Vol. 34, No. 1 (2024) | DOI: 10.37190/ord240109

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Operations Research and Decisions

www.ord.pwr.edu.pl

OPERATIONS RESEARCH AND DECISIONS QUARTERLY

A heuristic approach to minimizing the waiting time of jobs in two-stage flow shop scheduling

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Abstract

The paper presents the influence of the waiting time of jobs in a 2 machine k-job Flow Shop Scheduling (FSS) problem. The main intention of the study is to find a sequence of jobs that delivers the least sum of the time of waiting for jobs. A heuristic approach has been adopted to achieve the desired objective. The experiments are conducted for more than 2000 problems of various sizes for the problems with special structures and problems with random times of processing. The weighted mean absolute error (WMAE) for the average of the sum of the waiting times of jobs is computed for both kind of problems after comparing with the optimal solutions. WMAE has been obtained less than 0.0075 for problems with special structures and less than 0.087 for problems with random times of processing. The WMAE is also reducing significantly with the increase in job size. The results demonstrate that the presented step-by-step procedure of the heuristic delivers significantly close to optimal solutions.

Keywords: *special structures, FSS, waiting time, processing time, heuristic*

1. Introduction

FSS examines the machine (service provider)–job (client) models for several objectives where all the jobs have to move in a pre-defined order of the service providers/machines. Several studies have been available in the past to optimize the total completion time for n-job m-machine FSS problems but the objective to minimize the sum of the times of waiting for jobs has been paid less attention. The present paper explores the FSS models for the 2-machine k-job problem where consideration is minimizing the sum of the times of waiting for jobs. A heuristic is anticipated to attain a sequence of jobs that will deliver an optimal or near-optimal solution to the desired objective.

If the times of processing the jobs satisfy a definite condition then FSS problems are considered as specially structured problems. The initial study for 2 machines n-job specially structured FSS problems

Received 24 March 2023, accepted 23 January 2024, published online 28 March 2024
ISSN 2391-6060 (Online)/

The costs of publishing this issue have been co-financed by the program *Development of Academic Journals* of the Polish Ministry of Education and Science under agreement RCN/SP/0241/2021/1

was carried out by Bhatnagar et al. [3]. The authors proposed an algorithm to optimize the sum of the times of waiting for jobs in FSS where the times of processing the jobs are not on the entire random but fulfill a certain condition. Numerous special structures in FSS have been examined by Gupta [10] however to lessen the makespan/total completion time. The FSS problems considering aim to minimize the sum of the times of waiting time of jobs with the association of probabilities with the processing times but in a specially structured manner has been deliberated by Gupta and Goyal [8]. The study has been furthermore protracted by Gupta and Goyal [9] after making an allowance for the set-up times of machines detached from times of processing the jobs. Further Goyal et al. [5] made an addition to the study by making an allowance for the notion of job block and merited the recommended algorithm by making a comparative analysis with the prevailing tactics.

The FSS problem under uncertainties has also gained countless significance in the past for various objectives. Khalifa [12] studied the single machine scheduling with uncertain times of processing to reduce the total cost of the penalty. Alburiakan et al. [1] investigated a novel three-stage flow shop scheduling problem with an uncertain time of processing to minimize the fuzzy processing time of the machines subject to the rental policy. Zhou et al. [21] studied n-job flow shop scheduling with fuzzy piece-wise quadratic times of processing and three machines to minimize the leasing cost of equipment with the use of a fuzzy style and an inventive algorithm. Khalifa et al. [13] studied constrained multistage machines flow-shop scheduling problems to minimize the total piece-wise quadratic fuzzy elapsed time. Alharbi and Khalifa [2] investigated the flow-shop problem under a fuzzy environment in which the processing time of jobs is represented by pentagonal fuzzy numbers. The objective of this study is to reduce the rental cost of the machines. The authors rather than converting the times of processing into crisp values, provided a close interval approximation to the fuzzy processing times. Goyal and Kaur [6],[7] further explored the FSS models with times of processing in a fuzzy environment considering the aim to optimize the sum of the times of waiting of jobs.

The heuristic tactic in exploring FSS models has been demonstrated as a very operative tactic in research of scheduling. Nawaz et al. [16] introduced a well-known heuristic (NEH) algorithm to improve the total elapsed time for the m-machine n-job FSS problem. Nailwal et al. [15] developed two heuristics, one constructive and the other an upgrading heuristic procedure attained in an FSS for n-job, m–machine problem under no-wait constraint with the consideration to lessen the makespan. Chakraborty and Laha [4] established a heuristic for the FSS problem to attain an optimal schedule to minimize total elapsed time/makespan. Yilmaz and Yilmaz [18] proposed a genetic algorithm (GA) with the makespan objective for the hybrid flow shop scheduling problem by considering equal and consistent subplots under machine capability and limited waiting time constraints. Recently the FSS problems in which the time of processing the jobs was linearly dependent on the waiting time of the job was studied by Yang and Kuo [17] to develop a heuristic for the minimization of the makespan/total elapsed time. Liang et al. [14] made a computationally proficient optimization through uniting NEH and NEH-NGA methods.

As per the literature review, it has been discovered that the aim of attaining the minimum of the sum of all the waiting time of jobs has been paid attention only for the problems with special structures by Gupta and Goyal [8],[9]. The objective to minimize the sum of the waiting times of jobs for problems with arbitrary processing times has not been studied so far. The present paper provides a heuristic to obtain an optimal or near-optimal schedule for which the objective of minimizing the total waiting time of jobs can be achieved. The problem of optimizing the sum of the times of waiting for jobs is NP-Hard. So, a heuristic algorithm is proposed to optimize the total waiting time of jobs. To decrease the sum of the time of waiting, a step-by-step procedure that can provide a near-optimal job schedule has been presented and the error analysis has also been deliberated.

1.1. Significance, novelty, and highlights of the proposed work

The significance of the proposed objective can be detected in every single service provider organization/industry because client contentment is of extreme significance to every executive. In today's fastest-rising world, everyone desires to get the service deprived of waiting for too much time. Therefore, a service executive continuously pays attention to deliver service well-timed without making the client wait for a long period.

The majority of the work has been done in the past for various objectives such as minimizing the elapsed time, and minimizing the rental cost of machines. The objective proposed in the present study has been paid no attention by the researchers in the past for random times of processing. The novelty of the research work lies in the study of the objective and in the presentation of the proposed heuristic to minimize the times of waiting for jobs for FSS problems with randomly generated times of processing.

The novelty of the research work lies in the study of the objective function and in the presentation of the proposed heuristic to minimize the times of waiting for jobs for FSS problems with randomly generated times of processing. The major novel aspects of the work are:

1. The approach of the study is to minimize the sum of times of waiting under random times of processing.
2. The attention towards the client's contentment as most of the authors focus on the manager's/ industry satisfaction.
3. The proposed heuristic is novel as no heuristic is present in the literature that focuses on minimizing the sum of time of waiting for jobs.

The major highlights of the study are (i) a novel heuristic to minimize the times of waiting for jobs for FSS problems with random times of processing, (ii) a heuristic that optimizes the waiting time without significantly affecting the elapsed time, (iii) the computational experiments for a larger number of problems of various job sizes.

2. Mathematical formulation of the problem

2.1. Nomenclature

k – number of jobs
A_i – ith machine, i = 1, 2
t_{ij} – time of processing jth job on machine A_i, i = 1, 2
T_{ji} – time of initial processing of jth job on machine A_i
F_{ji} – time of finishing the jth job on machine A_i
W_j – time of waiting of jth job on machine A_2
W – sum of the times of waiting of$k$-jobs on machine A_2

W_{best} – most accurate achieved value of W
W_{heu} – achieved value of W obtained by executing recommended process
$W_{\max}$ – maximum of all possible values of W
W_{PIH} – achieved value of W obtained by executing heuristic PIH [15]
C_{heu} – makespan obtained by executing recommended process
C_{PIH} – makespan obtained by executing heuristic PIH [15]

2.2. FSS problem having arbitrary times of processing

Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order A_1, A_2. Let t_{1j}, $\mathrm{t}_{2\mathrm{j}}$ are the times of processing the jth job on machines A_1 and A_2 respectively (Table 1).

Table 1. Problem formulation in a matrix form

Machine	Job						
	1	2	3	...	k–2	k–1	k
A_1	t_{11}	t_{12}	t_{13}	...	$t_{1(k-2)}$	$t_{1(k-1)}$	t_{1k}
A_2	t_{21}	t_{22}	t_{23}	...	$t_{2(k-2)}$	$t_{2(k-1)}$	t_{2k}

Let $W_{\gamma_{\mathrm{m}}}$ be the time of waiting for job γ_{m} on machine A_2. The purpose is to attain a sequence of jobs that optimizes the sum of times of waiting of jobs W.

2.3. FSS problem with special structures

In the above problem, if times of processing the k number of jobs on machines A_1 and A_2 satisfies the condition

$$\max\{\mathrm{t}_{1\mathrm{j}}\} \preceq \min\{\mathrm{t}_{2\mathrm{j}}\} \tag{1}$$

Then, the FSS problem is recognized as 2-machine k-job with special structures FSS problem.

2.4. Assumptions

1. Passing of jobs is not to be done.
2. Each process once underway must perform till end.
3. Jobs are self-regulating.
4. Job is not to be processed by more than one machines at a time.
5. The time to set up a machine is supposed to be incorporated in times of processing the job.

2.5. Theorems

Lemma 1. Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order $A_1 A_2$. Let t_{1j}, t_{2j} are the times of processing the jth job on machines A_1 and A_2, respectively. Let F_{j2} is the time of finishing of jth job on machine A_2, then for job sequence S: γ_1, γ_2, γ_3, $\ldots$, γ_k of jobs

$$F_{\gamma_n 2} = \max_{1 \le v \le n} \left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{n} t_{2\gamma_j} \right), \text{ where } n \in \{1, 2, \ \ldots, \ k\} \tag{2}$$

Proof. Let us smear the induction principle on $S(n)$, where

$$S(n) = F_{n2} = \max_{1\le v\le n}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{n} t_{2\gamma_j}\right)$$

Since $F_{\gamma_1 2} = t_{1\gamma_1} + t_{2\gamma_1}$, consequently, $S(1)$ holds true.
Take up that $S(n)$ holds true for $n = m$, now for $n = m + 1$

$$F_{\gamma_{m+1}2} = \max(F_{\gamma_{m+1}1},\ F_{\gamma_m 2}) + t_{2\gamma_{\mathrm{m+1}}}$$

Using hypothesis of Induction

$$\begin{aligned}
F_{\gamma_{m+1}2} &= \max\left(t_{1\gamma_1} + t_{1\gamma_2} + ... + t_{1\gamma_{m+1}},\ \max_{1\le v\le m}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{m} t_{2\gamma_j}\right)\right) + t_{2\gamma_{m+1}}\\
&= \max\left(\sum_{i=1}^{m+1} t_{1\gamma_i} + t_{2\gamma_{m+1}},\ \max_{1\le v\le m}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{m+1} t_{2\gamma_j}\right)\right)\\
&= \max_{1\le v\le (m+1)}\left(\sum_{i=1}^{v} t_{1\gamma_i} + \sum_{j=v}^{m+1} t_{2\gamma_j}\right)
\end{aligned}$$

Hence for $n = m+1$, $S(m+1)$ holds true. $S(n)$ comes out to be true for $n = 1$, $n = m$ and $n = m+1$ and m being arbitrary. Hence for any k-job sequence S: $\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma$k, time of finishing of the job γ_n is certain to be obtained from equation (2). □

Lemma 2. Following from Lemma 1, the same assumptions and notations, for the sequence S: γ_1,γ_2, $\gamma_3, \ldots, \gamma_k$ of jobs $W_{\gamma_1} = 0$ and, for $2 \le n \le k$

$$W_{\gamma_n} = \max\left(0,\ \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1\,\le\, v\,\le\,(n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right) \tag{3}$$

where $W_{\gamma n}$ is the time of waiting for job γ_n on machine A_2.

Proof. Since for any k-job sequence S: $\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k$, time of waiting $W_{\gamma_n} = T_{\gamma_n 2} - F_{\gamma_n 1}$ for $1 \le n \le k$. For $n = 1$, W_{γ_1}= $T_{\gamma_1 2} - F_{\gamma_1 1}$= $t_{1\gamma_1} - t_{1\gamma_1}$=0 and for $2 \le n \le k$,

$$W_{\gamma_n} = T_{\gamma_n 2} - F_{\gamma_n 1} = \max(F_{\gamma_n 1},\ F_{\gamma_{n-1}2}) - F_{\gamma_n 1} = \max(0,\ \ F_{\gamma_{n-1}2} - F_{\gamma_n 1})$$

From equation (2),

$$\begin{aligned}
W_{\gamma_{\mathrm{n}}} &= \max\left(0,\ \max_{1\,\le\, v\,\le\,(n-1)}\left(\sum_{i=1}^{n} t_{1\gamma_i} + \sum_{j=v}^{n-1} t_{2\gamma_j}\right) - \sum_{i=1}^{n} t_{1\gamma_i}\right)\\
&= \max\left(0,\ \max_{1\,\le\, v\,\le\,(n-1)}\left(\sum_{i=1}^{n} t_{1\gamma_i} - \sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{n-1} t_{2\gamma_j} - \sum_{j=1}^{v-1} t_{2\gamma_j}\right) - \sum_{i=1}^{n} t_{1\gamma_i}\right)
\end{aligned}$$

$$= \max\left(0, \max_{1\le v\le (n-1)}\left(-\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{n-1} t_{2\gamma_j} - \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)$$

$$= \max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} + \max_{1\le v\le (n-1)}\left(-\sum_{i=v+1}^{n} t_{1\gamma_i} - \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)$$

$$= \max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1\le v\le (n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)$$

□

Theorem 1. Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order A_1 A_2. Let t_{1j} and t_{2j} are the times of processing the k-jobs on machines A_1 and A_2, correspondingly. For any sequence S: γ_1,γ_2, γ_3, $\ldots$, γ_k, the sum of the times of waiting of jobs W is agreed by

$$W = \sum_{n=2}^{k}\left(\max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1\le v\le (n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)\right) \tag{4}$$

Proof. For sequence of k-jobs γ_1,γ_2, $\gamma_3, \ldots, \gamma_k$,

$$W = \sum_{n=1}^{k} W_{\gamma_n}$$

From lemma 2,

$$W = \sum_{n=2}^{k}\left(\max\left(0, \sum_{j=1}^{n-1} t_{2\gamma_j} - \min_{1\le v\le (n-1)}\left(\sum_{i=v+1}^{n} t_{1\gamma_i} + \sum_{j=1}^{v-1} t_{2\gamma_j}\right)\right)\right)$$

□

2.6. Inference of results for problems with special structures

Suppose that k-jobs are under process on two machines A_1 and A_2. All the jobs must be processed in the order A_1 A_2. Let t_{1j} and t_{2j} are the times of processing the k-jobs on machines A_1 and A_2, correspondingly, which satisfies the relation assumed in equation (1), $\max\{t_{1j}\} \preceq \min\{t_{2j}\}$, for any sequence $S : \gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_k$, the time of finishing the job $F_{\gamma_n 2}$ deducted from Lemma 1 is agreed by

$$F_{\gamma_n 2} = t_{1\gamma_1} + t_{2\gamma_1} + t_{2\gamma_2} + \ldots + t_{2\gamma_n} \tag{5}$$

k-job sequence S: γ_1, γ_2, γ_3, $\ldots$, γ_k, $W_{\gamma_1} = 0$, for $2 \le n \le k$, W_{γ_n} deducted from Lemma 2 is agreed by

$$W_{\gamma_n} = t_{1\gamma_1} + \sum_{s=1}^{n-1} y_{\gamma_s} - t_{1\gamma_n} \tag{6}$$

where $y_{\gamma_s} = t_{2\gamma_s} - t_{1\gamma_s}$ and $\gamma_s \in \{1, 2, \ldots, k\}$.

For k-job sequence S: $\gamma_1,\ \gamma_2,\ _3,\ ,\ldots,\ \gamma_k$, the sum of the times of waiting W deducted from Theorem 1 is agreed by

$$W = kt_{1\gamma_1} + \sum_{s=1}^{k-1}(k-s)y_{\gamma_s} - \sum_{j=1}^{k} t_{1j} \tag{7}$$

where $y_{\gamma_s} = t_{2\gamma_s} - t_1\gamma_s$ and $\gamma_s \in \{1,\ 2,\ \ldots,\ k\}$.

Theorem 2. For $n \in \mathbb{N}$ and $y_1,\ y_2,\ y_3,\ \ldots,\ y_n \in \mathbb{R}$ such that $y_1 \le y_2 \le y_3 \le \cdots \le y_n$, the value $ny_1 + (n-1)y_2 + (n-2)y_3 + \cdots + 2y_{n-1} + y_n$ is minimum.

Proof. Using induction principle on n, pettily result holds true for $n = 1$.
Take up that the result approaches true for up to n terms. Now,

$$\begin{aligned} &ny_1 + (n-1)y_2 + (n-2)y_3 + \cdots + 2y_{n-1} + y_n \\ &= (n-1)y_1 + (n-2)y_2 + (n-3)y_3 + \cdots + y_{n-1} + \sum_{i=1}^{n} y_i \end{aligned}$$

The term $\sum_{i=1}^{n} y_i$ is constant. Consequently following the assumption to the hypothesis $ny_1 + (n-1)y_2 + (n-2)y_3 + \cdots + 2y_{n-1} + y_n$ is minimum. □

3. Algorithms

3.1. Algorithm for problems with special structures

The method proficient in making the ideal solution for a 2-machine k-job with special structures FSS problem has been deliberated in recent times by Gupta and Goyal [8, 9]. The algorithm is as follows:

Step 1. Authenticate the structural relationship $\max\{t_{1j}\} \preceq \min\{t_{2j}\}$.

Step 2. Compute $y_j = t_{2j} - t_{1j}$ for every j, where $j \in \{1, 2,\ \ldots,\ k\}$.

Step 3. Organize the jobs in arising direction of values of y_j. Suppose $S_1 = (\gamma_1,\ \gamma_2, \gamma_3,\ \ldots,\ \gamma_k)$ is the sequence obtained after the arrangement.

Step 4. If $t_{1\gamma_1} = \min\{t_{1\mathrm{j}}\}$, at that juncture the sequence acquired in the 3rd step is the requisite sequence which delivers ideal solution, or else move on to 5th step.

Step 5. Find $(k-1)$ altered sequences represented as S_j for $2 \le j \le k$ by injecting jth job in the first sequence to the very first place and leaving the left over sequence unaffected.

Step 6. Calculate the sum of the times of waiting of jobs, W for each and every sequence $S_1,\ S_2,\ \ldots,\ S_k$ with the help of equation (7).

Theorem 2 justifies that the sequence with least W is the requisite sequence.

3.2. Proposed step-by-step heuristic

Following is the step-by-step description of algorithm:

Step 1. Assemble the jobs according to the increasing order of times of processing t_{2j} of machine A_2. Assume the sequence attained is $\{\gamma_n\}$, $n = 1,\ 2,\ \ldots, k$.

Step 2. Consider the initial two jobs from the obtained sequence and assemble them in both feasible ways to compute the times of waiting of jobs using the formula given in equation (4) and pick the sequence which provides least time of waiting for the initially selected two jobs.

Step 3. For $n = 3$ to k, go to 4th step and further to 5th step.

Step 4. Put in the nth job in the attained sequence of $(n - 1)$ jobs at the n probable places begin to insert from 1st spot then to the 2nd spot and repeating the procedure.

Step 5. Compute the sum of the times of waiting of the jobs by means of formula obtained in equation (4) for the sequences obtained in the 4th step and pick the sequence which delivers the least sum of the times of waiting.

Tie breaking condition. If a tie persists among more than one partial sequences for the least sum of the times of waiting, select the sequence where the injection of the nth job comes at the far position.

3.3. Pseudo code for proposed step-by-step heuristic

Sort the jobs in increasing order of processing times t_{2j} of machine A_2, getting the sequence
$S := \{\gamma_1, \gamma_2, \ldots, \gamma_\mathrm{k}\}$;
Select the first and second jobs of the sequence S.

Calculate the time of waiting for two jobs by allocating them to both possible positions:

begin
$\Pi := \Pi'$ a partial sequence formed by selected jobs.
for $n = 3$ to k **do**
select job γ_n from sequence S;
insert the job γ_n in all possible positions of Π, generating n partial sequence with n jobs;
$\Pi' :=$ select the best generated sequence;
end for
end

3.4. Significance of the proposed algorithm and heuristic approach

The heuristic anticipated is stimulated by the prominent NEH [16] heuristic which works on the principle of breaching the problem into smaller size and then optimizing it. Afterward injecting tasks/jobs one by one while optimizing at each stage upto the whole generalization of the problem. The heuristic approach has been demonstrated a very effective approach in the past [4, 16].

In the study by Yilmaz [19], warehouse supplies the products to supermarkets which, in turn, allocates the tasks to its workers in a U shaped line. The problem focuses on to minimize the operational cost which includes cost of transportation and the station opening cost. The stations handle the tasks to minimize the maximum workload imbalance. The AUGMECON 2 method has been applied to optimize the objectives. The principle used in the heuristic can also be applied to such combinatorial problems. So that the larger problem can be broken into smaller size problems. Then injecting the tasks one after the other and permuting the tasks to obtain the optimal solution at each stage.

The study by Gursoy and Soner Kara [11] focuses on the network design problem which includes just in time (JIT) delivery. The paper aims to minimize the total supply chain cost while passing through the four stages of supply chain network – suppliers, manufacturers, distribution counters and retailers.

Ensuring the JIT in network design problem with minimizing the cost and maximizing the raw material quality which is directly related to customer satisfaction. Our heuristic proposed can also be effective in this problem as the proposed study focuses on the measure of the objective of customer satisfaction.

The study on MODLP from a different point of view to minimize the overall cost, the cycle time, and maximum workload imbalance has been made by Yilmaz and Yaziki [20]. The NSGA-II algorithm has been applied to accomplish the anticipated objective. The problem has been considered as an NP-hard problem. The Heuristic approach has been significantly effective on problems that are NP hard. So the approach adopted in the present paper also seems to be effective while considering this multi-objective real-time problem.

4. Results and analysis of the experiments

In various trials conducted, approximately 2100 problems ranging from $k = 4$ to $k = 200$ (for apiece size 100 problems and 21 dissimilar sizes) are being produced and tested. The times of processing are made to run in uniform distribution with the range of $[1, 99]$. The results obtained in Table 2 demonstrate that the sum of the waiting times attained after applying the proposed heuristic provides the nearly optimal solution (experiment conducted for 100 problems for each k). The results given in Table 2) are plotted in Figure 1 which also demonstrates the efficiency of the proposed step-by-step procedure of the heuristic.

Table 2. Optimum sum of the waiting times for FSS problems with special structures

Job size (k)	Average W_{best}	Average W_{heu}
5	85.32	85.95
10	474.12	476.06
15	1224.8	1229.3
20	2386.2	2395.4
30	7211.6	7224.8
40	12,379	12,409
50	18717	18,770
60	27,268	27,313
70	35,115	35,194
80	49,223	49,304
90	58,111	58,250
100	76,525	76,703
120	111,230	111,400
140	156,930	157,200
160	208,440	208,680
180	228,782	228,962
200	281,850	282,010

Taking k number of jobs, weighted mean absolute error (WMAE)

$$e = \frac{\sum\limits_{i=1}^{100} |W_{\text{best}} - W_{\text{heu}}|}{\sum\limits_{i=1}^{100} W_{\text{best}}}$$

is given in Table 3 where W_{heu} is optimal of the sum of the waiting times obtained by employing the heuristic and W_{best} is optimal of the sum of waiting times of all jobs for all probable permutation of the schedules.

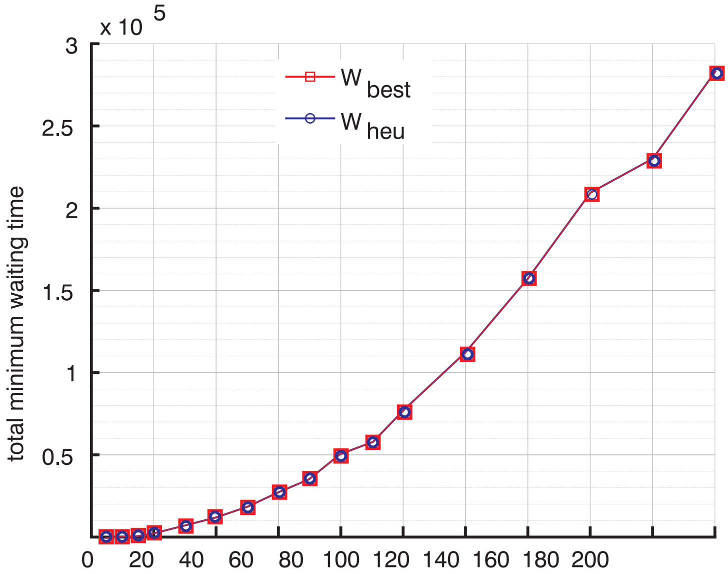

Figure 1. Comparison of the average of the proposed results with the averages of optimal solution

Table 3. Error analysis for FSS problems with special structures

Job size (k)	WMAE	Job size (k)	WMAE
5	0.0073	80	0.0016
10	0.0053	90	0.0023
15	0.0035	100	0.0024
20	0.0038	120	0.0014
30	0.0019	140	0.0017
40	0.0024	160	0.0013
50	0.0027	180	0.0007
60	0.0017	200	0.0005
70	0.0021		

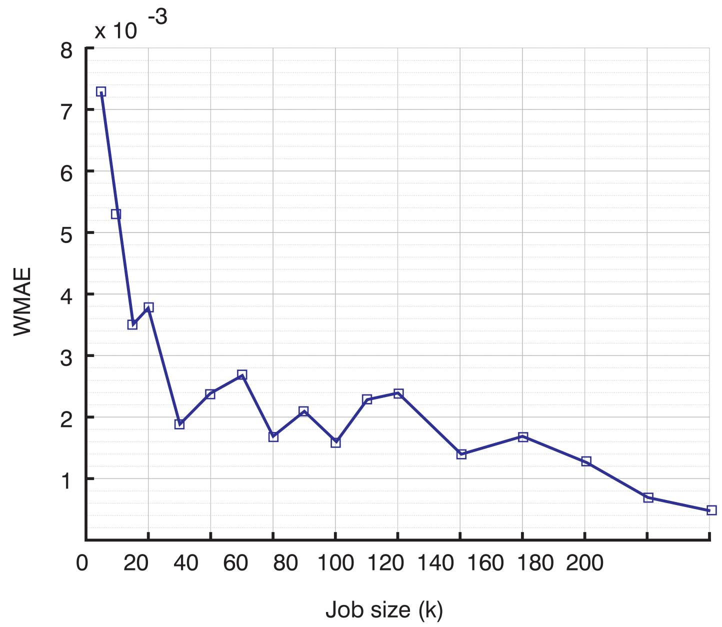

Figure 2. WMAE for FSS problems with special structures

The results shown in Table 3 are presented in Figure 2 which demonstrates that the error reduces accordingly to the increase of the size of the job. Therefore, the presented heuristic provides a near-optimal solution.

Table 4. Average of the sum of time of waiting of jobs for FSS problems with arbitrary times of processing

Job size (k)	Average W_{best}	Average W_{heu}	Average W_{max}
4	11.97	13.01	91.17
5	26.54	28.79	196.54
6	31.37	34	333.07
7	33.32	36.11	275.91

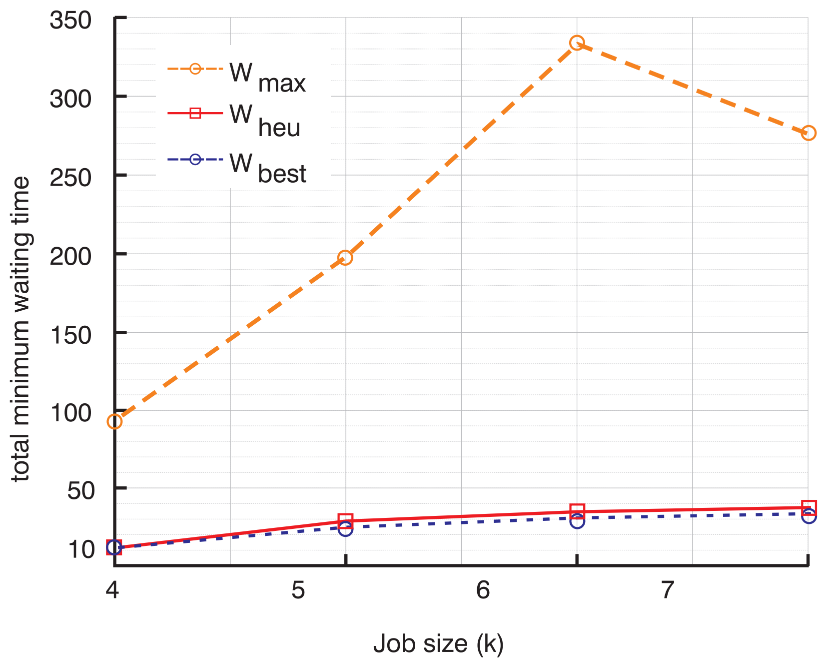

Figure 3. Averages of the sum of the times of waiting for FSS problems with arbitrary times of processing

Various trials were conducted for the problems with arbitrary processing times, approximately 400 problems ranging from $k = 4$ to $k = 7$ jobs were produced and tested. The results obtained after applying the proposed heuristic are then compared with the actual least summation of the waiting times of jobs in Table 4. Also, a maximum possible waiting time is computed to further prove the effectiveness of the presented heuristic. It is significantly proved that the average W_{heu} is very much lower than average W_{max} Figure 3.

Table 5. WMAE computation for problems with arbitrary times of processing

Job size	4	5	6	7
WMAE	0.0868	0.0847	0.0839	0.0838

The WMAEs are also given in Table 5 for the proposed heuristic which prove that the presented step-by-step procedure is highly effective to reach near the optimal solution. Due to the NP-hardness complexity of the problem, actual results can only be generated up to job size $k = 7$ but they are enough to demonstrate that the presented heuristic is providing lesser WMAE as the job size is increasing. Figure 4 shows the data generated and presented in (Table 5).

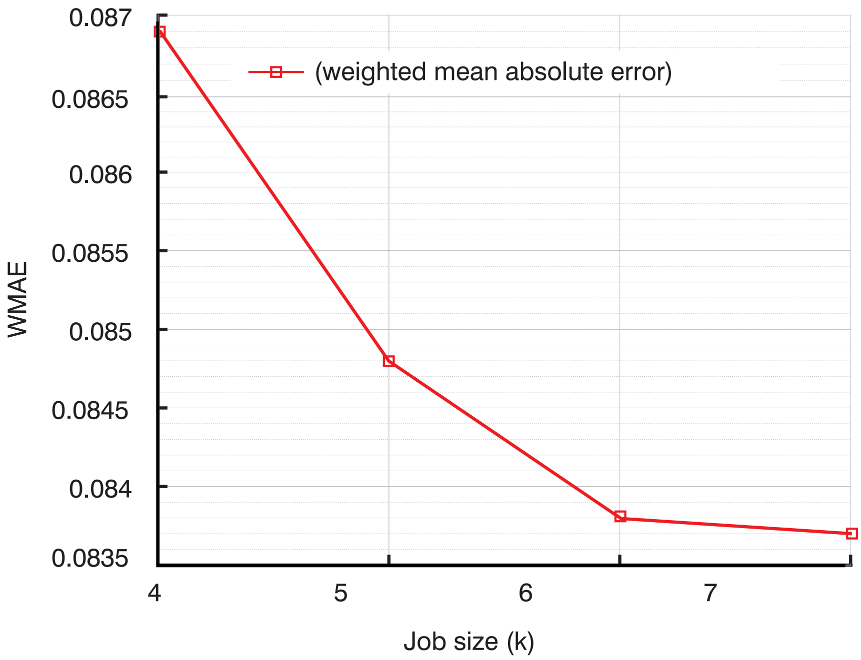

Figure 4. WMAE computation for problems with arbitrary times of processing

5. Comparative study of the proposed heuristic with existing makespan PIH approach [15]

The total waiting time and makespan are obtained for the randomly generated FSS problems of various job sizes for the sequences achieved by applying the proposed heuristic and PIH heuristic given by Nailwal et al. [15]. The problems with random processing times and the problems with special structures have been studied separately for the comparative study.

Table 6. Averages [h] for total waiting time of jobs for problems with random processing times

Job size (k)	W_{heu}	W_{PIH}
5	18.77	47.18
10	118.73	323.73
15	189.02	629.82
20	300.03	1119.17
25	547.67	2141.03
30	779.38	2888.50
35	2575.25	8963.78
40	1492.45	5486.82

In Table 6, computed averages for times of waiting of jobs for problems with random processing times (observed on 60 problems for each job size k), and in Table 7, averages for times of waiting of jobs for specially structured problems have been given using proposed heuristic and existing makespan heuristic PIH given by Nailwal et al. [15]. The data generated in Table 6 has been figured out in Figure 5 and data generated in Table 7 in Figure 6.

Table 7. Averages [h] for total waiting time of jobs for problems with special structures

Job size (k)	W_{heu}	W_{PIH}
5	289.8	356
10	1341.8	1706.23
15	3146.58	3896.55
20	5571.58	6950.8
25	8945.42	11,147.48
30	12,853.62	16022.67
35	18,165.25	22,460.38
40	23,487.18	29,153.45
50	37,277.03	46,074.57
60	54,192.78	66.591.4
70	73,243.1	90, 440.77
80	96,480	118,991.5

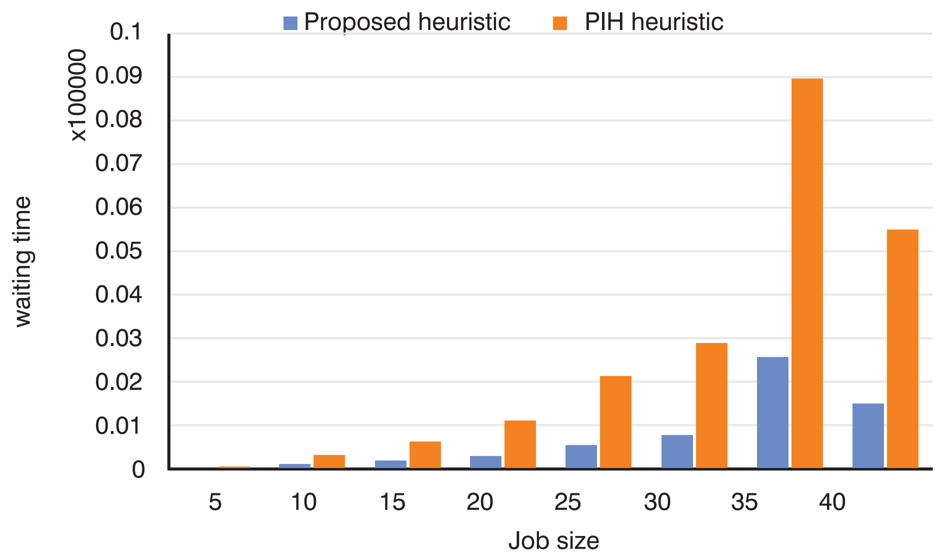

Figure 5. Waiting times of sequence obtained by proposed heuristic and PIH heuristic of problems with random processing times

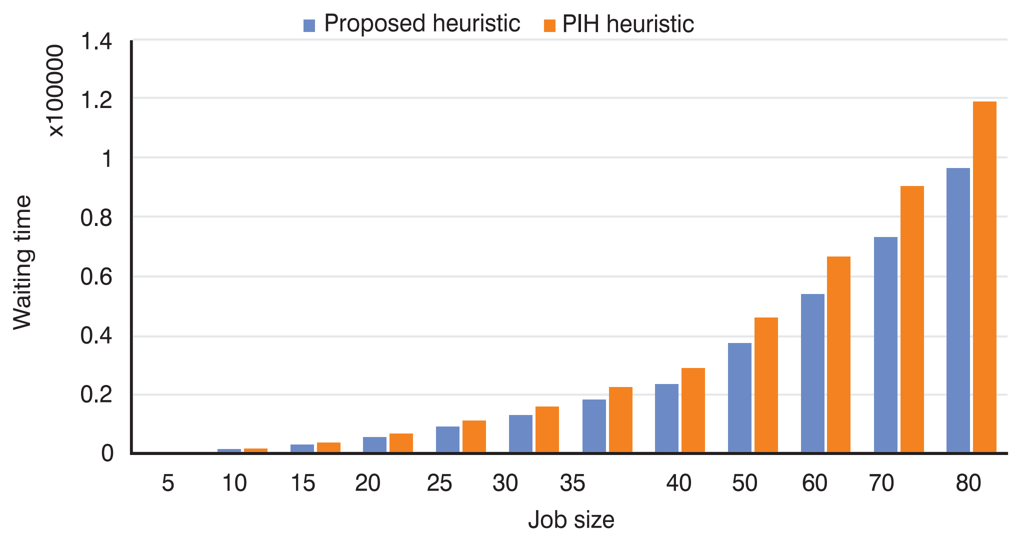

Figure 6. Waiting times of sequence obtained by proposed heuristic and PIH heuristic of problems with special structures

Based on Table 6 and Figure 5, it has been shown that on applying the makespan heuristic PIH, the waiting time has been significantly increased for problems with random processing times, and the client's dissatisfaction may affect the system. Based on Table 7 and Figure 6, it has been shown that the waiting time has been increased for makespan heuristic PIH for problems also with special structures, and it is approaching farther and farther as the job size is increasing.

Table 8. Averages [h] for makespan of problems with random processing time

Job size (k)	C_{heu}	C_{PIH}
5	312	284.43
10	626.62	589.75
15	918.67	877.47
20	1198.93	1155.98
25	1500.32	1447.47
30	1805.57	1753.00
35	2083.30	2040.53
40	2378.48	2321.38

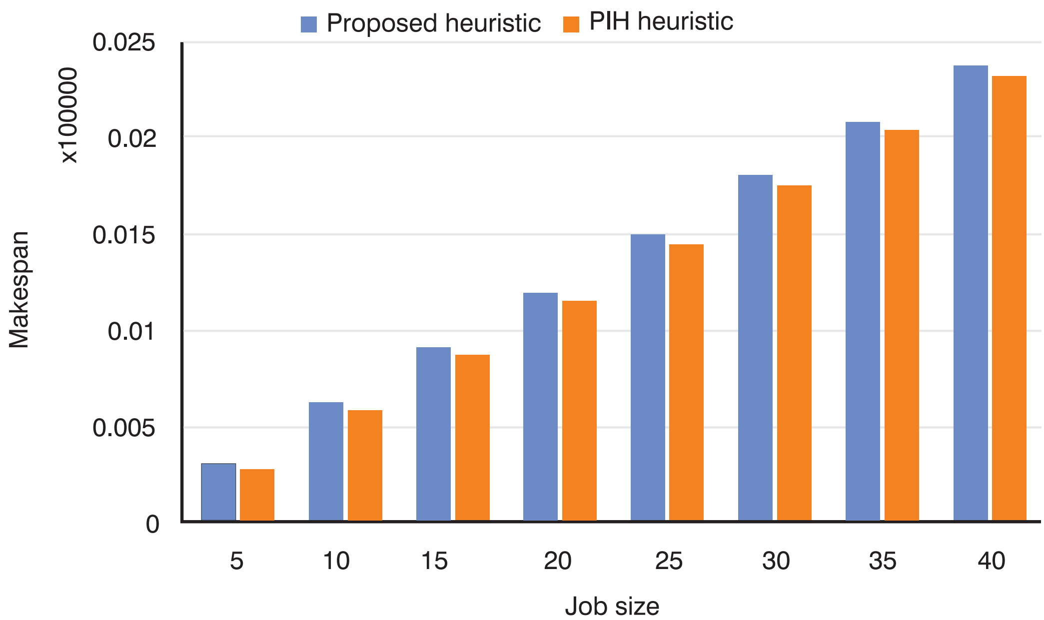

Figure 7. Makespan obtained employing proposed heuristic and PIH heuristic for problems with random processing times

In Table 8, averages for makespan of problems with random processing times and in Table 9, averages for makespan of specially structured problems (Observed on 60 problems for each job size k) have been computed using proposed heuristic and existing makespan heuristic PIH given by Nailwal et al. [15]. The data generated in Table 8 has been figured out in Figure 7 and data generated in Table 9 has been figured out in (Figure 8). From Figures 7 and 8, it is clear that the proposed heuristic also performs better in minimizing the makespan. The proposed heuristic also provides a near-optimal solution to minimize the makespan.

Hence it can be said that the proposed heuristic provides the near optimal solutions for the times of waiting of jobs without affecting makespan significantly whereas the makespan heuristics do not consider expressively the client's contentment as far as the objective of waiting times is concerned.

Table 9. Averages [h] for makespan of problems with special structures

Job size (k)	C_{heu}	C_{PIH}
5	388.68	383.85
10	757.07	750.30
15	1152.92	1146.27
20	1511.22	1502.80
25	1881.53	1874.07
30	2260.28	2250.77
35	2645.02	2635.90
40	3016.12	3008.30
50	3759.45	3752.03
60	4521.72	4512.57
70	5248.67	5239.68
80	6001.78	5993.82

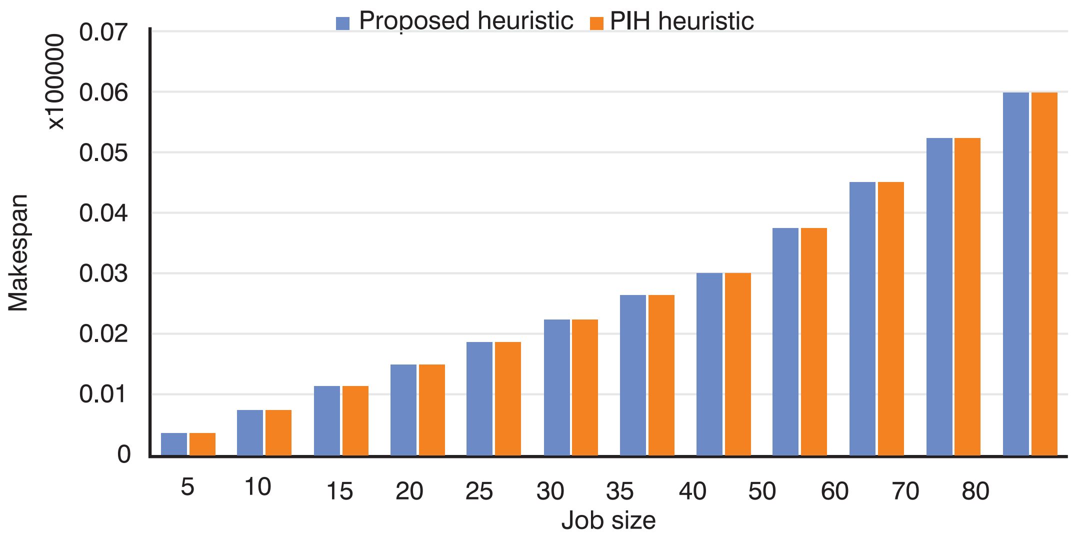

Figure 8. Makespan obtained employing proposed heuristic and PIH heuristic for problems with special structures

6. Concluding remarks

A heuristic approach has been implemented to optimize the total waiting time of jobs. The algorithm developed based on the heuristic approach is defensible by performing computational trials. The results found by trials are also compared with the optimal results for several problems. The method with special structures given by Gupta and Goyal [8, 9] provides a minimum waiting time. The proposed heuristic algorithm has been constructed basically to apply to the problems with randomly generated processing times of both machines. The computational trials and results show that the proposed algorithm delivers optimal or near-optimal resolution to problems with special structures as well. The proposed algorithm has been verified to be operative not only for problems with special structures but also for those with arbitrary processing times. Also by the comparative study, it can be observed that minimizing the desired objective does not affect the makespan significantly.

In today's real world, no client wants to wait for a long time to get the service. The executives of the organization/industry have insights into the study since client contentment is a matter of copious importance. Therefore, the study has been proved to be of prodigious importance for a manager to reduce the client's/job waiting time.

The study may have some limitations. The optimization of the waiting time objective may increase the utilization time of machines. If the machines are hired on rent it may lead to an increase in rental cost. It has been observed that the desired objective leads to an insignificant increase in elapsed time. Besides, its limitations the study yet has been proven to be very effective from the viewpoint of customer contentment.

The conclusion of the study advises to improve the effectiveness of the proposed heuristic. The WMAE can be reduced in the future study. A heuristic can be designed which can optimize both the waiting time and total elapsed time. Times of set up of machines from the times of processing the jobs can be detached. Two or more jobs can be grouped. The problem can be extended with uncertain times of processing.

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