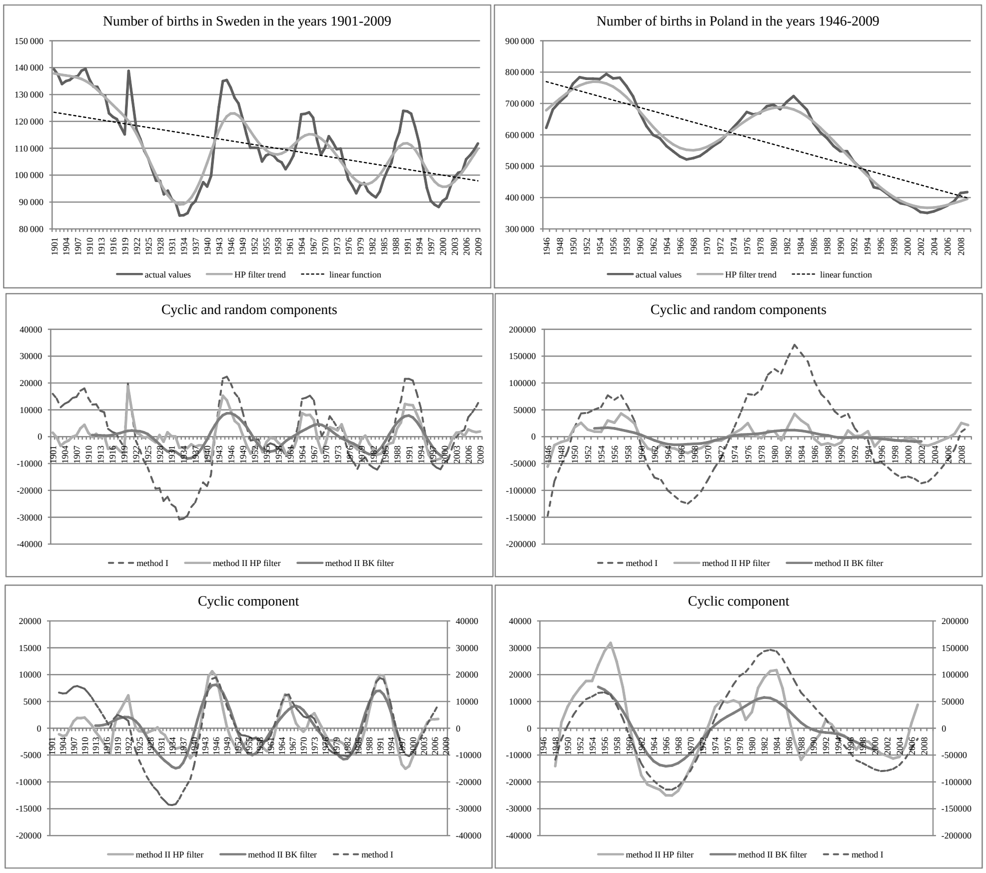

Fig. 2. Decomposition of the time series of the number of births in Sweden between 1901-2009 (left) and in Poland between 1946-2009 (right)

Source: author's own calculations based on data from the Human Mortality Database and the Central Statistical Office.

[8] In the case of quarterly or monthly data the TRAMO/SEATS procedure smooths the series, eliminating seasonal and random fluctuations. The smoothing process seems here necessary due to a large proportion of random fluctuations, in particular in the Sweden series.

Because of the significant amount of the numerical material obtained in the study, attention was paid only to certain properties of cyclical fluctuations, namely the turning points, cycle lengths and amplitudes. The obtained results are presented in Table 1.

The applied methods of extracting cyclical fluctuations allowed fluctuations to be identified and also measured in a manner corresponding to that used to measure business cycles. However, there exist some differences in the obtained morphological properties of the extracted cyclical fluctuations. Firstly, the number of identified turning points differs, the biggest number of turning points was obtained using method II with the HP filter (particularly for the Sweden series for 1801-1914 – 16 such points). However in the overwhelming majority of cases the turning points calculated by means of the two methods fall at the same points in time or differ by one to three years.

Secondly, a bigger number of cycles has been identified using method II involving the HP filter, which is obviously a result of a large number of the distinguished turning points. Thirdly, the lengths of the identified cycles obtained by means of the two methods are identical or similar as regards the number of observations, which in turn is a result of the fact that the periods of occurrence of the turning points coincide. However the ranges of the lengths of the fluctuation cycles vary, which is noticeable above all in the Sweden series for 1749-1949 (in the case of method I the smallest length is 10 years and the greatest length 32 years, in the case of method II involving the HP filter, it is 9 and 31 years, respectively, and involving the BK filter 16 and 30 years, respectively). The simple use of the HP filter produced cycles with relatively small lengths. For the Sweden series for 1901-2009 and the Poland series for 1946-2009 the cycles obtained by means of method I and method II with the HP or BK filters had very similar lengths. Fourthly, the cycles have amplitudes in different orders of magnitude. The biggest cycle amplitudes in terms of the absolute value were obtained by using method I to extract cyclical fluctuations. Fifthly, the random fluctuations occurring in the original series (relatively strong in the Sweden series for 1749-1849 and 1801-1914) make the use of the HP filter rather ineffective, which means that some fluctuation cycles should be deemed as spurious.

Table 1. Selected morphological properties of the cyclical fluctuations of the number of births in Sweden and Poland in the studied periods

Turning points			Cycles			Cycle lengths			Cycle amplitude		
Method I	Method II HP filter	Method II BK filter	Method I	Method II HP filter	Method II BK filter	Method I	Method II HP filter	Method II BK filter	Method I	Method II HP filter	Method II BK filter
Sweden 1749-1849											
1753	**1753**		**1753-1762**	**1753-1769**		**10**	**17**		**290**	**274**	
1759	1759		1759-1771	1759-1771		13	13		4312	1835	
1762	**1769**	**1765**	**1762-1779**	**1769-1779**	**1765-1780**	**18**	**11**	**16**	**-1544**	**-1542**	**-216**
1771	1771	1771	1771-1785	1771-1785	1771-1787	15	15	17	-2566	-1405	-132
1779	**1779**	**1780**	**1779-1794**	**1779-1794**	**1780-1794**	**16**	**16**	**15**	**-371**	**945**	**-326**
1785	1785	1787	1785-1808	1785-1807	1787-1807	24	23	21	3303	-427	-85
1794	**1794**	**1794**	**1794-1824**	**1794-1824**	**1794-1823**	**31**	**31**	**30**	**-4464**	**-864**	**-307**
1808	1807	1807	1808-1839	1807-1829	1807-1837	32	23		-1382	142	-470
1824	**1824**	**1823**		**1824-1835**			**12**			**585**	
	1829			1829-1839			11			390	
	1835			**1835-1843**			**9**			**1210**	
1839	1839	1837									
	1843										
Sweden 1801-1914											
1811	1807		1811-1838	1807-1829		28	23		9738	33	
1824	**1824**	**1823**	**1824-1864**	**1824-1835**	**1823-1861**	**41**	**12**	**39**	**-6586**	**555**	**-2225**
1838	1829		1838-1870	1829-1839		33	11		-6458	-1911	
1864	**1835**		**1864-1877**	**1835-1843**		**14**	**9**		**4375**	**1109**	
1870	1839		1870-1894	1839-1855		25	17		-1889	58	
1877	**1843**	1842	**1877-1900**	**1843-1860**	1842-1870	**24**	**18**	29	**6827**	**-1172**	1106
1894	1855			1855-1870			16			4487	
1900	**1860**	**1861**		**1860-1875**	**1861-1879**		**16**	**19**		**-147**	**2268**
	1870	1870		1870-1882	1870-1894		13	25		-5300	-1331
	1875			**1875-1886**			**12**			**359**	
	1882	**1879**		1882-1891			10			278	
	1886			**1886-1900**			**15**			**1115**	
	1891	1894		1891-1905			15			-788	
	1900			**1900-1909**			**10**			**-82**	
	1905										
	1909										
Sweden 1901-2009											
1908	**1908**		**1908-1946**	**1908-1922**		**27**	**15**		**-3162**	**-4208**	
	1917			1917-1939			23			689	
	1922	**1921**		**1922-1945**	**1921-1946**		**24**	**26**		**-4498**	**-6015**
1934	1939	1935	1934-1959	1939-1965	1935-1956	26	22	22	-23774	-1310	-2472
1946	**1945**	**1946**	**1946-1966**	**1945-1983**	**1946-1968**	**21**	**21**	**23**	**6315**	**4318**	**3971**
1959	1960	1956	1959-1982	1960-1983	1956-1981	24	24	26	5494	497	786
1966	**1965**	**1968**	**1966-1991**	**1965-1991**	**1968-1991**	**26**	**27**	**24**	**-6149**	**-3612**	**-2846**
1982	1983	1981	1982-1999	1983-1998		18	16		-21	2743	
1991	**1991**	**1991**									
1999	1998										
Poland 1946-2009											
1956	**1957**	**1955**	**1956-1983**	**1957-1984**	**1955-1982**	**28**	**28**	**27**	**-79279**	**10121**	**3940**
1967	1967	1966	1967-2001	1967-1988	1966-2000	35	22	35	-34678	-13312	-6152
1983	**1984**	**1982**		**1984-1992**			**9**			**18835**	
	1988			1988-2003			16			-449	
	1992										
2001	2003	2000									

Legend: Peaks, cycles identified on the basis of peaks and their properties are marked in bold.

Source: author's own calculations.

The division of the Sweden time series into three periods for the purposes of the cyclicity analysis of the number of births resulted in certain fragments overlapping. The periods 1801-1849 and 1901-1914 were repeated. It is important that the turning points occurred in the same years in period I and in period II (obviously in the overlapping 1801-1849 fragments). Only in the first examined period the turning points identified by method II with the HP filter occur in the following years: 1807, 1824, 1828, 1835, 1839, 1843, by method I – 1824, and by method II with the BK filter – 1832, and fall in the same years for the variable in the second studied period for Sweden.

5. Recapitulation

The applied methods of extracting cyclical fluctuations not only allowed fluctuations to be identified, but also measured in a manner corresponding to that used to measure business cycles. Differences were observed in the obtained morphological properties of the distinguished cyclical fluctuations, i.e. the number of turning points, cycle lengths and cycle amplitudes. A comparison of the findings indicates that the use of the Baxter-King filter produces results very similar to those obtained when the classical method of extracting cyclical fluctuations is applied (method I based on the modified Harvard method), albeit at a cost of a significant loss of information. The use of the Hodrick-Prescott filter is reasonable if the time series does not contain any significant random fluctuations, otherwise the series should be smoothed.

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