NON-AGGREGATED INDICATORS OF ENVIRONMENTAL SUSTAINABILITY

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Abstract: In this paper, we propose a new way to assess and compare the sustainability level of countries, using concepts and tools from the theory of partially ordered sets. Using so-called average height and the newer concept of embedded scales, we show how one can develop synthetic sustainability indicators, without aggregating attribute scores, through composite indicators. In particular, the paper shows how rankings of countries can be obtained out of multidimensional data systems, paving the way to more comprehensive sustainability studies, where the illustrated posetic methodology, and other tools from partial order theory, can be employed fruitfully. While setting the stage for the application of partial order theory, we also show the use of tools for dimensionality reduction, namely Non-negative Matrix Factorization, which to our knowledge is not yet so widely employed in environmental data analysis.

Keywords: sustainability, partially ordered set, poset, synthetic indicator.

1. Introduction

In this paper, we show how the theory of partially ordered sets (*poset theory*) can be used to analyze complex multi-indicator systems and to compute synthetic, yet non-aggregated, indicators of environmental sustainability. For exemplification purposes, we consider a subset of data from the Notre Dame Global Adaptation Initiative (ND-GAIN) dataset, freely available on the web, and provide an analysis of it, from data dimensionality assessment to indicators computation. Poset theory has been recently advocated as a potentially powerful tool in data analysis, particularly when ordinal multidimensional data are of concern, both in social and environmental sciences [Bachtrogler et al. 2014; Carlsen, Brugemann 2016; Fattore, Maggino, Greselin 2011; Fattore, Maggino 2014; Fattore 2016; Fattore, Arcagni 2016; Fattore, Maggino, Arcagni 2016; Fattore, Bruggemann (eds.) 2017; Iglesias et al. 2016; Patil, Taillie 2004]. More generally, the theory of partial

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orders can be employed whenever one has to deal with complex systems of attributes and when rankings and multidimensional comparisons are of interest, overcoming the limitations of classical composite indicators, which often prove inappropriate and ineffective to support decision-making. In particular, and in view of sustainability evaluation, here we introduce an alternative posetic approach to the computation of non-aggregated synthetic indicators, combining *average height* [Bruggemann, Patil 2011], with the newer concept of *embedded scale*. As it will be seen, this procedure allows to build rankings and to partially quantify countries' sustainability levels, in quite a natural way. The paper is organized as follows. Section 2 provides a brief description of the ND-GAIN dataset, namely of the Readiness and Vulnerability dimensions. Section 3 performs a dimensionality analysis of sustainability data, employing Singular Value Decomposition and Non-negative Matrix Factorization. Section 4 introduces the posetic tools for indicator construction, i.e. average height and embedded scales. Section 5 exemplifies the posetic evaluation procedure on a subset of the ND-GAIN data. Section 6 concludes. A final Appendix reports the codes of the countries analyzed in the paper.

2. The data

The data used to illustrate the posetic evaluation procedure are extracted from the ND-GAIN datasets, for years 1995-2014. Quoting from the website *index.gain.org*: ”The Notre Dame Global Adaptation Initiative (ND-GAIN) is part of the Climate Change Adaptation Program of the University of Notre Dame's Environmental Change initiative (ND-ECI). The ND-GAIN Country Index follows a data-driven approach to show which countries are best prepared to deal with global changes brought about by overcrowding, resource-constraints and climate disruption. The Index aims to unlock global adaptation solutions in corporate and development communities to save lives and improve livelihoods while strengthening market positions. It demonstrates strategic and operational decisions using data from 1995, to create a rank of 181 countries.” Sustainability is described in terms of Readability and Vulnerability, defined as “a country's ability to leverage investments and convert them to adaptation actions” and “a country's exposure, sensitivity and capacity to adapt to the negative effects of climate change”, respectively. Overall Readiness is measured by considering three pillars, namely

Economic Readiness, Governance Readiness and Social Readiness. Overall Vulnerability is instead measured in terms of six life-supporting sectors (food, water, health, ecosystem service, human habitat, and infrastructure), which are then aggregated into three pillars, *Capacity* (the availability of social resources for sector-specific adaptation), *Exposure* (the degree to which a system is exposed to significant climate change from a biophysical perspective) and *Sensitivity* (the extent to which a country is dependent upon a sector negatively affected by climate hazard, or the proportion of the population particularly susceptible to a climate change hazard). Given the aim of the paper, we focus just on the three pillars of Readiness and Vulnerability and do not consider the lower level indicators. We thus deal with a multi-indicator system comprising six variables.

3. Dimensionality analysis of ND-GAIN data

To check whether it is possible to represent Readiness and Vulnerability data in a synthetic way, we first try to measure the "intrinsic dimension" of the data. By intrinsic dimension, we mean that sustainability profiles, even if embedded into a six-dimensional vector space, can possibly belong to a lower-dimensional subspace whose dimension determines the real complexity of the data. For each available year we computed the Singular Value Decomposition (SVD) of the corresponding ND-GAIN dataset. Table 1 reports the relative cumulative sums of the six singular values for each year. As it can be seen, almost invariably, unidimensional projections of ND-GAIN data approximate (in the Frobenius norm) original data up to around 60% and two-dimensional projections up to around 80%. We thus consider the intrinsic data dimension as at least 2 and focus on bidimensional approximations to ND-GAIN data. To provide an interpretation of this bidimensional space, we could proceed as in principal component analysis, considering the loadings of the original attributes in the two linear combinations spanning the approximating subspace. From the theory of non-negative matrices, however, it is well-known that, apart from the first eigenvector, elements of SVD eigenvectors have both positive and negative signs; this leads to interpretation difficulties, not being easy to give a "name" to components. A way out from this problem could be to seek for a suitable change of basis, in the same spirit of rotations in factor analysis. To help interpretation, however, two conditions should be met: (i) each component should be "linked" to a small subset of input attributes and (ii) the loadings of such attributes should be positive. Only in this case, in fact, one can interpret the components as representing "parts" of the latent construct, which is recovered combining them in an additive way. The main tool for such "reconstruction by parts" is the so-called *Non-negative Matrix Factorization* (NMF, [15]); since this decomposition procedure is much less known than the more common SVD, we briefly introduce it.

NMF aims at best approximating (in the Frobenius norm) a non-negative matrix $M_{n\times k}$ of rank k $\left(k \leq n\right)$ as the product of two non-negative matrices $W_{n\times p}$ and $H_{p\times k}$, where $p \leq k$ is the rank of W and H:

$$M \approx W \cdot H. \tag{1}$$

The value of p determines to what extent the NMF reduces the dimensionality of the data and should be fixed based on trials, checking the degree of fit of the decomposition. The rows of H are linear combinations of the rows of M and represent "prototypical" profiles; rows of W contains coefficients, used to combine the rows of H to reconstruct, approximately, the rows of M. The constraint of non-negativity on W and H leads the rows of H (and the column of W, as well) to have loadings concentrated on a few input variables, as desired. As a consequence, rows of M get reconstructed as the "positive superposition" of components (in fact, the coefficients in W are non-negative), each of which captures a "part" of the latent construct of interest.

In the case of ND-GAIN data, we selected $p = 2$, given the results of the SVD analysis (notice, however, that the subspace spanned by the two resulting NMF components is different from the "best" two-dimensional SVD subspace; consequently, the degree of approximation of the NMF solution must be checked independently). Table 2 reports the NMF components (i.e. the two rows of matrix H), for each year in the dataset. Almost invariably, the two NMF components are related to Vulnerability and Readiness respectively, as can be checked by looking at the elements of the respective vectors. Indeed, there are some oscillations in the components, so this interpretation must be considered with some care. Figure 1 gives an overall picture of the profiles corresponding to the rows of H, over the years. By direct inspection, one can see that the two NMF components are quite separate, even if some of their loadings are close to one another. Only year 2012 presents a peculiar shape of the Vulnerability component, which has smaller loadings than usual. In all the years considered, the relative errors of approximation[1] of the NMF reconstructions of the original data matrices are quite small and are comprised between 14.9% and 17.2%. In addition to assessing the global fit of the solutions, we have also checked for the approximations of single countries' profiles. We do not provide figures here (they are too many), but report that in all the years the sustainability of the countries' profiles are well approximated; the median approximation errors are between 13% and 16% and the maximum relative error is about 35% (this is achieved by an outlier, in the ND-GAIN dataset pertaining to the year 2003). In summary, we can accept the two-dimensional NMF solution as a satisfactory representation of ND-GAIN data. Finally, we have analyzed the correlation between the ND-GAIN Readability and Vulnerability

Table 1. Cumulative sums (in quotas) of the six singular values of the Vulnerability-Readiness dataset, for years 1995-2014.

Year	1	2	3	4	5	6
1995	0.612	0.790	0.856	0.914	0.966	1.000
1996	0.611	0.790	0.857	0.914	0.966	1.000
1997	0.611	0.791	0.857	0.915	0.966	1.000
1998	0.610	0.791	0.858	0.915	0.966	1.000
1999	0.609	0.790	0.858	0.915	0.966	1.000
2000	0.608	0.791	0.858	0.915	0.966	1.000
2001	0.607	0.791	0.859	0.915	0.966	1.000
2002	0.607	0.792	0.859	0.915	0.966	1.000
2003	0.607	0.793	0.859	0.915	0.965	1.000
2004	0.612	0.798	0.864	0.922	0.966	1.000
2005	0.616	0.801	0.869	0.926	0.967	1.000
2006	0.619	0.804	0.870	0.928	0.968	1.000
2007	0.620	0.804	0.870	0.928	0.968	1.000
2008	0.622	0.804	0.869	0.927	0.968	1.000
2009	0.624	0.806	0.870	0.927	0.969	1.000
2010	0.625	0.807	0.870	0.927	0.968	1.000
2011	0.628	0.808	0.871	0.927	0.969	1.000
2012	0.629	0.808	0.871	0.927	0.968	1.000
2013	0.631	0.809	0.872	0.928	0.969	1.000
2014	0.633	0.810	0.873	0.929	0.969	1.000

Source: author's computations.

[1] The approximation error is computed as

$$\frac{\| M - W \cdot H \|_F}{\| M \|_F},$$

where M is the target matrix, $W \cdot H$ is the NMF approximation and $\|\cdot\|_F$ is the Frobenius norm.

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indexes and the corresponding components derived from the NMF procedure. In all the years, the correlations are very high, as shown in Figure 2 for year 2014 (to allow for a visual comparison of the indexes, we have normalized all of them, prior to plotting).

4. Building rankings and partial quantifications through poset theory

In the previous section, we have proved that sustainability data can be approximately described in a two-dimensional linear space, spanned by two vectors that can be roughly interpreted as Readiness and Vulnerability. However, in many cases, and sustainability is no exception, the final goal is to obtain a ranking of countries on the dimensions of interest. Usually, rankings are worked out by computing composite indicators and then by comparing the scores of statistical units. This poses one main problem. When combining different indicators into one composite, one unavoidably sums up "apples and oranges", implicitly assuming a common scale of measurement, whose existence is at least questionable. As a result, one gets a ranking based on scores that are not easily interpretable and which are also affected by compensations among attributes. The problem of how to get a ranking arises from the multidimensionality of both the Readiness and Vulnerability indicator systems.

Table 2. NMF components 1995-2014 (CA = Capacity; EX = Exposure; SE = Sensitivity; EC = Economic; GO = Governance; SO = Social)

Year	NMF cmp.	CA	EX	SE	EC	GO	SO
1	2	3	4	5	6	7	8
1995	$H1$	0.016	0.178	0.051	0.535	0.527	0.346
	$H2$	1.087	0.759	0.85	0.238	0.319	0.048
1996	$H1$	0.144	0.335	0.168	0.775	0.771	0.501
	$H2$	0.834	0.582	0.651	0.178	0.241	0.030
1997	$H1$	0.153	0.343	0.171	0.783	0.781	0.524
	$H2$	0.935	0.660	0.740	0.213	0.283	0.036
1998	$H1$	0.151	0.344	0.169	0.791	0.788	0.548
	$H2$	0.760	0.540	0.608	0.177	0.234	0.026
1999	$H1$	0.151	0.339	0.168	0.773	0.774	0.557
	$H2$	0.903	0.639	0.725	0.201	0.265	0.016
2000	$H1$	0.029	0.181	0.056	0.516	0.516	0.408
	$H2$	1.086	0.785	0.877	0.286	0.357	0.041
2001	$H1$	0.118	0.263	0.130	0.593	0.600	0.469
	$H2$	1.019	0.742	0.822	0.278	0.341	0.043
2002	$H1$	0.148	0.342	0.165	0.782	0.793	0.646
	$H2$	0.791	0.566	0.636	0.180	0.227	0.000

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1	2	3	4	5	6	7	8
2003	*H*1	0.157	0.333	0.171	0.722	0.734	0.608
	*H*2	0.837	0.605	0.668	0.207	0.256	0.016
2004	*H*1	0.133	0.289	0.143	0.609	0.632	0.548
	*H*2	0.891	0.646	0.718	0.253	0.278	0.016
2005	*H*1	0.177	0.364	0.186	0.721	0.777	0.679
	*H*2	0.764	0.552	0.617	0.245	0.225	0.009
2006	*H*1	0.129	0.291	0.139	0.607	0.635	0.586
	*H*2	1.012	0.738	0.825	0.347	0.321	0.022
2007	*H*1	0.074	0.263	0.102	0.624	0.650	0.635
	*H*2	0.944	0.684	0.765	0.324	0.285	0.014
2008	*H*1	0.128	0.323	0.149	0.708	0.724	0.713
	*H*2	1.022	0.741	0.829	0.363	0.306	0.021
2009	*H*1	0.114	0.336	0.138	0.772	0.778	0.796
	*H*2	0.830	0.603	0.680	0.302	0.244	0.015
2010	*H*1	0.180	0.398	0.197	0.842	0.835	0.848
	*H*2	0.866	0.629	0.707	0.315	0.250	0.022
2011	*H*1	0.100	0.286	0.124	0.651	0.643	0.670
	*H*2	1.023	0.743	0.839	0.388	0.292	0.029
2012	*H*1	0.224	0.459	0.241	0.945	0.910	0.937
	*H*2	0.582	0.423	0.479	0.217	0.159	0.013
2013	*H*1	0.167	0.367	0.186	0.783	0.745	0.778
	*H*2	0.763	0.554	0.625	0.288	0.205	0.018
2014	*H*1	0.161	0.367	0.183	0.799	0.760	0.794
	*H*2	0.829	0.602	0.679	0.323	0.222	0.022

Source: author's computations.

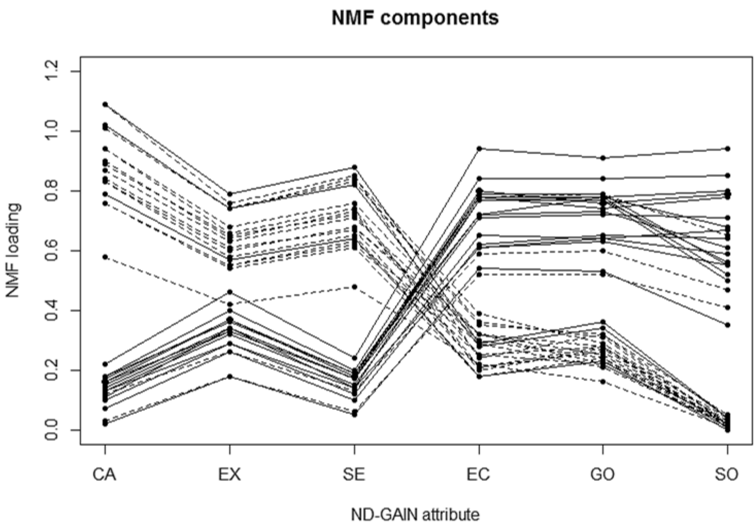

Fig. 1. Shapes of NMF components (CA = Capacity; EX = Exposure; SE = Sensitivity; EC = Economic; GO = Governance; SO = Social)

Source: author's computations.

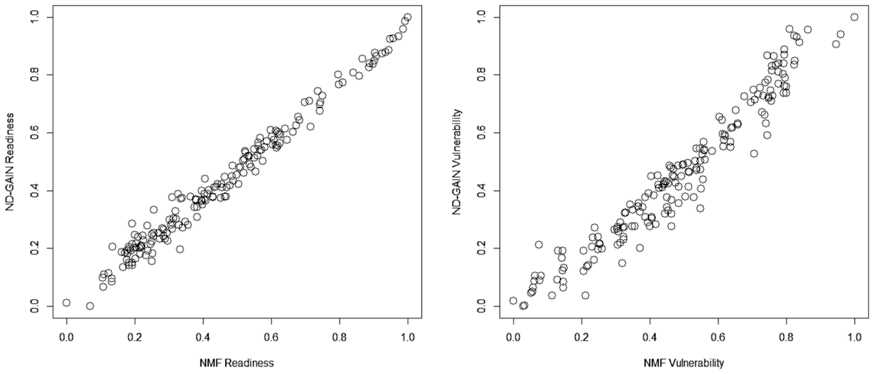

Fig. 2. Correlation between ND-GAIN indexes and NMF components, for year 2014

Source: author's computations.

Countries' profiles cannot be ranked, since conflicting scores among them exist, so that country A may have a higher score than country B on (say) the Governance attribute of the Readiness dataset, but a lower one on the Social attribute. Technically, the set of countries' profiles defines a *partially ordered set or a poset*, for short [Neggers, Kim 1998; Schroder 2002]. In it, some pairs of countries can be ordered, since their profiles do not have conflicting scores; other pairs, instead, cannot, so explaining the term "partially". When the number of elements is not too high, posets can be easily depicted as Hasse diagrams, which are a kind of acyclic directed graph built according to two simple rules: (i) if element *b* is greater than element *a* (i.e. if $a \leq b$, where $\leq$ denotes the ordering criterion), than the corresponding node is placed higher in the graph and (ii) if no element *c* exists such that $a \leq c \leq b$, then an edge links node *b* to node *a*. Figure 3 depicts the Hasse diagram of the Readiness poset for the South-American countries. Looking at the diagram, one can easily realize the existence of both comparabilities, reflecting higher or lower Readiness levels, and incomparabilities, reflecting the existence of conflicting scores and the impossibility to directly extract a ranking out of the poset.

4.1. Average height

In posetic terms, the problem of building a ranking from a poset π is the same as the problem of picking a specific *linear extension* out of the set $\Omega(\pi)$ of its linear extensions. A linear extension of a poset is, in fact, a complete order of all of its elements, obtained by turning incomparabilities into comparabilities [Neggers, Kim 1998]. This can be done in a number of different ways, each of which defines a complete order (and hence a ranking) compatible with the input poset. In this perspective, a composite indicator can be seen as a way to list poset elements based on their composite scores, resolving incomparabilities and getting the final ranking (however, possibly, with ties). To avoid the aforementioned drawbacks of composite indicators, other posetic approaches to ranking extraction have been recently proposed [Bruggemann, Patil 2011; Patil, Taillie 2004]. Their main feature is to cast the problem in terms of multidimensional comparisons and to avoid any aggregation among attributes, exclusively drawing on the relational structure of the input partial order. The simplest, and more natural way, of pursuing this idea is to associate to each poset element its *average height* in the set of linear extensions. The idea is in itself very simple. Consider again the Readiness poset π depicted in Figure 3. Given a linear extension l of it, to each South-American country it is associated the corresponding

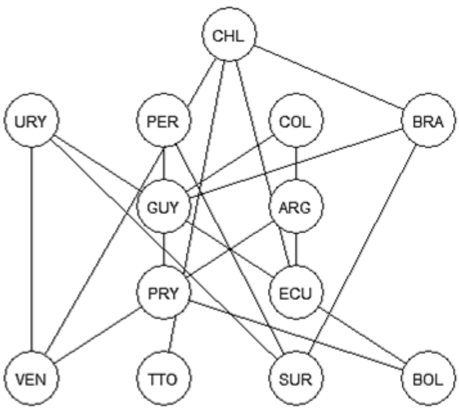

Fig. 3. South-American countries: Readiness Hasse diagram for year 2014

Source: author's computations.

height, i.e. the number of elements below it, in l, plus 1 (in practice, the "position" in the l ranking). The height of an element, in general, depends upon the selected linear extension, so that to each element of the poset one finally associates the distribution of heights, over $\Omega(\pi)$. It is then natural to compute the average height $avh(\cdot)$ for each poset element and to employ it to build the final ranking. Notice that:

1. The average height does not involve any aggregation of attribute scores. Attributes determine countries' profiles and these, in turn,

determine the structure of the partial order; it is such a structure to serve as an input to the computation of the average height.

2. The average height is order-preserving, in that if $a \leq b$ in π, then necessarily $avh(a) < avh(b)$.

3. Technically, the average height need not produce a linear extension of the input poset, since ties may occur; this, for example, happens when the Hasse diagram of the poset has special symmetries (however, in the examples shown in this paper, no ties are produced).

Average height has been recently used in different contexts, where composite indicators are not effective, see for example [Bachtrogler et al. 2014] and [Bruggemann, Patil 2011].

4.2. Average height and embedded scales

The average height may solve the problem of getting a ranking out of a partial order, but loses any information on the attribute scores of countries' profiles. As a consequence, for example, we cannot compare the average heights, across different groups of countries. To somehow anchor the average height computation to a "common reference system", we introduce the concept of *embedded scale*. Consider again the Readiness domain and suppose one can identify some countries (or some prototypical Readiness profiles, possibly not observed in the dataset) which represent Readiness benchmarks. Just for exemplification purposes, in this paper we have defined 11 benchmark profiles, having constant scores on the three Readiness attributes (Economic, Governance and Social): BNC1 =(0.0,0.0,0.0), BNC2 = (0.1,0.1,0.1), ..., BNC10 = (0.9,0.9,0.9), BNC11 = (1.0,1.0,1.0). Adding these prototypical profiles, which form a *scale* of increasing Readiness levels *embedded* in the original poset, to the input dataset of South-American countries, we obtain the Hasse diagram depicted in Figure 4. The benchmarks spread across the Hasse diagram, providing points that help anchoring both the comparisons between profiles and the average heights to a reference scale. Suppose, in fact, to exogenously associate to each benchmark profile a global Readiness score (for example, BNC1→0, BNC2→0.1..., but other quantification criteria may well be employed), then:

1. Each country gets an interval of Readiness scores (for example, Peru lies between BNC4 and BNC8, so its Readiness level must consistently be between 0.3 and 0.7, according to the previous "toy" quantification).

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2. The quantification induced by the benchmarks, however, is only partial; the Readiness intervals of different elements may have different width, showing that the quantification power of an embedded scale depends upon the structure of the partial order (for example, Venezuela lies between BNC3 and BNC6, i.e. its Readiness level would be between 0.2 and 0.5).

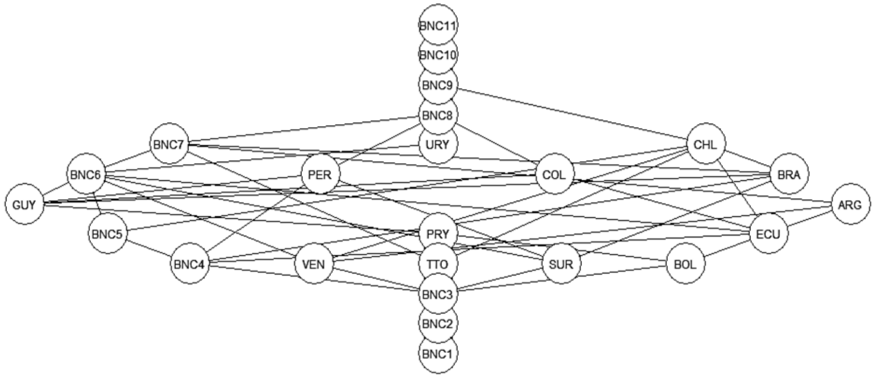

Fig. 4. South-American countries: Readiness Hasse diagram with benchmarks for 2014

Source: author's computations.

Moreover, benchmark profiles get their average heights as well, so that reference points are introduced into the final ranking, partly quantifying it. In practice, through what we have called embedded scales (the original concept has been borrowed from a completely different discipline [Knuth, Bahreyni 2014]), one quantifies reference profiles and injects into the evaluation procedure a minimum amount of exogenous information, which is then spread across the poset, consistently with the structure of the partial order relation. No attribute aggregation is required and no composite indicator is computed; as a result, due to incomparabilities, quantification is to some degree uncertain (in general, just intervals are associated to poset elements). This is not a limitation of the procedure; rather, it reveals the essential complexity of the data.

Before turning to examples, a short comment on computational aspects is due. Computing all of the linear extensions of a poset is, in most cases, unfeasible. In practice, one employs sampling algorithms and perform approximated computations. Further details can be found in [Arcagni, Fattore 2014], where the R package used for working out the following examples is described.

5. Readiness and Vulnerability rankings

For exemplification purposes, we have applied the above ranking methodology to South-American countries, computing the average heights for both Readiness and Vulnerability in 2014. In both cases, benchmarks have been defined as stated in the previous paragraph. In Figures 5 and 6 we report the rankings computed based on the average height, with benchmarks. Vertical lines show the ranges of heights across the set of linear extensions of the posets, so as to give an idea of the uncertainty involved in the ranking exercise. Dashed horizontal lines correspond to the benchmarks and are drawn to help with visualizing such reference levels.

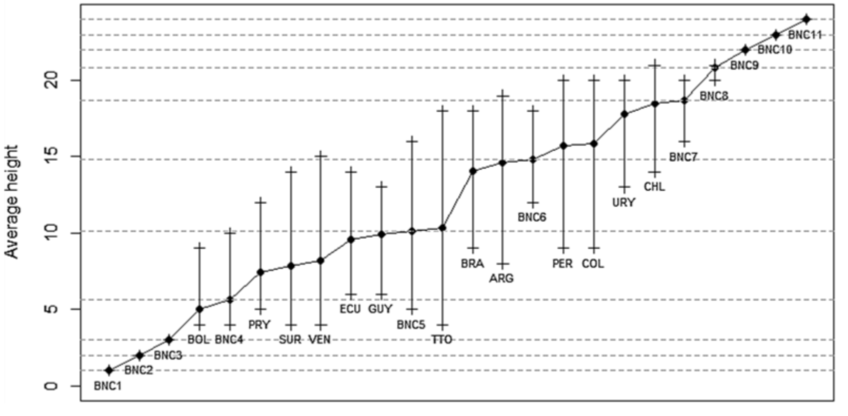

Fig. 5. South American countries: Readiness ranking for 2014

Source: author's computations.

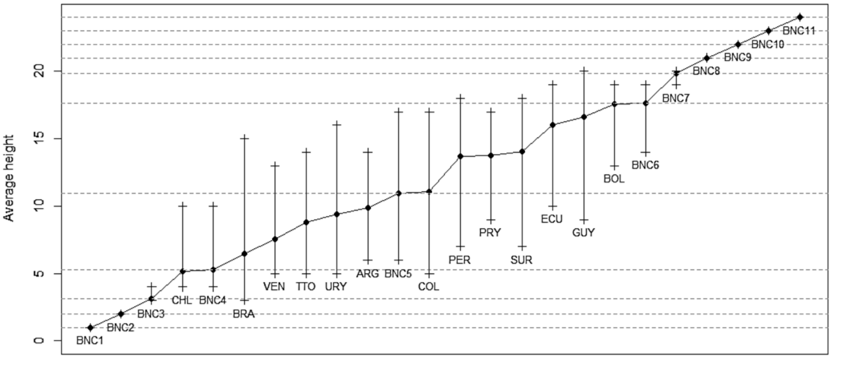

Fig. 6. South American countries: Vulnerability ranking for 2014

Source: author's computations.

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Given the methodological aim of the paper, we limit ourselves to some general comments on the results.

1. First of all, as anticipated, the resulting rankings do not involve any aggregation of attribute scores, showing that achieving synthesis does not mean computing composites. This is a crucial point, since this also allows for working with ordinal data (which is not the case here), where summing scores is not possible.

2. Second, the height intervals are quite large, particularly in the middle of the rankings. This is a direct consequence of the high number of incomparabilities among countries' profiles, characterizing both the Readiness and the Vulnerability posets. One thus realizes how getting a ranking out of a multi-indicator system may imply non-negligible information losses or, in a sense, some hypersimplification of data complexity. At the extremes of the rankings, intervals are obviously non-symmetric; this may suggest to use other indicators to build the rankings, such as the median height, and to define height intervals differently. At present, however, only average height is implemented in the available software routines (e.g. [Arcagni, Fattore 2014]).

3. From Figures 5 and 6, one can see that the benchmark profiles are not uniformly distributed along the rankings; as a consequence, equal differences in the average heights may correspond to different Readiness/Vulnerability gaps, in a non-linear fashion. This is consistent with the usual behavior of measurement devices, whose response flattens at the extremes of the measurement range.

Although simple, the provided examples show the benefits of the posetic approach which is capable to extract information respecting the complex and multidimensional nature of the data. Clearly, in real applications the choice of the embedded scale is essential, since it affects all of the subsequent evaluation process. The set of benchmarks should be composed of profiles that can be assessed in a clear way and should neatly reflect the "point of view" assumed in the evaluation exercise. Using a different terminology, an embedded scale can be considered as an *observer* which assesses, to different precision degrees, poset elements. Different observers, i.e. different embedded scales, may obviously provide different assessment results. What is important is to clearly and carefully motivate the choice of the embedded scale and of its quantification.

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6. Conclusion

In this paper we have shown how sustainability comparisons across countries can be effectively performed using some basic concepts of partial order theory, together with up-to-date data mining tools such as the Non-negative Matrix Factorization. In particular, we have introduced the concept of embedded scale as a way to anchor the computation of average heights to some reference benchmarks, in order to provide interpretable synthetic, yet non-aggregated, sustainability indicators. Although the use of posetic tools faces some computational limitations, what described and cited in the paper should be enough to attest the effectiveness of poset theory as a general resource for evaluation exercises in environmental and social sciences.

As repeatedly mentioned, all of the evaluation results depend upon the structure of the input poset. When attributes are measured on continuous scales, as in ND-GAIN data, casual measurement errors may affect the resulting partial order relation. This is a subtle issue that deserves more research, in order to develop procedures to "estimate" the true input poset. At the moment, however, this is still an open problem.

Speaking more generally, the analysis of complex systems of multidimensional indicators is becoming a major issue in many different scientific fields. Composite indicators are being increasingly criticized, since they tend to oversimplify data and are not so easy to interpret. Interest is currently turning to softer ways to perform evaluation, where data complexity is simplified, but not collapsed into too trivial indicators. Partial order theory, together with other tools for multidimensional data analysis, represents a potential resource in this perspective.

7. Appendix – Countries' codes

Table 3. South American countries

Code	Country
1	2
ARG	Argentina
BOL	Bolivia, Plurinational State of
BRA	Brazil
CHL	Chile
COL	Colombia

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1	2
GUY	Guyana
PER	Peru
PRY	Paraguay
SUR	Suriname
TTO	Trinidad and Tobago
URY	Uruguay
VEN	Venezuela, Bolivarian Republic of

Source: ND-GAIN dataset.

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NIEAGREGOWANE WSKAŹNIKI ZRÓWNOWAŻONEGO ROZWOJU ŚRODOWISKOWEGO

Streszczenie: W niniejszym artykule proponujemy nowy sposób pomiaru poziomu zrównoważonego rozwoju kraju, opierając się na teorii porządków częściowych. Stosując tak zwaną średnią wysokość oraz pojęcie skal zagnieżdżonych, rozwijamy syntetyczne wskaźniki rozwoju bez konieczności agregowania zmiennych. W szczególności pokazujemy, jak skonstruować ranking krajów na podstawie wielowymiarowych zmiennych, torując drogę do bardziej kompleksowych analiz dotyczących rozwoju, w których narzędzia z teorii porządków częściowych mogą być z powodzeniem stosowane. Pokazujemy również, jak teoria ta może być użyta do redukcji wymiarów, mianowicie nieujemnej faktoryzacji macierzy, co wedle naszej najlepszej wiedzy nie jest jeszcze stosowane w analizie danych środowiskowych.

Słowa kluczowe: zrównoważony rozwój, porządek częściowy, zbiór częściowo uporządkowanych, wskaźnik.