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IMPROVING GLOBAL ELASTICITY OF BONUS-MALUS SYSTEM

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Abstract: We optimize transition rules of bonus-malus system to achieve possibly best premium elasticity as defined by Loimaranta [1972] and later generalized as global elasticity by De Pril [1978]. We use premium scale given by Norberg [1976]. This issue constitutes a nonlinear nonconvex discrete optimization problem. To solve this problem, we apply improved greedy optimization algorithm, similar to one proposed by Morlock [1984]. We analyse systems of different size for portfolios characterized by inverse Gaussian risk structure function with various parameters. We also propose alternative measures of global elasticity.

Keywords: bonus-malus system, transition rules, optimization, premium elasticity, automobile insurance

INTRODUCTION

Bonus-malus systems (BMS) are used as a tools of a posteriori premiums differentiation in risk assessment process in automobile insurance. While tools of systems analysis and premium calculation criteria are well-described in the literature, relatively little space is devoted to the optimization of transition rules between classes of a bonus-malus system. We try to optimize transition rules in order to address two issues described below.

Goal of the research

Bonus-malus systems have been criticised because:
- bonus-malus systems have low premium elasticity,
- policyholders tend to cluster in 'better classes'.

Considering above disadvantages of bonus-malus systems our research question is:

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- Can we eliminate these disadvantages by optimizing transition rules in order to achieve higher premium elasticity?

RISK

Risk process is modelled typically for this kind of problems, so we assume that:

- claim amount and number of claims are independent,
- expected claim amount equals 1 (claim rate λ is a measure of risk of a single insured),
- policyholders form a heterogeneous portfolio (insured differ by claim rate λ) with overdispersion,
- there is no bonus hunger.

We distinguish two random variables:

K – number of claims ~ Poisson(λ),

Λ – claim rate ~ Inverse Gaussian IG(μ, θ),

furthermore

$u(\lambda)$ – is probability density function of Λ, so called risk structure function.

Conditional probability of reporting *k* claims in unitary period (one year)

$$P_k(\lambda) = P(K = k|\Lambda = \lambda) = \frac{e^{-\lambda}\lambda^k}{k!} \tag{1}$$

Unconditional probability of *k* claims in unitary period (one year)

$$P_k = P(K = k) = \int_0^\infty \frac{e^{-\lambda}\lambda^k}{k!} u(\lambda) d\lambda = \int_0^\infty \frac{e^{-\lambda}\lambda^k}{k!} dU(\lambda) \tag{2}$$

With above assumptions we have expected value of number of claims and its variance given by:

$$EK = \mu, \;\; VarK = \mu + \mu\theta \tag{3}$$

and expected claim rate and variance of claim rate given by:

$$E\Lambda = \mu, \;\; Var\Lambda = \mu\theta \tag{4}$$

BONUS-MALUS SYSTEM (BMS)

We assume that bonus-malus system consists of [Lemaire 1985]:

- finite number of classes $i \epsilon S, S = \{1, 2, \dots, s\}$ such as insured belongs to one and only one class in unitary period and the class in the next period depends only on the class and the number of claims reported in the current period according to transition rules,
- premiums b_i specified for each class,

- specified starting class for those who insure for the first time (unnecessary condition for stationary state analysis).

Additionally, we assume that:

- the best class is class number 1 (best class means class with the lowest premium and the most favourable transition rules),
- the worst class is class number *s*.

Transition rules can be represented by a transition table or transition matrix $\boldsymbol{T} = [t_{ik}]$, which shows to which class insured passes after reporting *k* claims in class *i*.

Example of transition table

	k =	0	1	2	3+
class	1	1	2	3	5
	2	1	3	5	5
	3	2	5	6	6
	4	3	6	6	7
	5	4	6	7	7
	6	5	7	7	8
	7	6	7	8	8
	8	7	8	8	9
	9	8	9	9	10
	10	9	10	10	10

Example of transition matrix

$$\boldsymbol{T} = [t_{ik}] = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 1 & 3 & 5 & 5 \\ 2 & 5 & 6 & 6 \\ 3 & 6 & 6 & 7 \\ 4 & 6 & 7 & 7 \\ 5 & 7 & 7 & 8 \\ 6 & 7 & 8 & 8 \\ 7 & 8 & 8 & 9 \\ 8 & 9 & 9 & 10 \\ 9 & 10 & 10 & 10 \end{bmatrix}$$

MODEL OF A BONUS-MALUS SYSTEM

As bonus-malus system possess Markov property (class in the next period depends only on the class and the number of claims in the previous period) it is usually modelled by suitable Markov chain [Lemaire 1985, 1995].

Transformation matrix is a matrix $\boldsymbol{T}_k = [t_{ij}(k)]$, where:

$$t_{ij}(k) = \begin{cases} 1 \text{ for } t_{ik} = j \\ 0 \text{ for } t_{ik} \neq j \end{cases} \tag{5}$$

Probability of transition from class *i* to class *j* (depending on claim rate λ)

$$p_{ij}(\lambda) = \sum_{k=0}^{\infty} p_k(\lambda) t_{ij}(k) \tag{6}$$

The transition probability matrix of Markov chain

$$\boldsymbol{P}(\lambda) = [p_{ij}(\lambda)] = \sum_{k=0}^{\infty} p_k(\lambda) \boldsymbol{T}_k \tag{7}$$

For regular transition probability matrix, after sufficient time the chain tends to stationary state [Kemeny 1976] with stationary distribution:

$$\boldsymbol{e}(\lambda) = [e_1(\lambda), \dots, e_s(\lambda)] \tag{8}$$

$$\begin{cases} \boldsymbol{e}(\lambda)\boldsymbol{P}(\lambda) = \boldsymbol{e}(\lambda) \\ \boldsymbol{e}(\lambda)\boldsymbol{1} = 1 \end{cases} \tag{9}$$

Unconditional stationary distribution is given by

$$\boldsymbol{e} = [e_1, \dots, e_s] = [\int_0^\infty e_1(\lambda)u(\lambda)d\lambda, \dots, \int_0^\infty e_s(\lambda)u(\lambda)d\lambda] \tag{10}$$

PERMISSIBLE SYSTEMS

We limit ourselves to systems which fulfil below conditions:

- elements in rows of transition matrix $\boldsymbol{T}$ are non-decreasing (weak monotonicity in rows) - in each class penalty* for more claims is no less than for fewer claims
- elements in columns of transition matrix $\boldsymbol{T}$ are non-decreasing (weak monotonicity in columns) - penalty[1] for the same number of claims in the worse class cannot be less than in the better class (with the exception of the worst class)
- systems are irreducible (are modelled by an irreducible Markov chain) - none of elements of stationary distribution equal zero
- systems are ergodic (are modelled by an ergodic Markov chain) - stationary distribution does not depend on starting class

Systems which fulfil above conditions are called permissible systems.

PREMIUMS

We use Norberg criterion of premiums calculation [Norberg 1976]

$$Q(\boldsymbol{b}) = \int_0^\infty \sum_{j=1}^{s} \left(b_j - \lambda\right)^2 e_j(\lambda)u(\lambda)d\lambda \rightarrow min \tag{11}$$

Which gives so called *Q*-optimal premiums (where b_j is a premium to be paid in class j)

$$b_j = \frac{\int_0^\infty \lambda e_j(\lambda)u(\lambda)d\lambda}{\int_0^\infty e_j(\lambda)u(\lambda)d\lambda} = \frac{\int_0^\infty \lambda e_j(\lambda)u(\lambda)d\lambda}{e_j} \tag{12}$$

For *Q*-optimal premiums, system is financially balanced, that is stationary premium equals expected claim rate for the portfolio (and equals μ for IG risk structure function)

$$\sum_{j=1}^{s} e_j b_j = E\Lambda = EK = \mu \tag{13}$$

[1] Penalty is understood in terms of transition to a worse class, we do not address premiums yet.

CHARACTERISTICS OF A BONUS-MALUS SYSTEM

In order to monitor performance of bonus-malus systems we use characteristics which describe quality of particular system over different dimensions (different aspects).

Stationary premium [Loimaranta 1972]

$$b_e = \sum_{j=1}^{s} e_j b_j \tag{14}$$

shows expected premium after sufficient number of periods. Can be interpreted as average income from one policy in stationary state. It is further used in many other measures of quality of BMS.

Volatility coefficient of the stationary premium [Lemaire 1985, 1995]

$$V_{b_e} = \frac{\sqrt{\sum_{j=1}^{s}\left(b_j - b_e\right)^2 e_j}}{b_e} \tag{15}$$

shows how on average stationary premium differs for randomly chosen policyholder. Can be interpreted as a measure of financial toughness of BMS. Higher values show that relatively high part of the risk is transferred to policyholder. Low values show relatively low system ability to risk differentiation. Some authors [Lemaire, Zi 1994] indicate that values higher than 1 can be hard to accept by customers. To compromise, most preferred values are close to 1.

RSAL – Relative stationary average level [Lemaire 1985, 1995]

$$RSAL = \frac{b_e - b_1}{b_s - b_1} \tag{16}$$

takes values from 0 to 1 and indicates position of stationary premium over the distance between the lowest and the highest possible premium. Values closer to 0 suggest clustering of policyholders in better (cheaper) classes.

Elasticity of the stationary premium [Loimaranta 1972]

$$\eta(\lambda) = \frac{\partial b_e}{b_e} \Big/ \frac{\partial \lambda}{\lambda} = \frac{\partial b_e}{\partial \lambda} \frac{\lambda}{b_e} \tag{17}$$

also called point elasticity, shows reaction of stationary premium for the change of claim rate λ. Namely, 1% change in claim rate is associated with $\eta(\lambda)\%$ change in stationary premium. Ideal value of elasticity is 1, which means that stationary premium reacts exactly proportionally for the change in the risk.

Global elasticity of the stationary premium [De Pril 1978]

$$\eta = \int_0^{\infty} \eta(\lambda) u(\lambda) d\lambda \tag{18}$$

can be interpreted as portfolio elasticity, that is elasticity weighted by the risk structure function.

As for majority of systems global elasticity takes values much lower than one, global elasticity becomes main point of our interest and farther on we try to arrange transition rules of bonus-malus system in the way that would lift up global elasticity.

Measure of goodness of risk assessment [Topolewski & Bernardelli 2015]

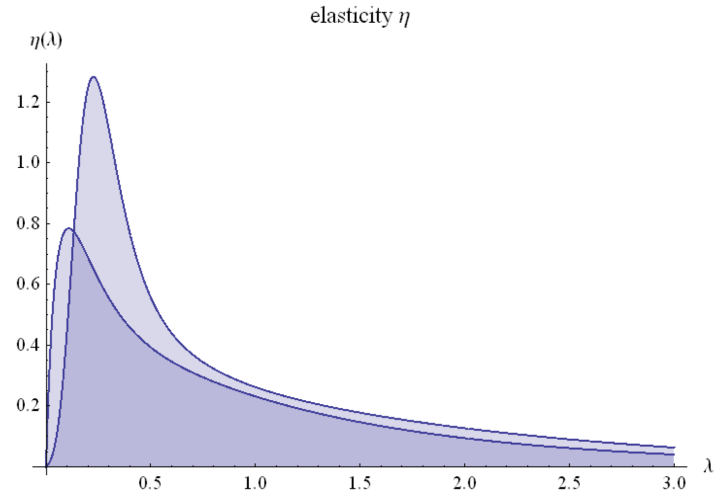
$$QN = \frac{\sum_{j=1}^{s} e_j b_j^2 - E^2\Lambda}{E\Lambda^2 - E^2\Lambda} \tag{19}$$

This measure is adequate only for systems with premiums given by (20). It is normalized measure that takes values from 0 to 1 and shows goodness of risk assessment of system with Norberg premiums for particular portfolio given by risk structure function. Values closer to 1 reflect better fit.

RESEARCH

Simple maximization of global elasticity may lead to spurious results, as lifting up its value may be achieved by lifting point elasticity too high, that is producing system which overreacts. Though we may get system with higher global elasticity, its point elasticity may be too high and we only substitute one imperfection with another. see figure 1.

Figure 2. Examples of too low and too high elasticity of stationary premium with respect to claim ratio λ



Source: own preparation

To overcome this problem, we propose optimization criteria that allow to keep global elasticity of BMS possibly close to one, namely we minimize distance between $\eta(\lambda)$ and 1, weighted by risk structure function.

Our objective function becomes (respectively):

- to maximize global elasticity η (typical approach – may give overreacting system)

$$\eta = \int_0^\infty \eta(\lambda)u(\lambda)d\lambda \rightarrow max \tag{I}$$

- to minimize mean absolute error (mean absolute distance from 1)

$$MAE = \int_0^\infty |1 - \eta(\lambda)|u(\lambda)d\lambda \rightarrow min \tag{II}$$

- to minimize root square mean error (root mean square distance from 1)

$$RMSE = \sqrt{\int_0^\infty [1 - \eta(\lambda)]^2 u(\lambda)d\lambda} \rightarrow min \tag{III}$$

Optimization of transition rules with respect to above functions is nonlinear and nonconvex discrete optimization problem. To solve this problem, we have to use adequate algorithm.

The algorithm

We use greedy algorithm similar to one used by [Morlock 1985] but with some alteration:

- We consider stationary state (stationary distribution)
- We impose weak monotonicity conditions, both in rows and in columns in the table of bonus-malus system (permissible systems)
- We limit ourselves to irreducible and ergodic systems
- We use different directions of optimization (rows, columns, diagonals)

Subsequently for each element t_{ik} of transition matrix $\boldsymbol{T}$ we change its value (taking into account the conditions for irreducibility, ergodicity and monotonicity of the system), for each value of t_{ik} we calculate premiums $\boldsymbol{b}$ and global elasticity and we choose t_{ik} which optimizes global elasticity. After optimization of all elements of $\boldsymbol{T}$ matrix procedure is repeated and we compare the results with the previous iteration. If in two subsequent iterative steps algorithm shows the same solution, we stop the procedure. We apply above algorithm in three ways, changing values of t_{ik} elements in rows, columns and by 'diagonals' starting from different initial systems (different $\boldsymbol{T}$ matrices). Solutions may differ – this is a greedy algorithm and may not always give globally optimal solution for each way.

Portfolios

We study systems of 10 classes that count up to 3 claims (more than 3 is treated as 3) and operate on different portfolios. Portfolios differ by parameters of risk structure function, IG(μ, θ), to screen portfolios with low and high claim rate and claim variance. We designate nine portfolios (nine sets of parameters) that reflect portfolios which can be meet in practice. Values of parameters have been chosen in the way, that they are close to parameters of claims distributions from Willmot 1987. Parameters of portfolios can be seen in Table 1.

Table 1. Portfolios characterised by parameters

Portfolio 1	Portfolio 2	Portfolio 3
μ = 0.05 θ = 0.01	μ = 0.05 θ = 0.05	μ = 0.05 θ = 0.15

Portfolio 4	Portfolio 5	Portfolio 6
μ = 0.15 θ = 0.01	μ = 0.15 θ = 0.05	μ = 0.15 θ = 0.15

Portfolio 7	Portfolio 8	Portfolio 9
μ = 0.30 θ = 0.01	μ = 0.30 θ = 0.05	μ = 0.30 θ = 0.15

Source: own preparation

It is worth notice, that majority of real portfolios would be more like portfolios 1 to 6 from Table 1 (will have average claim rate closer to 0.05 – 0.15), than like portfolios 7 to 9 (having very high average claim rate 0.3). But to have more complete portfolio review we decided to include also high claim rate portfolios.

RESULTS

Transition rules of systems given by algorithm as optimal for different portfolios and subsequent optimisation criteria are shown respectively in Tables 2, 3 and 4.

Table 2. Systems given by the algorithm as optimal by criterion (I) $\eta \to \max$

$\eta \to$ max											
Portfolio 1				Portfolio 2				Portfolio 3			
1	8	10	10	1	5	10	10	1	6	10	10
1	10	10	10	1	10	10	10	1	10	10	10
2	10	10	10	2	10	10	10	2	10	10	10
3	10	10	10	3	10	10	10	3	10	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 4				Portfolio 5				Portfolio 6			
1	9	10	10	1	5	9	10	1	3	7	9
1	10	10	10	1	9	10	10	1	7	9	10
2	10	10	10	2	9	10	10	2	7	9	10
3	10	10	10	3	9	10	10	3	9	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 7				Portfolio 8				Portfolio 9			
1	9	10	10	1	5	9	10	1	2	7	9
1	10	10	10	1	9	10	10	1	7	9	9
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	9	10
5	10	10	10	5	10	10	10	5	9	10	10
6	10	10	10	6	10	10	10	6	9	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10

Source: own preparation

Table 3. Systems given by the algorithm as optimal by criterion (II) MAE → min

MAE → min											
Portfolio 1				Portfolio 2				Portfolio 3			
1	8	10	10	1	5	10	10	1	6	10	10
1	10	10	10	1	10	10	10	1	10	10	10
2	10	10	10	2	10	10	10	2	10	10	10
3	10	10	10	3	10	10	10	3	10	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 4				Portfolio 5				Portfolio 6			
1	9	10	10	1	5	9	10	1	3	7	9
1	10	10	10	1	9	10	10	1	7	9	10
2	10	10	10	2	9	10	10	2	7	9	10
3	10	10	10	3	9	10	10	3	9	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 7				Portfolio 8				Portfolio 9			
1	10	10	10	1	6	10	10	1	4	9	10
1	10	10	10	1	10	10	10	1	9	9	10
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	9	10
5	10	10	10	5	10	10	10	5	9	9	10
6	10	10	10	6	10	10	10	6	9	9	10
7	10	10	10	7	10	10	10	7	9	10	10
8	10	10	10	8	10	10	10	8	9	10	10
9	10	10	10	9	10	10	10	9	10	10	10

Source: own preparation

Table 4. Systems given by the algorithm as optimal by criterion (III) RMSE → min

RMSE → min											
Portfolio 1				Portfolio 2				Portfolio 3			
1	10	10	10	1	6	10	10	1	6	10	10
1	10	10	10	1	10	10	10	1	10	10	10
2	10	10	10	2	10	10	10	2	10	10	10
3	10	10	10	3	10	10	10	3	10	10	10
4	10	10	10	4	10	10	10	4	10	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 4				Portfolio 5				Portfolio 6			
1	10	10	10	1	8	10	10	1	4	9	9
1	10	10	10	1	10	10	10	1	9	9	10
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	10	10
5	10	10	10	5	10	10	10	5	10	10	10
6	10	10	10	6	10	10	10	6	10	10	10
7	10	10	10	7	10	10	10	7	10	10	10
8	10	10	10	8	10	10	10	8	10	10	10
9	10	10	10	9	10	10	10	9	10	10	10
Portfolio 7				Portfolio 8				Portfolio 9			
1	10	10	10	1	9	10	10	1	4	9	10
1	10	10	10	1	10	10	10	1	9	9	10
2	10	10	10	2	10	10	10	2	9	9	10
3	10	10	10	3	10	10	10	3	9	9	10
4	10	10	10	4	10	10	10	4	9	9	10
5	10	10	10	5	10	10	10	5	9	9	10
6	10	10	10	6	10	10	10	6	9	9	10
7	10	10	10	7	10	10	10	7	9	10	10
8	10	10	10	8	10	10	10	8	9	10	10
9	10	10	10	9	10	10	10	9	10	10	10

Source: own preparation

We can observe that for majority of portfolios optimal systems are rather tough in terms of rules (sending policyholder to the worst or almost worst class for any reported claim).

To monitor properties of systems given as optimal for subsequent optimization criteria we calculate system's characteristics which are given in Tables 5, 6, 7. Systems are ranked according to the values of underling criterion.

Table 5. Ranking of optimal systems by criterion (I) η → max

Portfolio	μ	θ	*QN*	V_{be}	*RSAL*	***η***	*ME*	*MAE*	*RMSE*
8	0.3	0.05	0.321419	1.388710	0.201847	**0.599605**	0.400395	0.430068	0.528553
7	0.3	0.01	0.201232	2.457020	0.106629	**0.596285**	0.403715	0.438937	0.533067
9	0.3	0.15	0.466695	0.966121	0.226400	**0.576430**	0.423570	0.479767	0.574299
4	0.15	0.01	0.309714	2.155390	0.100166	**0.510879**	0.489121	0.489121	0.565899
5	0.15	0.05	0.427321	1.132240	0.164996	**0.487513**	0.512487	0.512487	0.590434
6	0.15	0.15	0.450998	0.671564	0.191713	**0.426207**	0.573793	0.573793	0.634645
1	0.05	0.01	0.389539	1.395600	0.083622	**0.355104**	0.644896	0.644896	0.687815
2	0.05	0.05	0.269839	0.519460	0.108728	**0.230867**	0.769133	0.769133	0.786754
3	0.05	0.15	0.117212	0.197663	0.135810	**0.112706**	0.887294	0.887294	0.889157

Source: own preparation

Table 6. Ranking of optimal systems by criterion (II) MAE → min

Portfolio	μ	θ	*QN*	V_{be}	*RSAL*	*η*	*ME*	***MAE***	*RMSE*
7	0.3	0.01	0.170241	2.259920	0.124114	0.594975	0.405025	**0.405025**	0.452337
8	0.3	0.05	0.307132	1.357500	0.210503	0.595962	0.404038	**0.407535**	0.506089
9	0.3	0.15	0.515548	1.015430	0.170585	0.561343	0.438657	**0.438657**	0.483195
4	0.15	0.01	0.309714	2.155390	0.100166	0.510879	0.489121	**0.489121**	0.565899
5	0.15	0.05	0.427321	1.132240	0.164996	0.487513	0.512487	**0.512487**	0.590434
6	0.15	0.15	0.450998	0.671564	0.191713	0.426207	0.573793	**0.573793**	0.634645
1	0.05	0.01	0.389539	1.39560	0.0836215	0.355104	0.644896	**0.644896**	0.687815
2	0.05	0.05	0.269839	0.51946	0.1087280	0.230867	0.769133	**0.769133**	0.786754
3	0.05	0.15	0.117212	0.197663	0.1358100	0.112706	0.887294	**0.887294**	0.889157

Source: own preparation

Table 7. Ranking of optimal systems by criterion (III) RMSE → min

Portfolio	μ	θ	*QN*	V_{be}	*RSAL*	η	*ME*	*MAE*	***RMSE***
7	0.3	0.01	0.170241	2.259920	0.124114	0.594975	0.405025	0.405025	**0.452337**
9	0.3	0.15	0.515548	1.015430	0.170585	0.561343	0.438657	0.438657	**0.483195**
8	0.3	0.05	0.291295	1.322030	0.215832	0.559857	0.440143	0.440143	**0.485149**
4	0.15	0.01	0.254498	1.953830	0.121405	0.510505	0.489495	0.489495	**0.524757**
5	0.15	0.05	0.355297	1.032420	0.198442	0.462336	0.537664	0.537664	**0.568946**
6	0.15	0.15	0.401352	0.633524	0.209115	0.414519	0.585481	0.585481	**0.617525**
1	0.05	0.01	0.285054	1.193850	0.125651	0.347099	0.652901	0.652901	**0.672276**
2	0.05	0.05	0.25877	0.508695	0.118548	0.230662	0.769338	0.769338	**0.783442**
3	0.05	0.15	0.117212	0.197663	0.135810	0.112706	0.887294	0.887294	**0.889157**

Source: own preparation

Analysing Tables 5, 6, 7 we can clearly see that for portfolios with low and medium claim rate (portfolio 1 to portfolio 6) the highest possible level of premium elasticity is rather low for any optimization criterion.

CONCLUSIONS

- Level of global elasticity depends on portfolio (claim rate and claim variance).
- For portfolios with low and medium claim rate (most typical portfolios) we have lower values of global elasticity.
- Systems optimal in sense of elasticity are tough in terms of transition rules
- For particular claim rate, systems with high elasticity are financially tough – rather high volatility coefficient V_{b_e}
- For particular claim rate, high elasticity tends to go together with concentration in better classes – low RSAL
- Better elasticity does not go together with better risk assessment – see QN

It is easier to achieve higher elasticity (for any optimization criterion) for portfolios with higher claim rate, but high elasticity generally requires a tough system. Considering other characteristics of BMS, optimization of elasticity does not make them necessarily better.

For most typical portfolios elasticity has rather low level and for majority of portfolios policyholders tend to cluster in better (cheaper) classes.

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