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THE FUZZY TOPSIS METHOD AND ITS IMPLEMENTATION IN THE R PROGRAMME

ROZMYTA METODA TOPSIS I JEJ IMPLEMENTACJA W PROGRAMIE R

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Summary: The TOPSIS method (*Technique for Order Preference by Similarity Ideal Solution*) suggested by Hwang and Yoon [1981], belongs to the group of pattern linear ordering methods of multidimensional objects. A characteristic feature of this method is a way to evaluate a synthetic criterion's values, which takes into consideration the distance of an evaluated object from a positive-ideal solution as well as from a negative-ideal solution. The fuzzy TOPSIS method enables the linear ordering of objects described through linguistic variables, whose values are expressed in the form of triangular fuzzy numbers. In this article, a way of synthetic measurement estimation in environment R was presented, according to the assumptions of the fuzzy TOPSIS method proposed by Chen [2000]. Scripts, which are included in the article make the accomplishment of this particular method's stages possible.

Keywords: linear ordering, fuzzy TOPSIS, fuzzy number, linguistic variable, R programme.

Streszczenie: Metoda TOPSIS (*Technique for Order Preference by Similarity Ideal Solution*), zaproponowana przez Hwanga i Yoona (1981 r.), należy do wzorcowych metod porządkowania liniowego obiektów wielowymiarowych. Jej cechą charakterystyczną jest sposób obliczania wartości kryterium syntetycznego, które uwzględnia odległość ocenianego obiektu zarówno od wzorca, jak i antywzorca rozwoju. Rozmyta metoda TOPSIS umożliwia porządkowanie liniowe obiektów opisanych za pomocą zmiennych lingwistycznych, których wartości wyrażone są w postaci trójkątnych liczb rozmytych. W artykule zaprezentowany został sposób estymacji miary syntetycznej w środowisku R zgodnie z założeniami rozmytej metody TOPSIS zaproponowanej przez Chena (2000 r.). Zamieszczone w artykule skrypty umożliwiają realizację poszczególnych etapów metody.

Słowa kluczowe: porządkowanie liniowe, rozmyta metoda TOPSIS, liczby rozmyte, rozmyte wagi, modele IRT.

1. Introduction

A lot of socio-economic phenomena have a complex character, therefore it is impossible to describe them by a single variable. The objects evaluation from the view of such phenomena is possible through the construction of a synthetic variable, aggregating fragmentary information included in particular criteria. Assessments of single criteria are often expressed in the form of linguistic values, which are the expression emerging from a natural language. Statistical analysis of this kind of information is possible, among others, through the substitution of linguistic expression by triangular fuzzy numbers, which constitute an exceptional case of fuzzy sets. For such prepared data, one can apply the fuzzy modification of linear ordering methods. One of them is a fuzzy TOPSIS method which finds the application in, among others, a situation when assessments and/or weights of criteria are expressed in the form of triangular fuzzy numbers.

The aim of this article is to discuss a fuzzy TOPSIS method in the issue of linear ordering objects and also in a presentation of some parts of authorial scripts in environment R enabling the accomplishment of this procedure. Scripts' functionality was illustrated by an example of the fuzzy TOPSIS method application, in the research on the satisfaction of the employees of administrative and territorial administrative offices.

2. Fuzzy TOPSIS method

The ordering of objects from the best one to the worst one considering an assumed synthetic measure, which is not subjected to a direct measurement, belongs to the task of linear ordering. The tool of these methods is the synthetic measure of development, constituting a certain function, aggregating fragmentary information included in particular variables. According to the way of synthetic measures construction of aggregation formula development, one can divide them into pattern and non-pattern ones. Pattern formulas are based on different type of distances of assessed objects from a pattern object [Walesiak 2006].

Hellwig's method [1968] and the TOPSIS method suggested by Hwang and Yoon [1981], can be included into the basic pattern linear ordering methods [Wysocki 2010]. The TOPSIS method can be treated as a modification of Hellwig's method. The difference is in the way of evaluating the synthetic criterion value. In the TOPSIS method, the formula which serves to evaluate this criterion takes into account, apart from the distance of the assessed object, also a distance from non-pattern development.

The fuzzy TOPSIS method was proposed by Chen [2000]. The difference in relation to the primary version of this method is in expressing assessments and/or criteria's weights in the form of triangular fuzzy numbers. An example of applying this method can be found, among others, in the studies of: Chang and Tseng [2008], Uyun and Riadi [2011], Madi and Tap [2011], Yayla et al. [Yayla, Yildiz, Özbek 2012], Jannatifar et al. [Jannatifar, Shahi, Moradi 2012], Erdoğan et al. [2013], Ataei [2013], Kia et al. [Kia, Danaei, Oroei 2014].

Let us assume that a certain set of objects $A = \{A_i | i = 1,\dots,n\}$ and a set of criteria $C = \{C_j | j = 1,\dots,m\}$, where $\widetilde{X} = \{\widetilde{x}_{ij} | i = 1,\dots,n; j = 1,\dots,m\}$ stand for a set of fuzzy evaluation criterion and $\widetilde{W} = \{\widetilde{w}_j | j = 1,\dots,m\}$ a set of fuzzy weights. The linear ordering of objects with the application of the fuzzy TOPSIS method with the above outlined assumptions requires the accomplishment of the following steps [Chen 2000]:

Step 1. Calculation of normalized fuzzy evaluation criteria:

$$\widetilde{z}_{ij} = \frac{\widetilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n} \widetilde{x}_{ij}^2}}, \ i = 1,\dots,n; \ j = 1,\dots,m . \tag{1}$$

Step 2. Calculation of weighted normalized fuzzy evaluation criteria:

$$\tilde{v}_{ij} = \tilde{w}_j \tilde{z}_{ij} . \tag{2}$$

Step 3. Appointing positive-ideal solution A^+ and negative-ideal solution A^- development:

$$\tilde{A}^+ = \left\{\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_m^+\right\} = \left\{(\max_i \tilde{v}_{ij} \,|\, j \in J_1), (\min_i \tilde{v}_{ij} \,|\, j \in J_2) \,|\, i = 1,\dots,n\right\}, \tag{3}$$

$$\tilde{A}^- = \left\{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_m^-\right\} = \left\{(\min_i \tilde{v}_{ij} \,|\, j \in J_1), (\max_i \tilde{v}_{ij} \,|\, j \in J_2) \,|\, i = 1,\dots,n\right\}, \tag{4}$$

where J_1 and J_2 are respectively the benefit criterion and the cost criterion.

Step 4. Calculation for each object of a distance from positive-ideal solution d_i^+ and negative-ideal solution d_i^- (in the original work it is an Euclidean distance).

Step 5. Calculation of a synthetic measure:

$$CC_i^+ = \frac{d_i^-}{d_i^+ + d_i^-}, \ \ i = (1,\dots,n) . \tag{5}$$

Measure values (5) are normalized in an interval $< 0;1 >$. The smaller the distance of an object from a positive-ideal solution and the bigger from a negative-ideal solution, the closer the value of a synthetic measure is to cohesion.

Step 6. Establishing the objects' ranking. The best object has the biggest value of a synthetic measure.

3. The assessment of employees' satisfaction with an application of the fuzzy TOPSIS method

The fuzzy TOPSIS method was applied to linear ordering of nine selected commune offices of the Zachodniopomorskie (West Pomeranian) Voivodship in respect of the employees' satisfaction[1]. For the above mentioned purpose, special scripts had been worked out, which can be downloaded from the website of the Department of Econometrics and Information Technology, Wroclaw University of Economics[2]. The opinions of 611 employees were taken into account. The detailed characteristics of the research material was introduced in the study of Błoński and Jefmański [2013]. The criteria were grouped into dimensions, according to the SERVQUAL model: C_1-C_2 – reliability, C_3-C_7 – responsiveness, C_8-C_{11} – assurance, C_{12}-C_{16} – empathy, C_{17}-C_{23} – tangibles. The names of the criteria were distinguished in Table 1 below.

Table 1. Criteria of employee's satisfaction

Symbol	Criteria
C_1	Timely handling of cases between co-workers at the office
C_2	Reliable handling of cases between co-workers at the office (no errors)
C_3	Desire to help from the other office staff
C_4	Cooperation in handling of cases by customers with other office staff
C_5	Desire to help from the other office staff in emergencies and crisis situations
C_6	Desire to help from the superior
C_7	Identifying the employees with the office
C_8	Confidentiality (non-commenting) of customer cases by the office staff
C_9	Adjust the level of knowledge and skills to the position held
C_{10}	Mutual respect and kindness at work
C_{11}	Sense of job security
C_{12}	Desire to share information helpful in handling of customer cases
C_{13}	Transmission of information between employees in a meaningful way
C_{14}	Adapting working time to the needs of customers
C_{15}	Efficient flow of information between employees and superiors
C_{16}	Clarity in commands formulated by the superior
C_{17}	Decor
C_{18}	Functionality of the workplace (space, lighting, etc.)
C_{19}	Availability of working facilities (fax, telephone, computer, copier)
C_{20}	Financial motivation
C_{21}	Non-financial motivation
C_{22}	Training
C_{23}	Opportunity for professional development

Source: [Błoński, Jefmański 2013].

[1] The research was part of the task: "Customer and Local Government Employees Satisfaction" carried out in the framework of the project: "Implementation of management improvements in local government units in the area of Zachodniopomorskie (Western Pomerania) province". Project manager: Prof. T. Lubińska, PhD, Szczecin University; task manager: Prof. Jolanta Witek, PhD.

[2] http://wgrit.ae.jgora.pl/ad/materialy/r/topsis_chan_procedure.r.

In the criteria's assessment, a rating scale was applied with the following items: 1 – very low, 2 – low, 3 – medium, 4 – high, 5 – very high. The ranges of triangular fuzzy numbers representing particular items were determined in accordance with the method using the Partial Credit Model belonging to the family of IRT models [Jefmański 2015]. In Table 2, the parameters' values of fuzzy numbers were distinguished, they are described as: a – left spread of fuzzy number, b – the centre of the fuzzy number area, for which the values of function's membership equals value 1, c – right spread of fuzzy number.

Table 2. Parameters of triangular fuzzy numbers for particular items

Criteria	Categories														
	very low			low			medium			high			very high		
	A	*B*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
C_{1}	-10	-10	-3.07	-3.07	-1,90	-0,72	-0,72	2,27	3,82	3,82	5,91	7,99	7,99	10	10
C_{2}	-10	-10	-4.41	-4.41	-2,63	-0,84	-0,84	2,96	5,08	5,08	6,96	8,84	8,84	10	10
C_{3}	-10	-10	-2,19	-2,19	-1,18	-0,17	-0,17	1,52	2,87	2,87	4,06	5,24	5,24	10	10
C_{4}	-10	-10	-4,09	-4,09	-2,48	-0,87	-0,87	1,84	2,8	2,8	4,29	5,78	5,78	10	10
C_{5}	-10	-10	-1,85	-1,85	-1,10	-0,35	-0,35	1,35	2,35	2,35	3,55	4,75	4,75	10	10
C_{6}	-10	-10	-1,29	-1,29	-0,89	-0,48	-0,48	1,06	1,63	1,63	2,83	4,02	4,02	10	10
C_{7}	-10	-10	-1,76	-1,76	-0,99	-0,22	-0,22	1,65	3,07	3,07	4,42	5,76	5,76	10	10
C_{8}	-10	-10	-1,67	-1,67	-0,92	-0,16	-0,16	1,01	1,85	1,85	3,12	4,38	4,38	10	10
C_{9}	-10	-10	-2,11	-2,11	-1,62	-1,13	-1,13	1,41	1,68	1,68	3,69	5,69	5,69	10	10
C_{10}	-10	-10	-1,28	-1,28	-1,11	-0,93	-0,93	1,39	1,85	1,85	2,92	3,99	3,99	10	10
C_{11}	-10	-10	-1,6	-1,6	-1,04	-0,48	-0,48	1,41	2,33	2,33	3,83	5,32	5,32	10	10
C_{12}	-10	-10	-3,08	-3,08	-2,02	-0,95	-0,95	1,69	2,43	2,43	4,15	5,87	5,87	10	10
C_{13}	-10	-10	-1,76	-1,76	-1,37	-0,98	-0,98	1,79	2,59	2,59	4,17	5,75	5,75	10	10
C_{14}	-10	-10	-1,36	-1,36	-1,24	-1,12	-1,12	1,52	1,92	1,92	3,81	5,7	5,7	10	10
C_{15}	-10	-10	-1,6	-1,6	-1,00	-0,4	-0,4	1,46	2,52	2,52	4,12	5,72	5,72	10	10
C_{16}	-10	-10	-1,77	-1,77	-1,20	-0,63	-0,63	1,23	1,82	1,82	3,35	4,88	4,88	10	10
C_{17}	-10	-10	-1,29	-1,29	-0,87	-0,44	-0,44	1,15	1,86	1,86	2,57	3,27	3,27	10	10
C_{18}	-10	-10	-1,57	-1,57	-1,04	-0,5	-0,5	0,96	1,42	1,42	2,4	3,38	3,38	10	10
C_{19}	-10	-10	-2,17	-2,17	-1,86	-1,55	-1,55	0,87	0,18	0,18	1,32	2,46	2,46	10	10
C_{20}	-10	-10	-1	-1	-0,62	-0,23	-0,23	1,4	2,57	2,57	3,5	4,42	4,42	10	10
C_{21}	-10	-10	-1,23	-1,23	-0,64	-0,05	-0,05	1,16	2,26	2,26	3,25	4,23	4,23	10	10
C_{22}	-10	-10	-1,03	-1,03	-0,86	-0,69	-0,69	1,13	1,57	1,57	2,9	4,22	4,22	10	10
C_{23}	-10	-10	-1,05	-1,05	-0,78	-0,5	-0,5	1,15	1,8	1,8	2,86	3,92	3,92	10	10

Source: own computations.

For each of the objects, the average assessment was calculated from the criterion in accordance with the principles of fuzzy numbers arithmetic. The average criteria's assessments will also have the form of triangular fuzzy numbers. The suggested script requires reading the data below in the following way:

```
          v1        v2         v3         v4
 [1,]  1.820000  2.100000  1.3653846  0.8215385
 [2,]  3.896154  4.292692  3.1765385  3.0396154
 [3,]  5.326923  5.916923  4.1769231  4.2069231
 [4,]  1.142222  1.438889  1.0038889  0.4669444
 [5,]  3.549306  4.175000  2.6065278  2.6956944
 [6,]  5.164722  6.053333  3.7841667  3.8683333
 [7,] -0.747500 -0.786875 -0.4981250 -1.2415625
 [8,]  1.517344  1.977812  0.8482812  0.8232812
 [9,]  3.450312  4.199375  2.5021875  2.3331250
[10,]  1.830417  2.464583  1.5191667  1.1366667
[11,]  4.216875  5.060625  2.8862500  3.0872917
[12,]  6.063333  7.026667  4.1258333  4.3854167
[13,]  1.737018  2.298947  1.4364912  1.0166667
[14,]  4.090263  4.867018  2.8579825  3.0106140
[15,]  5.877719  6.795789  4.0707018  4.2894737
[16,]  2.558889  3.435556  2.0255556  1.7805556
[17,]  4.895278  5.848889  3.3508333  3.6080556
[18,]  6.831667  7.795556  4.5816667  4.9522222
[19,]  0.978500  1.349500  0.9450000  0.4370000
[20,]  3.515750  4.280750  2.3990000  2.6012500
[21,]  5.261000  6.288000  3.6660000  3.8085000
[22,]  0.255625  0.584375  0.2293750 -0.2656250
[23,]  2.605938  3.300937  1.5818750  1.7462500
[24,]  4.669375  5.526875  3.2525000  3.2575000
[25,]  2.945625  3.835000  2.2581250  2.0687500
[26,]  5.252187  6.150000  3.7928125  4.0331250
[27,]  7.073125  7.972500  4.9450000  5.2987500
```

The extract of the read by script data file includes the values of the four first criteria for the nine objects which were analysed. The first three lines include parameters a, b and c of the fuzzy assessments of the first four criteria characterizing the first object. The next threesomes of lines include the parameters of the fuzzy assessments for further objects. It has to be emphasised that a script assumes that all criteria influence stimulatingly a synthetic criterion. Therefore, in cases when in the criteria's set there are a cost criterion, they should be previously transformed into benefit criterion. Formulas enabling such a treatment for triangular fuzzy numbers were introduced among others in the study of Wysocki [2010].

The fuzzy TOPSIS method assumes in the first step of procedure, the normalization of fuzzy numbers according to the formula of linear scale transformation. An extract of the code responsible for the normalization of triangular fuzzy numbers is introduced below:

```
normalization.Chen<-function(fuzzyData,type="n0"){
  toReturn<-fuzzyData;
```

```
  for(j in 1:dim(fuzzyData)[2]){
    if(type=="n0"){
      toReturn[,j,]<-fuzzyData[,j,]/max(fuzzyData[,j,3]);
    }
    else{
      toReturn[,j,]<-fuzzyData[,j,]/min(fuzzyData[,j,1]);
    }
  }
  toReturn;
}
```

According to the second step of the fuzzy TOPSIS method's procedure, the weights of particular criteria can be expressed in the form of triangular fuzzy numbers. For this purpose, one should give in the script the coordinates of a vector introduced in the following way: the first three ones constitute the following parameters a, b and c of the triangular fuzzy number representing the weight of the first criterion. In this example the same system of weights has been assumed for all variables, therefore the parameters' values of fuzzy numbers representing weights are the same and come to 1. Because the example takes into account 23 criteria, a weight vector should include 69 elements:

```
weightsRaw<-c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,
1,1,1,1,1,1,1,1);
```

Further stages of the fuzzy TOPSIS method are responsible for determining patterns and non-patterns, the calculation of Euclidean distances of each object from a pattern and non-pattern, and a calculation for each object synthetic criterion's value. The distinguished activities are illustrated in the extract of a code below:

```
for(i in 1:n){
  for(j in 1:m){
    for(k in 1:3){
      data1[i,j,k]<-dataRaw[k+(j-1)*3+(i-1)*m*3]
      weights[j,k]<-weightsRaw[k+(j-1)*3];
    }
  }
}
data3<-data2<-normalization.Chen(data1)
for(i in 1:n){
  data3[i,,]<-data2[i,,]*weights;
}

distancePlus<-rep(0,n);
distanceAnti<-rep(0,n);
for(i in 1:n){
  for(j in 1:m){
    distancePlus[i]<-distancePlus[i]+sqrt(1/3*
                                          ((data3[i,j,1]-
1)^2+(data3[i,j,2]-1)^2+(data3[i,j,3]-1)^2));
    distanceAnti[i]<-distanceAnti[i]+sqrt(1/3*
```

```
                                              ((da-
ta3[i,j,1])^2+(data3[i,j,2])^2+(data3[i,j,3])^2));
  }
}
```

The last code line of the studied script:

```
print(distanceAnti/(distancePlus+distanceAnti));
```

allows to show the values of synthetic measure for each of the objects:

```
[1] 0.5557600 0.4643094 0.2476754 0.5169039 0.5113564 0.5944460
[7] 0.4320480 0.3391906 0.6540052
```

The highest level of employees' satisfaction was observed in the last form of the analysed office. The values of synthetic measure for this object came to, approximately 0,65. The lowest value of synthetic measure was (0,25), which in other words means that the lowest level of employees' satisfaction was noted for the thir office in line.

4. Conclusion

The linear ordering of objects described through linguistic variables is possible through expressing the values of these variables in the form of fuzzy numbers, and then the application of a certain method of linear ordering. One such method often applied in the subject literature is the fuzzy TOPSIS method. In this article, the extracts of the studied script were presented, for a classical version of this method suggested by Chen [2000]. It assumes that the criteria of objects' assessment, as well as the weight ordered for each of the criteria, are expressed in the form of triangular fuzzy numbers.

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