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IMPROVED SEPARATE RATIO EXPONENTIAL ESTIMATOR FOR POPULATION MEAN USING AUXILIARY INFORMATION

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ABSTRACT

This paper advocates the improved separate ratio exponential estimator for population mean $\bar{Y}$ of the study variable y using the information based on auxiliary variable x in stratified random sampling. The bias and mean squared error (MSE) of the suggested estimator have been obtained upto the first degree of approximation. The theoretical and numerical comparisons are carried out to show the efficiency of the suggested estimator over sample mean estimator, usual separate ratio and separate product estimator.

Key words: Study variable, auxiliary variable, stratified random sampling, separate ratio estimator, separate product estimator, bias and mean squared error.

1. Introduction

In sampling theory the use of the proper auxiliary information always increases the precision of an estimator. Stratification is one of the design tools which yield increased precision. Stratified sampling entails first dividing the whole population of N units into non-overlapping subpopulations of $N_1, N_2, ..., N_L$ units, respectively, called strata that together comprise the entire population, so that $N_1+N_2+ ... + N_L=N$ and then drawing an independent samples of size $n_1, n_2, ..., n_L$ from each stratum. If the sample in each stratum is a simple random sample, the whole procedure is described as stratified random sampling. We can stratify the population in such a manner that (i) within each

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stratum there is as uniformity as possible and (ii) among various strata the difference are as great as possible.

To obtain the full benefits from stratification, the values of the N_h must be known. We use this technique because when we divide heterogeneous population into relatively more homogenous sub population, it reduces heterogeneity and hence increases precision of the estimator. This technique is also preferred because of its administrative convenient in carrying out the survey.

The ratio estimate of the population mean $\bar{Y}$ can be made in two ways. One is to make a separate ratio estimate of the total of each stratum and add these totals. An alternative estimate is derived from a single combined ratio. Many authors Kadilar and Cingi (2003), Singh and Vishwakarma (2006), Singh and Vishwakarma (2010), Koyuncu and Kadilar (2010) etc. have suggested the estimators of population parameters in stratified random sampling.

Let the population of size N is equally divided into L strata with N_h elements in the h^{th} stratum such that $N=\sum_{h=1}^{L} N_h$. Let n_h be the size of the sample drawn from h^{th} stratum of size N_h by using simple random sampling without replacement (SRSWOR) such that sample size $n=\sum_{h=1}^{L} n_h$. Let y and x be the study and the auxiliary variables, respectively, assuming values y_{hi} and x_{hi} for the i^{th} unit in h^{th} stratum.

Let $W_h=\left(N_h/N\right)$ be the stratum weight, $f_h=\left(n_h/N_h\right)$ be the sampling fraction, $\left[\bar{Y}_h=\left(1/N_h\right)\sum_{i=1}^{N_h} y_{hi},\ \bar{X}_h=\left(1/N_h\right)\sum_{i=1}^{N_h} x_{hi}\right]$ and $\left[\bar{y}_h=\left(1/n_h\right)\sum_{i=1}^{n_h} y_{hi},\ \bar{x}_h=\left(1/n_h\right)\sum_{i=1}^{n_h} x_{hi}\right]$ be the population means and sample means of the study variate y and the auxiliary variate x respectively. Our purpose is to estimate the population mean $\bar{Y}=\sum_{h=1}^{L} W_h\bar{Y}_h=\bar{Y}_{st}$ of the study variable y.

When the population mean $\bar{X}_h$ of the h^{th} stratum of the auxiliary variable x is known then the usual separate ratio, product and regression estimators for population mean $\bar{Y}$ are respectively given as

$$\bar{y}_{rs}=\sum_{h=1}^{L} W_h\left(\frac{\bar{y}_h}{\bar{x}_h}\bar{X}_h\right) \tag{1.1}$$

$$\bar{y}_{ps} = \sum_{h=1}^{L} W_h \left(\bar{y}_h \frac{\bar{x}_h}{\bar{X}_h} \right) \quad (1.2)$$

$$\bar{y}_{lrs} = \sum_{h=1}^{L} W_h \left[\bar{y}_h + b_h \left(\bar{X}_h - \bar{x}_h \right) \right] \quad (1.3)$$

where $b_h = \left(s_{yxh} / s_{xh}^2 \right)$ is the sample regression coefficient of y on x of the h^{th} stratum, $s_{yxh} = \left\{ 1/\left(n_h - 1 \right) \right\} \sum_{i=1}^{n_h} \left(y_{hi} - \bar{y}_h \right)\left(x_{hi} - \bar{x}_h \right)$ is the sample covariance between y and x, $s_{yh}^2 = \left\{ 1/\left(n_h - 1 \right) \right\} \sum_{i=1}^{n_h} \left(y_{hi} - \bar{y}_h \right)^2$ is the sample mean square/variance of y and $s_{xh}^2 = \left\{ 1/\left(n_h - 1 \right) \right\} \sum_{i=1}^{n_h} \left(x_{hi} - \bar{x}_h \right)^2$ is the sample mean square/variance of x in the h^{th} stratum respectively.

We know that

$$\mathrm{Var}\left(\bar{y}_{st} \right) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 \quad (1.4)$$

where $S_{yh}^2 = \left\{ 1/\left(N_h - 1 \right) \right\} \sum_{i=1}^{N_h} \left(y_{hi} - \bar{Y}_h \right)^2$ is the population mean square/variance of the study variate y.

The mean squared error of the estimators $\bar{y}_{rs}$, $\bar{y}_{ps}$ and $\bar{y}_{lrs}$ are respectively given by

$$\mathrm{MSE}\left(\bar{y}_{rs} \right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + R_h^2 S_{xh}^2 - 2 R_h S_{yxh} \right] \quad (1.5)$$

$$\mathrm{MSE}\left(\bar{y}_{ps} \right) = \sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2 + R_h^2 S_{xh}^2 + 2 R_h S_{yxh} \right] \quad (1.6)$$

$$\mathrm{Var}\left(\bar{y}_{lrs} \right) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{yh}^2 \left(1 - \rho_h^2 \right) \quad (1.7)$$

where $\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h} \right)$, $R_h = \left(\bar{Y}_h / \bar{X}_h \right)$

and

$\rho_{hyx} = \left(S_{hyx} / S_{yh} S_{xh} \right)$.

In this paper, we suggested an improved separate ratio exponential estimator of the population mean $\bar{Y}$ of the study variable y using the supplementary information of the auxiliary variable x. The bias and mean squared error have been obtained upto the first degree of approximation.

2. An improved separate ratio exponential estimator

Motivated by Upadhyaya et al (2011), we suggested a separate ratio exponential estimator $t_{RS}^{(a)}$ of the population mean $\bar{Y}$ of the study variable y is defined as

$$t_{RS}^{(a)}=\sum_{h=1}^{L}W_h\bar{y}_h\exp\left[\frac{\bar{X}_h-\bar{x}_h}{\bar{X}_h+\left(a_h-1\right)\bar{x}_h}\right] \qquad (2.1)$$

To obtain the bias and mean square error (MSE) of the estimator $t_{RS}^{(a)}$ at (2.1), we write

$\bar{y}_h=\bar{Y}_h\left(1+e_{0h}\right)$ and $\bar{x}_h=\bar{X}_h\left(1+e_{1h}\right)$

such that $E\left(e_{0h}\right)=E\left(e_{1h}\right)=0$

and ignoring the finite population correction (fpc) term, we have

$$E\left(e_{0h}^2\right)=\frac{1}{\bar{Y}_h^2}\gamma_h S_{yh}^2,\quad E\left(e_{1h}^2\right)=\frac{1}{\bar{X}_h^2}\gamma_h S_{xh}^2,\quad E\left(e_{0h}e_{1h}\right)=\frac{1}{\bar{Y}_h\bar{X}_h}\gamma_h S_{yxh} \qquad (2.2)$$

Expressing (2.1) in terms of e's, we have

$$t_{RS}^{(a)}=\sum_{h=1}^{L}W_h\bar{Y}_h\left(1+e_{0h}\right)\exp\left[-\frac{e_{1h}}{a_h}\left\{1+\left(\frac{a_h-1}{a_h}\right)e_{1h}\right\}^{-1}\right]$$

$$=\sum_{h=1}^{L}W_h\bar{Y}_h\left(1+e_{0h}\right)\left[1-\frac{e_{1h}}{a_h}\left\{1+\left(\frac{a_h-1}{a_h}\right)e_{1h}\right\}^{-1}+\frac{e_{1h}^2}{2a_h^2}\left\{1+\left(\frac{a_h-1}{a_h}\right)e_{1h}\right\}^{-2}-...\right]$$

$$=\sum_{h=1}^{L}W_h\bar{Y}_h\left(1+e_{0h}\right)\left[1-\frac{e_{1h}}{a_h}\left\{1-\left(\frac{a_h-1}{a_h}\right)e_{1h}+\left(\frac{a_h-1}{a_h}\right)^2e_{1h}^2-...\right\}+\frac{e_{1h}^2}{2a_h^2}\left\{1-2\left(\frac{a_h-1}{a_h}\right)e_{1h}+3\left(\frac{a_h-1}{a_h}\right.\right.$$

$$=\sum_{h=1}^{L}W_h\bar{Y}_h\left(1+e_{0h}\right)\left[1-\frac{e_{1h}}{a_h}+\frac{\left(a_h-1\right)}{a_h^2}e_{1h}^2+\frac{1}{2a_h^2}e_{1h}^2+...\right]$$

$$=\sum_{h=1}^{L} W_h \bar{Y}_h \left[1+e_{0h}-\frac{e_{1h}}{a_h}-\frac{e_{0h}e_{1h}}{a_h}+\frac{e_{1h}^2}{a_h^2}\left(a_h-\frac{1}{2}\right)+...\right]$$

Neglecting the terms of e's having power greater than two, we have

$$\left(t_{RS}^{(a)}-\bar{Y}\right)=\sum_{h=1}^{L} W_h \bar{Y}_h \left[e_{0h}-\frac{e_{1h}}{a_h}+\frac{e_{1h}^2}{a_h^2}\left(a_h-\frac{1}{2}\right)-\frac{e_{0h}e_{1h}}{a_h}\right] \tag{2.3}$$

Taking expectation on both sides of (2.3), we have the bias of $t_{RS}^{(a)}$ upto the first degree of approximation as

$$B\left(t_{RS}^{(a)}\right)=\sum_{h=1}^{L} W_h^2 \gamma_h \frac{1}{\bar{X}_h}\left[\frac{1}{a_h^2}\left(a_h^2-a_h+\frac{1}{2}\right)R_h S_{xh}^2+\frac{S_{yxh}}{a_h}\right] \tag{2.4}$$

Squaring both sides of (2.3) and neglecting the terms having power greater than two, we have

$$\left(t_{RS}^{(a)}-\bar{Y}\right)^2 \cong \left[\sum_{h=1}^{L} W_h^2 \gamma_h \left(e_{0h}-\frac{e_{1h}}{a_h}\right)\right]^2$$

$$=\sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left(e_{0h}-\frac{e_{1h}}{a_h}\right)^2+\sum_{h\neq h'=1}^{L} W_h W_{h'} \bar{Y}_h \bar{Y}_{h'}\left(e_{0h}-\frac{e_{1h}}{a_h}\right)\left(e_{0h'}-\frac{e_{1h'}}{a_{h'}}\right)$$

$$\left(t_{RS}^{(a)}-\bar{Y}_h\right)^2=\sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left(e_{0h}^2+\frac{e_{1h}^2}{a_h^2}-2\frac{e_{0h}e_{1h}}{a_h}\right) \tag{2.5}$$

[since sampling from stratum to stratum is independent from each other]

Taking expectation on both sides of (2.5), we have the mean squared error of $t_{RS}^{(a)}$ upto the first degree of approximation as

$$MSE\left(t_{RS}^{(a)}\right)=\sum_{h=1}^{L} W_h^2 \gamma_h \left[S_{yh}^2+\frac{R_h^2}{a_h^2}S_{xh}^2-2\frac{R_h}{a_h}S_{yxh}\right] \tag{2.6}$$

where $R_h=\left(\bar{Y}_h/\bar{X}_h\right)$

The MSE of $t_{RS}^{(a)}$ is minimized at

$$a_h=\left(\frac{R_h}{\beta_h}\right)=a_{h0} \quad \text{(say)} \tag{2.7}$$

Thus the resulting minimum MSE of $t_{RS}^{(a)}$ is given by

$$\text{min.MSE}\left(t_{RS}^{(a)}\right)=\sum_{h=1}^{L}W_h^2\gamma_h S_{yh}^2\left(1-\rho_h^2\right) \tag{2.8}$$

which is also equal to the variance of the separate regression estimator $\bar{y}_{lrs}=\sum_{h=1}^{L}W_h\left[\bar{y}_h+b_h\left(\bar{X}_h-\bar{x}_h\right)\right]$.

3. Efficiency comparison

<u>Case I</u>. When the scalar a_h does not coincides at its exact optimum value a_{h0}

From (1.4), (1.5), (1.6) and (2.6), we have

(i) $\text{Var}\left(\bar{y}_{st}\right)-\text{MSE}\left(t_{RS}^{(a)}\right)>0$ if

$$\left[\sum_{h=1}^{L}W_h^2\gamma_h S_{yh}^2-\sum_{h=1}^{L}W_h^2\gamma_h\left(S_{yh}^2+\frac{R_h^2}{a_h^2}S_{xh}^2-2\frac{R_h}{a_h}S_{yxh}\right)\right]>0$$

Since $$\sum_{h=1}^{L}W_h^2\gamma_h\left(\frac{R_h^2}{a_h^2}S_{xh}^2-2\frac{R_h}{a_h}S_{yxh}\right)<0 \tag{3.1}$$

Therefore, $$\left(\frac{R_h^2}{a_h^2}S_{xh}^2-2\frac{R_h}{a_h}S_{yxh}\right)<0$$

$$a_h\geq\frac{R_h}{2\beta_h}\quad\text{and}\quad R_h\leq 0 \tag{3.2}$$

(ii) $\text{MSE}\left(t_{rs}\right)-\text{MSE}\left(t_{RS}^{(a)}\right)>0$ if

$$\left[\sum_{h=1}^{L}W_h^2\gamma_h\left(S_{yh}^2+R_h^2S_{xh}^2-2R_hS_{yxh}\right)-\sum_{h=1}^{L}W_h^2\gamma_h\left(S_{yh}^2+\frac{R_h^2}{a_h^2}S_{xh}^2-2\frac{R_h}{a_h}S_{yxh}\right)\right]>0$$

$$\sum_{h=1}^{L}W_h^2\gamma_h\left[R_h^2S_{xh}^2-\frac{R_h^2}{a_h^2}S_{xh}^2-2R_hS_{yxh}+2\frac{R_h}{a_h}S_{yxh}\right]>0 \tag{3.3}$$

either $R_h\left(1-\frac{1}{a_h}\right)>0$ or $\left[\left(1+\frac{1}{a_h}\right)R_hS_{xh}^2-2S_{yxh}\right]>0$

$$\left.\begin{array}{ll}\text{either} & 1 < a_h < \left(\dfrac{R_h}{2\beta_h - R_h}\right) \\ \text{or} & \left(\dfrac{R_h}{2\beta_h - R_h}\right) < a_h < 1\end{array}\right\} \tag{3.4}$$

or, equivalently

$$\text{min.}\left\{1, \left(\frac{R_h}{2\beta_h - R_h}\right)\right\} < a_h < \text{max.}\left\{1, \left(\frac{R_h}{2\beta_h - R_h}\right)\right\} \tag{3.5}$$

(iii) $MSE\left(t_{ps}\right) - MSE\left(t_{RS}^{(a)}\right) > 0$ if

$$\left[\sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + R_h^2 S_{xh}^2 + 2R_h S_{yxh}\right) - \sum_{h=1}^{L} W_h^2 \gamma_h \left(S_{yh}^2 + \frac{R_h^2}{a_h^2} S_{xh}^2 - 2\frac{R_h}{a_h} S_{yxh}\right)\right] > 0$$

$$\sum_{h=1}^{L} W_h^2 \gamma_h \left[R_h^2 S_{xh}^2 - \frac{R_h^2}{a_h^2} S_{xh}^2 + 2R_h S_{yxh} + 2\frac{R_h}{a_h} S_{yxh}\right] > 0 \tag{3.6}$$

$$\text{either} \quad R_h\left(1 + \frac{1}{a_h}\right) > 0 \quad \text{or} \quad \left[\left(1 - \frac{1}{a_h}\right) R_h S_{xh}^2 + 2S_{yxh}\right] > 0$$

$$\left.\begin{array}{ll}\text{either} & -1 < a_h < \left(\dfrac{R_h}{2\beta_h + R_h}\right) \\ \text{or} & \left(\dfrac{R_h}{2\beta_h + R_h}\right) < a_h < -1\end{array}\right\} \tag{3.7}$$

or, equivalently

$$\text{min.}\left\{-1, \left(\frac{R_h}{2\beta_h + R_h}\right)\right\} < a_h < \text{max.}\left\{-1, \left(\frac{R_h}{2\beta_h + R_h}\right)\right\} \tag{3.8}$$

<u>Case II</u>. When the scalar a_h coincides at its exact optimum value a_{h0}

From (1.4), (1.5), (1.6) and (2.8), we have

(iv) $\mathrm{Var}\left(\bar{y}_{st}\right)\text{-min.MSE}\left(t_{RS}^{(a)}\right) > 0$ if

$$\left[\sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2 - \sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2\left(1-\rho_h^2\right)\right] > 0$$

$$\text{Since } \sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2\rho_h^2 > 0 \tag{3.9}$$

$$\text{Therefore, } \quad \rho_h^2 > 0 \tag{3.10}$$

(v) $\mathrm{MSE}\left(t_{rs}\right)\text{-min.MSE}\left(t_{RS}^{(a)}\right) > 0$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[S_{yh}^2 + R_h^2 S_{xh}^2 - 2R_h S_{yxh} - S_{yh}^2\left(1-\rho_h^2\right)\right] > 0 \tag{3.11}$$

$$\rho_h^2 > -R_h^2\frac{S_{xh}^2}{S_{yh}^2}\left(1-2\frac{\beta_h}{R_h}\right) \tag{3.12}$$

(vi) $\mathrm{MSE}\left(t_{ps}\right)\text{-MSE}\left(t_{RS}^{(a)}\right) > 0$ if

$$\sum_{h=1}^{L} W_h^2\gamma_h\left[S_{yh}^2 + R_h^2 S_{xh}^2 + 2R_h S_{yxh} - S_{yh}^2\left(1-\rho_h^2\right)\right] > 0 \tag{3.13}$$

$$\rho_h^2 > -R_h^2\frac{S_{xh}^2}{S_{yh}^2}\left(1+2\frac{\beta_h}{R_h}\right) \tag{3.14}$$

4. Empirical study

To judge the merits of the proposed estimator over usual unbiased estimator $\bar{y}_{st}$, we considered a population data set whose description is given in the Table 4.1.

Table4.1. Data Statistics I [Source: Kadilar and Cingi (2005)]

In this data set, Y is the apple production amount and X is the number of apple trees in 854 villages of Turkey in 1999. The population information about this data set is given as:

$N_1=106$	$N_2=106$	$N_3=94$
$N_4=171$	$N_5=204$	$N_6=173$
$n_1=9$	$n_2=17$	$n_3=38$
$n_4=67$	$n_5=7$	$n_6=2$
$\bar{X}_1=24375$	$\bar{X}_2=27421$	$\bar{X}_3=72409$
$\bar{X}_4=74365$	$\bar{X}_5=26441$	$\bar{X}_6=9844$
$\bar{Y}_1=1536$	$\bar{Y}_2=2212$	$\bar{Y}_3=9384$
$\bar{Y}_4=5588$	$\bar{Y}_5=967$	$\bar{Y}_6=404$
$C_{x1}=2.02$	$C_{x2}=2.10$	$C_{x3}=2.22$
$C_{x4}=3.84$	$C_{x5}=1.72$	$C_{x6}=1.91$
$C_{y1}=4.18$	$C_{y2}=5.22$	$C_{y3}=3.19$
$C_{y4}=5.13$	$C_{y5}=2.47$	$C_{y6}=2.34$
$S_{x1}=49189$	$S_{x2}=57461$	$S_{x3}=160757$
$S_{x4}=285603$	$S_{x5}=45403$	$S_{x6}=18794$
$S_{y1}=6425$	$S_{y2}=11552$	$S_{y3}=29907$
$S_{y4}=28643$	$S_{y5}=2390$	$S_{y6}=946$
$\rho_1=0.82$	$\rho_2=0.86$	$\rho_3=0.90$
$\rho_4=0.99$	$\rho_5=0.71$	$\rho_6=0.89$
$\beta_2(x_1)=25.71$	$\beta_2(x_2)=34.57$	$\beta_2(x_3)=26.14$
$\beta_2(x_4)=97.60$	$\beta_2(x_5)=27.47$	$\beta_2(x_6)=28.10$
$\gamma_1=0.102$	$\gamma_2=0.049$	$\gamma_3=0.016$
$\gamma_4=0.009$	$\gamma_5=0.138$	$\gamma_6=0.006$
$\omega_1^2=0.015$	$\omega_2^2=0.015$	$\omega_3^2=0.012$
$\omega_4^2=0.04$	$\omega_5^2=0.057$	$\omega_6^2=0.041$

For the purpose of the efficiency comparison of the proposed estimator, we have computed the percent relative efficiencies (PREs) of the estimators with respect to the usual unbiased estimator $\bar{y}_{st}$ using the formula:

$$PRE\left(t,\bar{y}_{st}\right)=\frac{MSE\left(\bar{y}_{st}\right)}{MSE\left(t\right)}X100; \quad \text{where } t=\left(\bar{y}_{st}, \bar{y}_{rs}, \bar{y}_{ps} \text{ and } t_{RS}^{(a)}\right)$$

The findings are given in the Table 4.2.

Table4.2. PREs of the estimators $\left(\bar{y}_{st}, \bar{y}_{rs}, \bar{y}_{ps} \text{ and } t_{RS}^{(a)}\right)$ with respect to the usual unbiased estimator $\bar{y}_{st}$

S. No.	Estimator	$PRE\left(., \bar{y}_{st}\right)$
1.	$\bar{y}_{st}$	100.00
2.	$\bar{y}_{rs}$	423.2052
3.	$\bar{y}_{ps}$	37.6085
4.	$t_{RS}^{(a)}$	629.0317

From Table 4.2, it is clear that the suggested improved separate ratio exponential estimator $t_{RS}^{(a)}$ is more efficient than the unbiased sample mean estimator $\bar{y}_{st}$, usual separate ratio estimator $\bar{y}_{rs}$ and the usual separate product estimator $\bar{y}_{ps}$.

5. Conclusion

It is observed that the suggested improved separate ratio exponential estimator $t_{RS}^{(a)}$ is more precise than $\bar{y}_{st}$, $\bar{y}_{rs}$ and $\bar{y}_{ps}$. Thus the use of proposed estimator is justified in practice.

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