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USEFUL GOVERNMENT EXPENDITURE INFLUENCE ON THE SHADOW ECONOMY

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Abstract: This paper contains the attempt to describe the phenomenon of shadow economy as a zero-sum non-cooperative, normal form game between households and the government. In the model government spending can be treated as a government consumption or as an expenses that contribute to increased social welfare and for the provision of public goods and services. We conduct sensitivity analysis of Nash equilibrium in models with two different types of government expenditure and examine whether proposed models indicate a various mechanisms and determinants of the undeclared economic activity.

Keywords: shadow economy, useful government expenditure, game theory, Nash equilibrium

INTRODUCTION. SHADOW ECONOMY

According to the latest estimates of the shadow economy in 31 European Countries informal sector is from 7,1% (Switzerland) to 31,2% (Bulgaria) of official GDP[1]. Therefore the examination of driving forces of the shadow economy seems to be necessary and extremely important.

In economic literature tax burden and social security contributions are mentioned among the most common determinants of the shadow economy, e.g. [Kozyra–Cybulska et al. 2010], [Schneider 2006], [Patera et al. 2007]. Also [Smuga et al. 2005] highlight complex, confusing and inflexible regulations and poor detection of undeclared activities. The results of 22 different empirical studies are summarized in paper [Schneider, Williams 2013]. Authors note that there are

[1] Schneider F. (2013) Size and Development of the Shadow Economies of Portugal and 35 other OECD Countries from 2003 to 2013: Some New Facts, working paper, p. 5.

several factors which explain 78-98% of the variance of the shadow economy. Taxation and social security contributions, quality of public institutions, public services, regulations of labor market, transfer payments and tax morale are mentioned among those driving forces of the shadow economy. [Kabaj 2009] suggests that one determinant of the shadow economy is high unemployment rate. This is consistent with increase in size of the shadow economy which happened in most OECD countries in 2009 during the economic crisis. [Rosser et al. 2000] show positive relationship between income inequality and the size of the shadow economy in transition economies.

Another determinant is mentioned by [Schneider, Dreher 2006]. According to their research the level of corruption has a significant influence on the shadow economy. This influence is ambiguous and depends on level of economic development. In high-developed countries the shadow economy and corruption are mutually substitutable, while in the case of developing countries - complementary.

Other researches show high correlation between size of the shadow economy and fiscal illusion [Buehn et al. 2012], minimal wage [Maloney, Mendez 2004] or rule of law and quality of institutions [Aruoba 2010].

The purpose of this paper is to analyze in a simple theoretical model impact of useful government expenditures on size of the shadow economy. This paper is organized as follows. Section 1 contains description of the model of shadow economy. This model is an extended version of model presented in [Malaczewska 2013] and was enriched with useful government expenditures. In section 2 the solution of the model is provided and detailed sensitivity analysis is conducted. The last section concludes with a discussion of possible extensions and directions for the future research.

MODEL OF SHADOW ECONOMY

The following model is an extension of the basic model of the shadow economy presented in [Malaczewska 2013], which has been enriched with the idea of useful government spending[2].

We consider a model with two different economic entities: households and government. Households have time endowment $\bar{L}$, which can be divided into activity in the shadow economy (L_s) or into activity in formal sector ($\bar{L} - L_s$). As a result, shadow economy is created by households. However, choice of the size of informal sector made by households depends on economic conditions which are determined by the government. Government has two instruments of economic policy - the average tax rate (τ) and the effectiveness of government's control institutions (by determining the amount of expenses for their activities, denoted by

[2] The concept of wasteful government spending has been applied among others in paper Marattin L., Palestini A. (2010) Edgeworth Dependence and Government Spending Multipliers: a Theoretical Analysis, working paper.

W_K). If households are caught on working in the informal sector, they must pay a fine (P) proportional to the amount of income received in the informal sector. Revenues from taxes and fines government spend on the control institutions or government spending (G). Government spending can be treated as socially useful expenses (useful government expenditures) or as government consumption (from household point of view - wasteful government expenditures). We assume balanced budget, so government revenues are equal to expenses. To sum up, government's budget constraint can be written as:

$$w_r(\bar{L} - L_s)\tau + P = W_K + G \tag{1}$$

where $\tau \in (0,1)$ and w_r (w_s) denotes the average wage per hour of work in formal sector (informal sector). We assume that expected revenues from fines P are given by the equation:

$$\mathrm{P} = \left[\beta\, \mathrm{A}_1\left(1 - \mathrm{e}^{-\phi\, \mathrm{w_k}}\right) + (1-\beta)\mathrm{A}_2 \frac{\mathrm{L_s}}{\bar{\mathrm{L}} - \mathrm{L_s}}\right] \mathrm{aw_s} \tag{2}$$

where $\phi \in \mathrm{R}_+$, $\beta, \mathrm{A}_1, \mathrm{A}_2 \in [0,1]$, and $a > 0$ denotes ratio or multiplicity of wage obtained in shadow economy, which determines the amount of fine.

Household income comes from activity in shadow economy ($\mathrm{w_s L_s}$) and from wages paid in formal sector, but decreased by taxation ($\mathrm{w_r}(\bar{\mathrm{L}} - \mathrm{L_s})(1-\tau)$). All revenues households can spend on consumption C or to pay penalties P. No borrowing is allowed. Summarizing, households' budget constraint is as follows:

$$\mathrm{w_r}(\bar{\mathrm{L}} - \mathrm{L_s})(1-\tau) + \mathrm{w_s L_s} = \mathrm{C} + \mathrm{P} \tag{3}$$

Both the government and households maximize their own utility function. We assume that utility function of government depends on two factors – government spending G and level of social support of government. The government, as an entity elected for the term, must take care about popularity among households, to ensure the possibility of reelection. Thus function of social support of government S was created, which depends on tax burden τ (negative relation) and size of formal sector (positive relation):

$$\mathrm{S} = \mathrm{D}_1 \sqrt{1-\tau} + \mathrm{D}_2(\bar{L} - \mathrm{L_s})^2 \tag{4}$$

where $\mathrm{D}_1, \mathrm{D}_2 \in \mathrm{R}_+$.

The utility function of government is assumed to have the normal properties of being concave with respect to law of diminishing marginal utility and can be written as:

$$\mathrm{U_g} = \frac{(\mathrm{G} + \gamma\, \mathrm{S})^{1-\alpha} - 1}{1-\alpha} \tag{5}$$

where $\gamma \in \mathrm{R}_+, \alpha \in (0, \infty)$.

Similarly, the utility of households is a function of two factors: consumption C and useful government expenditures (ψG):

$$\mathrm{U_h} = \frac{(\mathrm{C} + \psi\, \mathrm{G})^{1-\delta} - 1}{1-\delta} \tag{6}$$

where $\delta \in (0,\infty), \psi \in [0,1)$. Parameter ψ represents how much taxes and penalties paid by households are returned to them in the form of socially useful government spending. When $\psi = 0$ we consider basic model presented in [Malaczewska 2013] without useful government expenditures. In this case, all government expenditures G constitute government consumption and are only used to meet the government needs. When $\psi \neq 0$ some part of government spending contributes to the welfare of society and is used to provide public goods and services. In this paper, we analyze the case when $\psi \neq 0$. We will examine whether extension of the analysis of the shadow economy of useful government expenditures will change significantly equilibrium state of the model and its sensitivity. Then both cases will be compared and appropriate conclusions drawn.

Based on a previous discussion, government maximizes its utility function

$$\mathrm{U_g} = \frac{\left(\mathrm{G}+\gamma\left[\mathrm{D_1}\sqrt{1-\tau}+\mathrm{D_2}(\bar{L}-\mathrm{L_s})^2\right]\right)^{1-\alpha}-1}{1-\alpha} \tag{7}$$

subject to constraint

$$w_r(\bar{L}-L_s)\tau + \left[\beta\,\mathrm{A_1}\left(1-\mathrm{e}^{-\phi\,\mathrm{W_k}}\right) + (1-\beta)\mathrm{A_2}\frac{\mathrm{L_s}}{\bar{\mathrm{L}}-\mathrm{L_s}}\right]\mathrm{aw_s} = W_K + G \tag{8}$$

by choosing tax rate τ and amount of expenses for government's control institutions W_K. Similarly, households choose size of the shadow economy L_s which maximizes their utility function

$$\mathrm{U_h} = \frac{(\mathrm{C}+\psi\,\mathrm{G})^{1-\delta}-1}{1-\delta} \tag{9}$$

subject to the constraint

$$\mathrm{w_r}(\bar{\mathrm{L}}-\mathrm{L_s})(1-\tau) + \mathrm{w_s}\mathrm{L_s} = \mathrm{C} + \left[\beta\,\mathrm{A_1}\left(1-\mathrm{e}^{-\phi\,\mathrm{W_k}}\right) + (1-\beta)\mathrm{A_2}\frac{\mathrm{L_s}}{\bar{\mathrm{L}}-\mathrm{L_s}}\right]\mathrm{aw_s} \tag{10}$$

where $\tau \in (0,1)$, $\psi \in [0,1)$, $\beta, \mathrm{A_1}, \mathrm{A_2} \in [0,1]$, $\phi, \mathrm{D_1}, \mathrm{D_2}, \gamma, \mathrm{a}, \alpha, \delta \in \mathrm{R_+}$.

SOLUTION AND SENSITIVITY ANALYSIS OF THE MODEL

The model has been solved using the method of Lagrange multipliers. The first order conditions can be written as[3]:

$$\begin{cases} \frac{\partial\,\mathrm{U_h}}{\partial\,\mathrm{L_s}} = 0 \Rightarrow -\mathrm{w_r}[1-\tau(1-\psi)] + \mathrm{w_s} = (1-\beta)\mathrm{A_2}\mathrm{aw_s}\frac{\bar{\mathrm{L}}}{(\bar{\mathrm{L}}-\mathrm{L_s})^2}(1-\psi) \\ \frac{\partial\,\mathrm{U_g}}{\partial\,\tau} = 0 \Rightarrow \mathrm{w_r}(\bar{\mathrm{L}}-\mathrm{L_s}) = \frac{\gamma\,\mathrm{D_1}}{2\sqrt{1-\tau}} \\ \frac{\partial\,\mathrm{U_g}}{\partial\,\mathrm{W_K}} = 0 \Rightarrow \phi\,\beta\,\mathrm{A_1}\,\mathrm{aw_s}\,\mathrm{e}^{-\phi\,\mathrm{W_k}} = 1 \end{cases} \tag{11}$$

After several calculations stationary point of the Lagrange function is obtained. Sufficient conditions are fulfilled, so this point is both optimal solution and Nash equilibrium of considered model. The analytical solution of the model, when $\psi = 0$ is as follows:

[3] All calculations are available from the author on request.

$$\begin{cases} W_K^* = \frac{1}{\phi}(\ln\phi + \ln\beta + \ln A_1 + \ln a + \ln w_s) \\ \tau^* = 1 - \frac{w_s}{w_r + (1-\beta)A_2 a w_s \bar{L}\frac{4w_r^2}{\gamma^2 D_1^2}} \\ L_s^* = \bar{L} - \frac{\gamma D_1}{2w_r}\sqrt{\frac{w_r + (1-\beta)A_2 a w_s \bar{L}\frac{4w_r^2}{\gamma^2 D_1^2}}{w_s}} \end{cases} \tag{12}$$

while the analytical solution of the model, when $\psi \neq 0$ can be written as:

$$\begin{cases} W_K^* = \frac{1}{\phi}(\ln\phi + \ln\beta + \ln A_1 + \ln a + \ln w_s) \\ \tau^* = 1 - \frac{-w_r\,\psi + w_s}{(1-\psi)\left[w_r + (1-\beta)A_2 a w_s \bar{L}\frac{4w_r^2}{\gamma^2 D_1^2}\right]} \\ L_s^* = \bar{L} - \sqrt{\frac{(1-\beta)(1-\psi)A_2 a w_s \bar{L} + \frac{1}{4}\cdot\frac{1}{w_r}(1-\psi)\gamma^2 D_1^2}{-w_r\,\psi + w_s}} \end{cases} \tag{13}$$

Following tables 1 and 2 contains results of sensitivity analysis of Nash equilibrium due to changes in parameters values in both cases. Positive (negative) value of the first derivative of any decision variable with respect to given parameter informs us that in a case of two exact economies that are different only in a size of given parameter, the one with greater level of that parameter also has greater (lower) level of decision variable of interest.

Table 1. Sensitivity analysis of Nash equilibrium due to changes in parameters values, when $\psi = 0$

Variable/parameter	β	a	w_S	w_r
W_K^*	$\frac{\partial W_k^*}{\partial \beta} > 0$	$\frac{\partial W_k^*}{\partial a} > 0$	$\frac{\partial W_k^*}{\partial w_s} > 0$	no relation
τ^*	$\frac{\partial \tau^*}{\partial \beta} < 0$	$\frac{\partial \tau^*}{\partial a} > 0$	$\frac{\partial \tau^*}{\partial w_s} < 0$	$\frac{\partial \tau^*}{\partial w_r} > 0$
L_s^*	$\frac{\partial L_s^*}{\partial \beta} > 0$	$\frac{\partial L_s^*}{\partial a} < 0$	$\frac{\partial L_s^*}{\partial w_s} > 0$	$\frac{\partial L_s^*}{\partial w_r} > 0$

Source: own calculations

Table 2. Sensitivity analysis of Nash equilibrium due to changes in parameters values, when $\psi \neq 0$

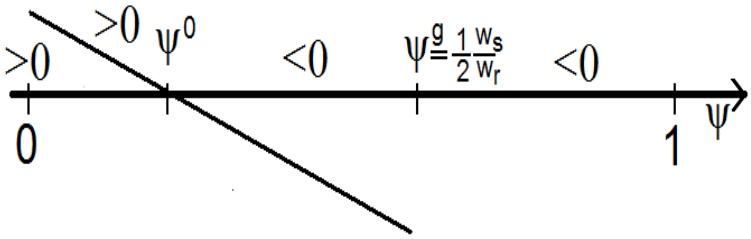
Variable/parameter	β	a	w_S	w_r
W_K^*	$\frac{\partial W_k^*}{\partial \beta} > 0$	$\frac{\partial W_k^*}{\partial a} > 0$	$\frac{\partial W_k^*}{\partial w_s} > 0$	no relation
τ^*	$\frac{\partial \tau^*}{\partial \beta} < 0$	$\frac{\partial \tau^*}{\partial a} > 0$	$\frac{\partial \tau^*}{\partial w_s} < 0$	$\frac{\partial \tau^*}{\partial w_r} > 0$
L_s^*	$\frac{\partial L_s^*}{\partial \beta} > 0$	$\frac{\partial L_s^*}{\partial a} < 0$	$\frac{\partial L_s^*}{\partial w_s} > 0$	$\frac{\partial L_s^*}{\partial w_r} = ?$

Source: own calculations

Analysis of tables 1 and 2 leads us to following conclusions:

- If the probability of detection of activity in the shadow economy dependent on W_K will grow (greater level of β), then it can be expected that consequently the government will raise the level of expenditures on control institutions (hence $\frac{\partial W_K}{\partial \beta} > 0$).
- The increase in w_s makes the work in the shadow economy more attractive. As a result, greater number of households work in shadow economy and the size of informal sector is growing.
- Increase in parameter a leads to increase in optimal level of W_K^*. Increasing a is equivalent to increasing the penalties for activities in the shadow economy. In this case, it is profitable for government to raise expenditures W_K, because it will be compensated by increase of revenues from penalties.
- The rest of the results of the sensitivity analysis, except $\frac{\partial L_s^*}{\partial w_r}$, is consistent with standard economic theory.
- The sign of the partial derivative $\frac{\partial L_s^*}{\partial w_r}$ depends on the value of the parameter ψ, which is depicted on figure 1.

Figure 1. The sign of the derivative $\frac{\partial L_s^*}{\partial w_r}$ depending on the value of the parameter ψ



Source: based on own calculation

By ψ^0 on Figure 1 we denote value of the parameter ψ for which the size of the shadow economy is insensitive to the wage rate in formal sector ($\frac{\partial L_S^*}{\partial w_r} = 0$), and by ψ^g – value of the parameter ψ from which we can unambiguously determine the sign of the partial derivative $\frac{\partial L_S^*}{\partial w_r}$. Level of ψ^0 is given by equation:

$$\psi^0 = \frac{\frac{w_S}{4w_r^2}\gamma^2 D_1^2}{\frac{1}{2w_r}\gamma^2 D_1^2 + w_S(1-\beta)A_2 a\bar{L}} \tag{14}$$

For ψ lower than ψ^0 (in particular for $\psi = 0$) we have $\frac{\partial L_S^*}{\partial w_r} > 0$, so with greater level of w_r greater levels of L_S are associated. When ψ is greater than ψ^0 (in particular for $\psi > \psi^g$) we have $\frac{\partial L_S^*}{\partial w_r} < 0$. Only in this case increase in level of wages in formal sector leads to decrease in the size of the shadow economy.

In model, when $\psi \neq 0$, we can also determine partial derivatives $\frac{\partial L_S^*}{\partial \psi}$ i $\frac{\partial \tau^*}{\partial \psi}$. The results are ambiguous and depend on the relationship between earnings in the informal sector and the shadow economy:

- if $w_S > w_r > w_r(1-\tau) + \tau\, w_r\psi$, then $\frac{\partial L_S^*}{\partial \psi} > 0$ i $\frac{\partial \tau^*}{\partial \psi} < 0$

This result is surprising, because the increasing share of government spending on public goods and services which meet the needs of households (increase of ψ) contributes to the growth of the shadow economy and to the decline in the tax burden. Probably, wages from the shadow economy are so large, that even improved care about needs of society and the tax cuts will not lead to a decrease in the shadow economy, but on the contrary – will contribute to its growth. This requires further studies.

- if $w_r > w_S > w_r(1-\tau) + \tau\, w_r\psi$, then $\frac{\partial L_S^*}{\partial \psi} < 0$ i $\frac{\partial \tau^*}{\partial \psi} > 0$

This result is consistent with our economic knowledge. Households encouraged by useful government expenditures (increase of ψ) leave shadow economy and return to formal sector despite tax increases. Apparently in this case households notice the benefits of paying taxes – bigger part of tax revenue returns as public goods and services provided by the government.

SUMMARY AND CONCLUSIONS

In this paper model describing the shadow economy as a result of interaction between households and government has been created. Each of economic entities maximize their own utility function – households by choosing optimal size of the shadow economy, and government by choosing tax rate and level of expenditures on control institutions. Additionally, model has been extended by useful government expenditures to analyze their influence on the size of shadow

economy. Model with useful government expenditures has been described, solved and compared with basic model. Analysis of the model leads to following conclusions:

1. Extension of the model of useful government expenditures does not change the effect of increase of parameter β, a and w_s on the equilibrium values L_S^*, τ^* and W_K^*.
2. Increasing wages in the formal sector always lead to an increase in taxation.
3. Increasing wages in the formal sector have ambiguous impact on the size of the shadow economy depends on the value of parameter ψ. When the share of useful government expenditures is low (low ψ or $\psi = 0$), then increase in wages in the formal sector leads to increase of the size of the shadow economy and taxation. Therefore, if the government intends to use this instrument (increasing wages in formal sector, e.g. by increasing minimum wage[4]) in order to reduce the shadow economy, it brings the opposite result. On the other hand, when the share of useful government expenditures is high (high ψ), then increase wages in the formal sector lead to decrease the size of the shadow economy, despite an increase in taxation.
4. Also ambiguous relationship is obtained for the effect of changing parameter ψ on the size of the shadow economy and tax burden. It appears that if wage in the shadow economy is significantly larger than the gross wage in the formal sector, then increase of the share of useful government spending leads to the increase of the shadow economy, despite the lower taxes. In this case, increasing ψ will not decrease the size of shadow economy. On the other hand, when wage in the shadow economy is smaller than gross wage in formal sector (but higher than net wage), the increase of ψ will lead to a decline in the shadow economy, even while the taxation has been increased.

Presented model and analysis are not faultless and require some improvements. First of all, it is necessary to extend the model to another economic entity – firms and, as a result, create labor market. Also other theoretical determinants of the shadow economy should be included in the model, such as corruption, fiscal illusion etc. Moreover, the assumption of balanced budget is doubtful and counterfactual. These observations will be developed in further research.

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