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# The energy of interval valued neutrosophic matrix in decision-making to select the manager for the company project

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**Abstract**

The concept of energy in graphs and matrices is used effectively in all application areas. The energy of the matrix is an extension of graph energy. The usage of the energy idea in neutrosophic matrices makes it more flexible and applicable in multi-criteria decision-making environments. In this paper, we propose the energy approach in neutrosophic matrices with interval values. We determined the given energy's upper and lower bounds. The energy is used of the interval-valued neutrosophic matrix to address the MCDM problem. A new strategy has been introduced called the interval-valued neutrosophic energy method to solve this problem. We look at the problem of choosing a qualified manager for a business project. A team of professionals in the company evaluates the options using neutrosophic numbers with interval values, and the energy method is then used to calculate the result. The result has been compared with the TOPSIS method results to show that the outcomes are similar.

**Keywords:** *neutrosophic set, matrix energy, interval-valued neutrosophic matrix, multi-criteria decision-making*

## 1. Introduction

In 1978, Gutman [11] was the first to bring out the idea of energy. It is described as the sum of the absolute values of the eigenvalues of the adjacency matrix of a graph. The energy of the graph is extended to uncertain surroundings. In 2013, Mathew and Anjali [1] introduced a fuzzy graph's energy. In an intuitionistic fuzzy graph, the concept of energy was created by Praba and Deepa [20]. In this study, they defined the adjacency matrix, the energy, and the bounds of an intuitionistic fuzzy graph. The matrix's energy equation was developed by DiStefano et al. The energy of a matrix is a generalization of the energy of a graph. Nikiforov [18] published the concept of energy in graphs and matrices. Then Bravoa et al. [4] presented a study on energy of matrices.

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Fuzzy sets and fuzzy logic were introduced by Zadeh [28] in 1965. The intuitionistic fuzzy set was proposed by Atanassov [2] in 1986. In 1998, Smarandache [23] introduced the neutrosophic set. Neutrosophic sets have truth, indeterminacy, and false as their membership functions. Smarandache and Kandasamy [12] introduced fuzzy and neutrosophic relational maps. They also added square neutrosophic matrices to this and developed the neutrosophic matrix and related algebraic operations [7]. Wang et al. [27] along with many other properties, functions, and relationships of interval neutrosophic sets, demonstrate the compactness of interval-valued neutrosophic sets. In 2014, Kharal [14] presented a method of MCDM based on neutrosophic sets. Neutrosophic sets are being offered to the MCDM community for the first time. Novel operators on interval-valued neutrosophic sets have been presented by Saha and Broumi [22]. They defined a few newer IVNS operators and looked at their characteristics. Mao et al. [15] proposed a neutrosophic-based method in data envelopment research with inappropriate outcomes. The suggested method has a basic construction and is focused on the aggregation operator. In 2020, Das [6] presented neutrosophic fuzzy matrices and several algebraic functions, defining some their properties. Vidhya et al. [26] defined interval-valued neutrosophic fuzzy matrix and its characteristics. Karaaslan [13] defined the determinant and adjoint of the interval-valued neutrosophic matrices based on the permanent function. Martin et al. [16] presented a new pithogenic sub-cognitive mapping technique with mediating effects of elements in the COVID-19 treatment model. This new strategy is more practical since it considers the mediating effects as well as the degree of element contradiction. Polymenis [19] provided an approach for conducting a neutrosophic student's t-type statistical test that concerns the population means. Veeramani et al. [25] used the decision-making trial and evaluation laboratory (DEMATEL) approach in a neutrosophic area to determine the relative significance of the financial ratios of two categories, accounting-based financial measures (AFM) and economic value-based financial measures (EFM). Edalatpanah [10] provided the introduction to the neutrosophic and plithogenic sets for science and engineering: theory, models, and applications in 2023. In the same year, Stanimirovic [24] suggested examining ways to improve line search techniques for dealing with unrestricted nonlinear optimization models.

The TOPSIS approach using interval-valued neutrosophic sets for employee selection was introduced by Vu Dung et al. [9]. Chou et al. [5] proposed interval-valued neutrosophic sets to create a multi-criteria decision-making strategy for renewable energy selection. Using the connectivity of the analytic hierarchical process method with the TOPSIS method under a neutrosophic environment, this study suggests an extension of the MCDM technique to assess renewable energy sources. Ramesh et al. [21] applied the TOPSIS method to group decision-making situations and computed the signless Laplacian energy of an intuitionistic fuzzy graph. Zavadskas et al. [29] This work introduces a new multi-criteria decision-making technique, MULTIMOORA, in multi-objective optimization by ratio analysis under interval-valued neutrosophic sets. Interval-valued neutrosophic AHP with potential degree method was worked by Bolturk et al. [3]. In this study, we use the representation power of the neutrosophic set to rank interval numbers using the potential degree method in our neutrosophic AHP methodology. In 2022, Deepa and Jeni [17] presented operations on multi-valued neutrosophic matrices and their application to the neutrosophic simplified TOPSIS method. They also described the determinant, adjoint, and different operations of the multi-valued neutrosophic fuzzy matrix. The neutrosophic, simplified TOPSIS approach was used to numerically illustrate the use of the proposed matrices.

In this paper, we defined the energy of the interval-valued neutrosophic matrix and its application to the MCDM problem. In Section 2, the basic definitions were given. Section 3 presents the energy of an interval-valued neutrosophic matrix, and some propositions were given. In Section 4, we proposed a new method for solving the MCDM problem using our definition. The proposed method is numerically demonstrated in Section 5. We compared our method to the TOPSIS method in Section 6. Results and discussions are given in Section 7. Finally, the conclusion was given.

## 2. Basic definitions

**Definition 1.** Energy of matrix [1]. Let $M_n(\mathbb{C})$ denote the space of $n \times n$ matrices with entries in $\mathbb{C}$ and $P$ be a matrix in $M_n(\mathbb{C})$. Then the energy of matrix $P$ is defined as

$$E(P) = \sum_{i=1}^{n} |\lambda_i - \mu| \tag{1}$$

where, $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the eigenvalues of $P$ and $\mu$ is the mean of eigenvalues. If $\mu = 0$ or $P$ is the adjacency matrix of graph $G$, then $E(P)$ is precisely the energy of the graph $G$.

**Definition 2.** Neutrosophic set $(NS)$ [6]. Let $U$ be the universal set and every element $a \in U$ has a degree of truth, indeterminacy, and falsity membership in a neutrosophic set. It is denoted by $S$. Then it can be defined as

$$S = \{\langle a, T_S(a), I_S(a), F_S(a)\rangle, \quad a \in U\} \tag{2}$$

where, $0 \leq T_S(a) + I_S(a) + F_S(a) \leq 3$ and $T_S$ is the truth membership function, $I_S$ is the indeterminacy membership function, $F_S$ is the false membership function, every function lies between [0, 1] in U.

**Definition 3.** Neutrosophic fuzzy matrix [7]. A neutrosophic fuzzy matrix $P$ of the order $m \times n$ is defined as

$$P = [\langle T_{ijp}, I_{ijp}, F_{ijp}\rangle]_{m\times n} \tag{3}$$

where, $T_{ijp},\ I_{ijp},\ F_{ijp}$ are called truth, indeterminacy and false membership of $ij$th in $P$, which satisfying the condition $0 \leq T_{ijp} + I_{ijp} + F_{ijp} \leq 3$. For simplicity, we write $[P_{ij}]_{m\times n}$. where $P_{ij} = \langle T_{ijp},\ I_{ijp},\ F_{ijp}\rangle$.

**Definition 4.** Interval-valued neutrosophic set [27]. Let $U$ be a nonempty set with generic elements in $U$ denoted by a. The interval-valued neutrosophic set A in $U$ is as follows

$$A = \{a : \langle a, T_A(a), I_A(a), F_A(a)\rangle ; a \in U\} \tag{4}$$

where interval truth membership function $T_A(a) = [T_A^L,\ T_A^U]$, interval indeterminacy membership function $I_A(a) = [I_A^L,\ I_A^U]$, interval false membership function $F_A(a) = [F_A^L,\ F_A^U]$ for each point $a \in U$ and $T_A(a),\ I_A(a),\ F_A(a) \in [0,\ 1]$

**Definition 5.** Interval-valued neutrosophic fuzzy matrix [26]. An interval-valued neutrosophic fuzzy matrix (IVNFM) $Q$ of order $m \times n$ is defined as

$$Q = [\langle T_{ijq},\ I_{ijq},\ F_{ijq}\rangle]_{m\times n} \tag{5}$$

where, $T_{ijq}$, $I_{ijq}$, and $F_{ijq}$ are truth, indeterminacy, and false membership elements which are the subset of [0, 1]. It is denoted by $T_{ijq} = [T^L_{ijq}, T^U_{ijq}]$, $I_{ijq} = [I^L_{ijq}, I^U_{ijq}]$ and $F_{ijq} = [F^L_{ijq}, F^U_{ijq}]$ with the condition $0 \leq T^L_{ijq} + I^L_{ijq} + F^L_{ijq} \leq 3$ and $0 \leq T^U_{ijq} + I^U_{ijq} + F^U_{ijq} \leq 3$ for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

**Definition 6.** Energy of neutrosophic matrix [8]. Let $P(N)$ be the square neutrosophic matrix. It can be expressed as truth, indeterminacy and false matrices, which contain the elements of truth membership values $a_{ij}$, indeterminacy membership values $b_{ij}$, and false membership values $c_{ij}$. It is denoted as $P(N) = \langle P(T_{ij}), P(I_{ij}), P(F_{ij})\rangle_{n\times n}$ and $a_{ij} \in P(T_{ij})_{n\times n}$, $b_{ij} \in P(I_{ij})_{n\times n}$ and $c_{ij} \in P(F_{ij})_{n\times n}$.

The neutrosophic matrix's energy is defined as

$$E[P(N)] = \langle E[P(T_{ij})], \quad E[P(I_{ij})], \quad E[P(F_{ij})]\rangle$$

$$E[P(N)] = \left\langle \sum_{i=1}^{n} |\lambda_i - \mu_\lambda|, \sum_{i=1}^{n} |\zeta_i - \mu_\zeta|, \sum_{i=1}^{n} |\eta_i - \mu_\eta| \right\rangle \tag{6}$$

where $\lambda_i$, $\zeta_i$ and $\eta_i$, $i = 1, 2, \ldots, n)$, are the eigenvalues of truth, indeterminacy, and false membership values, respectively, and $\mu_\lambda, \mu_\zeta$, and $\mu_\eta$ are the mean values of $\lambda_i$, $\zeta_i$ and $\eta_i$, respectively.

# 3. The energy of interval-valued neutrosophic matrix

This section introduces the matrix's energy of the interval-valued neutrosophic structure. The values of every element of the matrix are neutrosophic interval values. In some situations, single values are not reliable; in these cases, interval values are a better way to predict the outcomes. It would be more practical and convenient to evaluate the issue as compared to other structures. So, in this section, we define the interval-valued neutrosophic matrix energy as well as its lower and upper bounds.

Let $Q$ be the interval-valued neutrosophic matrix with $n \times n$ order. It is defined as

$$Q = \left[\left\langle (T^L_{ijq}, T^U_{ijq}), (I^L_{ijq}, I^U_{ijq}), (F^L_{ijq}, F^U_{ijq}) \right\rangle\right]_{n\times n} \tag{7}$$

It can be expressed as six matrices, the first two contain the elements of lower and upper limits of truth values,the second two matrices contain the elements of lower and upper limits of indeterminacy values and the last two matrices contain the elements of lower and upper limits of false values

$$\left[(a^L_{ij}, a^U_{ij}), (b^L_{ij}, b^U_{ij}), (c^L_{ij}, c^U_{ij})\right]$$

where $a^L_{ij} \in T^L_{ijq}$, $a^U_{ij} \in T^U_{ijq}$, $b^L_{ij} \in I^L_{ijq}$, $b^U_{ij} \in I^U_{ijq}$, $c^L_{ij} \in F^L_{ijq}$, and $c^U_{ij} \in F^U_{ijq}$.

Then the energy of the interval-valued neutrosophic matrix is defined as

$$E[Q] = \left[(E[T^L_{ijq}], E[T^U_{ijq}]), (E[I^L_{ijq}], E[I^U_{ijq}]), (E[F^L_{ijq}], E[F^U_{ijq}])\right]$$

$$E[Q] = \left[ \left( \sum_{i=1}^{n} \left| \lambda_i^L - \mu_{\lambda^L} \right|, \sum_{i=1}^{n} \left| \lambda_i^U - \mu_{\lambda^U} \right| \right), \left( \sum_{i=1}^{n} \left| \zeta_i^L - \mu_{\zeta^L} \right|, \sum_{i=1}^{n} \left| \zeta_i^U - \mu_{\zeta^U} \right| \right), \right.$$
$$\left. \left( \sum_{i=1}^{n} \left| \eta_i^L - \mu_{\eta^L} \right|, \sum_{i=1}^{n} \left| \eta_i^U - \mu_{\eta^U} \right| \right) \right] \tag{8}$$

where $\lambda_i^L$, $\lambda_i^U$, $\zeta_i^L$, $\zeta_i^U$, $\eta_i^L$, and $\eta_i^U$ are the eigenvalues of lower and upper limit values of truth, indeterminacy and false matrices. $\mu_{\lambda^L}$, $\mu_{\lambda^U}$, $\mu_{\zeta^L}$, $\mu_{\zeta^U}$, $\mu_{\eta^L}$, and $\mu_{\eta^U}$ are the mean values of the respected eigenvalues.

**Example**. Let $Q$ be the interval-valued neutrosophic matrix with $3 \times 3$ order.

$$Q = \begin{pmatrix} \langle(.1,.2),(.5,.6),(.8,.9)\rangle & \langle(.3,.4),(.5,.7),(.7,.8)\rangle & \langle(.6,.7),(.3,.4),(.2,.3)\rangle \\ \langle(.3,.5),(.8,.9),(.1,.2)\rangle & \langle(.4,.5),(.6,.7),(.3,.4)\rangle & \langle(.1,.2),(.3,.5),(.4,.6)\rangle \\ \langle(.7,.8),(.1,.2),(.4,.5)\rangle & \langle(.5,.6),(.1,.3),(.5,.7)\rangle & \langle(.4,.5),(.2,.4),(0.8,0.9)\rangle \end{pmatrix}$$

$$T_{ijq}^L = \begin{pmatrix} 0.1 & 0.3 & 0.6 \\ 0.3 & 0.4 & 0.1 \\ 0.7 & 0.5 & 0.4 \end{pmatrix} \qquad I_{ijq}^L = \begin{pmatrix} 0.5 & 0.5 & 0.3 \\ 0.8 & 0.6 & 0.3 \\ 0.1 & 0.1 & 0.2 \end{pmatrix} \qquad F_{ijq}^L = \begin{pmatrix} 0.8 & 0.7 & 0.2 \\ 0.1 & 0.3 & 0.4 \\ 0.4 & 0.5 & 0.8 \end{pmatrix}$$

$$T_{ijq}^U = \begin{pmatrix} 0.2 & 0.4 & 0.7 \\ 0.5 & 0.5 & 0.2 \\ 0.8 & 0.6 & 0.5 \end{pmatrix} \qquad I_{ijq}^U = \begin{pmatrix} 0.6 & 0.7 & 0.4 \\ 0.9 & 0.7 & 0.5 \\ 0.2 & 0.3 & 0.4 \end{pmatrix} \qquad F_{ijq}^U = \begin{pmatrix} 0.9 & 0.8 & 0.3 \\ 0.2 & 0.4 & 0.6 \\ 0.5 & 0.7 & 0.9 \end{pmatrix}$$

The eigenvalues of lower truth matrix $\lambda_i^L = 1.1203, -0.4023, 0.1820$ and mean $\mu_{\lambda^L} = 3$

$$E[T_{ijq}] = |1.1203 - 3| + |-0.4023 - 3| + |0.1820 - 3| = 1.6406$$

The other energy values are calculated in the same way.

$$E[Q] = [(1.6406, 2.1148), (1.6186, 2.1295), (1.4961, 2.0352)]$$

**Theorem 1.** Let $Q$ be the interval-valued neutrosophic matrix. If $\lambda_i^L$, $\lambda_i^U$, $\zeta_i^L$, $\zeta_i^U$, $\eta_i^L$, and $\eta_i^U$, $i = 1, 2, \ldots, n$ are the eigenvalues of lower and upper limits of truth $T_{ijq}^L$, $T_{ijq}^U$, indeterminacy $I_{ijq}^L$, $I_{ijq}^U$ and false $F_{ijq}^L$, $F_{ijq}^U$ membership values respectively. The eigenvalues are real or complex. Then,

$$\sum_{i=1}^{n}(\lambda_i^L - \mu_{\lambda^L}) = \sum_{i=1}^{n}(a_{ii}^L - \mu_{\lambda^L}) = \sum_{i=1}^{n}(\lambda_i^U - \mu_{\lambda^U}) = \sum_{i=1}^{n}(a_{ii}^U - \mu_{\lambda^U}) = 0$$
$$\sum_{i=1}^{n}(\zeta_i^L - \mu_{\zeta^L}) = \sum_{i=1}^{n}(b_{ii}^L - \mu_{\zeta^L}) = \sum_{i=1}^{n}(\zeta_i^U - \mu_{\zeta^U}) = \sum_{i=1}^{n}(b_{ii}^U - \mu_{\zeta^U}) = 0$$
$$\sum_{i=1}^{n}(\eta_i^L - \mu_{\eta^L}) = \sum_{i=1}^{n}(c_{ii}^L - \mu_{\eta^L}) = \sum_{i=1}^{n}(\eta_i^U - \mu_{\eta^U}) = \sum_{i=1}^{n}(c_{ii}^U - \mu_{\eta^U}) = 0$$

$$\sum_{i=1}^{n}(\lambda_i^L - \mu_{\lambda^L})^2 = \sum_{i=1}^{n} a_{ii}^{L^2} + 2\sum_{1\le i<j\le n} a_{ij}^L a_{ji}^L - n\mu_{\lambda^L}^2$$

$$\sum_{i=1}^{n}(\lambda_i^U - \mu_{\lambda^U})^2 = \sum_{i=1}^{n} a_{ii}^{U^2} + 2\sum_{1\le i<j\le n} a_{ij}^U a_{ji}^U - n\mu_{\lambda^U}^2$$

$$\sum_{i=1}^{n}(\zeta_i^L - \mu_{\zeta^L})^2 = \sum_{i=1}^{n} b_{ii}^{L^2} + 2\sum_{1\le i<j\le n} b_{ij}^L b_{ji}^L - n\mu_{\zeta^L}^2$$

$$\sum_{i=1}^{n}(\zeta_i^U - \mu_{\zeta^U})^2 = \sum_{i=1}^{n} b_{ii}^{U^2} + 2\sum_{1\le i<j\le n} b_{ij}^U b_{ji}^U - n\mu_{\zeta^U}^2$$

$$\sum_{i=1}^{n}(\eta_i^L - \mu_{\eta^L})^2 = \sum_{i=1}^{n} c_{ii}^{L^2} + 2\sum_{1\le i<j\le n} c_{ij}^L c_{ji}^L - n\mu_{\eta^L}^2$$

$$\sum_{i=1}^{n}(\eta_i^U - \mu_{\eta^U})^2 = \sum_{i=1}^{n} c_{ii}^{U^2} + 2\sum_{1\le i<j\le n} c_{ij}^U c_{ji}^U - n\mu_{\eta^U}^2$$

where $a_{ii}^L$, $a_{ii}^U$, $b_{ii}^L$, $b_{ii}^U$, $c_{ii}^L$, and $c_{ii}^U$ are the diagonal entries of truth $T_{ijq}^L$, $T_{ijq}^U$, indeterminacy $I_{ijq}^L$, $I_{ijq}^U$ and false $F_{ijq}^L$, $F_{ijq}^U$ matrices respectively.

**Theorem 2.** Let $Q = \left[\left\langle (T_{ijq}^L, T_{ijq}^U), (I_{ijq}^L, I_{ijq}^U), (F_{ijq}^L, F_{ijq}^U) \right\rangle\right)$ be the interval-valued neutrosophic matrix. Then,

$$\left(\left(\sum_{i=1}^{n}\left|\lambda_i^L - \mu_{\lambda^L}\right|\right)^2 - 2\sum_{1\le i<j\le n}\left|\lambda_i^L - \mu_{\lambda^L}\right|\left|\lambda_j^L - \mu_{\lambda^L}\right| + n(n-1)\left(\left|Q - \mu_{\lambda^L}\right|\right)^{2/n}\right)^{1/2}$$
$$\le E(T_{ijq}^L) \le \left(n\left(\left(\sum_{i=1}^{n}\left|\lambda_i^L - \mu_{\lambda^L}\right|\right)^2 - 2\sum_{1\le i<j\le n}\left|\lambda_i^L - \mu_{\lambda^L}\right|\left|\lambda_j^L - \mu_{\lambda^L}\right|\right)\right)^{1/2}$$

$$\left(\left(\sum_{i=1}^{n}\left|\lambda_i^U - \mu_{\lambda^U}\right|\right)^2 - 2\sum_{1\le i<j\le n}\left|\lambda_i^U - \mu_{\lambda^U}\right|\left|\lambda_j^U - \mu_{\lambda^U}\right| + n(n-1)\left(\left|Q - \mu_{\lambda^U}\right|\right)^{2/n}\right)^{1/2}$$
$$\le E(T_{ijq}^U) \le \left(n\left(\left(\sum_{i=1}^{n}\left|\lambda_i^U - \mu_{\lambda^U}\right|\right)^2 - 2\sum_{1\le i<j\le n}\left|\lambda_i^U - \mu_{\lambda^U}\right|\left|\lambda_j^U - \mu_{\lambda^U}\right|\right)\right)^{1/2}$$

$$\left(\left(\sum_{i=1}^{n}\left|\zeta_i^L-\mu_{\zeta^L}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\zeta_i^L-\mu_{\zeta^L}\right|\left|\zeta_j^L-\mu_{\zeta^L}\right|+n(n-1)\left(\left|Q-\mu_{\zeta^L}\right|\right)^{2/n}\right)^{1/2}$$

$$\le E(I_{ijq}^L)\le\left(n\left(\left(\sum_{i=1}^{n}\left|\zeta_i^L-\mu_{\zeta^L}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\zeta_i^L-\mu_{\zeta^L}\right|\left|\zeta_j^L-\mu_{\zeta^L}\right|\right)\right)^{1/2}$$

$$\left(\left(\sum_{i=1}^{n}\left|\zeta_i^U-\mu_{\zeta^U}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\zeta_i^U-\mu_{\zeta^U}\right|\left|\zeta_j^U-\mu_{\zeta^U}\right|+n(n-1)\left(\left|Q-\mu_{\zeta^U}\right|\right)^{2/n}\right)^{1/2}$$

$$\le E(I_{ijq}^U)\le\left(n\left(\left(\sum_{i=1}^{n}\left|\zeta_i^U-\mu_{\zeta^U}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\zeta_i^U-\mu_{\zeta^U}\right|\left|\zeta_j^U-\mu_{\zeta^U}\right|\right)\right)^{1/2}$$

$$\left(\left(\sum_{i=1}^{n}\left|\eta_i^L-\mu_{\eta^L}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\eta_i^L-\mu_{\eta^L}\right|\left|\eta_j^L-\mu_{\eta^L}\right|+n(n-1)\left(\left|Q-\mu_{\eta^L}\right|\right)^{2/n}\right)^{1/2}$$

$$\le E(F_{ijq}^L)\le\left(n\left(\left(\sum_{i=1}^{n}\left|\eta_i^L-\mu_{\eta^L}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\eta_i^L-\mu_{\eta^L}\right|\left|\eta_j^L-\mu_{\eta^L}\right|\right)\right)^{1/2}$$

$$\left(\left(\sum_{i=1}^{n}\left|\eta_i^U-\mu_{\eta^U}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\eta_i^U-\mu_{\eta^U}\right|\left|\eta_j^U-\mu_{\eta^U}\right|+n(n-1)\left(\left|Q-\mu_{\eta^U}\right|\right)^{2/n}\right)^{1/2}$$

$$\le E(F_{ijq}^U)\le\left(n\left(\left(\sum_{i=1}^{n}\left|\eta_i^U-\mu_{\eta^U}\right|\right)^2-2\sum_{1\le i<j\le n}\left|\eta_i^U-\mu_{\zeta^U}\right|\left|\eta_j^U-\mu_{\eta^U}\right|\right)\right)^{1/2}$$

# 4. The interval-valued neutrosophic energy method

In this part, we provide a new method for MCDM to select the best alternative using interval-valued neutrosophic matrix energy. Consider the set of r alternatives and $m$ criteria. A group of $n$ decision-makers examine the alternatives. So we set $DM = \{DM_1, DM_2, \ldots, DM_n\}$, $C = \{C_1, C_2, \ldots, C_m\}$ and $A = \{A_1, A_2, \ldots, A_r\}$

**Step 1.** Each decision-maker provided the weighted values of $m$ criteria and the rating values for each alternative on each criterion. We use a matrix to represent each alternative rating and weight value. The interval-valued neutrosophic numbers are used to express the rating values. So we got the neutrosophic matrix with interval values.

As a $m \times n$ matrix for weight $W$, consider the ratings of $m$ criteria provided by $n$ decision-makers.

$$W = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_m \end{array} \begin{array}{c} \begin{array}{ccc} DM_1 & \cdots & DM_n \end{array} \\ \begin{pmatrix} \left\langle(\alpha_{11}^L, \alpha_{11}^U), (\beta_{11}^L, \beta_{11}^U), (\gamma_{11}^L, \gamma_{11}^U)\right\rangle & \cdots & \left\langle(\alpha_{1n}^L, \alpha_{1n}^U), (\beta_{1n}^L, \beta_{1n}^U), (\gamma_{1n}^L, \gamma_{1n}^U)\right\rangle \\ \left\langle(\alpha_{21}^L, \alpha_{21}^U), (\beta_{21}^L, \beta_{21}^U), (\gamma_{21}^L, \gamma_{21}^U)\right\rangle & \cdots & \left\langle(\alpha_{2n}^L, \alpha_{2n}^U), (\beta_{2n}^L, \beta_{2n}^U), (\gamma_{2n}^L, \gamma_{2n}^U)\right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle(\alpha_{m1}^L, \alpha_{m1}^U), (\beta_{m1}^L, \beta_{m1}^U), (\gamma_{m1}^L, \gamma_{m1}^U)\right\rangle & \cdots & \left\langle(\alpha_{mn}^L, \alpha_{mn}^U), (\beta_{mn}^L, \beta_{mn}^U), (\gamma_{mn}^L, \gamma_{mn}^U)\right\rangle \end{pmatrix} \end{array}$$

As a $n \times m$ matrix for alternative $A_1$, consider the ratings provided by $n$ decision-makers for $m$ criteria.

$$A_1 = \begin{array}{c} \\ DM_1 \\ DM_2 \\ \vdots \\ DM_n \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_m \end{array} \\ \begin{pmatrix} \left\langle(a_{11}^L, a_{11}^U), (b_{11}^L, b_{11}^U), (c_{11}^L, c_{11}^U)\right\rangle & \cdots & \left\langle(a_{1m}^L, a_{1m}^U), (b_{1m}^L, b_{1m}^U), (c_{1m}^L, c_{1m}^U)\right\rangle \\ \left\langle(a_{21}^L, a_{21}^U), (b_{21}^L, b_{21}^U), (c_{21}^L, c_{21}^U)\right\rangle & \cdots & \left\langle(a_{2m}^L, a_{2m}^U), (b_{2m}^L, b_{2m}^U), (c_{2m}^L, c_{2m}^U)\right\rangle \\ \vdots & \ddots & \vdots \\ \left\langle(a_{n1}^L, a_{n1}^U), (b_{n1}^L, b_{n1}^U), (c_{n1}^L, c_{n1}^U)\right\rangle & \cdots & \left\langle(a_{nm}^L, a_{nm}^U), (b_{nm}^L, b_{nm}^U), (c_{nm}^L, c_{nm}^U)\right\rangle \end{pmatrix} \end{array}$$

**Step 2.** Calculate the weights of decision-makers for interval-valued neutrosophic numbers. The weights of each decision-maker can be evaluated using the formula below. The weight of $j$th decision-maker is

$$w_j = \frac{1 - \left(\left((1 - T^L(x))^2 + (1 - T^U(x))^2 + (I^L(x))^2 + (I^U(x))^2 + (F^L(x))^2 + (F^U(x))^2\right)/6\right)^{1/2}}{\sum_{i=1}^{n}\left(1 - \left(\{(1 - T^L(x))^2 + (1 - T^U(x))^2 + (I^L(x))^2 + (I^U(x))^2 + (F^L(x))^2 + (F^U(x))^2\}/6\right)^{1/2}\right)}$$

where $\sum_{j=1}^{n} w_j = 1$

**Step 3.** Aggregation of the weighted interval-valued neutrosophic decision matrix. The process multiplies each interval-valued neutrosophic element for each matrix by the weights of the respective decision-makers.

$$wW = \Big\langle (WT^L, WT^U), (WI^L, WI^U), (WF^L, WF^U) \Big\rangle$$
$$= \Big\langle (1-(1-T^L)^w, 1-(1-T^U)^w), ((I^L)^w, (I^U)^w), ((F^L)^w, (F^L)^w) \Big\rangle$$

$$wA_1 = \Big\langle (wT^L, wT^U), (wI^L, wI^U), (wF^L, wF^U) \Big\rangle$$
$$= \Big\langle (1-(1-T^L)^w, 1-(1-T^U)^w), ((I^L)^w, (I^U)^w), ((F^L)^w, (F^L)^w) \Big\rangle$$

Similarly, for the weighted matrix and alternative matrix, each element is multiplied by the weights of respective decision-makers.

$$W(T^L, T^U) = \begin{pmatrix} (1-(1-\alpha_{11}^L)^{w_1}, 1-(1-\alpha_{11}^U)^{w_1}) & \dots & (1-(1-\alpha_{1n}^L)^{w_n}, 1-(1-\alpha_{1n}^U)^{w_n}) \\ (1-(1-\alpha_{21}^L)^{w_1}, 1-(1-\alpha_{21}^U)^{w_1}) & \dots & (1-(1-\alpha_{2n}^L)^{w_n}, 1-(1-\alpha_{2n}^U)^{w_n}) \\ \vdots & \ddots & \vdots \\ (1-(1-\alpha_{n1}^L)^{w_1}, 1-(1-\alpha_{n1}^U)^{w_1}) & \dots & (1-(1-\alpha_{mn}^L)^{w_n}, 1-(1-\alpha_{mn}^U)^{w_n}) \end{pmatrix}$$

$$W(I^L, I^U) = \begin{pmatrix} ((\beta_{11}^L)^{w_1}, (\beta_{11}^U)^{w_1}) & ((\beta_{12}^L)^{w_2}, (\beta_{12}^U)^{w_2}) & \dots & ((\beta_{1n}^L)^{w_n}, (\beta_{1n}^U)^{w_n}) \\ ((\beta_{21}^L)^{w_1}, (\beta_{21}^U)^{w_1}) & ((\beta_{22}^L)^{w_2}, (\beta_{22}^U)^{w_2}) & \dots & ((\beta_{2n}^L)^{w_n}, (\beta_{2n}^U)^{w_n}) \\ \vdots & \vdots & \ddots & \vdots \\ ((\beta_{m1}^L)^{w_1}, (\beta_{m1}^U)^{w_1}) & ((\beta_{m2}^L)^{w_1}, (\beta_{m2}^U)^{w_2}) & \dots & ((\beta_{mn}^L)^{w_n}, (\beta_{mn}^U)^{w_n}) \end{pmatrix}$$

$$W(F^L, F^U) = \begin{pmatrix} ((\gamma_{11}^L)^{w_1}, (\gamma_{11}^U)^{w_1}) & ((\gamma_{12}^L)^{w_2}, (\gamma_{12}^U)^{w_2}) & \dots & ((\gamma_{1n}^L)^{w_n}, (\gamma_{1n}^U)^{w_n}) \\ ((\gamma_{21}^L)^{w_1}, (\gamma_{21}^U)^{w_1}) & ((\gamma_{22}^L)^{w_2}, (\gamma_{22}^U)^{w_2}) & \dots & ((\gamma_{2n}^L)^{w_n}, (\gamma_{2n}^U)^{w_n}) \\ \vdots & \vdots & \ddots & \vdots \\ ((\gamma_{m1}^L)^{w_1}, (\gamma_{m1}^U)^{w_1}) & ((\gamma_{m2}^L)^{w_1}, (\gamma_{m2}^U)^{w_2}) & \dots & ((\gamma_{mn}^L)^{w_n}, (\gamma_{mn}^U)^{w_n}) \end{pmatrix}$$

These three matrices can be divided into two matrices each. The lower, upper limit matrices of truth values, lower, upper limit matrices of indeterminacy values, and lower, upper limit matrices false values are the six matrices that are formed in the following stages.

$$w(T^L,T^U)=\begin{pmatrix}(1-(1-a_{11}^L)^{w_1},1-(1-a_{11}^U)^{w_1}) & \dots & (1-(1-a_{1m}^L)^{w_1},1-(1-a_{1m}^L)^{w_1})\\(1-(1-a_{21}^L)^{w_2},1-(1-a_{21}^U)^{w_2}) & \dots & (1-(1-a_{2m}^L)^{w_2},1-(1-a_{2m}^L)^{w_2})\\\vdots & \ddots & \vdots\\(1-(1-a_{n1}^L)^{w_n},1-(1-a_{n1}^U)^{w_n}) & \dots & (1-(1-a_{nm}^L)^{w_n},1-(1-a_{nm}^L)^{w_n})\end{pmatrix}$$

$$w(I^L,I^U)=\begin{pmatrix}((b_{11}^L)^{w_1},(b_{11}^U)^{w_1}) & ((b_{12}^L)^{w_1},(b_{12}^U)^{w_1}) & \dots & ((b_{1m}^L)^{w_1},(b_{1m}^U)^{w_1})\\((b_{21}^L)^{w_2},(b_{21}^U)^{w_2}) & ((b_{22}^L)^{w_2},(b_{22}^U)^{w_2}) & \dots & ((b_{2m}^L)^{w_2},(b_{2m}^U)^{w_2})\\\vdots & \vdots & \ddots & \vdots\\((b_{n1}^L)^{w_n},(b_{n1}^U)^{w_n}) & ((b_{n2}^L)^{w_n},(b_{n2}^U)^{w_n}) & \dots & ((b_{nm}^L)^{w_n},(b_{nm}^U)^{w_n})\end{pmatrix}$$

$$w(F^L,F^U)=\begin{pmatrix}((c_{11}^L)^{w_1},(c_{11}^U)^{w_1}) & ((c_{12}^L)^{w_1},(c_{12}^U)^{w_1}) & \dots & ((c_{1m}^L)^{w_1},(c_{1m}^U)^{w_1})\\((c_{21}^L)^{w_2},(c_{21}^U)^{w_2}) & ((c_{22}^L)^{w_2},(c_{22}^U)^{w_2}) & \dots & ((c_{2m}^L)^{w_2},(c_{2m}^U)^{w_2})\\\vdots & \vdots & \ddots & \vdots\\((c_{n1}^L)^{w_n},(c_{n1}^U)^{w_n}) & ((c_{n2}^L)^{w_n},(c_{n2}^U)^{w_n}) & \dots & ((c_{nm}^L)^{w_n},(c_{nm}^U)^{w_n})\end{pmatrix}$$

Similarly, these three matrices also can be divided into two matrices each.

**Step 4.** We change the non-square matrix into a square matrix in this stage.

$$A_1(T^L)_{n\times m} * W(T^L)_{m\times n}=\begin{pmatrix}a^L\alpha_{11}^L & a^L\alpha_{12}^L & \dots & a^L\alpha_{1n}^L\\a^L\alpha_{21}^L & a^L\alpha_{22}^L & \dots & a^L\alpha_{2n}^L\\\vdots & \vdots & \ddots & \vdots\\a^L\alpha_{n1}^L & a^L\alpha_{n2}^L & \dots & a^L\alpha_{nn}^L\end{pmatrix}_{n\times n}$$

$$A_1(T^U)_{n\times m} * W(T^U)_{m\times n}=\begin{pmatrix}a^U\alpha_{11}^U & a^U\alpha_{12}^U & \dots & a^U\alpha_{1n}^U\\a^U\alpha_{21}^U & a^U\alpha_{22}^U & \dots & a^U\alpha_{2n}^U\\\vdots & \vdots & \ddots & \vdots\\a^U\alpha_{n1}^U & a^U\alpha_{n2}^U & \dots & a^U\alpha_{nn}^U\end{pmatrix}_{n\times n}$$

In the same way, we convert upper and lower limit matrices of indeterminacy and false values.

**Step 5.** Calculate the matrix's energy using the interval-valued neutrosophic matrix energy formula. For one alternative, we got the energies for the lower and upper matrices of truth, indeterminacy, and false.

$$E[A_1]=[(E[A_1(T^L)],E[A_1(T^U)]]),(E[A_1(I^L)],E[A_1(I^U)]]),(E[A_1(F^L)],E[A_1(F^U)]])]$$

**Step 6.** Proceed with the remaining $r$ alternatives. We gained the interval-valued neutrosophic energies of choices $E(A_1),E(A_2),\dots,E(A_r)$.

**Step 7.** We have two energies for truth values, so we evaluate the average value of upper and lower energies of truth value. The alternatives are ranked based on their truth values. The best option will be the one with the maximum truth energy value.

Figure 1 shows the flow chart of the proposed method. The method applies to all situations. The difficulty of this approach is manually determining the energy value of the matrix.

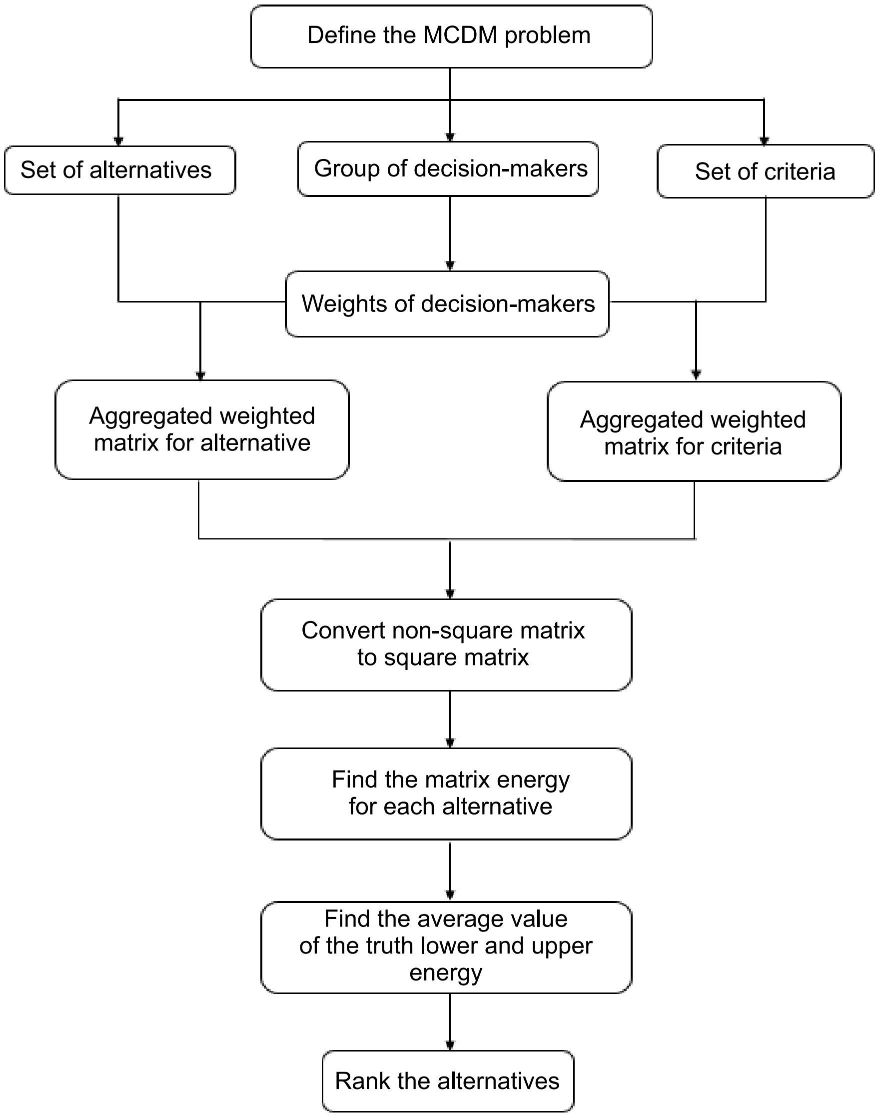


**Figure 1.** Flow chart of the proposed method

# 5. Numerical example

An example of the proposed interval-valued neutrosophic energy method was illustrated when selecting the project manager for the company. There are four staff members of the company taken as alternatives. The criteria for the selection process are $C_1$ – project idea, $C_2$ – cost estimation, and $C_3$ – experience. From these criteria and alternatives, the group of decision-makers who are the higher officials in the company will select one project manager for the company project. $DM_1$, $DM_2$, $DM_3$, and $DM_4$ are the decision-makers of the problem. The ratings of decision-makers are in terms of linguistic variables.

Table 1 shows the corresponding linguistic variables for interval-valued neutrosophic numbers [9].

**Table 1.** Terms for IVNNs

| Code (rating) | Term | Code (weight) | Term | IVNNs |
|---|---|---|---|---|
| VB | very bad | VU | very unimportant | $\langle [0.1, 0.2], [0.6, 0.7], [0.7, 0.8] \rangle$ |
| B | bad | U | unimportant | $\langle [0.2, 0.3], [0.5, 0.6], [0.6, 0.7] \rangle$ |
| A | average | FI | fairly important | $\langle [0.4, 0.6], [0.4, 0.5], [0.4, 0.5] \rangle$ |
| G | good | I | important | $\langle [0.7, 0.8], [0.3, 0.4], [0.2, 0.3] \rangle$ |
| VG | very good | VI | very important | $\langle [0.8, 0.9], [0.2, 0.3], [0.1, 0.2] \rangle$ |

**Step 1.** Weights of each criterion given by the 4 decision-makers are shown in Table 2. It was considered as $3 \times 4$ order of matrix.

**Table 2.** Weights of criteria

| Weight | $DM_1$ | $DM_2$ | $DM_3$ | $DM_4$ |
|---|---|---|---|---|
| $C_1$ | I | VI | FI | I |
| $C_2$ | FI | I | I | VI |
| $C_3$ | VI | I | VI | I |

Rating of alternatives given by 4 decision-maker for each criterion are taken as $4 \times 3$ matrix of each alternative. The rating are shown in Table 3.

**Table 3.** Ratings of alternatives

| | $A_1$ | | | $A_2$ | | | $A_1$ | | | $A_2$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $C_1$ | $C_2$ | $C_3$ | $C_1$ | $C_2$ | $C_3$ | $C_1$ | $C_2$ | $C_3$ | $C_1$ | $C_2$ | $C_3$ |
| $DM_1$ | A | G | G | VG | G | VG | G | A | A | B | A | G |
| $DM_2$ | G | VG | A | G | VG | G | B | A | VB | A | VG | G |
| $DM_3$ | A | B | A | G | A | VG | G | G | A | VB | G | VG |
| $DM_4$ | G | A | B | G | VG | A | VG | G | B | A | A | G |

**Step 2.** Determine the weights of decision-makers. Every decision-maker has its individual weights $DM_1$ – very important, $DM_2$ – important, $DM_3$ – important, and $DM_4$ – very important.

$$w_1 = \frac{1 - \left((1-0.8)^2 + (1-0.9)^2 + (0.2)^2 + (0.3)^2 + (0.1)^2 + (0.2)^2/6\right)^{1/2}}{4 - 0.196 - 0.292 - 0.292 - 0.196}$$

therefore, $w_1 = 0.266$, $w_2 = 0.234$, $w_3 = 0.234$, and $w_4 = 0.266$. Here $\sum_{j=1}^{4} w_j = 1$

**Step 3.** Aggregation of the weighted interval-valued neutrosophic decision matrix. First, we convert the linguistic terms into interval-valued neutrosophic numbers. Then separate the matrices into six terms, which are truth, indeterminacy, and false values in the lower and upper matrices, respectively. Using the formula for finding the aggregated weight, we build an interval-valued neutrosophic decision matrix for criteria and alternatives. Tables 4 and 5 show the truth lower limit aggregated weighted matrix for criteria and the truth lower limit aggregated weighted matrix for alternative $A_1$.

**Table 4.** Truth lower limit
aggregated weighted matrix for criteria

| $W(T^L)$ | $DM_1$ | $DM_2$ | $DM_3$ | $DM_4$ |
|---|---|---|---|---|
| $C_1$ | 0.2740 | 0.3138 | 0.1127 | 0.2740 |
| $C_2$ | 0.1271 | 0.2455 | 0.2455 | 0.3483 |
| $C_3$ | 0.3483 | 0.2455 | 0.3138 | 0.2740 |

**Table 5.** Truth lower limit
aggregated weighted matrix for $A_1$

| $A_1(w(T^L))$ | $C_1$ | $C_2$ | $C_3$ |
|---|---|---|---|
| $DM_1$ | 0.1271 | 0.2740 | 0.2740 |
| $DM_2$ | 0.2455 | 0.3138 | 0.1127 |
| $DM_3$ | 0.1127 | 0.0509 | 0.1127 |
| $DM_4$ | 0.2740 | 0.1271 | 0.0576 |

**Step 4.** Converting a non-square matrix into a square matrix.

$$A_1(w(T^L))_{4\times 3} * W(T^L)_{3\times 4} = A_1(T^L) = \begin{pmatrix} 0.1651 & 0.1744 & 0.1676 & 0.2053 \\ 0.1464 & 0.1818 & 0.1401 & 0.2074 \\ 0.0766 & 0.0755 & 0.0605 & 0.0795 \\ 0.1113 & 0.1313 & 0.0802 & 0.1351 \end{pmatrix}_{4\times 4}$$

**Step 5.** Find the energy of $A_1(T^L)$ matrix using the equation (1). Eigenvalues of $A_1(T^L)$, $\lambda_1 = 0.5404$, $\lambda_2 = -0.0211$, $\lambda_3 = 0.0232$, $\lambda_4 = 0$, and mean $\mu = 0.1356$

$$\begin{aligned} E(A_1(T^L)) =& |(0.5404 - 0.1356)| + |(-0.0211 - 0.1356)| \\ &+ |(0.0232 - 0.1356)| + |(0 - 0.1356)| = 0.8096 \end{aligned}$$

Eigenvalues of $A_1(T^U)$, $\lambda_1 = 1.0186$, $\lambda_2 = 0.0359$, $\lambda_3 = 0$, $\lambda_4 = -0.0277$ and mean $\mu = 0.2567$

$$\begin{aligned} E(A_1(T^U)) =& |(1.0186 - 0.2567)| + |(0.0359 - 0.2567)| \\ &+ |(0 - 0.2567)| + |(-0.0277 - 0.2567)| = 1.5239 \end{aligned}$$

Similarly, we calculate the energies for the lower and upper matrices of indeterminacy and false.

$$E[A_1] = [(E[A_1(T^L)], E[A_1(T^U)]), (E[A_1(I^L)], E[A_1(I^U)]), (E[A_1(F^L)], E[A_1(F^U)])]$$

$$E[A_1] = [(0.8096, 1.5239), (10.1105, 11.6354), (8.8062, 10.6248)]$$

**Step 6.** Proceed with the remaining 3 alternatives. We got the interval-valued neutrosophic energies of choices $E(A_2)$, $E(A_3)$, and $E(A_4)$.

$$E[A_2] = [(1.2333, 2.1862), (9.3959, 11.0112), (7.5928, 9.5903)]$$

$$E[A_3] = [(0.7343, 1.4067), (10.1882, 11.7087), (8.8881, 10.7114)]$$

$$E[A_4] = [(0.8933, 1.6457), (10.0321, 11.5713), (8.6597, 10.5045)]$$

**Step 7.** We evaluate the average value of the upper and lower energies of truth value for ranking the alternatives. The ranking order of alternatives is shown in Table 6.

**Table 6.** Ranking of alternatives

| Alternative | Truth energy $(T^L, T^U)$ | Average energy | Rank |
|---|---|---|---|
| $A_1$ | (0.8096, 1.5239) | 1.1667 | III |
| $A_2$ | (1.2333, 2.1862) | 1.7097 | I |
| $A_3$ | (0.7343, 1.4067) | 1.0705 | IV |
| $A_4$ | (0.8933, 1.6457) | 1.2695 | II |

As a result, alternative $A_2$ is the best one. The ranking order is $A_2 > A_4 > A_1 > A_4$.

# 6. Comparison of the proposed method to the TOPSIS method

In 2018, Vu Dung et al. [9] used an interval-valued neutrosophic set for solving a decision-making problem. The problem is about selecting the best personnel for an organization. There are 4 alternatives, 4 decision-makers, and 6 criteria. They solved this issue with the TOPSIS method. Here we solve this same problem with our proposed interval-valued neutrosophic energy method.

The steps are done with the same procedure for this problem. We get the square matrix of the lower truth matrix for the first alternative.

$$A_1(w(T^L))_{4\times 6} * W(T^L)_{6\times 4} = A_1(T^L) = \begin{pmatrix} 1.310 & 1.430 & 1.510 & 1.370 \\ 1.200 & 1.350 & 1.380 & 1.270 \\ 1.430 & 1.510 & 1.630 & 1.450 \\ 1.430 & 1.610 & 1.650 & 1.510 \end{pmatrix}_{4\times 4}$$

Therefore, the energy of the lower truth matrix = 8.6211. The remaining energies are calculated for each alternative.

$$E[A_1] = [(8.6211, 15.4294), (4.2474, 7.6695), (2.4684, 4.7941)]$$

$$E[A_2] = [(10.2958, 17.0397), (3.7200\, 16.6002), (2.0799, 4.1541)]$$

$$E[A_3] = [(10.0446, 16.8285), (3.8662, 6.8170), (2.1088, 4.2540)]$$

$$E[A_4] = [(10.0786, 16.7297), (4.0481, 7.0207), (2.1635, 4.3224)]$$

Now we determine the average energy of truth values and rank the alternatives. $A_1 = 12.0252$, $A_2 = 13.6677$, $A_3 = 13.4365$, and $A_4 = 13.4041$

# 7. Results and discussion

We compare the neutrosophic TOPSIS result with our proposed neutrosophic energy result. It is shown in Table 7. The ranking order is the same, making it easy to compare our method to the neutrosophic TOPSIS method. The advantages and disadvantages of our proposed method are given in Table 8.

**Table 7.** Comparison and results

| Alternative | TOPSIS | Rank | Enery | Rank |
|---|---|---|---|---|
| $A_1$ | 0.349 | IV | 12.0252 | IV |
| $A_2$ | 0.408 | I | 13.6677 | I |
| $A_3$ | 0.404 | II | 13.4365 | II |
| $A_4$ | 0.399 | III | 13.4041 | III |

**Table 8.** Advantages and disadvantages of the proposed method compared to other methods

| Advantages |
|---|
| 1. Applicable for all real-world MCDM problems and used to rank more alternatives. |
| 2. The steps of the proposed method are short and easy to understand. |
| 3. Using Matlab code, we can quickly obtain the result. |
| 4. The method gives more importance to the individual matrix. |
| Disadvantages |
| 1. It does not put as much focus on measuring distance. |
| 2. Calculating the energy of the matrix manually can be challenging. |
| 3. The final ranking is only depending on truth energy. |

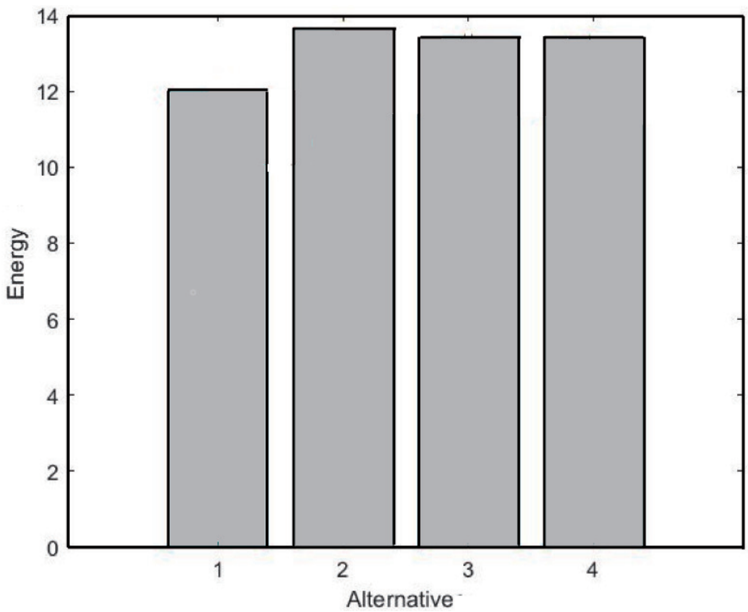


**Figure 2.** Barchat of energy result

As shown in Figure 2, the order of ranking for interval-valued neutrosophic TOPSIS method and interval-valued neutrosophic energy method is as follows $A_2 > A_3 > A_4 > A_1$.

# 8. Conclusions

The concept of energy is widely used in graphs and matrices. We developed a new neutrosophic matrix energy technique based on the interval-valued neutrosophic matrix and applied it to MCDM problems in our study. In the neutrosophic fuzzy multi-criteria decision-making area, there are many approaches to selecting the best alternative. However, our proposed energy method produced a very efficient result, and it also simplified the work and demonstrated the importance of the matrix. We compared the final result to the TOPSIS outcome, and we gave the advantages and disadvantages of our proposed method compared to existing MCDM methods. As a result, the neutrosophic energy technique is effective in identifying the best MCDM solution. Furthermore, we will apply this energy idea to several types of neutrosophic matrices, such as multi-valued and hesitant neutrosophic matrices. Then, it will be used for solving MCDM problems.

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## References

[1] Anjali, N. and Mathew, S. Energy of a fuzzy graph. *Annals of Fuzzy Mathematics and Informatics 6*, 3 (2013), 455–465.

[2] Atanassov, K. T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems 20*, 1 (1986), 87–96.

[3] Bolturk, E., and Kahraman, C. Interval-valued neutrosophic AHP with possibility degree method. *International Journal of the Analytic Hierarchy Process 10*, 3 (2018), 431–446.

[4] Bravo, D., Cubria, F., and Rada, J. Energy of matrices. *Applied Mathematics and Computation 312* (2017), 149–157.

[5] Chou S.-Y., Pham X. L., and Nguyen T. A. T. Interval-valued neutrosophic sets to develop multi-criteria decision-making approach for renewable energy selection. In *Transdisciplinary Engineering for Complex Socio-technical Systems. Proceedings of the 26th ISTE International Conference on Transdisciplinary Engineering, TE 2019*, (Kashiwa, Japan, July 30 - August 1, 2019), J. Stjepandić, N. Wognum, K. Hiekata, M. Inoue and B. R. Moser, Eds., vol. 10 of *Advances in Transdisciplinary Engineering*, IOS Press, 2019, pp. 179–185.

[6] Das, R., Smarandache, F., and Tripathy, B. C. Neutrosophic fuzzy matrices and some algebraic operations. Neutrosophic Sets and Systems 32, 1 (2020), 401–409.

[7] Dhar, M., Broumi, S., and Smarandache, F. A note on square neutrosophic fuzzy atrices. *Neutrosophic Sets and Systems 3* (2014), 37–41.

[8] Donbosco, J. S. M., and Ganesan, D. The Energy of rough neutrosophic matrix and its application to MCDM problem for selecting the best building construction site. *decision-making: Applications in Management and Engineering 5*, 2 (2022) 30–45.

[9] Dung, V., Thuy, L. T., Mai, P. Q., Van Dan, N., and Lan, N. T. M. TOPSIS approach using interval neutrosophic sets for personnel selection. *Asian Journal of Scientific Research 11*, 3 (2018), 434–440.

[10] Edalatpanah, S. A., and Smarandache, F. Introduction to the special issue on advances in neutrosophic and plithogenic sets for engineering and sciences: Theory, models, and applications. *Computer Modeling in Engineering & Sciences 134*, 2 (2023), 817–819.

[11] Gutman, I. The energy of a graph. *Berichte der Mathematisch-Statistischen Sektion im Forschungszentrum Graz 103* (1978), 1-22.

[12] Kandasamy, W. B. V., and Smarandache, F. Fuzzy relational maps and neutrosophic relational maps. Hexis, Church Rock 2004.

[13] Karaaslan, F., Hayat, K., and Jana, C. The determinant and adjoint of an interval-valued neutrosophic matrix. In *Neutrosophic Operational Research*, F. Smarandache and M. Abdel-Basset, Eds., Springer, Cham, 2021, pp. 127–151.

[14] Kharal A. A neutrosophic multi-criteria decision-making method, *New Mathematics and Natural Computation 10*, 2 (2014), 143–162.

[15] Mao, X., Guoxi, Z., Fallah, M., and Edalatpanah, S. A. A neutrosophic-based approach in data envelopment analysis with undesirable outputs. *Mathematical Problems in Engineering 2020* (2020), 7626102.

[16] Martin, N., Priya, R., and Smarandache, F. New Plithogenic sub cognitive maps approach with mediating effects of factors in COVID-19 diagnostic model. *Journal of Fuzzy Extension and Applications* 2, 1 (2021) 1–15.

[17] Martina, D. J. S., and Deepa, G. Operations on multi-valued neutrosophic matrices and its application to neutrosophic simplified-TOPSIS method. *International Journal of Information Technology & decision-making 22*, 1 (2023) 37–56.

[18] Nikiforov, V. The energy of graphs and matrices. *Journal of Mathematical Analysis and Applications 326*, 2 (2007), 1472–1475.

[19] Polymenis, A. A neutrosophic Students's t-type of statistic for AR (1) random processes. *Journal of fuzzy extension and application 2*, 4 (2021) 388–393.

[20] Praba, B., Chandrasekaran, V. M., and Deepa, G. Energy of an intuitionistic fuzzy. *Italian Journal of Pure and Applied Mathematics 32* (2014) 431–444.

[21] Ramesh, O., and Basha, S. S. Group decision-making of selecting partner based on signless laplacian energy of an intuitionistic fuzzy graph with TOPSIS method: Study on MATLAB programming. *Advances in Mathematics: Scientific Journal* 9 (2020), 5849–5859.

[22] Saha, A., and Broumi, S. New operators on interval valued neutrosophic sets. *Neutrosophic Sets and Systems 28* (2019), 128–137.

[23] Smarandache, F. *Neutrosophy. Neutrosophic Probability, Set, and Logic: Analytic Synthesis & Synthetic Analysis*. American Research Press, 1998.

[24] Stanimirović, P. S., Ivanov, B., Stanujkić, D., Katsikis, V. N., Mourtas, S. D., Kazakovtsev, L. A., and Edalatpanah, S. A. Improvement of unconstrained optimization methods based on symmetry involved in neutrosophy. *Symmetry 15*, 1 (2023) 250.

[25] Veeramani, C., Venugopal, R., and Edalatpanah, S. A. Neutrosophic DEMATEL approach for financial ratio performance evaluation of the NASDAQ Exchange. *Neutrosophic Sets and Systems* 51 (2022) 766–782.

[26] Vidhya, R., and Hepzipha, R. I. On interval valued neutrosophic fuzzy matrices. *Advances and Applications in Mathematical Sciences 20*, 4 (2021), 561–575.

[27] Wang, H., Madiraju, P., Zhang, Y., and Sunderraman, R. Interval neutrosophic sets, 2004. Preprint available from https://arxiv.org/abs/math/0409113.

[28] Zadeh, L. A. Fuzzy sets. *Information and control 8*, 3 (1965), 338–353.

[29] Zavadskas, E. K., Bausys, R., Lescauskiene, I., and Usovaite, A. MULTIMOORA under interval-valued neutrosophic sets as the basis for the quantitative heuristic evaluation methodology HEBIN. Mathematics 9, 1 (2020), 66.