Abstract

In this article we present a comparative case study of two master courses in mathematics organised respectively by a university in the UK and in Germany. The two cases are analysed in terms of the aspects of Autonomy, Mastery and Purpose which are related to the self-determination theory of Deci and Ryan and its concept of autonomous motivation. We reveal the differences between the two universities and consider their influence on the students behaviour and performance. Stronger features of Autonomy, Mastery and Purpose coincide with positive effects, which is in accordance with the predictions of self-determination theory. We propose Autonomy, Mastery and Purpose as a basis for establishing good practice in university teaching and make some suggestions for practical implementation.

Keywords: self-determination theory, autonomy, mastery, purpose, motivation, university teaching, course organization, module structure, assessment

Introduction

The initial idea for writing this article is closely related to certain events in the authors personal history. Between the years 2007 and 2011 the author had the chance to participate as a student in two courses. These courses were both at the master level and both in the area of mathematics. However, they have
been organised by two distinct universities one located in the UK and the other in Germany. The phenomena and behaviour of students which the author observed as well as the authors individual experience during the two master courses were dramatically different. In one of the universities the students were typically very motivated and engaged in the course, made significant progress and successfully completed the program, even though they started with various levels of mathematical proficiency. In the other university the students, despite being generally at a very high level, were usually less engaged with the subject of study, often showed signs of discouragement and many of them dropped out or required additional time to complete the course. For a long time the author found this disparity puzzling and could not account for the observed differences.

Eventually the author became familiar with the ideas of Autonomy, Mastery and Purpose. These have been recently described for example by D. Pink (Pink, 2010) but closely related concepts have previously appeared under other names and guises in public discourse and literature. In particular they are closely related to the psychological theory of self-determination formulated by R. Ryan and E. Deci (Ryan and Deci, 2000). These ideas were of particular significance to the author since they offered an explanation for the differences between the two master courses witnessed by the author. Hence the author decided to conduct a more careful analysis of the organisational features and circumstances characteristic of the two university courses. In particular to analyse the two environments from the perspective of the ideas of Autonomy, Mastery and Purpose.

This article is structured as follows. We start by outlining the development and the key elements of the self-determination theory of Deci and Ryan, and describe the related concepts of Autonomy, Mastery and Purpose. Then we describe how the master courses at the two universities were organised and account for other relevant features characteristic to the respective university environment. We then proceed to the analysis of the impact that these circumstances had upon the aspects of Autonomy, Mastery and Purpose. Subsequently, we report on the phenomena and behaviour observed among students participating in the two master courses. Finally, we suggest the ideas of Autonomy, Mastery and Purpose as guiding points
for establishing good practices in university teaching. In the final part of the article we propose how the Autonomy, Mastery and Purpose aspects can be realised in the university setting.

**EMERGENCE OF THE IDEAS OF AUTONOMY, MASTERY AND PURPOSE**

The ideas of Autonomy, Mastery and Purpose, although often appearing under different names and guises, have been slowly entering public discourse since the middle of the 20th century. The changes in business and daily life which in greater part have been brought on by the technological advancement meant that people were faced with new kinds of tasks. Those tasks increasingly involved dealing with novel situations, large amounts of information and complex problems. They required learning new skills, being creative and above all being able to act independently. In this setting many people, in particular in business, were able to recognise that the aspects of Autonomy, Mastery and Purpose are relevant to the productivity, engagement and well-being of their employees. Particularly in recent years due to fierce competition the companies that depend on their knowledge workers can no longer ignore those three aspects. The consideration for employees autonomy is reflected in the spreading of solutions such as flexible working time, home office or sabbatical. The recognition of the fact that employees want to improve, which is part of the Mastery aspect, inspired the mentoring programs and makes the companies stress the possibilities for professional development and personal growth in their job advertisements. Lastly, discovering the importance of the Purpose aspect is reflected in the fact that companies stress their values and mission, actively develop their internal culture and try to be seen as a community by their employees.

Similar ideas emerged in the scientific world. In the year 2000, following their earlier research and supported by the evidence collected by other scientists over the previous decades the psychologists E. Deci and R. Ryan formulated their self-determination theory. The self-determination theory is based on identifying autonomy, competence and relatedness, which correspond to the Autonomy, Mastery and Purpose aspects, as the basic needs of a person. It links the satisfaction of these needs to increased levels
of so-called autonomous motivation which in turn is conductive to good performance on complex tasks, especially those involving creativity and deep information processing.

We will now briefly discuss the development and the key points of the self-determination theory developed by Deci and Ryan (Ryan and Deci, 2000; Ryan and Deci, 2006). Further we will explain what is meant by the concepts of Autonomy, Mastery and Purpose which we will be discussing here and which have been described by D. Pink (Pink, 2010).

**Self-determination theory**

Before the self-determination theory emerged it was very common among scientists and practitioners to take into account only how much motivation people have for completing certain tasks. Whether a person becomes involved in an activity because they find it satisfying in itself, because they believe it is the right thing to do or because of the possible rewards and punishments was often not considered. Moreover, for a long time there existed a belief that higher levels of motivation would lead to better performance. This kind of thinking lead to the attempts to improve the productivity and efficiency of people, whether as employees, students or citizens, using external incentives such as deadlines, performance targets, testing, grades, performance-based renumeration, prizes and fines or other forms of reward and punishment. By now significant evidence has be collected that undermines this perspective. One such example is the experiment of S. Glucksberg (Glucksberg, 1962), who showed how in case of the so-called candle problem, a task which is relatively simple but requires non-schematic way of thinking, the performance was worse in case of individuals whose overall motivation was increased by offering an additional monetary incentive.

The self-determination theory stresses the importance of the quality of motivation that is present in an individual. In particular it distinguishes between autonomous motivation and controlled motivation. Autonomous motivation is present where a persons decision to complete a task comes from their willingness to do so and their free choice. Controlled motivation, on the other hand, describes a situation where an individual decides to complete a task under the experience of demand or pressure whose source
is seen by the individual as external. Deci and Ryan present various types of regulation resulting from the interaction between autonomous and controlled motivation on a scale of changing level of the internalisation. They divide them in terms of decreasing levels of internalisation starting from intrinsic regulation which is purely internal, through integrated and identified types of regulation to introjected regulation and finally external regulation which is non intrinsic. Intrinsic regulation refers to performing an activity because the activity is perceived as satisfying in itself. Integrated regulation means acting because of causes which are external to the activity itself but the causes are perceived as internal. Identified regulation is present where an individual performs an activity because of reasons which are external to that activity but which the individual perceives as justified and important. Introjected regulation means that a person accepts the existence of external demand and acts accordingly but does not recognize the demand itself as personally relevant, justified or important. Finally, external regulation refers to the situation in which a person acts solely due to external pressure. Intrinsic, integrated and identified regulations are forms of autonomous motivation whereas introjected and external regulations are forms of controlled motivation.

For a given activity or task the autonomous and controlled forms of motivation might coexist but they do not simply sum up but interact in a more complex way. For example an individual is internally motivated to engage in a certain activity, increasing the level of motivation by external incentives can decrease the level of internal motivation. This was shown by M. Lepper, D. Greene and R. Nisbett (Lepper, Greene and Nisbett, 1973) in the case of free drawing among children. Children who initially enjoyed drawing and did it often were encouraged to draw in order to receive a reward. Later when they were again free to draw only if they wanted to they expressed much less interest in the activity and spent less time performing it.

The quality of the motivation exhibited by an individual has consequences for the level of engagement in the activity and consequently on that individuals performance. The impact on performance is particularly vital in the case of tasks which are difficult, novel and complex, that require high level of mental engagement and deep information processing and involve creativity and
non-standard way of approaching problems. Numerous studies have shown that autonomous motivation promotes among others persistence (Vellerand and Bissonnette, 1992; Pelletier, Fortier, Vallerand and Brière, 2001), productivity (Fernet, Guay and Senecal, 2004), problem solving (Glucksberg, 1964; Amabile, 1996), conceptual understanding (Grolnick and Ryan, 1987), creativity (Koestner, Ryan, Bernieri and Holt, 1984; Amabile, 1996) but also general-well being and decreased vulnerability to burnout (Ryan, Rigby and King, 1993; Fernet, Guay and Senecal, 2004).

Self-determination theory connects the autonomous quality of motivation to the degree to which the need of autonomy, competence and relatedness of an individual are satisfied. These needs are considered to be natural human needs and independent of culture. Self-determination theory attributes increase of autonomous motivation to better satisfaction of these three needs. The need of autonomy is defined as the need for regulation by the self (Ryan and Deci, 2006). It can be understood as a persons need to determine their own behaviour and course of action, and to show initiative. The need of competence means that an individual should feel capable of performing the activities which he is involved with and dealing with challenges. The need of relatedness is understood as the desire to be involved with a group or community, in particular to contribute to that group and share its values.

For a detailed review of the research that supports the ideas of self-determination theory we refer the reader to the original papers of Deci and Ryan (Deci and Ryan, 2000; Deci and Ryan, 2007).

**Autonomy**

The concept of Autonomy consists of the concrete ways in which the need for autonomy is satisfied in the context of a persons professional activity and other forms of work including learning. In particular it refers to the persons ability to determine when to work and what tasks to work on at a given time. It also includes the choice of methods and means of completing the task also in terms of whether to work alone or who to collaborate with. Further, Autonomy incorporates the individuals influence regarding which projects and tasks he pursues.
Mastery

The term Mastery is related to the need of competence appearing in the self-determination theory. One of the components of Mastery is that an individual feels competent to deal with the tasks which they are faced with, which means that these tasks are accessible. Mastery also refers to the persons ability to work on tasks which are at the right level of difficulty for them, so that they are neither bored nor overwhelmed. In the ideal situation the person might experience a state of flow (Csikszentmihalyi, 2000) in which they function at the optimal level of mental and physical stimulation. An essential part of Mastery is also the persons opportunity to improve. The individual should be aware of the progress that they are making and experience their increasing ability and skill. Moreover, they should feel in control of this process and feel that they are responsible for their own achievement. A vital component of Mastery is also that the skills that the individual develops or knowledge which they gain should be relevant in a larger or more long-term context. In particular they should be meaningful and useful from that persons perspective and there should exist further levels of proficiency to which they can aspire.

Purpose

The aspect of Purpose is closely connected to the need of relatedness appearing in the self-determination theory. The main meaning of Purpose is that a person pursues goals which are of import not only with regard to the self-interest of that individual but for which there is a more enduring and larger cause. The activities that a person is involved in should allow them to perceive themselves as valuable members of a group, family or other community. In particular in this context the individuals work is seen as a contribution towards the goals and ideals present in that community. The goals that a person pursues should also lie in accordance with their values and moral standing.

Analysis of the two master courses

We now consider two university courses in which the author participated. These were both courses in the area of mathematics and both at the master level. One of them was organised at one of the leading universities in Great Britain
and the other one at one of the top universities in Germany. We will not identify the precise names of these institutions, since not all the information presented here shows them in a favourable light, but instead we will be referring to them as University A and University B. One should also point out that exchange with students and the materials available on university websites suggest that many of the practices described are actually common respectively in the other universities in the UK and in the other universities in Germany. However we will concentrate solely on the two courses in which the author participated, since no formal research was conducted to justify claims in a broader setting. We will describe the two master courses in terms of their formal organisation and other relevant characteristics of the university environment. Subsequently we will analyse these with regard to their influence on the Autonomy, Mastery and Purpose aspects. Finally we will describe the phenomena observed among students in terms of the learning strategies, engagement, working methods, achievement and general well-being.

**University A**

At University A the course consisted of sixteen to twenty learning modules and the preparation of a written project in the last academic year- the master thesis. The students could choose freely from a broad selection of courses at the master level. The prerequisites for these modules normally involved the completion of core undergraduate material and were designed to assure a certain level of mathematical maturity rather than the familiarity with particular volume of theory. Each student was assigned to a faculty member who took on the role of the students mentor and met with them at least once every semester. The student could seek advice on the choice of modules as well as receiving general advice and support regarding learning methods, learning progress, formal requirements and direction of their development.

The learning modules normally consisted of two to four hours of lectures and a one to two hours of tutorial each week. The students would be given every week a set of questions, which were discussed in the tutorial session of the following week. Once or twice in a semester there could be a written assessment or the students would have to submit a small written project. Every module ended with a written exam at the end of the semester. All
the information about the structure of the module, the way the students are assessed, as well as all the study materials including notes, slides, tutorial sets with solutions, assignments, list of recommended literature and exams from previous years were available through the university platform. The attendance to lectures and tutorials was not compulsory. The marks obtained from submitted solutions to tutorial questions and from completed assessments and projects counted towards the final grade making up between ten to forty percent of the final grade. However there was no formal requirement for the student to be admitted to the final written exam. Moreover the final exam was always in a written form and completely anonymised. Although the lecturers designed the exam questions they were not present at the examination of their own module, which was organised centrally by the university.

The material covered in the course was designed in such a way as to make it self-contained. This meant that all the notation and terminology as well as most of the methods and theory that appeared was also introduced in the course. Whenever it was not possible to include the relevant material there would be precise reference given to chapters of books or notes so that students could easily catch up in case they were not familiar with this material.

The tutorial questions were structured clearly in an increasing level of difficulty. The early questions would give the students a chance to make sure that they understand the main definitions and concepts. Further questions allowed them to apply the methods and theory introduced in the course. They gave the student the opportunity to develop and practice their skills and their ability to deal with problems in the area of study. Questions in this second part would range from easier to more complex questions at the level of the final exam. Finally the tutorial would include one or two questions designed for the student to deepen their understanding, either by encouraging experimenting with the material or requiring advanced application of the theory present in the course.

The university took means to instil pride in their students to be part of their university and to give the students a feeling that the university is a community of which they are a valuable part. This took on the form of making the history of the university known and present in the university town, celebrating various university traditions, organising graduation and
prize giving ceremonies as well as other communal activities, and giving
support to small groups and societies functioning within the university. The
members of the university staff consistently treated the students with respect.
The members of the mathematical department in speaking to students
made it clear that they see in them their followers the next generation of
scientists or future specialists. The students were treated as members of the
mathematical community. In particular they were encouraged to take part in
the weekly research seminars featuring both local speakers and invited guests.
They were welcome to take part in the weekly coffee meetings attended by
all the staff members of the mathematical department, which gave a chance
for informal conversation with lectures and tutors. Interested students could
also participate in the summer programs, where they would conduct small
pieces of research under the guidance of the professors. This was further
complemented by numerous events and lectures, some more scientific and
some more popular, which made the social life of the department outside of
the lecture rooms lively and engaging.

**University B**

At University B the course consisted of fourteen to twenty six modules
and seminars and the preparation of a master thesis. The students could also
choose their modules as was the case in University A. They could choose
from the available master courses or take up to two undergraduate modules
in a field they are not familiar with. In contrast to the situation at University A
the list of courses was revised every semester. Many modules although
appearing under the same name were each year thought by a different
member of faculty and followed a different program. Many of the subjects
were thought by temporary staff and guests and thus their availability
changed from semester to semester with many new modules appearing every
semester. In the case of non-standard modules there were usually no formal
prerequisites and the prerequisites for the standard modules usually included
two to four of the undergraduate or master courses available at University B.
Master students coming from universities other than University B were
usually also assigned a mentor. This mentor was usually chosen so that they
could be a potential master thesis supervisor for the student depending on
the interests expressed. The meetings with the mentor were to be initiated by the student but at least once every semester. The mentor was supposed to monitor the students progress in order to assess whether they are adequately prepared to start their master thesis project. This created a strange situation of dependency which made most students uncomfortable sharing their doubts and seeking advice from their mentor.

The learning modules consisted of two to four hours of lectures and two to three hours of tutorial each week. Every week the students were given a set of questions which they had to solve working in groups of two or three and submit to their tutor in written form within a week. There were usually no written assessments or projects. The learning modules usually ended with an oral exam conducted by the lecturer and his assistant, who was often one of the tutors. Some modules ended with a written exam which was prepared and conducted by the lecturer but the oral examination was much more common. The seminars consisted of two to four hours of weekly meetings during which students presented and discussed the assigned material, which they have studied and prepared independently. The students were assessed by the lecturer based on their presentation and contribution to discussions.

The information about the contents of the module and the form of assessment available through the university website was often incomplete and outdated. It was a common practice that no sets of notes or slides were made available to the students so that they had to rely on the notes taken during lectures. Some lecturers would refer students to materials which they prepared for other purposes or to their scientific articles. The exams from previous years were often unavailable or irrelevant, since they came from a course which covered different material. What is more the lecturers rarely communicated clearly what material they plan to cover, what knowledge and skills the students can master and develop throughout the course and what they would focus on and require in the examination.

The attendance to lectures was not compulsory, while it was required at the tutorials. The marks obtained from submitted solutions to tutorial questions were used as a basis for formal admission requirement to the final exam. Namely, in order to take part in the exam the student had to obtain a total of at least fifty per cent of points from all the tutorial question sets.
The material covered in the courses often was a continuation of the undergraduate level modules suggested as prerequisites. Importantly most of the terminology and notation that appeared in the earlier modules was not introduced or even commented upon. Other modules which did not build upon modules offered by University B often assumed acquaintance with certain specialised fields and applied their methods without clarifying which theories are used and without giving appropriate references. What was also very common was that the lecturers concentrated on presenting the available theory without explaining its significance to the field itself and related fields. The choice of material often did not provide students with the opportunity to learn any particular methods or develop any particular skills.

The tutorial questions given to students predominantly served the purpose of completing the arguments omitted during the lecture or providing additional material to the lectures. Providing solutions to these questions often used simple or technical methods that were not related to the studied field. On the other hand, solving them often required knowledge outside of the scope of the course and the material covered in the lectures was not sufficient or not relevant. The level of the tutorial questions was rather variable and inconsistent.

The university and the mathematical department focused mainly on the formal organisation of the courses and modules. There was little noticeable attempt of crating the experience of a community within the university, of presenting it as a unified body and something more than an institution. The students community was active and engaged in various activities within and outside the university but the student body appeared as separated from both the faculty and the administrative staff.

Within the mathematical department numerous talks, seminars and conferences were organised but the students attended them as audience rather than participants. Many of these events ended with a gathering intended only for the researchers. Most of the faculty had no contact with students and their research remained completely separate from the students activity within the university. This made a strong impression that the students are not part of the mathematical community of the university.
Impact on the aspect of Autonomy

Due to the fact that the learning modules at University A required a certain level of mathematical maturity rather than particular theoretical knowledge and were carefully planned in order for them to be largely self-consistent the students could indeed be free in their choice of modules. It was possible to follow their interests, experiment with new directions and with a bit of additional work smoothly enter a field of study in which they had little previous experience.

The situation was rather different in University B. Firstly the fact that the availability and contents of modules and seminars changed every semester meant that the students could not be entirely aware what exactly awaits them and hence what they are actually choosing. The strong dependence of the modules on either undergraduate modules or initial knowledge in a given field required the students to choose carefully and plan ahead. In order to participate in some of the courses that interested them they had to build up the necessary prerequisites already in their first semester. This meant that there was very little flexibility and the ability to experiment or enter a new field of study was virtually absent. As a result many students made their choices not based on their interests but simply depending on their previous background in order to minimise the risk of not being able to follow the material. Thus in University A the students had more influence on what they will study than in University B which contributes to a better fulfilment of the Autonomy aspect in University A.

At University A the clarity of the organisation of the modules and of how they are assessed allowed the students for individual long term planning of their work. They could adjust their own working schedule to fit with their personal needs and plans, their abilities and preferences. I was also possible to correct this plan depending on their learning progress. There remained a degree of flexibility to account for sickness or times of exhaustion and poor concentration. Further, no attendance requirement and the full availability of learning materials meant that the students could choose the learning methods and forms of study that best suited them. For example they could prepare before the lectures, work through the material with friends or work through the materials and literature on their own if they so desired. With the full
references given in the materials they could easily catch up on any background material that they were not familiar with or broaden their knowledge through suggested reading. In case some part of the material was particularly difficult for a student, no formal requirement for the exam admission made it possible for them to spend more time on that part of the material or go back to it at the end of the semester and still obtain a good result in the exam.

In contrast to University A, at University B the students had to rely predominantly on their own notes from the lectures and tutorials. This forced them to attend but those who required a slower pace or still struggled with some concepts in the earlier material would not be able to follow and thus would merely take notes. The formal requirement to obtain fifty percent of the points from tutorial questions in order to be admitted to the exam had significant impact on the students way of working. On one hand, since students had to submit solutions and attend lectures in each module the timetable determined a particular weekly and daily rhythm. This meant that in order to have a chance of fulfilling the formal requirements on a given day and at given time the students had to concentrate on a particular task within a particular module, with practically no flexibility. Also the need to satisfy the current demand of the course gave students no opportunity to spend additional time to the material which they found more difficult. They were also not in a position where they could decide to devote time to covering background material that would help them to understand the contents of the module and to catch up on the module work later, because they ran the risk of losing too many points from the tutorials. Thus the working schedule and ways of working were not determined by students and their needs but depended on the external conditions. Last thing which we would like to point out here is that the students choice of whether to work alone or with others and who to collaborate with was also restricted. Working in groups was clearly advantageous since it was easier to obtain points as a group than when working alone and students chose groups so as to maximise their marks.

The differences described above contributed further to the fact the conditions at University A were more advantageous in terms of the Autonomy aspect than in the case of University B.
Impact on the aspect of Mastery

The fact that the learning modules at University A aimed at being self-contained resulted in the material being easily accessible to the students. A particular role in terms of the Mastery aspect was played by the structure and content of the sets of tutorial questions in which the questions ranged from easy to difficult. Progressing through the subsequent levels allow the students to capture their own improvement. The easier questions helped them to identify any concepts or terminology that are still unclear. They could target these particular parts of material and then move to the next level of difficulty. The effect of this was that the students most of the time worked on problems that were neither too easy nor too difficult which aided the learning process and made progress possible. The tutorial questions thus gave the students the opportunity to truly engage with the material. Project work, which was included in some of the modules, gave the student further opportunity to become involved with the learning material and for them to apply the techniques presented in the module in a more natural and practical setting. This gave them a chance to test and experience their competence in the field of study.

Because of the clear structure of the modules and the availability of the materials it was also clear to the students what methods they can learn, what skills they can develop and what knowledge acquire during the course. Accordingly they were able to set their individual goals and at the end of the course be aware of their achievements. The fact that it was clearly communicated how the material of the module is related to other modules and how fits into the general framework of the field, helped the students appreciate how their learning might be useful in their own development. The potential applications, especially to real life situation, which were often pointed out by the lecturers put the students work in the long term perspective. The course work was more likely to be seen as a vital step on the way to becoming a competent specialist in the field. Stimulated in this way the students could establish their own long term goals and then strive towards them.

The practice of conducting examinations and assessments anonymously aided the understanding that the students are largely independent from the lecturer or tutor when it comes to their perception of the students abilities. This supported an atmosphere in which students felt free to ask questions and
seek advice from lecturers and tutors. They were inclined to be open and more likely to admit to having problems or not understanding something. Thus the faculty had more chance to interact with the students and act as their guides throughout the course.

Lastly, a clear and largely homogeneous way of grading across all the modules, allowed the students to test and improve their learning tactics. It also meant that students could focus on mastering the material of the modules instead of having to take into account the variability of the examination format.

The situation at University B had, in contrast, a very negative impact on the aspect of Mastery. The positive effects seen in University A were largely absent. The lack of clarity regarding the structure and aims of the modules hindered the students ability to determine the direction of their development, identify and plan their tasks, set goals or track their own learning progress. It was rarely clear how and if the modules are related to each other and how they fit into the general framework of the field. This was not conductive to seeing the learning process in a long time perspective.

Common, and often not explicitly stated, use of theories not covered in the course combined with the absence of essential reference materials made large parts of the course material and tutorial questions inaccessible to many students. The inconsistent level of tutorial exercises meant that the students rarely worked on problems at the appropriate level. Commonly they made the experience of working either on tasks that were routine and not-stimulating or on tasks that they were not properly equipped to deal with. So in many cases they had little influence on their success and were likely to fail at their attempts despite investing time and effort. This situation did not provide the students with the experience of improvement but rather that of powerlessness and left many feeling incompetent and frustrated. At the same time, the dependence on the lecturers and tutors as prospective examiners and supervisors did not allow the students to obtain proper support and guidance from them.

**Impact on the aspect of Purpose**

Through establishing a strong sense of community both within the university and within the mathematical department University A has very positively influenced the aspect of Purpose. Since the students tended to
identify with the university and the department they also identified with many of the values and goals which they represented and endorsed. There was a strong sense that the students are part of the university tradition and were inclined to carry on this tradition. Achievement in the academic work was a source of pride and reason for the students to feel that they are valuable members of these communities. Further, the feeling of being included in the mathematical community changed the character of the students studies. The mathematical community identified it as its mission to develop mathematical knowledge and to make this knowledge available to the general society. Thus the degree towards which the students worked could be seen by them not merely in terms of their own success and as something more than a cornerstone of their career. Instead, they could see it as preparation to make their own contributions to the cherished wealth of mathematical knowledge and to apply their skills outside of the academia. In particular being aware how their expertise is valuable to the general society they could look forward to put their abilities to service where they are required and hope to help in solving some of the practical problems. They would also see it as important to represent in the future the characteristic rigour of thought, in which the mathematical community prides itself.

At University B the tangible separation of the student community did not encourage identifying with the values represented by the university. On the contrary, the university was often seen as an organisation and to some extent a bureaucratic machine having different interests than those of the students. This situation had the effect that the academic contributions were seen rather in terms of personal achievement. The motives for it were attributed mostly to the desire to secure ones position in the society and a good level of financial affluence. The reasons for hard work were delivered primarily in the form of the expectations of the market or particular industries or companies. In this context obtaining the degree took on the character of a private endeavour, a goal pursued for personal gain merely with the help of the university and its faculty. At the same time the fact that the research community was independent from student community meant that unless a student had a clear prospect of joining the research community they were unlikely to identify with their the values and their goals. As a consequence, if a student strived
a career outside of the academia they might experience the inadequacy of what they encountered in their study to their own goals. Even their academic success within the mathematical department could be perceived as irrelevant to their plans.

**Observed consequences**

We will now describe the phenomena and behaviours that were observed among the students of the master courses at University A and University B. We would like to start by pointing out that in case of University A at the time of entering the program the level and scope of the students mathematical knowledge was varied. In University B the students were generally more advanced yet there existed differences between their backgrounds due to the fact that many of them previously attended different universities in other cities and other countries.

At University A the students generally put in a lot of effort into their studies and their motivation for succeeding in each module was high throughout the semester. It was visible that they were highly engaged with the course material. They devoted time to solving the tutorial problems, often asking questions in lectures and discussed the material among themselves even in casual conversations. Throughout the course in the case of most the planning and organisation of learning improved. Many would seek advice from the faculty, tried out new ways of working, developed their own techniques and cooperated with fellow students. The students generally did not doubt their abilities as mathematicians and were rarely worried if it will be possible for them to pass the exams and complete the course. There was an implicit assumption that their success depends on their involvement and hard work during the semester. Usually they were proud of their achievements and mistakes and disappointments were frequently seen as an impulse for improvement. Throughout the duration of the course most of the students clarified and developed their interests, from which often a concrete plan for the future emerged.

The general well-being of the students throughout the cause can be described as good. The majority of the students had a relatively healthy and regular lifestyle. There was little evidence of such problems as sleeping
deprivation, or problems with sleep, excessive drinking or other self-destructive behaviour. Most students also seemed to lead a normal social life and many were engaged in the local community or within the university.

At University A students rarely changed their choice of modules. During the course practically all students regardless of the initial level of knowledge made significant progress and were able to complete the program. With very few exceptions they completed the course in the standard amount of time.

At University B students also generally put a lot of effort into their studies but their motivation for succeeding in a module often dropped as the semester progressed. The change was often accompanied by doubts regarding their own ability and diminished interest in the topics discussed in the module. The students were generally less engaged in the course material and rarely discussed it in casual conversations. When working on the tutorial questions they would often divide the set among themselves, so that each person is responsible for a certain part. They would not work through all the questions or work together but simply explain the part they were responsible for to others. The weaker students would simply rewrite the answers from the better ones. It was also a common practice to look for ready answers in the literature or to allow the older students to explain how a problem should be solved. Frequently they experienced frustration at not being able to make enough progress with the tutorial problems.

Many of the students did not feel confident about their abilities and their learning progress. They often worried if they are able to prepare for the final examination. Indeed many of them failed or obtained lower grade than hoped they hoped for. Generally the students could not predict the outcome of their assessment and frequently felt that they might not succeed despite working very hard.

Many of the students failed to follow their interest and were not satisfied with the selection completed modules which often did not form an interesting and consistent path of development. Frequently they also questioned their general adequacy as mathematicians and were discouraged from pursuing a career in this direction.

It could be noticed that many students struggled to maintain a healthy lifestyle. They often complain of sleep deprivation or sleeplessness. Heavy
coffee use and missing meals was a widespread problem. Among numerous individuals one could observe physical neglect and withdrawal from social activity. Destructive behaviour was not very common but present.

At University B students frequently changed their choice of modules and dropped out of them. Those students for whom the offered modules were instantly accessible due to appropriate background, were often able to work completely independently and usually improved. The remaining students commonly experienced slower and limited improvement. Many of the latter required additional time to complete the program or did not complete the program at all.

**Supporting Autonomy, Mastery and Purpose in University Context**

We will now propose some ways in which the aspects of Autonomy, Mastery and Purpose can be supported in the university context in terms of the organisation, planning and realisation of courses and learning modules. We would like to stress that our list of suggestions is by no means complete and some of the ideas might not be applicable or suitable in a particular university setting. Thus what we propose here should only be seen as an inspiration. We suggest that the ideas Autonomy, Mastery and Purpose serve as a guide in establishing good practices at universities but the decision of how to best realise these three aspects remains in the hands of the faculty and university authorities. They will be the best judges of what is most suitable in the context of their own institution and hopefully will be able find even better solutions than those which we provide here.

**Supporting Autonomy**

One of the main forms of Autonomy is that a person is able to decide when they work and what task they work on at a given time. In order to support this kind of autonomy the students should be in a position where they plan and adjust their own individual daily, weekly and semester-long schedule. For this to be possible the students must have enough information to allow formulating a long-term plan that allows for a degree of flexibility in the short-term. In order to achieve this goal it is important that the structure
of the entire course and of particular modules are communicated clearly to
the students. In particular they should be made aware of all the requirements
which they have to satisfy by the end of the course and during each semester,
including the scope and format of the assessments and final examinations.
The means to guarantee this include making the study materials, especially
the module plans, notes and the past papers accessible to students. It is also
advisable that deadlines are given only when necessary and announced well
in advance ideally as early as possible. Here long-term deadlines would also
be preferable to short-term. In particular question sets, project details and
course material should be made available at the earliest opportunity.

Another feature of Autonomy is that a person can choose which methods
and means to use to complete their tasks. Here it is immediately relevant that in
the projects or tasks that the students are required to complete there a degree of
freedom in terms of the methods, format as well as who they work with. Further,
this feature of Autonomy can be facilitated by making all the course materials
readily available and preferably a choice of additional sources should be included.
The aim is that the students have access to a variety ways in which to approach
their learning. In particular, they should have the opportunity to truly interact
with lecturers, tutors and other students, be able to prepare before lectures and
revise after and choose between multiple visual and written mediums.

Lastly, the Autonomy aspect includes not only the ability choose what tasks
to work on at a given time but also deciding which tasks to pursue. On a large
scale this would mean guaranteeing that it is possible and realistic for the
students to choose the direction of their study and select among the available
learning modules. This would require that the students understand what they
will learn in each module and are actually able to participate in it. A way of
achieving this would be firstly to design the modules so that they are as self-
contained as possible. Secondly, a common curriculum should be established
by all the faculty members teaching a given module. Such a curriculum could
of course contain some optional parts which are are covered depending on the
lecturer but all such variable elements should be clearly marked in the course
materials. On a smaller scale the freedom to choose what tasks to pursue could
be supported by allowing the students to choose the topics of their project work
or to select questions in their assignments. They could also have some influence
regarding which topics should be covered in a module, in which form they should be discussed or which should be covered in more detail. One could include such optional components in an otherwise constant curriculum.

**Supporting Mastery**

A core feature of the Mastery aspect is that a person feels competent to deal with the tasks that they are working on. In order to achieve this it is vital that both the task and all the basic concepts that a student has to use are clear, including the formal terminology and notation. At every stage one should make sure that all the tools that the students need for dealing with their tasks are known and available to them, or if necessary point them in their direction and provide references.

For the Mastery aspect to be realised the tasks must not only be accessible but the students should be able to work on tasks that are at the right level of difficulty. They should not be forced to worked on tasks which are not stimulating enough so that they are not bored. Similarly they should not be faced with tasks for dealing with which they are not properly equipped, where they are likely to fail despite their efforts. Ideally the students should be able to experience the exactly right level of stimulation which is promotes learning and is also conductive to experiencing a sense joy in the activity, which is often described as the state of flow. In order to facilitate this we suggest that the material covered in a course features elements at various level of difficulty and that it is also made clear how that level changes. The discussion of each topic might for example take a very basic case as the starting point and then develop the ideas leading the student through the increasing stages of complexity and depth. Also the examples used to illustrate the material should be both elementary and more complicated. Further, it is advisable that the students are provided with a large amount of problems and questions to practice on which vary in difficulty and that it is clear how they fit with the material of the course. In case certain tedious tasks cannot be avoided it would be worthwhile to acknowledge this fact and then at least allow freedom regarding the precise execution of these tasks.

The appropriate difficulty discussed above is important for stimulating the learning process, and thus important for making progress. But being able to make progress and experiencing own improvement is another essential part of Mastery. The students should make the experience of improving. They
should be able to sense and track the change as they increase their expertise. In this context we would firstly advise that the course and the particular study modules include not only theory but also concrete methods that the students can learn and skills that they can develop. These aims of the course should be made explicit. Secondly, we would recommend giving feedback and regularly providing opportunities for testing the students state of knowledge and development of their skills. Yet, these measures should serve primarily as means of delivering the student information about his learning progress. Moreover, we suggest promoting an atmosphere in which the students feel safe to make mistakes, discuss and challenge ideas. They should feel free to raise doubts and ask questions to their lecturers and tutors, who should be seen as guides in the learning process.

Another feature of the Mastery aspect is that a person should be the active agent and have control over own learning process. The students should have the chance to experience their own competence and ability. Allowing this calls for limiting those tasks which are mechanised and instead stimulate and promote deeper understanding of the subject. A way of achieving this would be to incorporate in the course opportunities for the students to experiment and personally engage with the study matter. The students should be encouraged to be active, think for themselves, develop opinions and test their own ideas. Their own initiative should be acknowledged and supported. Lecturers and tutors should ask students questions that test and stimulate understanding of the subjects. Instead of immediately giving them full solutions they should try to guide them in finding the answer.

The Mastery aspect should not be restricted only to the perspective of a single learning module or university course. The students must understand how their own progress within the learning modules and throughout the course fits with the general direction of their development. They should be aware of the tasks that might lie ahead of them in the future and the levels of mastering the subject of study to which they can aspire. In order to allow the students to see their work in this bigger framework we recommend that the students come in contact with experienced practitioners in their field and witness how their field of study is applied at an advanced level. This could for example be achieved through participating in research
projects, attending conferences and presentations, visiting companies or laboratories, dealing with advanced literature.

**Supporting Purpose**

Many of the conditions which influence the Purpose aspect cannot be easily influenced by the university authorities and faculty members responsible for the organisation of a particular course or learning module. But there are still means of influencing this aspect.

We expect that it is beneficial to help the students realise what kind of relevance their expertise can have for the challenges which they will meet in their professional life. In particular what contributions they will be able to make in their working environment and in the context of the general society; in particular whom and how they will be able to help.

We would further recommend taking steps to stimulate the feeling of community within the particular classroom or university department. The aim should be that the students see themselves as important members of these groups as well as members of the respective professional or research community. The means of achieving this are multiple and in this case must be tailored to the particular environment. Some ideas would be to give opportunity for informal discussions and interaction, allowing everyone to participate and contribute independent of their level of expertise, initiating communal celebrations or establishing traditions.

Finally, an important contribution might come from establishing a relationship of trust and respect between tutors and students or the lecturer and students. In particular a relationship which allows the tutor or lecturer to be seen as a guide and source of valuable advice. This requires a level of involvement so that the faculty members are genuinely interested in providing assistance to the students and securing their progress. The tutors and lecturers are then likely to perceive the students achievement as their own success.

**Conclusions**

We have explained what role the Autonomy, Mastery and Purpose aspects play in promoting engagement in the course work, learning and general well being of students in view of the self-determination theory of
Deci and Ryan. Our analysis started with demonstrating the differences that existed between University A and University B in terms of the organisation of the courses and particular modules as well as in terms of other relevant factors. We have shown how these differences contributed to the disparity between University A and University B in terms of the realisation of the aspects of Autonomy, Mastery and Purpose. In particular the environment of University A was more advantageous in terms of these three aspects. We considered the consequences of the different degrees of Autonomy, Mastery and Purpose for the students, their learning process and behaviour. We described the positive effects such as genuine involvement with the course, good working ethic, persistence in pursuing learning goals and significant progress in University A. At the same time we could report on negative effects in University B which included limited efforts, poor performance, discouragement and decreased well-being.

Our comparative case study illustrates how a high degree of Autonomy, Mastery and Purpose can positively influence the students learning-outcome and well-being. Hence it supports the idea that Autonomy, Mastery and Purpose can form a base for good practice for organising university courses. Consequently, we would like to suggest these three concepts as useful guiding directions for improving the quality of the university teaching. We have provided some recommendations and suggestions for the measures that can be taken in practice. We would like these to be seen merely as a source of inspiration as we are confident that they are many ways of realising the Autonomy, Mastery and Purpose aspects. The faculty of a given university knowing the particular conditions of their institution and department will be able to devise the most suitable course of action.

References


