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CREATING AN ALGORITHM OF A REAL-LIFE SITUATION AS A FORM OF MATHEMATICAL MODELLING

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Abstract. Algorithmisation is one of the forms of mathematical modelling. The necessity to create an algorithm imposes specific forms of work that brings numerous didactical benefits. The research presented here concerns math students' (future teachers) abilities which relate to mathematical modelling (i.e. creating algorithms) of an everyday situation. The analysis of their work demonstrated their lack of appropriate knowledge in the field of mathematical modelling – more than the half of the models did not work properly. The application of the form of the algorithm did not improve the quality of the models as well. However, it created an impulse to apply different tools (conditional instructions) and to apply the block diagram. The research shown points at weak sides of the students' skills, but on the other hand, it emphasizes the sense of joining modelling with creating algorithms.

Keywords: mathematical modeling, algorithmisation, higher education.

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1. Algorithmisation as a form of mathematical modelling

One of main objectives of mathematics education is for students to attain skills to solve problems encountered in everyday life. Finding a solution to the problem forces the creation of a mathematical model of a given situation. What is meant by mathematical modelling?

Mogens Niss (2012), defines it in the following way: mathematical modelling is an ability to describe a real situation in the language of mathematics, to interpret and verify the results in natural language, to match ready-to-use mathematical models with real situations and to search for real situations that are specific to those models, to reflect upon, analyse and evaluate one's own and others' mathematical models. The construction of a mathemati-

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cal model of the situation requires the student to examine it, and then to distinguish the objects and relations between them.

In another place, Mogens Niss states succinctly: "When a mathematical model is introduced (selected, modified or constructed) from scratch to deal with aspects of an extra-mathematical context and situation, we say that *mathematical modelling* is taking place" (2012, p. 50).

These two statements present a broad spectrum of activities related to modelling. They point to the need to verify the once selected mathematical tool, to control various aspects included in the tool and the model, to verify its correctness – e.g. by matching the sample data, considering a variety of possible solutions to the problem and so on. Mathematical modelling is thus a process in which one cyclically returns to the initial real situation. The aim of this process is to realize which aspects of the reality have to be included in the model and which should be simplified.

Mathematical modelling is also one of several groups of mathematics teaching objectives included in the Polish core curriculum. According to the core curriculum students at the end of the fourth stage of education should possess the abilities for mathematical modelling. For students in mathematical education at advanced level, this requirement is formulated as follows: *The student creates a mathematical model of a given situation taking into account limitations and reservations* (Core curriculum with comments, 2009, p. 41). This goal is usually associated with the ability to use mathematical tools (e.g. systems of equations, linear function) to solve the so-called word problems. In my opinion, such an understanding is too narrow: the activities presented in the textbooks are usually closed because of the data, they do not involve selecting the information and often explicitly indicate a mathematical tool.

In didactical journals numerous examples can be found of how to create a model of the situation from the scratch, starting with collecting sample data on the basis of which the model is built by testing and verifying the created model. Such an approach does not appear in the textbooks, not to mention the attempt to create a general model by searching for, and the symbolic formulation of relations and dependencies that relate to the problem. Such forms of modelling appear only in certain fields of study, although creating models on one's own brings many teaching benefits (see: Rams 1982, pp. 138-140).

Mathematical modelling is differently perceived, and thus it can take various forms. One of them is algorithmisation. Creating a mathematical model in the form of the algorithm is associated with specific methods of work which result from the definition (nature) of the algorithm. According to Z. Krygowska: "An algorithm created for a *certain class of activities* that is based on a *certain,* known in advance *set of basic activities* is any *scheme of a finite sequence of activities* selected from this set such that *the execution of these activities in the planned order* with the data that specify a task of this class *leads to the solution to this task*" (1977, p. 144).

This definition ensures that the basic features of the algorithm are retained. One of them is the *elementariness of operations*. This means that each operation appearing in the scheme is controlled by the student. This feature is relative and depends on the skills of the student at a given level of education because some operations that are not elementary at a certain stage may become elementary in the further course of study. *Unambiguity* means that it should precisely define the sequence of operations leading to the result. Therefore, a student who has mastered the basic operations is able to get to the solution of a complicated task by doing the step-by-step activities planned in the scheme. The *effectiveness* of an algorithm guarantees that the resulting outcome is the correct solution of the task after a finite number of steps, whereas the *generality* condition means that an algorithm should comprise the whole class of tasks by working on parameters, the specification of which defines a given task. Of course, an algorithm always works in the same way for the same initial data. The feature of unambiguity imposes this.

Regardless of how an algorithm is presented (a verbal description, a list of steps, a block diagram or a programming language), its features impose specific working methods, which results in multiple teaching benefits (see: Pyzara 2012, pp. 51-68; Rams 1982, pp. 138-140). What is more, the algorithm for the solution of the situation considered is at the same time its mathematical model.

Of course, the issue of modelling is much broader than just creating algorithms, which is only one of the ways of representing a mathematical model. This kind of work with a student brings many teaching benefits. This results from the dual nature of mathematics: conceptual and algorithmic. These two natures of mathematics are closely related and they require an individual approach. Z. Krygowska (1977), T. Rams (1982) and M. Sysło (2008) among others, pay attention to it. They put the importance of the algorithmic approach to mathematics equally with conceptual approach. They also emphasize the advantages of using algorithmisation in teaching mathematics, in particular the unassisted creation of algorithms by students. T. Rams writes: "Creating block diagrams or tree algorithms by students in the process of learning mathematics supports and stabilizes the thought processes performed by them, influences the discipline of the student's mind while solving a mathematical problem and helps them to understand better the structure of the tasks" (1982, p. 139).

Despite the numerous benefits of unassisted creation of a model in the form of an algorithm, it is difficult to find in mathematics textbooks at the level of school education, proposals of such forms of solving problems. M. Sysło (2008, p. 14), even claims that: "Reading school mathematics textbooks gives few examples of integration of the conceptual and algorithmic approach". This is a worrying situation because it can make students perceive mathematics as a collection of isolated facts and concepts on the one hand and calculation methods on the other, which do not constitute a coherent field of knowledge. How then should mathematics be taught so that the acquired knowledge forms a whole? How to fulfil the objectives of teaching mathematics so as to include both the conceptual and algorithmic approaches to mathematics? Perhaps more attention should be paid to the problem of mathematical modelling, in particular to algorithmisation.

M. Sysło in the already quoted paper confirms the lack of algorithmic approach in teaching mathematics as: "Indeed, the algorithmic approach is not used for activating students in connection with problem approach or for solving real (realistic) problems, which generally require thoughtful and often unusual computational (algorithmic) methods" (2008, p. 14). Students do not have to analyse real life situations by themselves so as to find their mathematical solution because the instructions of the tasks give the necessary data to solve them. Creating a model is limited to providing the formula (function, equation) describing this situation without having to go through the entire process of creating the model, as presented by Z. Krygowska and M. Niss. The very concept of mathematical modelling is usually omitted (ignored), although modelling is one of the main mathematical competences in the requirements for students. Thus, there is an implicit assumption that students will acquire this skill themselves, and hence, will be able to create models of situations encountered in everyday life and obtain their solutions through the use of mathematical tools. Does it really happen so? Mogens Niss (2012, p. 51), claims (referring to papers by Ikeda andStephens, Stillman, Kaiser and Maass) that purely mathematical knowledge is necessary to create models, yet insufficient.

In his opinion, there is no guarantee that a translation (transfer) from mathematical to modelling skills will appear in students. Niss claims that many examples in literature can be found of how students with very good mathematical knowledge are not able to translate it into the skill of modelling. One reason is that the assumptions, simplifications and decisions made during the process of modelling are closely related to the extra-mathematical aspects of the considered situation. It is often necessary to acquire the data and measurements alone or to discover the dependencies describing a given phenomenon. Such skills are not exercised with students during traditional maths classes. One fact is comforting that modelling (and thus the above-mentioned skills) can be effectively taught, which is confirmed by M. Niss and other researchers in this field. It is difficult, though, to find information in literature about how students deal with one of the types of a model, which is the algorithm.

Therefore I decided to determine whether Polish students of mathematics (future teachers) can create a mathematical model of an everyday-life problem, in particular, whether they are able to create the algorithm for the solution of this situation. Despite the lack of an algorithmic approach in school education, do pupils (now students) possess sufficient skills in the field of modelling? This paper presents some of the results of the research.

2. Research methodology

2.1. The research objectives and the description of the research tool

The aim of the research was to explore students' skills that relate to mathematical modelling with the use of algorithms. The students were to create a scheme allowing them to calculate the cost of a school tour. I wanted to know whether the students could create an algorithm leading to the solution to the problem.

I tried to answer this question by analysing the written works of students, which were the product of the implementation of the instruction in the task (the research tool):

Create a scheme that will allow to calculate the cost of a one-day school tour. The scheme should be so general and clear that it can be used by another organizer while planning the same tour next year.

Input data:

- *it is a coach tour,*
- *there is one chaperone per 15 pupils,*
- *four classes go on the tour,*
- *there is a visit to a museum and a theatre in the tour programme.*

I expected that this task would be treated as a mathematical problem requiring the creation of a mathematical model of an everyday situation. The information given indicated what factors were to be considered while creating the model. The vague formulation of the instructions allowed the respondents a lot of freedom and creativity to supplement the scheme with their own additional assumptions which they considered important. Regardless of this vagueness, I expected that the assumptions made by the students, together with the data provided by me, would be included in the appropriate scheme of dependencies.

The research tool was prepared in two versions differing only in one key word. The word "scheme" from the version shown above was replaced by the word "algorithm". While preparing the research tool, I was in a dilemma about which word to choose. I expected that using the word "algorithm" might considerably affect the work of the respondents because in the course of the studies they became familiar with basic knowledge of algorithmisation and programming. Thus, I decided to prepare two forms of the instruction and see how it would affect the way of solving the task. The collected research material could be used to answer the following research questions:

Q1: Did the word "algorithm" considerably affect the work of the students?

Q2: What changes appeared in the presented models as the result of the implementation of the word "algorithm"?

Q3: How did the expression "algorithm" affect the form of presenting the model?

I had certain expectations with regard to the change of the formulation of the instruction in the research tool. According to the first part of the article, the word "algorithm" should show the way of solving the task a functional form: to force the student to clearly identify elementary actions, to design the next operational steps, to consider various methods (options) of proceeding, to determine assumptions. The application of various ways that may lead to the solution of the task is possible by using the conditional instruction, and it allows a student to consider several variants of the test situation, which promotes analytical thinking. Another tool made available by algorithms is the loop, which allows the multiple execution of the same operations. Creating an algorithm is connected with the necessity of testing its correctness by doing calculations on specific data. Such activity forces a respondent to choose the data independently so as to go through each way of calculation. Thus, creating examples should intrinsically become a goal of the students' activity. I expected that the respondents would test the algorithms and would show the symptoms of reflective thinking (according to the steps of the modelling process).

The need to provide an algorithm also suggests the use of the appropriate form of the solution. An algorithm can be graphically represented by means of block diagrams – boxes (blocks) with specific functions, such as: inputting and outputting data (parallelograms), performing calculations (rectangles), checking the conditions (diamonds), and initiating and terminating the algorithm (ovals – START, STOP). The other forms of an algorithm are the following: a list of steps, programming language and pseudo programming language (commands presented in the form of a verbal description combined by keywords analogous to the programming language such as if the condition is met, then execute the command 1, or else execute the command 2. Regardless of the choice of the form, an algorithm must maintain clarity, efficiency and generality.

2.2. Organising the research

The research was conducted in April 2013 on two groups of students of mathematics. The first group consisted of 11 female second year students, including 9 on the teaching specialization – *mathematics with computer science* – and 2 on the *financial mathematics and insurance* specialization. The second group consisted of 28 first year students of complementary master's studies with the teaching specialization – *mathematics with computer science*. The time to implement the task was 45 minutes, however the majority of the students finished their work earlier. The first group was given the instruction with the keyword "scheme", whereas the second group with the word "algorithm".

3. The analysis of the works of the respondents

The written work of the students was analysed. I will focus here on the analysis of the models with regard to algorithmisation. Recognition of the ability to create a mathematical model of the discussed situation has already been presented in the article "Mathematical modelling of a problem situation known from every-day life" (Pyzara 2014)[1]. Considerations will concern the influence of the presence of the word "algorithm" in the instruction.

[1] In this article (Pyzara 2014), the work from the first group is analysed (11 persons), where the word "scheme" appeared in the instruction. A detailed analysis of the work was carried out under the terms of the precision of thinking about relationships, the choice of the notation of the model and the data taken into account by the students and the way of denoting these data. Let me recapitulate the most important conclusions concerning the ability to create a mathematical model of a situation known from everyday life. The math students treated this problem more like an everyday situation than a mathematical problem. The analysis of the work clearly shows the lack of appropriate knowledge in the field of mathematical modelling. In none of the works the process of improving the model by testing and improving the working model is visible (see: Warwick 2007, p. 33). The

3.1. Example solution

Below I will show an example algorithm (see: Figure 1) to calculate the cost of a school tour per student. I assume that chaperones do not incur any costs while participating in the tour (they do not also receive any salary for the care given) – all the costs are borne by the students. The model does not include additional costs; it is based only on the information provided in the instruction.

Denotation of variables:

n1, n2, n3, n4 – number of students from each class (respectively) taking part in the tour,

n – number of all students taking part in the tour,

op – number of chaperones,

uczestnicy – number of all participants of the tour,

a_miejsc – number of seats in the coach,

a_cena – rental price of one coach per day,

a_liczba – number of coaches needed,

koszt – working variable that specifies the cost of the tour,

t_ucznia, t_opiekuna – price of the ticket to the theatre for a student / chaperone respectively,

m_ucznia, m_opiekuna – price of the ticket to the museum for a student / chaperone respectively,

k – number of persons in a group with which one chaperone may enter the museum free of charge,

gratis – working variable that specifies the number of chaperones who do not have to buy tickets to the museum,

Variables: n1, n2, n3, n4, n, op, uczestnicy, a_miejsc, a_liczba, gratis, k are natural numbers, whereas the other are real numbers.

With the above denotation we can calculate the cost of the tour per one participant using the algorithm shown in Figure 1. This is one of the proposals of an algorithm constituting a mathematical model of the solution to a situation known from everyday life.

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students limited their solutions to the first mathematical formulation of the model. Among 11 schemes from the first group only 4 work properly. This is a definitely disappointing result. One fact is comforting that in this group 7 out of 11 persons presented a scheme that is based on a well-thought out concept which precisely shows the dependencies between the data shown in the form of variables. They represent an attitude typical for a mathematician solving a problem using mathematical methods. Unfortunately, there are still people with a mathematical education who are way behind in understanding what mathematization is.

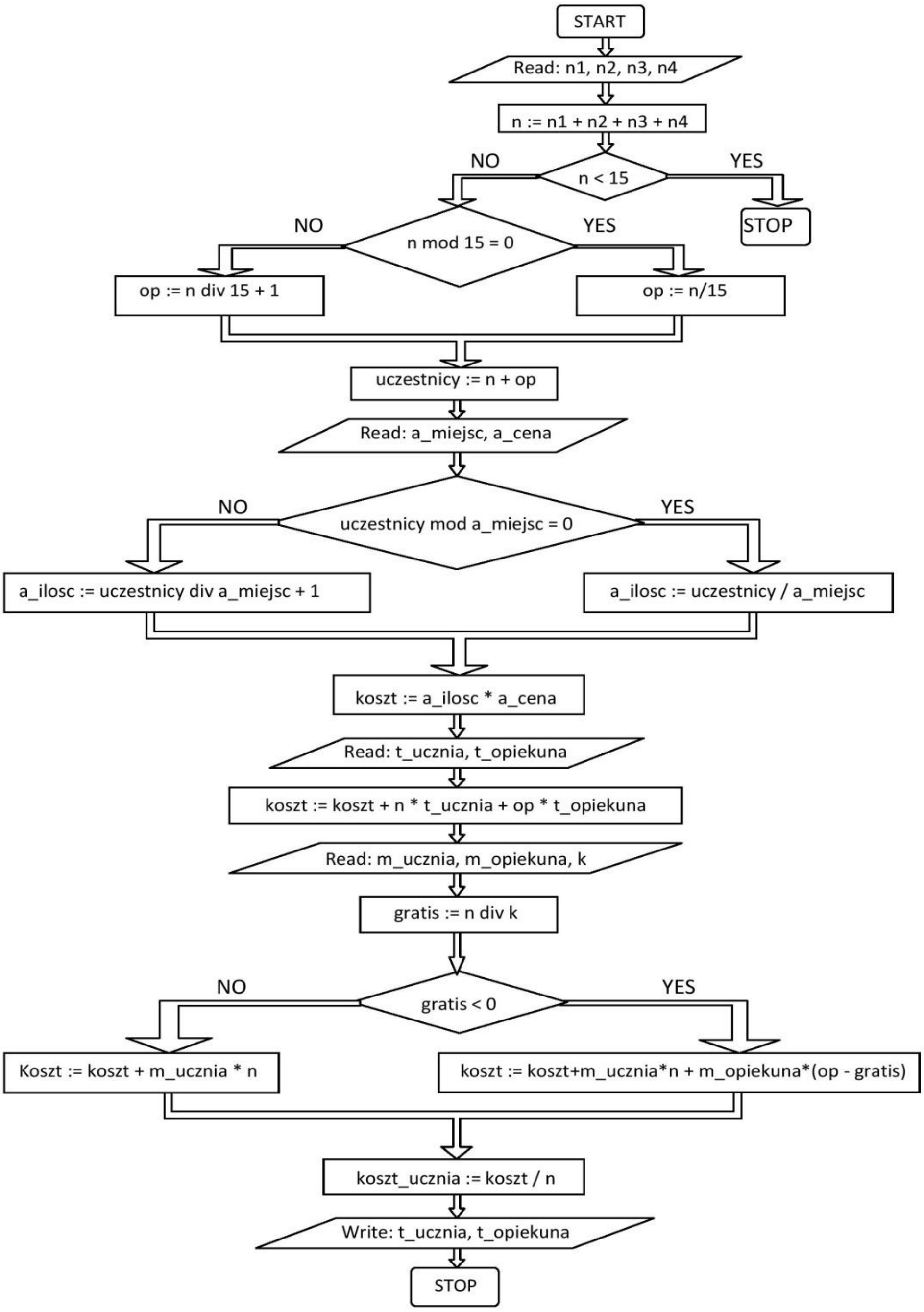

Fig. 1. Algorithm calculating the cost of participation in the school tour

Source: students' own works.

The starting point of this algorithm was determining the number of the tour participants. It was assumed that if the number of students is less than 15, the tour will not take place. A number above 15 defines the number of chaperones, which is determined by a conditional instruction. After calculating the number of participants, the number of coaches is determined, which depends on their capacity, and the cost of transport is determined. In further stages of the algorithm subsequent components of the tour cost are added, i.e. the entrance fee to the theatre and the museum.

The model takes into account the differentiation between the ticket prices for students and chaperones, and one chaperone may enter the museum free of charge with any *k*-number of students, and that is why it is checked whether the number of chaperones does not exceed the number of adults that can come free of charge. After adding up all the costs, the determined amount is divided between the students (we assume that all the financial burden associated with the tour is carried by the students) and in this way we get the cost of participation of one student in the tour. The most troublesome part of the algorithm concerns calculating the cost of transport. The number of coaches needed depends on their capacity and the number of participants, and the mathematical solution presented here is mathematically correct although not necessary economical. It may turn out that one of the coaches will take only a few persons and then the cost per person will increase significantly. It is not easy to create a simple scheme for calculating the cost of transport which will take into account the number of coaches and their capacity which will present an economical solution at the same time. This part of the algorithm helps to realize that the cost ineffectiveness appearing here has to be examined with real data, and depending on these data it has to be properly verified by changing the type of coach (of a different capacity).

It is possible to include such changes in the algorithm but in this way it will complicate and extend it significantly (it is difficult to present in this article the graphical version of the fully developed algorithm). Nevertheless, such a solution will be artificial because the discussed issue will always be examined in reality and verified e.g. by specifying the maximum number of the tour participants or completing the list of students willing to go in case there is a vacancy. I expected, though, that the students would test their models using realistic data and verify the complexity of the algorithm with their help.

3.2. Discussion of selected works of the research participants

Algorithmisation can be a method of improving the skills of mathematical modelling. Students of mathematics in the course of study become acquainted with the basic knowledge of algorithmisation and programming (normally there is no such subject as "modelling" in the Polish syllabus). So did the word "algorithm" significantly influence the method of students' work (Q1)? It turns out that the schemes from the second group are similar to those from the first group. Some changes appeared, though, but they are not significant for the understanding of what modelling is.

The analysis of the work showed that only 12 out of 28 algorithms work properly (only 4 out of 11 in group 1). In the presented solutions, a connection with the reality is repeatedly visible, e.g. by introducing unauthorized constants that limit the generality of the model (8). The majority of students included all the information given, whereas half of them extended it with additional costs which they considered important. Only 8 persons directly formulated the algorithm's assumptions, however the precision of formulating the dependencies between the considered variables is similar to the works from the first group.

Table 1. Forms of presenting the mathematical model

I group (11 persons)	II group (28 persons)
formula (4) formulas with description, commentary, instructions (3) diagram with list of steps (3) attempt to present the form of a spreadsheet on paper (1)	formula (10) description (3) list of steps (7) **block diagram (7)** commands connected with arrows (1)

Source: own elaboration.

The introduction of the word "algorithm" influenced most the form of presentation of the model, which forms the basis for referring back to one of the research questions (Q3). Table 1 shows the forms in which the students presented their solutions. Let us remember that the algorithm can be presented in various ways. The most commonly used forms are: list of steps, block diagram, pseudo-language and programming language. All groups used in similar proportions, forms like: formula, description, list of steps (see: Table 1). Additionally, a new form appeared in the second group, i.e.

a block diagram. The application of a block diagram reveals the clear influence of the change applied in the instruction. Also the arrows used to connect commands suggest an attempt to create an algorithm. Unfortunately, the form of the block diagram does not translate into the correct formulation of the algorithm.

Neither in this example nor in the other works did the presentation of the model in the form of a block diagram improve the precision of thinking about relationships and influence the way of mathematizing the problem. In these examples of work, unjustified constants appear, formulation is often unclear, ambiguous and not precise and colloquial statements appear as well. It turned out that none of these is a correct formulation of the algorithm. The outer form of the algorithm was not a sufficient factor for students to present their solutions in a precise, formal and at the same time understandable way for other users.

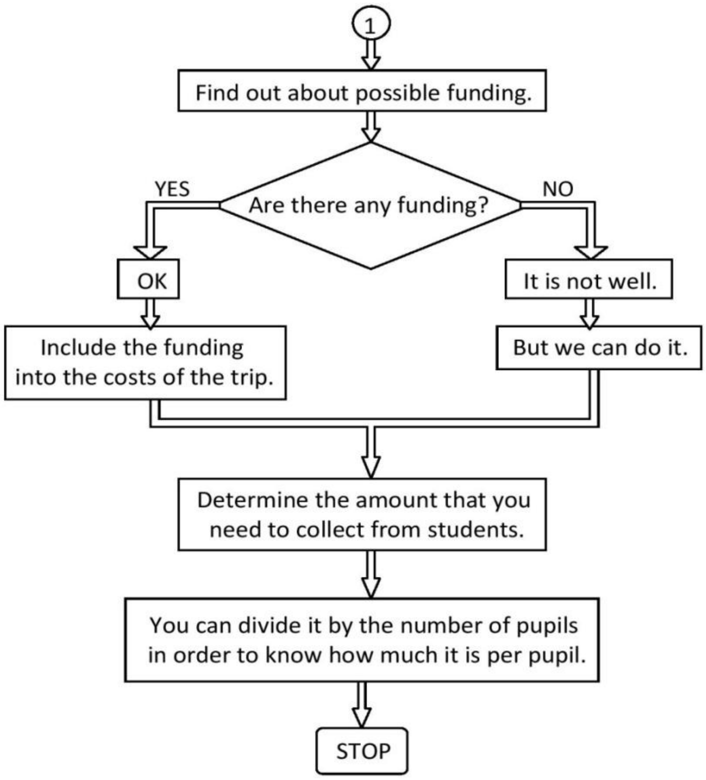

Fig. 2. A part of the algorithm written by student in informal language (scheme)

Source: students' own works.

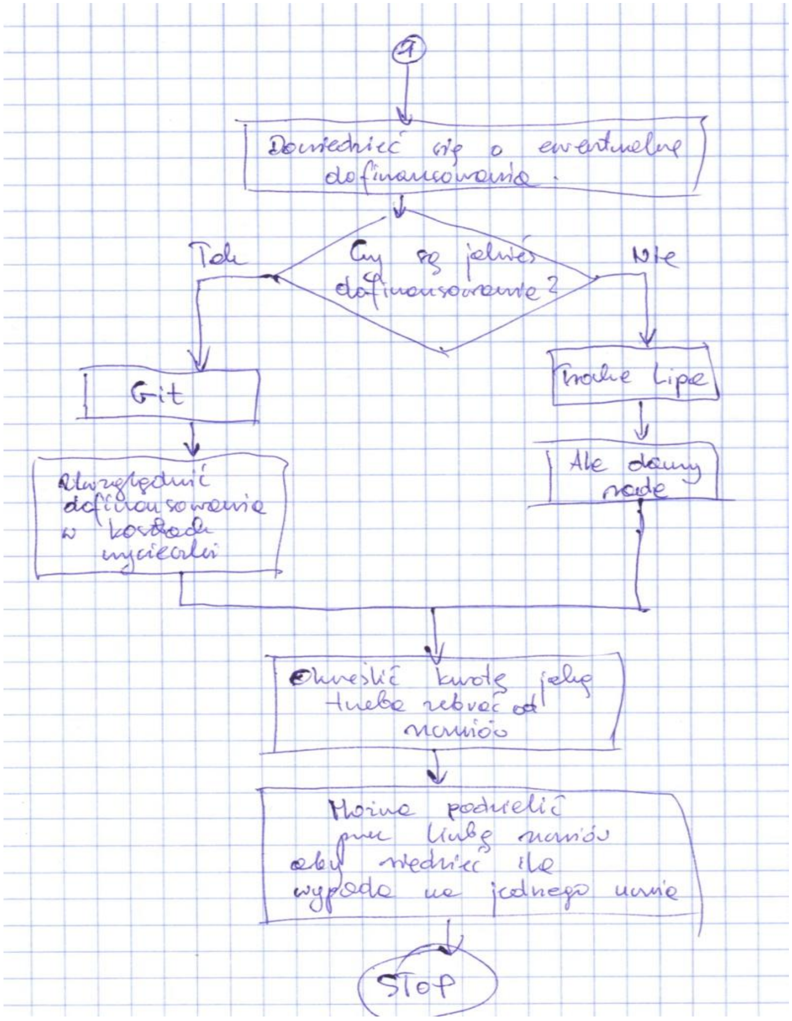

Fig. 3. A part of the algorithm written by a student in informal language (original)

Source: students' own works.

The analysis of the works under the terms of the second research question (Q2) showed that not all the possibilities offered by algorithmization were used. A typical element of an algorithm is a loop (iteration). This did not appear in any of the work. Although some of the conditions from the conditional instruction were not met (e.g. too few students, see Figure 4), which should lead to the termination of the flowchart, a "while" loop did not appear. Perhaps this is because the author of this work concluded that there was no point in organising a tour for such a small group.

Despite the word “algorithm”, the students from the second group formulated the assumptions that concern the operation of the model less frequently (8 out of 28, whereas 6 out of 11 in group I). This is quite a surprising situation because every algorithm is based on certain assumptions. If we do not know them, we cannot state in what situations a certain algorithm can be used. A lack of specification may lead to errors in using the algorithm because, e.g. a user may not know what a certain variable means. Perhaps the respondents limited themselves to the assumptions stated in the instructions. Nevertheless, it was necessary to provide information, the lack of which may lead to errors.

3.3. The beneficial effect of the word “algorithm” on the understanding of modelling

Is it the case that the change in the instruction did not influence the students' reasoning, but only the form of presenting the solution? Such a conclusion would be unfair and untrue. The word “algorithm” opened the students' awareness of the possibility of using new tools, and thus drew attention to those aspects of modelling which are associated with the tool. One of them is a conditional instruction, which now appears more frequently in their work (7/28) and is used in a wider range than in the first group. The conditional instruction applies there (2/11) only to rounding the number of chaperones, whereas in the second group it applies to various elements of the model, allowing for example to consider the possibility of obtaining funds for the tour or to stop calculating if the number of children who want to participate in the tour is too small.

The application of the form of the algorithm imposes a certain form of work but also it creates many opportunities. Let us look more closely at one of the works to have a better idea of how the students used algorithmisation to solve the task.

Figure 4 shows the graphical part of the student's solution (the solution was presented in two forms, the first of which was the list of steps). This algorithm can be treated as the first trial model which will be improved in further stages of creating the mathematical model. The presented solution is far from the example solution. What is problematic here is the precision of its presentation, but not the idea of how to solve the problem.

The presented solution is a block diagram with “start” and “stop” blocks – they appear only in some of the works. Processing blocks and input/output blocks are not precisely distinguished, although it can be clearly seen in the penultimate block that the respondent changed to the correct form.

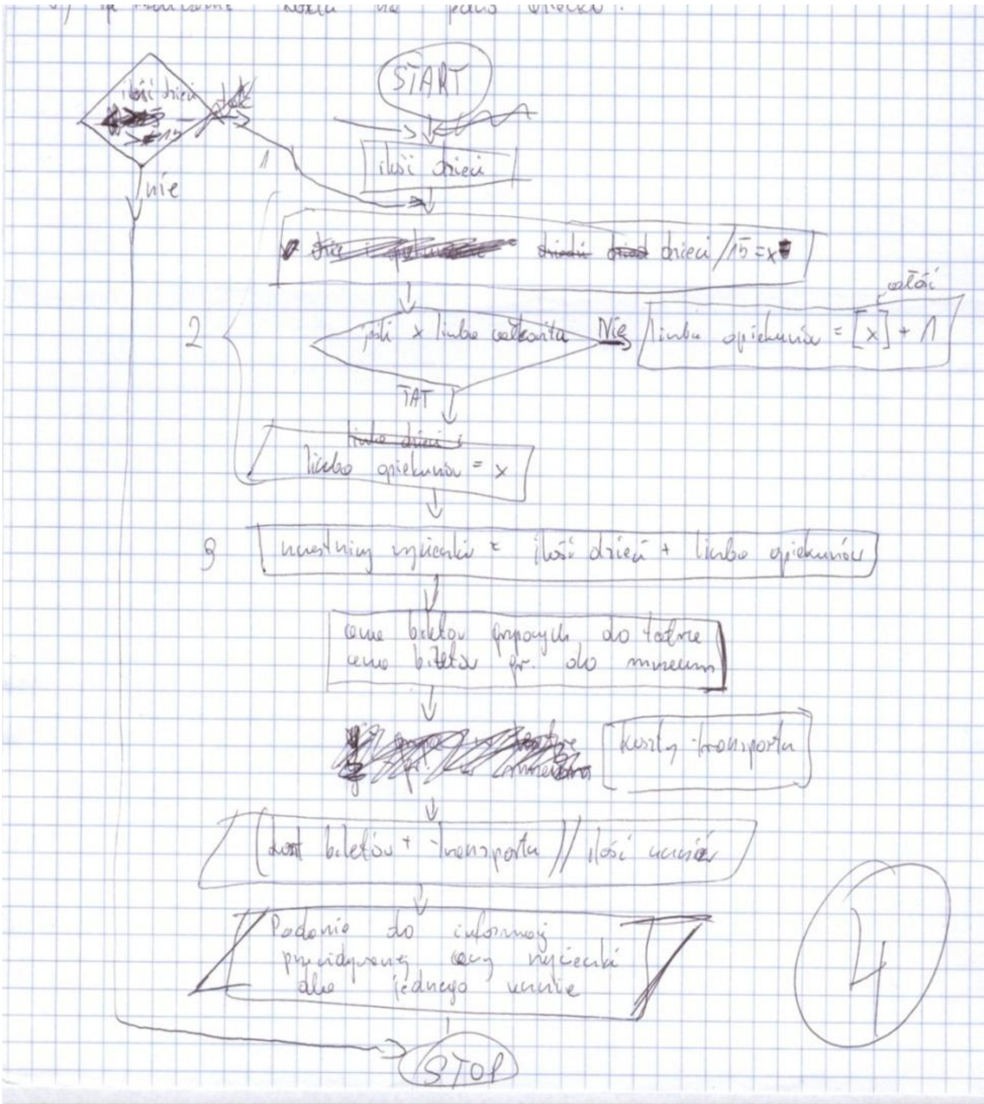

Fig. 4. Block diagram calculating the cost of the tour (original)

Source: students' own works.

In the algorithm, conditional instructions were used (one of them does not have all the connections). The author of this work did not formulate the assumptions and did not explain the symbols used, perhaps because the majority of the instructions were presented in the form of a verbal description. The descriptions are ambiguous, e.g. "costs of transport" can be interpreted as the overall cost of transport of the whole group or the cost per one participant. The algorithm does not precisely show the method of determining the considered elements because it was not specified, for example how to calculate the "cost of the tickets". The presented model indicates the elements which have to be considered while calculating the cost of the tour – they are only indicated but not precisely shown.

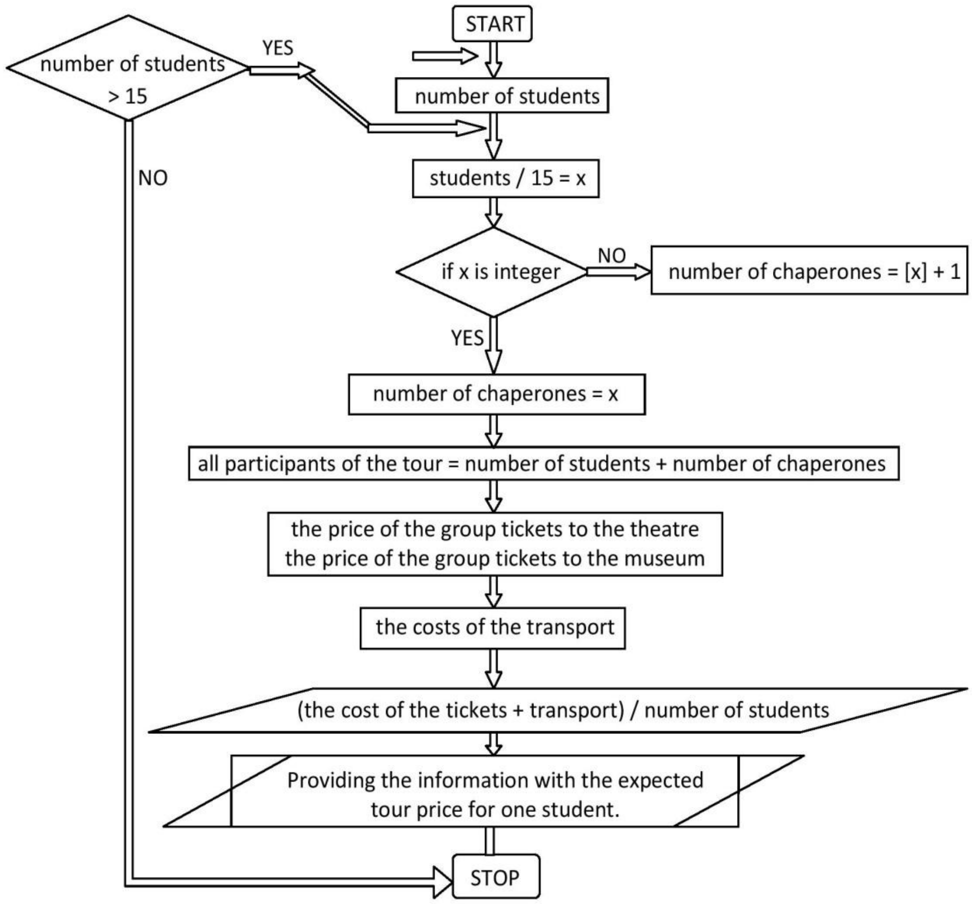

Fig. 5. Block diagram calculating the cost of the tour (original)

Source: students' own works.

4. Summary

Algorithmisation is one of the forms of creating a mathematical model of a situation known from everyday life. The application of this form should open the users' awareness to the opportunities that an algorithm can offer. In the surveyed group the introduction of the word "algorithm" did not result in significant changes in the presented models. The most visible change is the application of block diagrams, however this does not translate into the improved quality of the models in terms of their correctness and precision of formulation. Half of the presented models do not work well, which is a considerably unsatisfactory result. Recurrent errors were:

- perceiving the tasks only from the perspective of practical life,
- not noticing all the dependencies between the variables or their incorrect formulation,
- not considering all the possibilities, which with some data made the algorithm impossible to finish,
- not considering the necessity to round the number of chaperones when the result was not an integer number,
- assigning constant values to variables, which narrows a general situation to a specific case,
- using the same symbols for different variables,
- ambiguous, inaccurate, sometimes unclear and colloquial formulation of operations,
- incorrect formulation of the algorithm:
  - confused functions of block boxes,
  - lack of a beginning and end of the algorithm,
  - lack of all connections between the blocks (conditional blocks with only one way out because the second option is not included),
  - not using key words (key statements), i.e. until... follow...; if.... then... else.... (in descriptive forms),
- lack of specification of the algorithm.

As can be seen, despite some theoretical skills related to algorithmisation (acquired by students in computer science classes), students have numerous difficulties in applying this knowledge in an open situation. The core curriculum of teaching mathematics requires from teachers not only the ability of modelling, but most of all conducting classes during which students will make the effort to create models. Good preparation by a teacher in this respect is therefore a matter of urgency and we should not delude ourselves into believing that prospective teachers will learn this skill by themselves. One fact is comforting which is that the introduction of the word “algorithm” in the research had a positive impact on the understanding of modelling. The positives were the following:

- opening students' awareness to the possibility of using new tools – the more frequent use of conditional instructions,
- considering various versions (options) of solving the problem – conditional instructions concern different elements (aspects) of the model,
- marking the beginning and the end of the algorithm (start/stop),
- using the form of the block diagram with an attempt to show appropriate functions of block boxes.

The presented research indicates the weaknesses in students' skills, but on the other hand, it emphasizes the sense of connecting modelling with creating algorithms. It is worth paying more attention to improving students' algorithmisation skills by focusing not on the form of the presentation of the result, but on the correct, strict formulation of a mathematical model.

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