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Neutrosophic VIKOR approach for multi-attribute group decision-making

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Abstract

Uncertainty, imprecise, incomplete, and inconsistent information can be found in many real-life systems and may cause more complex problems. A neutrosophic set is an effective and useful tool to describe problems with Uncertainty, imprecise, incomplete, and inconsistent information. The neutrosophic set is characterized by three independent degrees namely the truth-membership degree (T), indeterminacy-membership degree (I), and falsity-membership degree (F). In this paper, we present an extension of the VIKOR method for the solution of multi-criteria decision-making problems, namely neutrosophic set-VIKOR (NS-VIKOR) in a refined neutrosophic environment. The weight of each decision-maker is considered a single-valued neutrosophic number. The criteria for the weight of every decision-maker are also considered neutrosophic numbers. An aggregation operator is used to combine all decision-makers' opinions into a single opinion for a rating between criteria and alternatives. Euclidean distances from the positive and negative ideal solutions are calculated to construct relative closeness coefficients. Lastly, an illustrative example of tablet selection is provided to show the applicability of the proposed VIKOR approach.

Keywords: *fuzzy sets, VIKOR, group decision-making, refined neutrosophic environment*

1. Introduction

Many problems in engineering, medical science, economics, and the social sciences involve uncertainty. Researchers proposed some theories such as the theory of fuzzy set [57], the theory of intuitionistic fuzzy set [8], the theory of rough set [35], and the theory of vague set [19], to resolve these problems.

In many cases, it is difficult for decision-makers to precisely express a preference when solving multiple criteria decision-making (MCDM) problems with inaccurate, uncertain, or incomplete information. Under these circumstances, Zadeh's fuzzy sets (FSs) [57], where the membership degree is represented by a real number between zero and one, is regarded as an important tool for solving MCDM problems

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[12, 51], fuzzy logic and approximate reasoning [58], and pattern recognition [36]. However, FSs cannot handle those cases in which it is hard to define the membership degree using one specific value [10, 18]. To overcome the lack of knowledge of non-membership degrees, Atanassov [8] introduced intuitionistic fuzzy sets (IFSs) – an extension of Zadeh's FSs. Furthermore, Gau and Buehrer [19] defined vague sets. Subsequently, Bustince [14] pointed out that vague sets and IFSs are mathematically equivalent objects. To date, IFSs have been widely applied in solving MCDM problems [29, 50], neural networks [25, 45], medical diagnosis [42], colour region extraction [15, 16], and market prediction [22]. IFSs simultaneously consider the membership degree, non-membership degree, and hesitation degree. Therefore, they are more flexible and practical when addressing fuzziness and uncertainty compared to traditional FSs. Moreover, in some actual cases, the membership degree, non-membership degree, and hesitation degree of an element in IFSs may not be a specific number; hence, they were extended to interval-valued intuitionistic fuzzy sets (IVIFSs) [7].

IFSs and IVIFSs can handle only incomplete information but not indeterminate or inconsistent information which commonly exists in real situations. For example, if we ask the opinion of an expert about a certain statement, he or she may state that the probability that the statement is true is between 0/5 and 0/7. The probability that the statement is false is between 0/2 and 0/4. The degree to which he or she is not sure is between 0/1 and 0/3. For neutrosophic notation, it can be expressed as $x \ \ ([0/5, 0/7], [0/1, 0/3], [0/2, 0/4])$. In another example, suppose there are 10 voters in a voting process. In time t_1, four vote yes, three vote no, and three are undecided. In neutrosophic notation, it can be expressed as x (0/4, 0/3, 0/3). If in time t_2, two vote yes, three vote no, two give up, and three are undecided, it can be expressed as $x \ (0/2, 0/3, 0/3)$. The given information is beyond the scope of the IFS. Thus, the notion of a neutrosophic set is more general and can overcome the aforementioned issues.

The neutrosophic set generalizes the aforementioned sets from a philosophical point of view. From a scientific or engineering point of view, the neutrosophic set and set-theoretic operators need to be specified. Otherwise, it will be difficult to apply what is in real applications.

Generally, in the description of the shortcomings of uncertainty approaches, it can be said that random numbers are used to explain the uncertainty of variable occurrence, but they lack the objectivity to explain the uncertainty of variable value; gray numbers are unable to explain the degree of correctness or incorrectness; Fuzzy logic is unable to deal with situations about which there is no knowledge so that the decision-maker is faced with a third state called uncertainty or doubt. Intuitionistic fuzzy logic can only deal with incomplete information. It does not provide a solution for dealing with uncertain and inconsistent information. This is even though in neutrosophic logic, the sum of the components, like classical and classical fuzzy logic, is not necessarily one, and every number is between 0^- and 3^+, and this feature allows neutrosophic logic to be able to deal with contradictions; That is, it can check propositions that are true at the same time and false at the same time. Fuzzy logic cannot do this because there the sum of components must be one [31].

Decision-making in today's complex world has become a challenge for managers and organizations in such a way that the multiplicity of decision-making indicators, the variety of quantitative and qualitative criteria, and the need to consider them at the same time, the importance of the effects and consequences of decisions and factors like that add to the complexity of decisions. Decision-making is not a linear and one-dimensional process, and therefore a successful decision-maker is someone who examines the issue of the decision from different aspects and uses several criteria jointly and simultaneously in order to choose the best option based on different criteria. In other words, multi-criteria decision-making is a promising framework for evaluating multi-dimensional, contradictory and inconsistent issues in a way that includes the range of personal and individual issues to large and macro issues, and the decision-maker tries to choose the best option based on various criteria among several choose the available option; In this regard, it can be concluded that the growing trend of using neutrosophic logic in the field of decision-making topics, including the use of multi-criteria decision-making techniques, shows its effectiveness in explaining uncertainty [31].

Therefore, Wang et al. [49] proposed a single-valued neutrosophic set (SVNS) and provided the set-theoretic operators and various properties of SVNSs. Recently, Ye [54] proposed similar measures among interval neutrosophic sets and applied them to multi-criteria decision-making problems under an interval neutrosophic environment. Also, Abdel-Baset and his colleagues have done many studies in the neutrosophic environment, including supplier selection with group TOPSIS technique under type-2 neutrosophic number [6], project selection with a hybrid neutrosophic multiple criteria group decision--making [2], evaluation hospital medical care systems based on plithogenic sets [3], selecting supply chain with a hybrid plithogenic decision-making approach [4], solve transition difficulties with Utilising neutrosophic theory [5], Evaluation of the green supply chain management practices [1].

An SVNS is an instance of a neutrosophic set, which give the additional possibility to represent uncertain, imprecise, incomplete, or inconsistent information existing in the real world. It would be more suitable to apply indeterminate and inconsistent information measures in decision-making. However, the connector in the fuzzy set is defined concerning T, i.e., membership only. Hence, the information on indeterminacy and non-membership is lost. The connectors in the IFS are defined concerning T and F, i.e., membership and non-membership only. Hence, the indeterminacy is what is left from 1. On the other hand, in the SVNS, they can be defined with respect to any of them (no restriction). So, the notion of SVNSs is more general and overcomes the aforementioned issues. On the other hand, SVNSs can be used for scientific and engineering applications, because SVNS theory is valuable for modeling uncertain, imprecise, and inconsistent information. Due to its ability to easily reflect the ambiguous nature of subjective judgments, the SVNS is suitable for capturing imprecise, uncertain, and inconsistent information in multi-criteria decision-making analysis [53].

The VIKOR (Vlse Kriterijumsk Optimizacija Kompromisno Resenje in Serbian) method was introduced as an applicable technique to be implemented within an MCDM problem. It was developed as a multi-attribute decision-making method to solve a discrete decision-making problem with non-commensurable (different units) and conflicting criteria [33, 34]. This method focuses on ranking and selecting from a set of alternatives and determining the compromise solution for a problem with conflicting criteria, which can help decision-makers reach a final solution.

The VIKOR method was developed to solve MCDM problems that had non-commensurable and conflicting criteria. Many researchers have extended this method to a variety of fuzzy environments. Opricovic [32, 34] proposed a fuzzy VIKOR method in which both the attribute values and weights could be triangular fuzzy numbers. Sayadi et al. [40] extended the VIKOR method to a decision-making problem with interval numbers. Ju and Wang [23] presented an extension of the VIKOR method for a multiple-criteria group decision-making problem based on the two-tuple linguistic model. Wan et al. [48] developed an extended VIKOR method for multi-attribute group decision-making using triangular intuitionistic fuzzy numbers. Jiang and Shang [21] extended the VIKOR method to group decision-making by employing an optimization-based method to integrate the decision-makers' judgments. Liao et al. developed a hesitant fuzzy linguistic VIKOR (HFL-VIKOR) method. Qin et al. [38] presented a novel extension of the VIKOR method under an interval type-2 fuzzy environment.

Over the last decade, the VIKOR method has been extensively researched and applied to a variety of problems. Bazzazi et al. [11] proposed a VIKOR method to derive a preference order for open-pit mining equipment. Jahan et al. [20] presented a comprehensive VIKOR method for material selection. Shemshadi et al. [41] developed a fuzzy VIKOR method with entropy measures for objective weighting in a best-supplier selection problem. Liou et al. [26] utilized a modified VIKOR method to improve domestic airline service quality. Yuenur and Demirel [56] proposed an extended VIKOR method to solve an insurance company problem under a fuzzy environment. Chang [17] developed a fuzzy VIKOR method to evaluate hospital service quality in Taiwan. Kim and Chung [24] proposed a fuzzy VIKOR model to assess the vulnerability of water supply to climate change and variability in South Korea. Safari et al. [39] identified and evaluated enterprise architecture risks using a failure mode and effects analysis (FMEA) and a fuzzy VIKOR method. Liu et al. [27] used the VIKOR method to solve a site selection problem in waste management. Liu et al. [28] further proposed an approach for FMEA based on the combination of weighting and a fuzzy VIKOR method. Those studies demonstrate the rapid development of the VIKOR method and its successful application to diverse MCDM problems. However, although defuzzification is important in the fuzzy VIKOR method, it has seldom been employed in previous research. For this reason, the defuzzification step should be explicitly considered in the fuzzy VIKOR method framework. The VIKOR method can efficiently determine compromise solutions to problems with conflicting criteria; hence, it is suitable for machine tool selection problems. Therefore, in this paper, we develop a new multi-criteria group decision-making (MCGDM) framework for machine tool selection, based on the fuzzy VIKOR method.

In the VIKOR method, the numerical measures of the relative importance of criteria and the performance of each alternative in terms of these criteria are crucial. It is difficult to precisely determine the exact data, as human judgment is often vague under certain situations and conditions. Fuzzy sets and other nonstandard fuzzy sets [52] are efficient in tackling such uncertainties present in the provided data. Therefore, the extension of the VIKOR method to the non-standard fuzzy environment is natural. Out of these non-standard fuzzy sets, IFSs are more efficient in dealing with uncertainty. In many situations, the available information is not sufficient for the exact definition of the degree of membership for a certain element. There may be some degree of hesitation between membership and non-membership. Thus, in many real-life problems, due to the insufficiency in information availability, IFSs with little-known membership grades are appropriate. IFSs are particularly useful in dealing with uncertainty. In this paper, criteria values, and criteria weights are considered linguistic variables.

The present paper is devoted to the extension of the VIKOR approach for multi-attribute group decision-making (MAGDM) in a refined neutrosophic environment.

2. Neutrosophic sets

Neutrosophic set is a part of neutrosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra [43]. It is a powerful general formal framework that generalizes the sets of fuzzy and intuitionistic fuzzy from the philosophical point of view. We present a brief review of the general concepts of neutrosophic set [9, 43]:

Definition 1. Let X be the space of the objects and $x \in X$. A neutrosophic set A in X is defined by three functions: truth-membership function $T_A(x)$, an indeterminacy membership function $I_A(x)$, and falsity-membership function $F_A(x)$. These functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are defined on real standard or real non-standard subsets of $]0-, 1+[$. In other words, $T_A(x) : X \to]0-, 1+[$, $I_A(x) : X \to]0-, 1+[$ and $F_A(x) : X \to]0-, 1+[$. We have no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$; thus, $0- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3+$ [46, 47].

Definition 2. The complement of a neutrosophic set A is denoted by A^c and is defined as $T^c_A(x) = \{1^+\} - T_A(x)$, $I^c_A(x) = \{1^+\} - I_A(x)$, and $F^c_A(x) = \{1^+\} - F_A(x)$ for every x in X [43].

Definition 3. A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$, if and only if $\inf T_A(x) \leq \inf T_B(x)$, $\sup T_A(x) \leq \sup T_B(x)$, $\inf I_A(x) \geq \inf I_B(x)$, $\sup I_A(x) \geq \sup I_B(x)$, $\inf F_A(x) \geq \inf F_B(x)$ and $\sup F_A(x) \geq \sup F_B(x)$ for every x in X.

2.1. Single-valued neutrosophic sets

An SVNS is an instance of a neutrosophic set that can be used in real scientific and engineering applications. In the following section, we introduce the definition of an SVNS [49].

Definition 4. Let X be a space of points (objects) with generic elements in X denoted by x. An SVNS A in X is characterized by the truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$

Therefore, an SVNS A can be written as:

$$A = \{\langle x, T_A(x), I_A(x), F_A(x)\rangle | x \in X\} \tag{1}$$

The following expressions are defined in [22] for SVNSs A, B:

1. $A \subseteq B$ If and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$ for any x in X.
2. $A = B$ if and only if $A \subseteq B$, $B \subseteq A$.
3. $A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle | x \in X\}$.

For convenience, an SVNS A is denoted by the simplified symbol $A = \{T_A(x), I_A(x), F_A(x)\}$ for any x in X. For two SVNSs A and B, the operational relations are defined by [49].

1. $A \cup B = \langle \max(T_A(x),\ T_B(x)),\ \min(I_A(x),\ I_B(x)), V \min(F_A(x),\ \ F_B(x))\rangle$ for any x in X.
2. $A \cap B = \langle \min(T_A(x),\ T_B(x)),\ \max(I_A(x),\ I_B(x)),\ \max(F_A(x),\ F_B(x))\rangle$ for any x in X.

3. $A \oplus B = \langle T_A(x) + T_B(x) - T_A(x) \cdot T_B(x), I_A(x) \cdot I_B(x), F_A(x) \cdot F_B(x)\rangle$ for any x in X.
4. $A \otimes B = \langle T_A(x) \cdot T_B(x), I_A(x) + I_B(x) - I_A(x) \cdot I_B(x), F_A(x) + F_B(x) - F_A(x) \cdot F_B(x)\rangle$ for any x in X.

Wang et al. [49] defined the two operators – truth-favourite ($\triangle$) and falsity-favourite ($\bigtriangledown$) to remove the indeterminacy in the SVNSs and transform them into IFSs or paraconsistent sets. These two operators are unique on SVNSs, which are given as [49].

1. $\triangle A = \langle \min(T_A(x) + I_A(x), 1), 0, F_A(x)\rangle$ for any x in X.
2. $\bigtriangledown A = \langle T_A(x), 0, \min(F_A(x) + I_A(x), 1)\rangle$ for any x in X.

2.2. Neutrosophic refined set

Let A be a neutrosophic refined set.

$$A = \{\langle x, (T_A^1(x_i), T_A^2(x_i), \ldots, T_A^m(x_i)), (I_A^1(x_i), I_A^2(x_i), \ldots, I_A^m(x_i)), (F_A^1(x_i), F_A^2(x_i), \ldots, F_A^m(x_i))\rangle : x \in X\} \tag{2}$$

where $T_A^j(x_i) : X \in [0,1]$, $I_A^j(x_i) : X \in [0,1]$, $F_A^j(x_i) : X \in [0,1]$, $j = 1, 2, \ldots, m$ such that $0 \leq \sup T_A^j(x_i) + \sup I_A^j(x_i) + \sup F_A^j(x_i) \leq 3$, $j = 1, 2, \ldots, m$ for any $x \in X$. Now, $(T_A^j(x_i), I_A^j(x_i), F_A^j(x_i))$ are the truth-membership, indeterminacy membership, and falsity-membership sequences of the element x, respectively. Also, m is called the dimension of neutrosophic refined sets A [44].

2.3. Euclidean distance between two SVNSs

Let $A = \{\langle x_i : T_A(x_i), I_A(x_i), F_A(x_i)\rangle,\ i = 1, 2, \ldots, n\}$ and $B = \{\langle x_i : T_B(x_i), I_B(x_i), F_B(x_i)\rangle,\ i = 1, 2, \ldots, n\}$ be SVNSs. Then the Euclidean distance between two SVNSs A and B can be defined as follows [30]:

$$E(A,B) = \left(\sum_{i=1}^{n} ((T_A(x_i) - T_B(x_i))^2) + ((I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2)\right)^{1/2} \tag{3}$$

The normalized Euclidean distance between two SVNSs A and B can be defined as follows:

$$E_N(A,B) = \left(\frac{1}{3n}\sum_{i=1}^{n} ((T_A(x_i) - T_B(x_i))^2) + ((I_A(x_i) - I_B(x_i))^2 + (F_A(x_i) - F_B(x_i))^2)\right)^{1/2} \tag{4}$$

2.4. Crispfication of A neutrosophic set

Let $A = \{\langle x_i : T_{A_j}, I_{A_j}(x_i), F_{A_j}(x_i)\rangle, j = 1, 2, \ldots, n\}$ be n SVNSs . The equivalent crisp number of each W_j can be defined as [13]:

$$W_j^c = \frac{1 - \left(\frac{1}{3}\left((1 - T_{A_j}(x_i))^2 + (I_{A_j}(x_i))^2 + (F_{A_j}(x_i))^2\right)\right)^{1/2}}{\sum_{i=1}^{n}\left(1 - \left(\frac{1}{3}\left((1 - T_{A_j}(x_i))^2 + (I_{A_j}(x_i))^2 + (F_{A_j}(x_i))^2\right)\right)^{1/2}\right)} \quad W_j^c \geq 0\,, \sum_{k=1}^{p} W_j^c = 1 \tag{5}$$

2.5. Aggregation operator

In the present problem, there are p alternatives. The aggregation operator [15] applied to neutrosophic refined set is defined as follows [55]:

$$F(D_1, D_2, \ldots, D_r) = \left\langle \prod_{i=1}^{r} (T_{ij}^k)^{w_i}, \prod_{i=1}^{r} (I_{ij}^k)^{w_i}, \prod_{i=1}^{r} (F_{ij}^k)^{w_i} \right\rangle$$
$$\tilde{d}_{kj} = \left\langle \prod_{i=1}^{r} (T_{ij}^k)^{w_i}, \prod_{i=1}^{r} (I_{ij}^k)^{w_i}, \prod_{i=1}^{r} (F_{ij}^k)^{w_i} \rangle \ or \ \tilde{d}_{kj} = \langle \tilde{T}_{kj}, \tilde{I}_{kj}, \tilde{F}_{kj} \right\rangle \tag{6}$$

where $i = 1, 2, \ldots, r;\ j = 1, 2, \ldots, q;\ k = 1, 2, \ldots, p.$

2.6. VIKOR approach

The VIKOR approach is employed to identify the best alternative based on the concept of the compromise solution. The best compromise solution reflects the shortest Euclidean distance from the positive ideal solution and the largest Euclidean distance from the negative ideal solution. The VIKOR approach can be presented as follows: Assume that $A = \{A_1, A_2, \ldots, A_m\}$ is the set of alternatives with the set C of n criteria, $C = \{C_1, C_2, \ldots, C_n\}$, $D = (d_{ij})_{m\times n}$ is the decision matrix (Table 1), and $W = \{W_1, W_2, \ldots, W_n\}$ is the weight vector of criteria. Here, $d_{ij},\ i = 1, 2, \ldots, m$, are all single-valued neutrosophic numbers. Here, λ is the vector of experts' weight, based on which the opinion of experts is aggregated.

Table 1. Single-valued neutrosophic set decision matrix $D = (d_{ij})_{m\times n}$

Alternatives	Criteria			
	C_1	C_2	$\ldots$	C_n
A_1	$\langle d_{11}\rangle$	$\langle d_{12}\rangle$	$\ldots$	$\langle d_{1n}\rangle$
A_2	$\langle d_{21}\rangle$	$\langle d_{22}\rangle$	$\ldots$	$\langle d_{2n}\rangle$
$\vdots$	$\vdots$	$\vdots$		$\vdots$
A_m	$\langle d_{m1}\rangle$	$\langle d_{m2}\rangle$	$\ldots$	$\langle d_{mn}\rangle$
W_j	w_1	w_2	$\ldots$	w_n

Here, $d_{ij},\ i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$ are all single-valued neutrosophic numbers. Here, λ is the vector of experts' weight, based on which the opinion of experts is aggregated.

Step 1. Aggregate the decision-makers (DMs') opinions to construct a neutrosophic decision matrix. Let $r_{ij}^k = (T_{ij}^k,\ I_{ij}^k,\ I_{ij}^k)$ be the neutrosophic number provided by DM_k on the assessment of A_i with respect to C_j. The aggregated neutrosophic rating of alternatives concerning each criterion is calculated based on the neutrosophic weighted averaging (NWA) operator as:

$$r_{ij}^k = NWA\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(l)}\right) = \left\langle 1 - \prod_{k=1}^{l}\left(1 - T_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(I_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(F_{ij}^{(k)}\right)^{\lambda_k} \right\rangle \tag{7}$$

Step 2. Determine the weights of the criteria. There are various ways to determine the weights of the criteria. Let $w_j^k = (T_j^k, I_j^k, F_j^k)$ be the weight of criterion C_j given by K^{th} decision-maker DM. The aggregated neutrosophic weights (w_j) of criteria are calculated by

$$w_j = \lambda_1 w_j^{(1)} \cup \lambda_2 w_j^{(2)} \cup \cdots \cup \lambda_k w_j^{(k)} = \left\langle 1 - \prod_{k=1}^{l} \left(1 - T_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l} \left(I_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l} \left(F_{ij}^{(k)}\right)^{\lambda_k} \right\rangle \quad (8)$$

where $w_j = (T_j, I_j, F_j),\ j = 1, 2, \ldots, n$

Step 3. Determine the neutrosophic positive ideal solutions (NPIS). $A_j^* = (T_j^*, I_j^*, F_j^*)$ and the neutrosophic negative ideal solutions (NNIS)

$$A_j^* = \begin{cases} \max\limits_i r_{ij} & \text{for benefit criteria} \\ \min\limits_i r_{ij} & \text{for cost criteria} \end{cases} \qquad i = 1, 2, \ldots, m \quad (9)$$

$$A_j^- = \begin{cases} \min\limits_i\ r_{ij} & \text{for benefit criteria} \\ \max\limits_i\ r_{ij} & \text{for cost criteria} \end{cases} \qquad i = 1, 2, \ldots, m \quad (10)$$

Step 4. Compute the normalized neutrosophic difference $(\bar{d}_{ij})$ using Euclidean distance.

$$\bar{d}_{ij} = \frac{d\left(A_j^*, r_{ij}\right)}{d\left(A_j^*, A_j^-\right)} \quad (11)$$

$$d\left(A_j^*, r_{ij}\right) = \left(\frac{1}{2}\left(\left(T_j^* - T_{ij}\right)^2 + \left(I_j^* - I_{ij}\right)^2 + \left(F_j^* - F_{ij}\right)^2\right)\right)^{1/2} \quad (12)$$

$$d\left(A_j^*, A_j^-\right) = \left(\frac{1}{2}\left(\left(T_j^* - T_j^-\right)^2 + \left(I_j^* - I_j^-\right)^2 + \left(F_j^* - F_j^-\right)^2\right)\right)^{1/2} \quad (13)$$

According to the calculated distance of each A^j, the normalized distance matrix is determined as follows:

$$\bar{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_n \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & \cdots & C_m \end{matrix} \\ \left|\begin{matrix} \bar{d}_{11} & \bar{d}_{12} & \cdots & \bar{d}_{1m} \\ \bar{d}_{21} & \bar{d}_{22} & \cdots & \bar{d}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \bar{d}_{n1} & \bar{d}_{n2} & \cdots & \bar{d}_{nm} \end{matrix}\right| \end{array}$$

Step 5. Compute the values S_i, R_i, and Q_i, $i = 1, 2, \ldots, m$. According to matrix $\bar{D}$, values of S_i, R_i, Q_i for each alternative are determined using the following equations:

$$S_j = \sum_{j=1}^{n} \bar{w}_j \bar{d}_{ij}, \quad R_j = \max_j(\bar{w}_j \bar{d}_{ij})$$

$$Q_j = \frac{v(S_j - S^*)}{(S^- - S^*)} + \frac{(1 - v)(R_j - R^*)}{(R^- - R^*)}$$

$$S^* = \min_j S_j, \quad S^- = \max_j S_j, \qquad R^* = \min_j R_j, \; R^- = \max_j R_j$$

where w_i is the weight of criteria, v are the weights of maximum group utility.

Step 6. Rank the alternatives. Sorting by the values S, R, and Q in increasing order. The three ranking lists represent the final result.

Step 7. Propose a compromise solution of the alternative $(a^{'})$. It is ranked the best by measure Q (minimum) if the two following conditions are satisfied [34]:

1. Acceptable advantage. $Q(a^{''}) - Q(a^{'}) \geq DQ$, where $(a^{''})$ is the alternative with second position in the ranking list by Q; DQ = $\dfrac{1}{(M-1)}$; M is the number of alternatives.
2. Acceptable stability in decision-making: Alternative $(a^{'})$ must also be ranked best by S and/or R. The best alternative ranked by Q is the one with the minimum value of Q. The main ranking result is the compromise ranking list of alternatives.,

3. Numerical example

In this section, we present a numerical example from the paper by Pramanik et al. [37] to illustrate how the proposed method can be used. Suppose that the owner of a small shop wants to buy a tab. After initial screening, three tabs from three different companies (A_1, A_2, A_3) are selected for further evaluation. A committee of four decision-makers, namely D_1, D_2, D_3, D_4, has been formed to choose the most suitable tablet concerning five main criteria $C_1 - C_5$. The criteria are as follows:

1) technical specifications (C_1), 2) quality (C_2), 3) supply chain reliability (C_3), 4) finances (C_3), 5) ecology (C_5). In the present problem, $r = 4, \; j = 1, \; 2, \; 3, \; 4, \; 5, \; i = 1, \; 2, \; 3$.

Step 1. Constructing the decision matrix. The results of the evaluation of alternatives by four experts, based on the criteria, are shown in Table 2.

Table 2. Single-valued neutrosophic set decision matrix

D_1	C_1	C_2	C_3	C_4	C_5
A_1	(0.7, 0.2, 0.1)	(0.8, 0.3, 0.3)	(0.4, 0.1, 0.2)	(0.5, 0.1, 0.1)	(0.6, 0.4, 0.1)
A_2	(0.6, 0.2, 0.1)	(0.7, 0.4, 0.2)	(0.3, 0.2, 0.1)	(0.3, 0.1, 0.2)	(0.8, 0.2, 0.2)
A_3	(0.7, 0.1, 0.2)	(0.6, 0.2, 0.2)	(0.4, 0.4, 0.4)	(0.6, 0.1, 0.1)	(0.7, 0.1, 0.1)
D_2	C_1	C_2	C_3	C_4	C_5
A_1	(0.8, 0.2, 0.1)	(0.7, 0.1, 0.2)	(0.5, 0.1, 0.1)	(0.6, 0.2, 0.3)	(0.5, 0.6, 0.1)
A_2	(0.7, 0.3, 0.2)	(0.6, 0.1, 0.1)	(0.6, 0.2, 0.3)	(0.5, 0.1, 0.2)	(0.4, 0.5, 0.2)
A_3	(0.6, 0.2, 0.2)	(0.8, 0.2, 0.1)	(0.6, 0.1, 0.2)	(0.7, 0.1, 0.1)	(0.5, 0.5, 0.1)
D_3	C_1	C_2	C_3	C_4	C_5
A_1	(0.9, 0.1, 0.1)	(0.5, 0.3, 0.2)	(0.6, 0.4, 0.1)	(0.2, 0.5, 0.3)	(0.4, 0.4, 0.4)
A_2	(0.8, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.5, 0.4, 0.1)	(0.4, 0.2, 0.1)	(0.5, 0.3, 0.2)
A_3	(0.8, 0.1, 0.2)	(0.7, 0.1, 0.1)	(0.6, 0.3, 0.2)	(0.4, 0.1, 0.1)	(0.6, 0.1, 0.2)
D_4	C_1	C_2	C_3	C_4	C_5
A_1	(0.6, 0.1, 0.1)	(0.8, 0.2, 0.1)	(0.9, 0.2, 0.3)	(0.7, 0.4, 0.3)	(0.7, 0.3, 0.4)
A_2	(0.7, 0.2, 0.01)	(0.7, 0.1, 0.3)	(0.7, 0.3, 0.1)	(0.6, 0.5, 0.1)	(0.6, 0.2, 0.4)
A_3	(0.7, 0.1, 0.2)	(0.6, 0.1, 0.2)	(0.6, 0.2, 0.1)	(0.7, 0.1, 0.3)	(0.7, 0.3, 0.2)

Step 2. The neutrosophic weights of decision-makers are considered

	K_1	K_2	K_3	K_4
W	(0.8, 0.1, 0.1)	(0.9, 0.2, 0.1)	(0.5, 0.4, 0.1)	(0.8, 0.2, 0.2)

Using Equation (5), the equivalent crisp weights are:

	K_1	K_2	K_3	K_4
W	0.261996	0.261996	0.22366	0.2523

Step 3. The aggregated decision matrix can be determined by applying the aggregated operator (6) and calculated as in Table 3:

Table 3. The aggregated neutrosophic decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.738, 0.144, 0.1)	(0.695, 0.203, 0.187)	(0.57, 0.162, 0.158)	(0.465, 0.244, 0.225)	(0.543, 0.414, 0.193)
A_2	(0.693, 0.222, 0.067)	(0.65, 0.184, 0.158)	(0.499, 0.259, 0.133)	(0.436, 0.175, 0.144)	(0.559, 0.278, 0.238)
A_3	(0.693, 0.12, 0.2)	(0.67, 0.144, 0.143)	(0.54, 0.219, 0.201)	(0.593, 0.1, 0.132)	(0.619, 0.201, 0.139)

Step 4. Determining the weights of the criteria. The weight matrix (Table 3) of the criteria described in this problem can be displayed as follows (Table 4).

Table 4. Weight matrix of criteria

	C_1	C_2	C_3	C_4	C_5
D_1	(0.9, 0.1,0.2)	(0.8,0. 2,0.3)	(0.5,0.4,0.3)	(0.5,0.2,0.15)	(0.5,0.4,0.4)
D_2	(0.8,0.2,0.1)	(0.7,0.1,0.3)	(0.6,0.3,0.3)	(0.8,0.25,0.1)	(0.6,0.3,0.4)
D_3	(0.6,0.3,0.2)	(0.5,0.3,0.2)	(0.8,0.2,0.1)	(0.7,0.2,0.1)	(0.4,0.4,0.4)
D_4	(0.6,0.1,0.2)	(0.6,0.1,0.2)	(0.6,0.2,0.3)	(0.5,0.1,0.2)	(0.3,0.2,0.1)

The aggregated weights for all criteria are presented below:

C_1	C_2	C_3	C_4	C_5
(0.725, 0.15, 0.166)	(0.653, 0.15, 0.25)	(0.604, 0.27, 0.241)	(0.608, 0.178, 0.133)	(0.444, 0.31, 0.281)

Step 5. The aggregated weighted neutrosophic (w_j) of criteria. Based on the obtained weights for the criteria and the aggregated evaluation of the matrix of alternatives (above table), the aggregated weighted neutrosophic matrix is as follows (Table 5):

Table 5. The aggregated weighted neutrosophic decision matrix

	C_1	C_2	C_3	C_4	C_5
A_1	(0.535, 0.272, 0.249)	(0.454, 0.323, 0.39)	(0.344, 0.389, 0.361)	(0.283, 0.378, 0.328)	(0.241, 0.595, 0.42)
A_2	(0.502, 0.339, 0.222)	(0.424, 0.306, 0.369)	(0.302, 0.459, 0.342)	(0.265, 0.322, 0.258)	(0.248, 0.502, 0.452)
A_3	(0.502, 0.252, 0.333)	(0.437, 0.272, 0.357)	(0.326, 0.43, 0.394)	(0.361, 0.26, 0.247)	(0.275, 0.449, 0.381)

Step 6. Since the present problem pertains to the decision to buy a tablet, the decision matrix is a profit-type matrix. Using (5) and (6), the NPIS and NNIS are presented below:

	C_1	C_2	C_3	C_4	C_5
A^+	$(0.535, 0.252, 0.222)$	$(0.454, 0.272, 0.357)$	$(0.344, 0.389, 0.342)$	$(0.361, 0.26, 0.247)$	$(0.275, 0.449, 0.381)$
A^-	$(0.502, 0.339, 0.333)$	$(0.424, 0.323, 0.39)$	$(0.302, 0.459, 0.394)$	$(0.265, 0.378, 0.328)$	$(0.241, 0.595, 0.452)$

Step 7. The distance between the weighted matrix and the positive ideal. The results are given in Table 6.

Table 6. The distance between the weighted matrix and the positive ideal

d_{ij}	C_1	C_2	C_3	C_4	C_5
A_1	0.0341	0.060	0.019	0.163	0.155
A_2	0.093	0.047	0.082	0.114	0.093
A_3	0.115	0.017	0.068	0	0

Also, the $\bar{d_{ij}}$ matrix is as follows (Table 7).

Table 7. The normalized neutrosophic distance

d_{ij}	C_1	C_2	C_3	C_4	C_5
A_1	0.053	0.093	0.029	0.251	0.240
A_2	0.144	0.072	0.127	0.177	0.143
A_3	0.178	0.026	0.106	0	0

Step 8. The results of applying the extended VIKOR method of neutrosophic numbers to solve this problem for the final argument are presented in Table 8.

Table 8. Ranking of alternatives by S, R, and Q in ascending order

	A_1	A_2	A_3	Ranking order
S_j	0.129	0.132	0.066	$A_3 > A_1 > A_2$
R_j	0.05	0.04	0.04	$A_2 > A_3 > A_1$
Q_j	0.178	0.026	0.106	$A_3 > A_1 > A_2$

The utility threshold value is $Q(a^{''}) - Q(a^{'}) \geq \dfrac{1}{M-1}$. Following the judgment standards and rules, it is found that A_3 is weakly superior to A_1, A_2. In other words, the final order relation is $A_3 > A_1 > A_2$, with A_3 the best supplier.

4. Conclusions

The information on rating values considered in multi-attribute decision-making (MADM) problems is imprecise, indeterminate, incomplete, and inconsistent. NS is a useful tool that can capture all these types of information in the MADM process. In this paper, we have investigated the MADM problem in which rating values are considered with NSs. To extend the VIKOR method for MADM, we first define the normalized Hamming distance of NS. Having defined the positive ideal solution (PIS) and the negative ideal solution (NIS), we calculate the distance between each alternative and the ideal alternatives (PIS and NIS). Then, we determine S, R, and Q values to obtain the ranking order of all the alternatives. Finally, we provide an illustrative example to show the validity and effectiveness of the proposed approach. This paper presents the VIKOR approach for MAGDM for a refined neutrosophic environment. In the future, we will extend the proposed approach to MADM under an SVNHFS environment with unknown weight information and MADM with the interval-valued neutrosophic hesitant fuzzy environment.

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