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Integrating queue theory and multi-criteria decision-making tools for selecting roll-over car washing machine

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Abstract

The study aims to develop a decision-making framework by integrating queuing theory and multi-criteria decision-making (MCDM) tools, namely TOPSIS, EDAS, CoCoSo, and TODIM to select a roll-over car washing machine for an oil station. The queue, technical and financial characteristics of the alternatives are added to the decision-making process. The decision matrix includes five criteria and five alternatives. One million weight sets are created randomly, and MCDM techniques are applied to interpret the results statistically. Results indicate that Alternative 3 is statistically superior to the others. The proposed procedure can help decision makers to make decisions when expert knowledge isn't available, and it can be applied for other purposes by making small changes.

Keywords: *queue theory, multi-criteria decision-making, TOPSIS, EDAS, CoCoSo, TODIM*

1. Introduction

Multi-criteria (attribute) decision-making (MCDM) tools are used to select the best of a set of alternatives by evaluating them concerning different attributes/criteria [2]. If there is only one criterion, the alternatives are sorted according to the criterion, and the best one can be selected. If there is only one alternative, there is no need to make any calculations for the selection. MCDM tools enable the selection process to be carried out rationally in cases with conflicting criteria and multiple alternatives. In the most general sense, the three main steps of MCDM tools are as follows [27]: present alternatives and attributes as a decision matrix (1), link numerical measures to the relative importance of different attributes and the impact of alternatives on these attributes and (2), calculate numerical measures to sort and rank different alternatives (3). Some of the MCDM tools are: CoCoSo (combined compromise solution) [29], EDAS (evaluation based on distance from average solution) [19], TOPSIS (a technique for order preference by similarity to ideal solution) [15], and TODIM [13]. Although not limited to those mentioned above, MCDM techniques are applied to portfolio selection [3], machine selection [7], and

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supplier selection [5]. These applications show that MCDM can be used as a decision support system when there are conflicting criteria and more than one alternative.

Queues or waiting lines are a natural occurrence in the everyday lives of consumers and the operations of every business. Queues are customers' first point of contact with the business. Therefore, the customer's experience in the queue determines their first impression of the business. The queue structure assists employees and managers in monitoring, prioritizing, and providing services and processes. For this reason, it provides the cornerstone of productivity for businesses. Inefficiencies in queues are undesired as they can result in substantial losses to a business, such as a bad reputation and loss of customers [1].

Queue theory attempts to provide numerical values regarding the characteristics of a queuing system. A queuing system consists of customers arriving at random times at some facility where they receive service and departing [14]. The goals of queue theory [25] are to determine some measures of effectiveness for a given process and to design an optimal system according to some criterion. To accomplish the first goal, formulas are developed for determining the average number of items in the queue (or in the system), average waiting time in the queue (or in the system), probability of the servers being idle and system utilization rate. These formulas are developed based on probability theory and employed for various purposes, such as traffic flow, scheduling, facility design etc. [25].

Studies have demonstrated the correlation between waiting time and customer satisfaction; the longer the waiting time, the lower the level of customer satisfaction is [1]. Therefore, applying queuing theory is important to help select the best alternative (machine or process) to minimize the waiting time in a queue. Selecting the most suitable roll-over car washing machine can be considered a complex decision-making problem that can be evaluated through analytical and mathematical models. This study aims to integrate queue theory and MCDM techniques to select a roll-over car washing machine for an oil station operating in Turkey. Previously, queue theory and TOPSIS have been implemented to evaluate ATMs for banks [4]. Using the two techniques together ensures that more criteria are involved in selecting alternatives. Incorporating more criteria into the decision-making process can provide objectivity in the evaluation process. To the best of the author's knowledge, this study is the first attempt to solve selecting the best roll-over car washing machine by integrating queue theory and MCDM techniques.

Since criteria weights directly influence the overall results, they play an important role in the process of MCDM [10]. Some studies employ other MCDM methods, such as AHP, BWM, and MACBETH, to determine the weights of the criteria. However, this effort requires an expert evaluation that may be biased, expensive, or unavailable. It is also possible to determine criteria weights by objective techniques such as CRITIC (criteria importance through intercritera correlation) [8] or SECA (simultaneous evaluation of criteria and alternatives) [18]. The common point of these techniques is that they can determine the criteria weights by using the decision matrix. These techniques determine a single set of weights containing the criteria weights. Thus for each alternative, a single score is calculated. If two alternatives have quite close scores, it is unclear whether there is a practically significant difference between them. Practical significant differences are always an issue of judgment and interpretation for the decision maker. However, the discipline of statistics can guide the issue of statistically significant differences [24]. In this study, one million weight sets consisting of random values were created to examine the statistical superiority of the alternatives from each other, and the techniques were run repeatedly for each weight set. The alternative that exhibits statistical superiority is determined using one million scores.

As stated above, various MCDM techniques have been developed. It is possible to evaluate by using only one of them. However, in this study, four different techniques (TOPSIS, EDAS, CoCoSo and TODIM) were used together, and the results were compared. The TOPSIS technique was chosen because of its popularity among MCDM techniques. The EDAS technique was chosen because of its stability in different criteria weights [19]. The CoCoSo technique was chosen because of its stability and using three aggregator strategies to form a complete measure [29]. The TODIM technique was chosen because the decision-making outcome is determined by computing the degree of gain or loss of an alternative relative to the rest, to better reflect the behavioural preference of the decision makers such as reference dependence and loss aversion [9]. This study examines the performance of these techniques in solving the machine selection problem.

The contribution of this study can be highlighted as follows:

- applying queue theory equations for automatic car washing machine selection;
- integrating queue characteristics with other criteria into a single decision matrix;
- applying four different MCDM techniques (TOPSIS, EDAS, CoCoSo and TODIM) to the problem;
- creating random weight sets to observe the effect of weights on scores;
- perform ANOVA to test the differences.

The paper is organized into five sections. In Section 2, the literature about queue theory and MCDM is summarized. Section 3 presents the equations for queue theory and MCDM techniques. The Section 4 applies the methodology to selecting a roll-over car washing machine. The conclusions are given in the Section 5.

2. Literature review

Queue theory is applied in various fields. It is possible to summarize the studies on queuing theory as follows. Kondrashova, optimized local car-wash services [20]. A single server queue system is considered in the paper. It is reported that it is possible to develop the proposed model further. Jia et al. employed queue theory to model airport taxi drivers' decisions [17]. The authors validated their model at an airport in China and report that it may help taxi drivers decide whether to wait to pick up passengers at the airport or ditch the airport and search elsewhere for their next trip. Ghomi et al. reviewed and classified papers on cloud computing modelling [16]. They selected 71 studies published between 2008 and 2017 as primary studies. After reporting a summary of queuing models, they presented detailed tables about the classification used in their study.

Farayibi examined the queue theory application in Nigeria's banking system [12]. The queuing characteristics of the banks were analyzed using a multi-server queuing model. The waiting time and operation costs are computed for two banks. Recommendations to improve the quality of the service are presented. Eze and Odunukwe employed queue models for customer management in the banking system [11]. The authors used a multiple-server single queue model to analyze the data obtained from a bank in Nigeria. They reported the average time in the queue, the probability of the system being idle, and the expected number of people in the queue. Then they calculated the financial effects of using the various number of servers.

Lakshmi and Iyer presented a detailed literature review about applying queuing theory in health care [21]. The authors reviewed and classified 141 papers published between 1952 and 2011. They report that queuing theory provides an effective and powerful modelling technique that can help managers improve the system's performance. Zak and Golda employed queue theory to analyze and evaluate the performance of logistics centers [30]. They presented a detailed technical framework and then a logistics centre case study. Xiao and Zhang applied queue theory in bank service optimization [28]. An optimal number of servers and the optimal service rate are investigated utilizing the queuing theory. The authors report that their proposed model is feasible, can reduce the average waiting time in queues, and increase customer satisfaction. Adeleke and colleagues applied queue theory to the waiting time of out-patients in a hospital in Nigeria [2]. The arrival rate and service rate are calculated by using the hospital's records. Queuing system characteristics are presented. Li et al. integrated queue theory and goal programming for a multi-objective decision-aiding model [22]. They presented the applications of their proposed model in a public hospital in China. Che Soh et al. modelled a multilane-multiple intersection based on queue theory [6]. The authors applied their proposed model on one of the busiest streets in Kuala Lumpur. They report that simulation results correlate well with the proposed models and real case studies. Singer and Donoso employed queue theory to assess the performance of private ambulance services in Chile [26]. They report that their study shows the applicability of queuing theory to support decision-making in the ambulance business.

The studies summarized above indicate that the queuing theory has been applied to many areas. However, this technique was previously not used in selecting the automatic car-washing machine. Moreover, the characteristics calculated with the help of the queuing theory were not considered in the roll-over car washing machine selection problem under the MCDM framework. Enriching the decision matrix with queue features will ensure that the alternatives are evaluated from a customer-oriented perspective. Such an approach will contribute to increasing customer satisfaction.

3. Methodology

3.1. Queue theory

The basic process assumed by the queuing models can be summarized as follows [14]. Vehicles requiring washing service are generated over time by an input source. These vehicles enter the queue and join a queue if no car is already washed. The first vehicle in the queue is selected for service. The required service is performed, after which the vehicle leaves the queuing system (Figure 1).

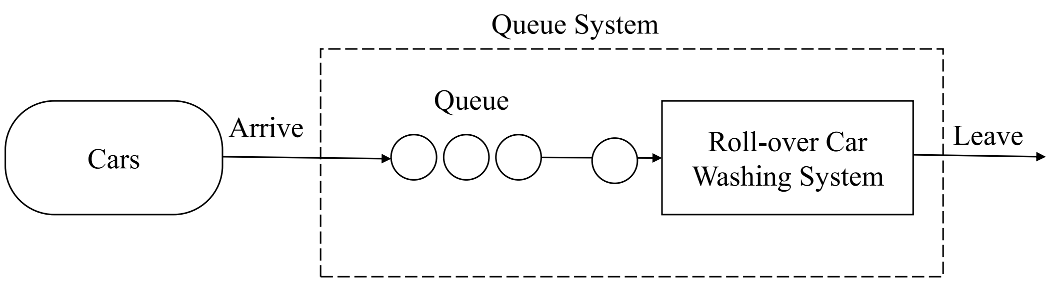

Figure 1. Single server queue system

The size of the calling population is assumed to be infinite. The maximum permissible number of customers in the queue is also assumed to be infinite. The first-come-first-served queue discipline is applied in the queue. Since there is only one roll-over vehicle washing machine, a single server machine is suitable for modelling the system. It is assumed that there will be no baulking (refusing to join the queue because it is too long), reneging (leaving the queue after joining), or blocking (preventing from joining the queue). This study assumes that all inter-arrival times are independent and identically distributed and that all service times are independent and identically distributed. This model is labelled as follows:

$$M/D/1 \tag{1}$$

where M is the distribution of inter-arrival times (Poisson arrival rate), D – distribution of service times (constant – deterministic service time), 1 – the number of servers.

A quantitative evaluation of a queuing system requires a mathematical characterization of the underlying process [25]. Some of the queue characteristics for the $M/D/1$ model can be calculated using equations [14]:

The average length of a queue

$$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)} \tag{2}$$

The average waiting time in a queue

$$W_q = \frac{\lambda}{2\mu(\mu - \lambda)} \tag{3}$$

where μ is the average number of arrivals per period, λ – average number of cars washed per period.

3.2. TOPSIS

Hwang and Yoon developed TOPSIS (technique for order preference by similarity to ideal solution) in 1981 [15]. The TOPSIS method's calculation steps can be summarized below [23].

Step 1. Define a decision matrix, where rows represent the alternatives, and columns represent the criteria. Dimensions of the matrix are $m \times n$, where m represents the number of alternatives and n represents the number of criteria:

$$x = \begin{bmatrix} x_{11} & \dots & x_{1n} \\ \dots & \dots & \dots \\ x_{m1} & \dots & x_{mn} \end{bmatrix} \tag{4}$$

Step 2. The dataset should be dimensionless before being used in the analysis. In other words, the data set should be normalized. With the normalization process, it is ensured that the variables take values in the same range. equation (5) is used for the normalization process.

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad j = 1, 2, \dots, n, \quad i = 1, 2, \dots, m \tag{5}$$

where x_{ij} is the old value and r_{ij} represents the normalized value of the ith alternative on jth criterion.

Step 3. By assigning different weights to criteria, their dominance in the analysis can be adjusted. With the help of equation (6), the normalized matrix in the previous step is weighted

$$v_{ij} = w_j r_{ij} \tag{6}$$

where w_j is the weight of the criterion j, r_{ij} is the normalized value, and v_{ij} is the weighted normalized value of ith alternative over jth criterion.

Step 4. Determine the ideal (v^+) and negative-ideal (v^-) solutions using equation

$$\begin{aligned} v^+ &= \{v_1^+, v_2^+, \dots, v_n^+\} = \{(\max_i v_{ij} | j \in J), (\min_i v_{ij} | j \in J')\} \\ v^- &= \{v_1^-, v_2^-, \dots, v_n^-\} = \{(\min_i v_{ij} | j \in J), (\max_i v_{ij} | j \in J')\} \end{aligned} \tag{7}$$

where J represents the set of benefit criteria (the-higher-the-better) which is a subset of $j = 1, 2, \dots, n$, and J' represents the set of cost criteria (the-lower-the-better). J is a complementary set of J'.

Step 5. Calculate the separation measures with equation

$$S_i^* = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2}, \qquad S_i^- = \sqrt{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2}, \qquad i = 1, 2, \dots, m \tag{8}$$

Step 6. Relative closeness to the ideal solution is calculated by the following equation

$$C_i = \frac{S_i^-}{S_i^- + S_i^+} \qquad i = 1, 2, \dots, m \tag{9}$$

where C_i represents the distance to the ideal solution. The value of the C_i is in the range [0, 1]. If $C_i = 1$, then the solution is equivalent to the positive ideal solution. If $C_i = 0$, then the solution is equivalent to a negative ideal solution.

3.3. EDAS

The EDAS steps can be summarized as follows [19]: Define the decision matrix as in equation (4). Determine the average solution according to all criteria with the following equation:

$$AV_j = \frac{\sum_{i=1}^{m} x_{ij}}{m} \tag{10}$$

Calculate the positive distance from average (PDA) and the negative distance from average (NDA) matrixes according to the type of criteria (benefit or cost), shown as follows:

If jth criterion is beneficial,

$$PDA_{ij} = \frac{\max(0, (x_{ij} - AV_j))}{AV_j}, \qquad NDA_{ij} = \frac{\max(0, (AV_j - x_{ij}))}{AV_j} \tag{11}$$

and if jth criterion is non-beneficial,

$$PDA_{ij} = \frac{\max(0, (AV_j - x_{ij}))}{AV_j}, \qquad NDA_{ij} = \frac{\max(0, (x_{ij} - AV_j))}{AV_j} \tag{12}$$

where PDA_{ij} and NDA_{ij} denote the positive and negative distance of ith alternative from the average solution in terms of jth criterion, respectively.

Determine the weighted sum of PDA and NDA for all alternatives, shown as follows:

$$SP_i = \sum_{j=1}^{n} w_j PDA_{ij}, \qquad SN_i = \sum_{j=1}^{n} w_j NDA_{ij} \tag{13}$$

where w_j is the weight of jth criterion. Normalize the values of SP and SN for all alternatives with the following equations:

$$NSP_i = \frac{SP_i}{\max_i SP_i}, \qquad NSN_i = 1 - \frac{SN_i}{\max_i SN_i} \tag{14}$$

Calculate the appraisal score (AS) for all alternatives with the following equation:

$$AS_i = \frac{1}{2}\left(NSP_i - NSN_i\right) \tag{15}$$

The alternative with the highest AS score is the best choice among the candidate alternatives.

3.4. CoCoSo

The CoCoSo method can be summarized as follows [29]. Suppose, there are m alternatives and n criteria. The initial decision matrix is defined in equation (4). The following normalization technique is applied for the benefit criterion:

$$r_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \tag{16}$$

The following normalization technique is applied for the cost criterion:

$$r_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}} \tag{17}$$

For each alternative the S_i and P_i values are calculated with the following equations:

$$S_i = \sum_{j=1}^{n} w_j r_{ij}, \qquad P_i = \sum_{j=1}^{n} r_{ij}^{w_j} \tag{18}$$

Three appraisal score strategies are used to generate relative weights of other options, which are derived using the following equations:

$$\xi_{ia} = \frac{P_i + S_i}{\sum_{i=1}^{m}(P_i + S_i)}, \qquad \xi_{ib} = \frac{S_i}{\min_i S_i} + \frac{P_i}{\min_i P_i}$$
$$\xi_{ic} = \frac{\lambda(S_i) + (1-\lambda)(P_i)}{\left(\lambda \max_i S_i + (1-\lambda)\max_i P_i\right)}, \qquad 0 \leq \lambda \leq 1 \tag{19}$$

In equation (19), λ is determined by decision-makers and usually set as 0.5. In this study, λ is also determined as 0.5. The final ranking of the alternatives is determined based on ξ_i values:

$$\xi_i = (\xi_{ia}\xi_{ib}\xi_{ic})^{1/3} + \frac{1}{3}(\xi_{ia} + \xi_{ib} + \xi_{ic}) \tag{20}$$

The higher the value of ξ_i, the better the solution is.

3.5. TODIM

The TODIM steps can be summarized as follows [13]. Let the decision matrix be defined as in equation (4). The decision matrix is normalized with the following equations.

If the criterion j is beneficial

$$r_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \tag{21}$$

If the criterion j is non-beneficial

$$r_{ij} = \frac{\min_i x_{ij}}{x_{ij}} \tag{22}$$

Calculate the measurement of the dominance of alternatives with the following equations:

$$\delta(A_i, A_j) = \sum_{c=1}^{n} \phi_c(A_i, A_j), \quad \forall(i,j)$$
$$\phi_c(A_i, A_j) = \begin{cases} \sqrt{\dfrac{w_{cr}(r_{ic} - r_{jc}}{\sum_{c=1}^{n} w_{cr}}} & \text{if } (r_{ic} - r_{jc}) > 0 \\ 0 \quad \text{if } (r_{ic} - r_{jc}) = 0 \\ -\dfrac{1}{\theta}\sqrt{\dfrac{\left(\sum_{c=1}^{n} w_{cr}\right)(r_{jc} - r_{ic})}{w_{cr}}} & \text{if } (r_{ic} - r_{jc}) < 0 \end{cases} \tag{23}$$

where θ is the attenuation factor of the losses. In this study, θ is set equal to 1. w_{cr} represents relative criteria weight and is calculated with $w_{cr} = w_c/w_r$, where w_c represents criteria weight and w_r represents the maximum of the criteria weights ($w_r = \max(w_c|c = 1, 2, \ldots, n)$).

To determine the overall value of alternative i through normalization of the corresponding measurements, the following equation is used

$$\tau_i = \frac{\sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)}{\max \sum_{j=1}^{n} \delta(A_i, A_j) - \min \sum_{j=1}^{n} \delta(A_i, A_j)} \tag{24}$$

The best alternative has the highest τ score.

4. Analysis

The conceptual framework of the study is presented in Figure 2. The steps of the study can be summarized as follows

- Observe the system to obtain information about the arrival rate of cars to the car washing machine
- Check if this arrival fits a Poisson distribution.
- Calculate queue characteristics (equations (2) and (3)).
- Obtain technical characteristics about the alternatives.
- Estimate the financial characteristics of the alternatives.
- Integrate queue, technical and financial characteristics in a decision matrix.
- Apply MCDM techniques to the decision matrix.
- Create one million weight sets and run the MCDM analysis with those weights.
- Analyze the results visually and statistically.

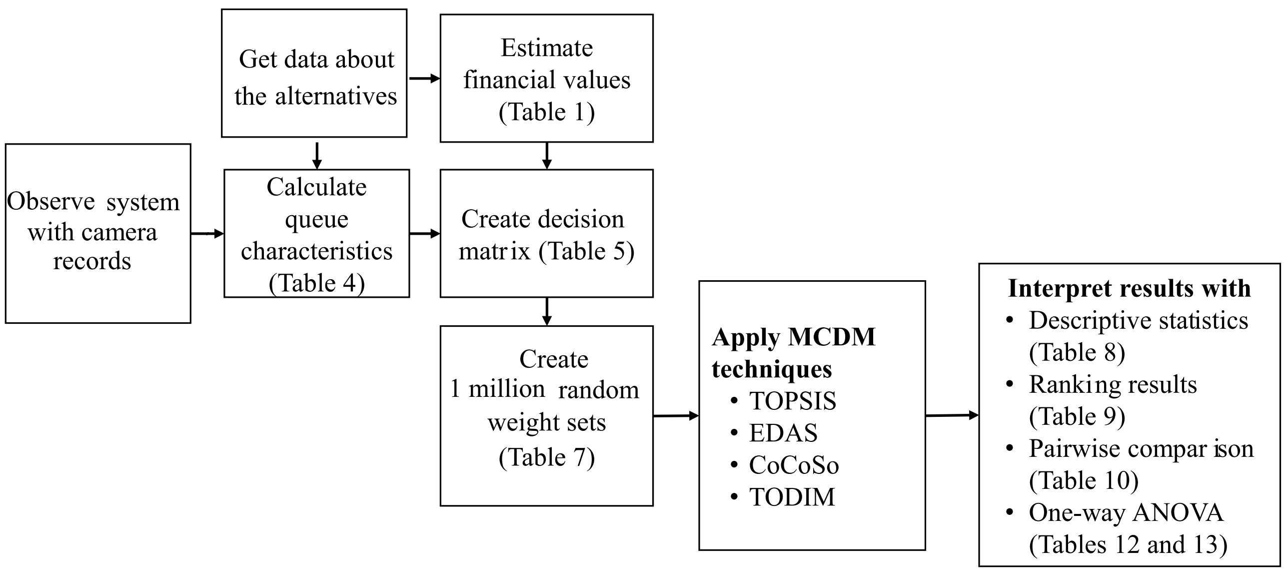

Figure 2. Cenceptual framework

All of the calculations are performed on the MATLAB platform. Since there is no available toolbox on MCDM techniques, all MCDM procedures are coded as a function on the MATLAB platform.

4.1. About firm

The oil station is one of the oldest businesses in the Kilis province of Turkey. It is located at the city's entrance and covers an area of approximately 5,000 m^2. A roll-over car wash system is currently used.

According to the firm, the advantages of a roll-over car wash system are that it is (1) suitable for most vehicles, (2) fast completion of the washing process, (3) low cost, and (4) recycling most of the water. However, the system has some disadvantages, such as (1) water spots on the vehicle and (2) a rough cleaning rather than a thorough one. The currently used system has approached the end of its economic life, and the cost of waste per unit vehicle has increased. The firm wants to replace the old system with a new one and is searching for analytical techniques to solve the system renewal problem.

4.2. About alternatives

To identify alternatives, 7 vendors were interviewed and received offers for 9 roll-over car washing machines (some companies may provide more than one model). As a result of the preliminary evaluation, it was determined that 4 options were not feasible for the firm and were eliminated. Each of the remaining 5 models is feasible. The names of the companies and the models will be kept confidential. Company X is a European company and provides two models, Alt 1 and Alt 2. Other models (Alt 3, Alt 4 and Alt 5) are of Chinese origin. The technical features and prices of the models will be given in the following sections.

4.3. About criteria

4.3.1. Mounting area

The currently used system covers an area of 30.20 m^2. The mounting area of each alternative is obtained from vendors and presented in Table 1.

4.3.2. Initial cost

The initial cost of the alternatives includes the following elements

- purchase price,
- insurance,
- transportation cost,
- setup and trial run.

Table 1. Calculation of annual operation and maintenance costs [$]

Alternative	Operation cost X	Maintenance cost Y	First year cost $X+Y$	Annual cost A
1	365 × 20×8×0.4 = 23,360	100	23,460	26,515.87
2	365×22×8×0.4 = 25,696	250	25,946	29,341.52
3	365×24×8×0.4 =28,032	95	28,127	31,790.79
4	365×21×8×0.4 =24,528	150	24,678	27,892.53
5	365×25×8×0.4 = 29,200	320	29,520	33,365.24

The initial cost of each alternative was determined through interviews with vendors. The initial costs of the alternatives are presented in Table 1.

4.3.3. Annual operation and maintenance cost

Annual operation and maintenance costs were calculated for each alternative by the following procedure:

- The machines are assumed to work 365 days a year and 8 hours a day (with maximum capacity). The number of vehicles (μ) that each alternative can wash in an hour is given in Table 4. The cost of water, electricity and liquid cleaner needed to wash a vehicle was \$0.4. The operation cost was calculated for the first year by multiplying the values (X).
- For the first year, regular maintenance and repair costs are estimated (Y).
- It is assumed that the sum of these two cost items (first year cost, $FYC = X + Y$) will increase by 3% each year.
- It is assumed that the annual capital cost of the firm is 6%, and will not change in the forthcoming years.

The economic life of each alternative is assumed to be 10 years. The annual operation and maintenance costs (A) is calculated (Table 1) with the help of the following equation.

$$\sum_{n=1}^{10} \frac{FYC(1.03)^{n-1}}{1.06^n} = A\left[\frac{1.06^{10} - 1}{0.06(1.06)^{10}}\right] \tag{25}$$

4.3.4. Calculating queue characteristics

The arrival rate (λ) of the vehicles to the system must be determined to calculate the queue characteristics. However, there is no data on how many cars arrive at the system per hour. To determine the arrival rate, the security camera records of the firm were examined. Since the process will be performed manually, examining all available records will take a long time. That is why only records for the year 2019 were taken into account. Two hours the company keeps the washing system operating (between 08:00 and 19:00) are randomly selected for each day. Camera records were used to figure out how many cars came to the system on the chosen days and hours. The number of vehicles arriving at the system varies in each time zone. For example, the system is less busy in the morning hours. For this reason, observing the system simultaneously every day will produce misleading results. For this reason, the data is collected at different times on different days. The collected data is firm-specific and not comparable.

There were 365 days in the year 2019. Since the number of arrivals in two hours was counted for each day, 730 counts were performed. A section from the dataset is presented in Table 2. The number of vehicles arriving each hour is included as the θ vector in the last column of the table. According to Table 2, on Tuesday, January 1, between 9 and 10 am, 9 vehicles arrived at the system. A histogram of the θ vector, which contains 730 rows, is presented in Figure 3. Descriptive statistics of the θ vector are also presented in the figure.

Table 2. A section from vehicle counts dataset

Row No.	Date	Time span	No. of arrivals θ
1	January 01, 2019, Tuesday	09:00–10:00	9
2	January 01, 2019, Tuesday	17:00–18:00	13
3	January 02, 2019, Wednesday	12:00–13:00	11
4	January 02, 2019, Wednesday	18:00–19:00	10
...			
729	December 31, 2019, Tuesday	13:00–14:00	13
730	December 31, 2019, Tuesday	18:00–19:00	12

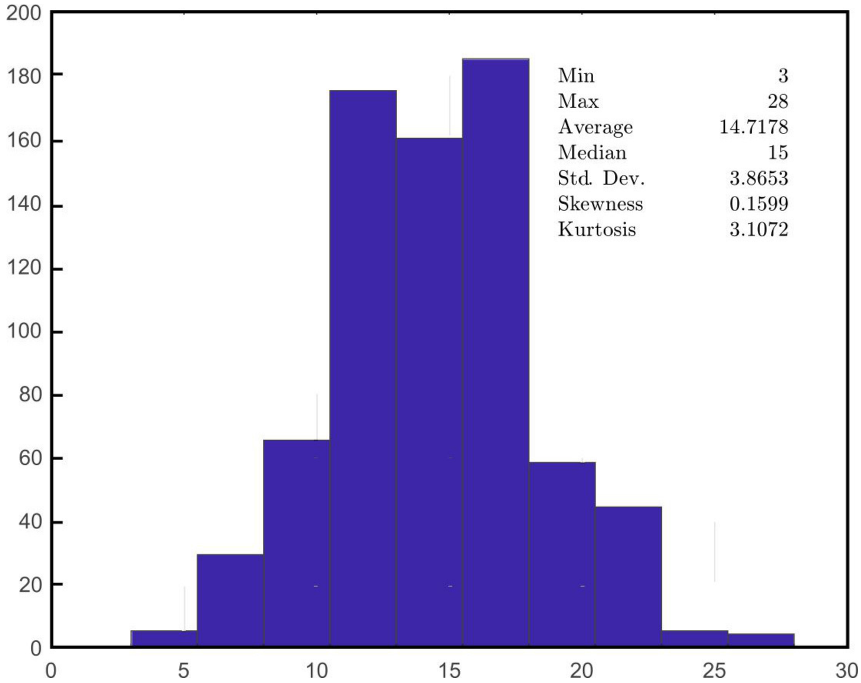

Figure 3. Histogram of vehicle counts (θ) vector in Table 6

Poisson, negative binomial, and binomial distributions are fitted to the data, and the goodness-of-fit results are presented in Table 3. As the results in the table indicate, the best theoretical fit is the Poisson distribution with $\lambda = 14.72$. Since the number of cars arriving at the station is discrete, we assumed that $\lambda = 15$.

Table 3. Goodness-of-fit data

	Parameter	Value	Lower	Upper	$N\log L$	BIC	AIC	AICc
Poisson	λ	14.7178	14.4395	14.9961	2022.1	4050.7	4046.1	4046.1
Negative binomial	success	1049.5	0.0	8912.4	2022.0	4057.2	4048.0	4048.1
	probability	0.9862	0.8840	1.0				
Binomial	number of trials	28	28	28	2175.8	4364.8	4355.7	4355.7
	probability	0.5256	0.5188	0.5325				

How many vehicles each alternative can wash in an hour (μ) was obtained from the vendors. The parameters needed to apply the formulas and calculations are presented in Table 4.

Table 4. Queue characteristics of alternatives

Alternative	Arrival rate λ	Service rate μ	Cars in a queue L_q (Eq. (2))	Time in a queue [h] W_q (Eq. (3))	Time in a queue [min] $60W_q$
1	15	20	1.125	0.75	4.5
2	15	22	0.731	0.0487	2.92
3	15	24	0.521	0.0347	2.08
4	15	21	0.893	0.0595	4.5
5	15	25	0.45	0.03	1.8

Financial and queue characteristics are collected together to construct the decision matrix as in Table 5. Equal weights (1/5 = 0.2) are assigned to each criterion, and each criterion has a cost (the lower, the better) characteristics.

Table 5. Decision matrix for selecting roll-over car washing machine problem

Alternative	Mounting area [m^2]	Initial cost [$]	Annual cost [$]	L_q (vehicles)	W_q [min]
1	26.24	65,700	26,515.87	1.125	4.50
2	29.43	67,800	29,341.52	0.731	2.92
3	31.62	64,450	31,790.79	0.521	2.08
4	34.86	67.300	27,892.53	0.893	3.57
5	44.56	66,150	33,365.24	0.450	1.80

4.4. MCDM application

After the calculations were made with equal weights assigned to each criterion through the four MCDM techniques, the results of Table 6 were derived. It is evident from Table 6 that TOPSIS and EDAS techniques produced the same rankings, and, similarly, CoCoSo and TODIM produced the same rakings. More clearly, Alternative 3 is the best roll-over car washing machine as per four techniques. While Alternative 5 is ranked as the second among the alternatives in relation to TOPSIS, TODIM and EDAS methods, it is ranked the fifth-best machine as per the CoCoSo method. Alternative 2 is ranked the third-best machine as per four techniques. Alternative 4 is ranked the fourth-best machine as per TOPSIS and EDAS techniques, while it is ranked as the second and fifth-best machine as per CoCoSo and TODIM techniques, respectively. Alternative 1 is ranked as the fifth-best machine as per TOPSIS and EDAS, while it is ranked as the fourth-best alternative as per CoCoSo and TODIM methods. Weight simulation is applied in the following sub-section to interpret the results statistically.

Table 6. Overall scores and ranking results of alternatives as per four MCDM methods

Alternative	TOPSIS		EDAS		CoCoSo		TODIM	
	C_i	Rank	AS_i	Rank	ξ_i	Rank	τ_i	Rank
Alt 1	0.325	5	0.148	5	1.542	4	0.356	4
Alt 2	0.618	3	0.564	3	1.859	3	0.482	3
Alt 3	0.811	1	0.859	1	2.315	1	1.000	1
Alt 4	0.391	4	0.342	4	2.008	2	0.000	5
Alt 5	0.675	2	0.854	2	1.612	5	0.524	2

4.5. Weight simulation

To determine whether the scores of the alternatives differ statistically from each other, random weights were created, and the MCDM analysis was rerun. The process of generating a random weight set can be summarized as follows:

- There are five criteria in the dataset. Create five random numbers in the range of [1, 100] (lower and upper bounds are determined arbitrarily).
- Normalize weights by dividing each number by the sum of the five numbers. With this process, all of the weights will be in the range of [0, 1], and the sum of the weights will be equal to 1.

A section from the weight simulation process is presented in Table 7. For example, the first random number set is generated as 24, 17, 66, 74, and 33. The sum of these random numbers is 214. The weight of the first criterion is calculated as 24/214 = 0.111, the weight of the second criterion is 17/214 = 0.08, etc. As a result, the first weight set is determined as 0.11, 0.08, 0.31, 0.35 and 0.15. Similarly, calculations were made for other criteria, and as a result, 1 million different weight sets were obtained.

Table 7. A section from random weight criterion process

	Cr 1	Cr 2	Cr 3	Cr 4	Cr 5	Sum
Random number set (RNS) 1	24	17	66	74	33	214
Weight set (WS) 1	0.11	0.08	0.31	0.35	0.15	
RNS 2	82	91	13	92	64	342
WS 2	0.24	0.27	0.04	0.27	0.19	
...						
RNS 999,999	10	28	55	96	97	286
WS 999,999	0.03	0.10	0.19	0.34	0.34	
RNS 1,000,000	96	93	43	10	12	254
WS 1,000,000	0.38	0.37	0.17	0.04	0.05	

Descriptive statistics of weight sets are presented in Table 8. The arithmetic average of the weights converges to 0.2 since there are 5 criteria, and uniformly distributed random numbers are generated.

Table 8. Descriptive statistics of weight sets

	Cr 1	Cr 2	Cr 3	Cr 4	Cr 5
Minimum	0.0026	0.0026	0.0026	0.0026	0.0025
Maximum	0.8889	0.8649	0.8851	0.8873	0.8462
Mean	0.2000	0.2002	0.1999	0.2000	0.1999
Median	0.2000	0.2000	0.2000	0.2000	0.2000
Standard deviation	0.1120	0.1119	0.1120	0.1121	0.1121
Skewness	0.3517	0.3551	0.3525	0.3565	0.3533
Kurtosis	2.9408	2.9592	2.9469	2.9537	2.9393

Table 9. Ranking results

	Alternative	Rank 5	Rank 4	Rank	Rank 2	Rank 1	Total
TOPSIS	1	780,722	101,138	89,491	15,323	13,326	1,000,000
	2	1	30	654,388	295,364	50,217	1,000,000
	3	0	6,288	23,726	115,880	854,106	1,000,000
	4	125,964	864,897	5,737	2,478	924	1,000,000
	5	93,313	27,647	226,658	570,955	81,427	1,000,000
	Total	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	
EDAS	1	745,099	146,985	66,377	13,207	28,332	1,000,000
	2	2	299	793,569	206,086	44	1,000,000
	3	0	1,412	14,442	210,527	773,619	1,000,000
	4	195,965	802,099	1,279	641	16	1,000,000
	5	58,934	49,205	124,333	569,539	197,989	1,000,000
	Total	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	
CoCoSo	1	434,394	474,831	54,762	29,798	6,215	1,000,000
	2	1,927	103,963	684,937	209,173	0	1,000,000
	3	0	2,897	3,317	10,227	983,559	1,000,000
	4	0	89,896	228,063	671,821	10,220	1,000,000
	5	563,679	328,413	28,921	78,981	6	1,000,000
	Total	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	
TODIM	1	124,515	608,381	161,271	67,562	38,271	1,000,000
	2	61	44,378	706,513	249,048	0	1,000,000
	3	0	7,524	11,529	47,745	933,202	1,000,000
	4	798,339	193,669	4,392	3,600	0	1,000,000
	5	77,085	146,048	116,295	632,045	28,527	1,000,000
	Total	1,000,000	1,000,000	1,000,000	1,000,000	1,000,000	

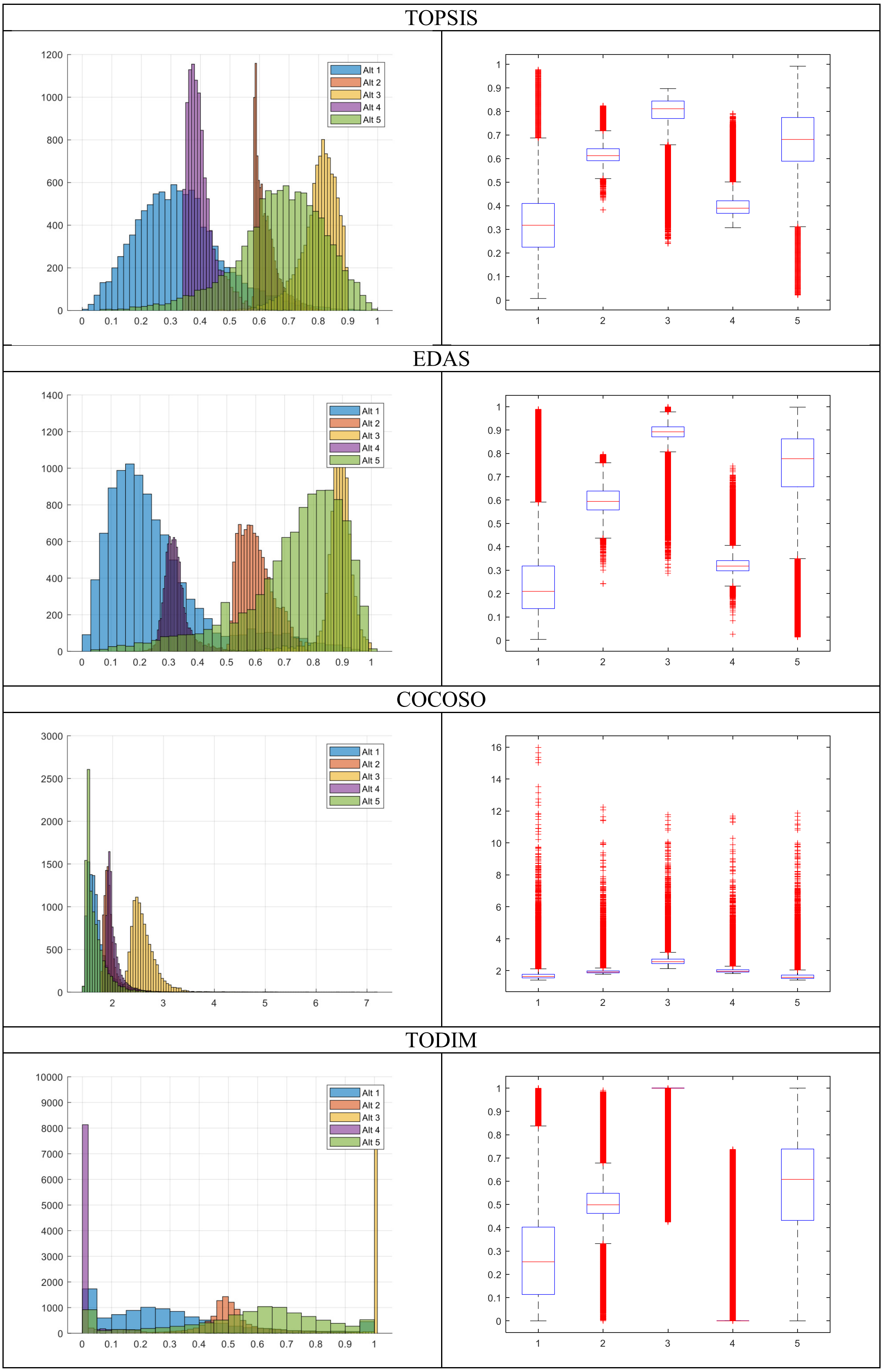

Figure 4. Histogram distributions and box plots of MCDM scores

MCDM score calculations were performed for each weight set. As a result, 1 million different MCDM values were obtained. Histogram distributions and box plots of MCDM scores are presented in Figure 4.

Both histograms and box plots indicate that Alternative 3 has obtained the highest scores in one million weighted sets.

A ranking table is prepared to obtain more insight into the one million MCDM scores (Table 9) This table provides information about the performance of alternatives under different weight sets. It includes the data about how many times each alternative is ranked 1st, 2nd, etc. According to Table 9 TOPSIS results section, Alternative 3 is ranked 85.4106% (854,106/1,000,000) as the best alternative (since there are a million weight sets, the figures in the table can also be seen as a percentage), also it never ranked as the worst alternative during the weight simulation process. This finding indicates that Alternative 3 would be among the best alternatives even if weights are determined randomly. The superiority of Alternative 3 over other alternatives can be validated with other MCDM techniques (Table 9).

To further investigate the results, pairwise comparisons of rankings are calculated (Table 10). This table includes information about how many times an alternative ranked in a higher degree on a million weight set compared with another alternative. For example, Alternative 1 is ranked in a better position (or had a higher TOPSIS score) than Alternative 2 in 13,925 weight sets (13,925/1,000,000=1.3925%). The superiority of Alternative 3 can be seen in this table, too. It ranked higher than the first alternative at 97.0457% (=970,457/1,000,000) of the weight sets and higher than the second alternative in 93.5590%, etc. Similar comparisons can be made with other MCDM techniques. The overall table indicates that Alternative 3 has superiority over other alternatives in most of the weight sets.

Table 10. Pairwise comparison of superiority among alternatives

TOPSIS	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5	Total
Alt 1	0	13,925	29,543	216,089	119,836	379,393
Alt 2	986,075	0	64,410	997,666	347,615	2,395,766
Alt 3	970,457	935,590	0	993,184	918,573	3,817,804
Alt 4	783,911	2,334	6,816	0	94,440	887,501
Alt 5	880,164	652,385	81,427	905,560	0	2,519,536
Total	3,620,607	1,604,234	182,196	3,112,499	1,480,464	10,000,000
EDAS	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5	Total
Alt 1	0	41,538	28,361	254,688	108,101	432,688
Alt 2	958,462	0	15,792	999,148	232,469	2,205,871
Alt 3	971,639	984,208	0	998,495	802,011	3,756,353
Alt 4	745,312	852	1,505	0	58,975	608,644
Alt 5	891,899	767,531	197,989	941,025	0	2,798,444
Total	3,567,312	1,794,129	243,647	3,193,356	1,201,556	10,000,000
CoCoSo	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5	Total
Alt 1	0	88,334	8,652	36,930	564,693	698,609
Alt 2	911,666	0	3,048	276,852	909,790	2,101,356
Alt 3	991,348	996,952	0	986,154	999,994	3,974,448
Alt 4	963,070	723,148	13,846	0	902,301	2,602,365
Alt 5	435,307	90,210	6	97,699	0	623,222
Total	3,301,391	1,898,644	25,552	1,397,635	3,376,778	10,000,000
TODIM	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5	Total
Alt 1	0	140,050	38,271	875,248	232,887	1,286,693
Alt 2	859,950	0	18,967	995,961	329,670	2,204,548
Alt 3	961,729	981,033	0	992,390	971,473	3,906,625
Alt 4	124,515	4,039	7,610	0	77,089	213,253
Alt 5	767,113	670,330	28,527	922,911	0	2,388,881
Total	2,713,307	1,795,452	93,375	3,786,747	1,611,119	10,000,000

Descriptive statistics of scores are presented in Table 11. Low standard deviation values for Alternatives 2–4 indicate that the ranking results of these alternatives are insensitive to the weights (i.e., using different weights will not cause significant changes in the scores of alternatives). Similarly, Alternative 1 and Alternative 5 have higher standard deviation scores indicating that these alternatives are sensitive to the weight sets. In other words, using different weights will cause significant changes in the MCDM scores.

Table 11. Descriptive statistics of scores

TOPSIS	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5
Minimum	0.008	0.383	0.241	0.308	0.023
Maximum	0.978	0.825	0.898	0.791	0.993
Mean	0.330	0.625	0.800	0.402	0.670
Median	0.318	0.613	0.812	0.390	0.682
Standard Deviation	0.148	0.042	0.064	0.047	0.680
Skewness	0.719	1.473	-1.437	1.538	-0.712
Kurtosis	3.791	5.240	6.717	6.157	3.777
EDAS	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5
Minimum	0.004	0.242	0.287	0.026	0.014
Maximum	0.989	0.796	0.999	0.746	0.998
Mean	0.527	0.603	0.886	0.325	0.734
Median	0.209	0.594	0.892	0.318	0.777
Standard Deviation	0.179	0.057	0.054	0.044	0.177
Skewness	1.501	0.616	-2.478	1.721	-1.252
Kurtosis	5.082	2.722	13.865	8.229	4.434
CoCoSo	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5
Minimum	1.414	1.778	2.131	1.821	1.415
Maximum	15.973	12.239	11.765	11.677	11.868
Mean	1.719	1.964	2.616	2.024	1.663
Median	1.641	1.918	2.562	1.963	1.585
Standard Deviation	0.298	0.199	0.271	0.202	0.256
Skewness	5.473	6.892	3.641	5.906	5.555
Kurtosis	89.888	125.952	46.546	99.124	82.389
TODIM	Alt 1	Alt 2	Alt 3	Alt 4	Alt 5
Minimum	0.000	0.000	0.425	0.000	0.000
Maximum	1.000	0.990	1.000	0.737	1.000
Mean	0.296	0.514	0.990	0.034	0.563
Median	0.254	0.499	1.000	0.000	0.608
Standard Deviation	0.250	0.118	0.053	0.102	0.263
Skewness	1.137	0.685	-7.082	4.068	-0.642
Kurtosis	4.035	5.647	57.158	21.348	2.821

One-way variance analysis is performed for each technique to test the null hypothesis that the score set samples are drawn from populations with the same mean. Results are presented in Table 12. The small p-value indicates that differences between column means are significant. Multiple comparisons (Tukey–Kramer) results statistically support the superiority of Alternative 3 over other alternatives (Table 13). Moreover, all of the pairwise comparisons have a significant p-value. This finding can be interpreted as follows: if an alternative has a higher rank over another alternative, this superiority is, at the same time, statistically significant.

Table 12. Results of one-way analysis of variance

TOPSIS					
Source	SS	df	MS	F	p
Columns	151,443.60	4	37,860.90	3,652,476.85	0.0
Error	51,829.03	4,999,995	0.01		
Total	203,272.63	4,999,999			
EDAS					
Source	SS	df	MS	F	p
Columns	285,434.91	4	71,358.73	4,980,188.42	0.0
Error	71,642.53	4,999,995	0.01		
Total	357,077.43	4,999,999			
CoCoSo					
Source	SS	df	MS	F	p
Columns	572,907.20	4	143,226.80	2,327,526.31	0.0
Error	307,679.99	4,999,995	0.06		
Total	880,587.19	4,999,999			
TODIM					
Source	SS	df	MS	F	p
Columns	501,046.23	4	125,261.56	3,946,450.74	0.0
Error	158,701.38	4,999,995	0.03		
Total	659,747.61	4,999,999			

Table 13. Results of multiple comparisons

TOPSIS						CoCoSo					
Alternatives		$\mu_A - \mu_B$	Lower	Upper	p	Alternatives		$\mu_A - \mu_B$	Lower	Upper	p
1	2	-0.295	-0.294	-0.294	0.0	1	2	-0.219	-0.218	-0.218	0.0
1	3	-0.470	-0.470	-0.470	0.0	1	3	-0.246	-0.245	-0.244	0.0
1	4	-0.072	-0.072	-0.072	0.0	1	4	-0.897	-0.896	-0.895	0.0
1	5	-0.340	-0.340	-0.339	0.0	1	5	-0.305	-0.304	-0.303	0.0
2	3	-0.176	-0.176	-0.175	0.0	2	3	0.055	0.056	0.057	0.0
2	4	0.222	0.222	0.223	0.0	2	4	-0.652	-0.651	-0.650	0.0
2	5	-0.046	-0.045	-0.045	0.0	2	5	-0.060	-0.060	-0.059	0.0
3	4	0.398	0.398	0.398	0.0	3	4	0.300	0.301	0.302	0.0
3	5	0.130	0.130	0.131	0.0	3	5	0.591	0.592	0.593	0.0
4	5	-0.268	-0.268	-0.267	0.0	4	5	0.951	0.952	0.953	0.0
EDAS						TODIM					
Alternatives		$\mu_A - \mu_B$	Lower	Upper	p	Alternatives		$\mu_A - \mu_B$	Lower	Upper	p
1	2	-0.346	-0.346	-0.345	0.0	1	2	-0.219	-0.218	-0.218	0.0
1	3	-0.629	-0.629	-0.628	0.0	1	3	-0.696	-0.695	-0.694	0.0
1	4	-0.068	-0.068	-0.067	0.0	1	4	0.260	0.261	0.262	0.0
1	5	-0.478	-0.477	-0.477	0.0	1	5	-0.268	-0.267	-0.266	0.0
2	3	-0.283	-0.283	-0.282	0.0	2	3	-0.477	-0.476	-0.476	0.0
2	4	0.278	0.278	0.279	0.0	2	4	0.479	0.480	0.480	0.0
2	5	-0.132	-0.131	-0.131	0.0	2	5	-0.049	-0.049	-0.048	0.0
3	4	0.561	0.561	0.562	0.0	3	4	0.955	0.956	0.957	0.0
3	5	0.151	0.152	0.152	0.0	3	5	0.427	0.428	0.429	0.0
4	5	-0.410	-0.410	-0.409	0.0	4	5	-0.529	-0.528	-0.527	0.0

4.6. Managerial implications

Results indicate that Alternative 3 ranks the highest according to all four techniques. This alternative is also the alternative with the lowest initial cost among the five alternatives. Although there are alternatives with lower annual costs, considering other criteria, the optimal alternative was determined as the third alternative. When this alternative is selected, there will be an average of 0.5 vehicles in the queue, and the vehicles will wait in the queue for an average of two minutes.

While multi-criteria decision-making techniques are used in solving problems in practical life, determining the weights of the criteria emerges as a problem that needs to be solved carefully. While applying multi-criteria decision-making techniques in the literature, there are studies in which weights are assigned to criteria with other multi-criteria decision-making techniques such as AHP, Entropy, and Fuzzy TOPSIS. While evaluating the techniques mentioned above, one or a group of experts is requested to evaluate the criteria. Thus, weights are assigned to the criteria according to their importance. Expert evaluation can be a time-consuming or costly process. In some cases, expert knowledge may not be available.

In this study, a simulation experiment is proposed in order to test whether the differences between the two alternatives are significant or not. Thus, the application of another analysis that requires expert knowledge is eliminated. In this study, alternatives were evaluated not only from the managers' point of view. Managers may tend to evaluate alternatives at the lowest cost and highest return.

In this study, customers' desire to wait for less in the queue was also included in the decision-making process. New variables were calculated and included in the analysis with the help of queuing theory developed in the field of operations research. The number of customers waiting in the queue and the waiting time of the customers will be necessary for the managers when evaluating the alternatives. By taking these variables into account, more customer-oriented choices can be made, leading to an increase in customer satisfaction.

5. Conclusions

Businesses have to select an alternative from among complex alternatives considering conflicting criteria all the time. In cases where there is more than one alternative, it is essential to rationally formulate the selection process among the alternatives so that the decision can add value to the business. This study integrated queuing theory and MCDM techniques for a roll-over car washing machine selection. The properties calculated with the help of queuing theory and the technical and financial properties of the alternatives are aggregated in a decision matrix. Four different MCDM techniques were applied to the decision matrix, and it was determined that the best alternative was the third alternative. In order to observe the change in ranking in different weight sets, one million weight sets consisting of random values were generated and MCDM analysis was run separately for each one. The resulting 1 million scores were analyzed for each technique. For this purpose, tables related to ranking values and pairwise performance comparisons were created. In addition, one-way ANOVA and post hoc analysis were applied to perform the statistical analysis. As a result, the superiority of the third alternative over the other alternatives has been proven statistically.

The contribution of this paper is to develop a decision support system where expert knowledge is not available to determine the weights of the criteria. Creating random weights and determining the superior alternatives has practical importance since the expert evaluation of criteria may be biased, expensive, or unavailable. The procedure in the study can be applied together with other MCDM techniques and can be easily adapted to solve different kinds of problems.

There are some limitations to this study. The research is limited in terms of the firm. A single business is considered in the study. A time-consuming process (recording data from security camera records) is followed to determine the queue characteristics. In future studies, assumptions about the arrival rates of the customers can be used. Four MCDM techniques are selected by considering their calculation steps. Some assumptions about cash flows are made.

In future studies, recently developed techniques may be employed along with the techniques considered in this study. Fuzzy evaluations may be applied, and those results may be compared with the findings of this study. In addition, multiple servers may be considered. Other variables developed in the queue theory (such as utilization rate) may be added to the decision matrix.

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