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The evolution of the balanced synthetic indicators

Zmiany wskaźnika syntetycznego rozwoju

Streszczenie: Zbadana została ewolucja wskaźników syntetycznych. Obliczamy wartości wskaźników syntetycznych. W tym celu stworzona została macierz prawdopodobieństw przejścia. Celem niniejszej pracy jest przewidywanie wartości wskaźników syntetycznych na następny rok.

Słowa kluczowe: wskaźnik syntetyczny, wzrost, spadek, macierz prawdopodobieństw przejścia

Abstract. The evolution of the synthetic indicators is explored. We calculate the values of the synthetic indicators. For this we built the matrix of the transition probabilities. The purpose of this work is to predict the values of the synthetic indicators for the next year.

Key words: synthetic indicator, increase, decrease, the matrix of the transition probabilities.

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Synthetic indicators are generalizated indicators in economics. They characterise an economic object and an economic situation in general. Hence, we have problems, with the prediction of values of the synthetic indicators for the next year, when we know them this year. The purpose of this work is to find the probability of change of the synthetic indicator for the next year.

We know that a synthetic indicator has a value from interspace [0; 1]. Every region in the country has its synthetic indicator. It allows to investi-gate the economic situation in different regions as well as in the country in general.

Let us consider a country. Supposing the administrative division in this country predicts n regions. And we know the synthetic indicators in all of them for three years, and these years are in sequence. During these three years, the synthetic indicator was changed twice. During this evolution the value can increase, decrease or be constant. In accordance, we can divide all regions into classes. This allows to deal with regions with a similar change of the synthetic indicators. Classification can be the following: the first class contains regions, where there is observed an increase and decrease of the synthetic indicator, the second – only decrease, the third - decrease and increase, the fourth – only increase, in the fifth class the synthetic indicator changed only in the third year, in comparison with the second year, moreo-ver, this change is decrease, in the sixth class in the first change the indicator is permanent and in the second change this indicator increases, in the sev-enth class we observe decrease, but later it becomes permanent, in the eight class firstly we can see increase and then the synthetic indicator is constant, and the last ninth class contains regions, where synthetic indica-tors are permanent during all three years.

There are several regions in every class. Supposing that class number i contain ki regions. Moreover the sum of all of them is equal n. All of this data is introduced in the following table 1.

In order to predict the situation for the next year, we need to find probability that the synthetic indicator will decrease, increase or remind permanent in every class. In the first class after the increase the synthetic indicator decreases. After the analysis, it is easy to see that the indicator decreases in the second year in the second, the third and the seventh classes.

Table 1. Matrix of classes of synthetic indicator

Class number	Number of elements	Change for 3 years
I	k_1	↗↘
II	k_2	↘↘
III	k_3	↘↗
IV	k_4	↗↗
V	k_5	=↘
VI	k_6	=↗
VII	k_7	↘=
VIII	k_8	↗=
IX	k_9	= =

Source: own authoring

Whereas in the third class after this decrease we observe increase, probability that the synthetic indicator in the first class will increase in the fourth year equals $k_3/(k_2+ k_3+ k_7)$. Accordingly, the probability of decrease is $k_2/(k_2+ k_3+ k_7)$, and the probability that indictors will be constant is $k_7/(k_2+ k_3+ k_7)$. In the second class in the second year we observe decrease too, therefore the probabilities are similar. Considering the third class, after decreasing in the second year the synthetic indicator increases in the third year, in comparison with the second year. The increase for the first period was in all regions of the first, the forth and the eighth classes. In that only in the forth class after the increase of the synthetic indicator, we observe an increase again, then probability that the synthetic indicator will increase in regions from the third class is $k_4/(k_1+ k_4+k_8)$. Accordingly, the probability of decrease equals $k_1/(k_1+ k_4+k_8)$, and probability that indicators will be constant is $k_8/(k_1+ k_4+k_8)$. Let us consider another class, where the synthetic indicator is constant during one of these periods. It can be the seventh class. In all regions of this class, after the decrease in the second year, in comparison with the first year, the synthetic indicator has a similar value in the next year. As during the first possible change the synthetic indicators from the fifth, the sixth and the nineth classes were the same, then probability of an increase for the synthetic indicators from the seventh class is $k_6/(k_5+ k_6+k_9)$. The probability of decrease and the probability that indicators will be constant are $k_5/(k_5+ k_6+k_9)$ and $k_9/(k_5+ k_6+k_9)$ accordingly. All another probabilities are in the following table 2 [Yarova 2014, 247-253].

Table 2. The probability of successive values of synthetic indicator

Class number	P(↗)	P(↘)	P(=)
I	$k_3/(k_2+k_3+k_7)$	$k_2/(k_2+k_3+k_7)$	$k_7/(k_2+k_3+k_7)$
II	$k_3/(k_2+k_3+k_7)$	$k_2/(k_2+k_3+k_7)$	$k_7/(k_2+k_3+k_7)$
III	$k_4/(k_1+k_4+k_8)$	$k_1/(k_1+k_4+k_8)$	$k_8/(k_1+k_4+k_8)$
IV	$k_4/(k_1+k_4+k_8)$	$k_1/(k_1+k_4+k_8)$	$k_8/(k_1+k_4+k_8)$
V	$k_3/(k_2+k_3+k_7)$	$k_2/(k_2+k_3+k_7)$	$k_7/(k_2+k_3+k_7)$
VI	$k_4/(k_1+k_4+k_8)$	$k_1/(k_1+k_4+k_8)$	$k_8/(k_1+k_4+k_8)$
VII	$k_6/(k_5+k_6+k_9)$	$k_5/(k_5+k_6+k_9)$	$k_9/(k_5+k_6+k_9)$
VIII	$k_6/(k_5+k_6+k_9)$	$k_5/(k_5+k_6+k_9)$	$k_9/(k_5+k_6+k_9)$
IX	$k_6/(k_5+k_6+k_9)$	$k_5/(k_5+k_6+k_9)$	$k_9/(k_5+k_6+k_9)$

Source: own authoring

There are the probabilities of change of the synthetic indicator in all regions in every class. Then, for the better full analysis of this situation, we need to make the matrix of transition probabilities [Feller, 388-390; Yarova, Yeleyko, Dziekański 2014]. In this case it must be the matrix with nine columns and nine strings. But, we do not know in which class we are. Supposing, we are in the first class with probability p_1, in the second class with probability $p_{2,}$ in the ninth class with probability $p_{9.}$ Moreover, $\sum_{i=1}^{9} p_i = 1$.

Now we can make matrix of transition probabilities for our case. Let us consider the first class. Synthetic indicators in this class decrease during the second change. And we observe decrease during the first year in the regions from the second, the third and the seventh classes. Consequently, the probability of an increase of the synthetic indicator will be the same as the transition probability in the third class, because in this class we observe consistent decrease and increase of the value of the synthetic indicator. Reciprocally, the probability of decrease of the synthetic indicator will be the same as the transition probability in the second class, and the probability that indicators will be constant in the third change, in comparison with the second change, is equal to the transition probability in the seventh class next year [Dziekański 2012, 229 – 242; Dziekański 2013, 180-209]. Observing other classes, the probabilityies of transition of the regions from the first class into the forth, the fifth, the sixth, the eighth and the ninth classes equal zero, because the decrease of the synthetic indicator, which is noticed during the second change in these classes is not observed. Therefore, the synthetic indicator of the regions from first class cannot change such as synthetic indicator regions from these classes. As for the first class, the indicator cannot stay there next year too, because there was an increase in the first change, but we do not know what change had been before this. It should be noted that the probability that the indicators will be constant in the fourth year is equal to the transition probability in the seventh class the next year. The matrix of the transition probabilities has the next view – table 3.

Table 3. The matrix of the transition probabilities

0	$\frac{p_1k_2}{k_2+k_3+k_7}$	$\frac{p_1k_3}{k_2+k_3+k_7}$	0	0	0	$\frac{p_1k_7}{k_2+k_3+k_7}$	0	0
0	$\frac{p_2k_2}{k_2+k_3+k_7}$	$\frac{p_2k_3}{k_2+k_3+k_7}$	0	0	0	$\frac{p_2k_7}{k_2+k_3+k_7}$	0	0
$\frac{p_3k_1}{k_1+k_4+k_8}$	0	0	$\frac{p_3k_4}{k_1+k_4+k_8}$	0	0	0	$\frac{p_3k_8}{k_1+k_4+k_8}$	0
$\frac{p_4k_1}{k_1+k_4+k_8}$	0	0	$\frac{p_4k_4}{k_1+k_4+k_8}$	0	0	0	$\frac{p_4k_8}{k_1+k_4+k_8}$	0
0	$\frac{p_5k_2}{k_2+k_3+k_7}$	$\frac{p_5k_3}{k_2+k_3+k_7}$	0	0	0	$\frac{p_5k_7}{k_2+k_3+k_7}$	0	0
$\frac{p_6k_1}{k_1+k_4+k_8}$	0	0	$\frac{p_6k_4}{k_1+k_4+k_8}$	0	0	0	$\frac{p_6k_8}{k_1+k_4+k_8}$	0
0	0	0	0	$\frac{p_7k_5}{k_5+k_6+k_9}$	$\frac{p_7k_6}{k_5+k_6+k_9}$	0	0	$\frac{p_7k_9}{k_5+k_6+k_9}$
0	0	0	0	$\frac{p_8k_5}{k_5+k_6+k_9}$	$\frac{p_8k_6}{k_5+k_6+k_9}$	0	0	$\frac{p_8k_9}{k_5+k_6+k_9}$
0	0	0	0	$\frac{p_9k_5}{k_5+k_6+k_9}$	$\frac{p_9k_6}{k_5+k_6+k_9}$	0	0	$\frac{p_9k_9}{k_5+k_6+k_9}$

Source: own authoring

Hence, from this matrix of transition probabilities we know all probabilities of change of the synthetic indicator for the next year.

For example, consider a country. It can be Poland. Poland is devided into administrative units, called powiats. And we know the synthetic indicators in thirteen of them for three years, and these years are in sequence. All this information is introduced in the following table 4.

During these three years, the synthetic indicator changed twice. During this evolution the value can increase, decrease or be constant. In accordance, we can divide all regions into classes. This allows to deal with regions with a similar change of the synthetic indicators. Classification can be the following: the first class contains regions where there is observed an increase and decrease of the synthetic indicator, the second – only decrease, the third - decrease and increase, the fourth – only increase, in the fifth class the synthetic indicator changed only in the third year, in comparison with the second year, moreover, this change is decrease, in the sixth class in the first change the indicator is permanent and in the second change this indicator increase, in the seventh class we observe decrease, but later it becomes permanent, in the eight class firstly we can see increase and then the synthetic indicator is constant, and the last ninth class contain regions, where synthetic indicators are permanent during all three years. But, in our situation we have only five first classes [Dziekański 2013 (a), 149-163; Satoła 2015, 115-123].

Table 4. Synthetic indicator of districts in the coming years

№	Powiat	First year	Second year	Third year
1	Opatowski	0,68	0,56	0,71
2	Starachowicki	0,57	0,61	0,47
3	Skarzyski	0,63	0,61	0,54
4	Buski	0,59	0,60	0,53
5	Sandomierski	0,53	0,58	0,45
6	Ostrowiecki	0,49	0,45	0,43
7	Konecki	0,54	0,50	0,43
8	Kielecki	0,44	0,53	0,37
9	Włoszczowski	0,40	0,38	0,35
10	Staszowski	0,44	0,44	0,36
11	Pińczowski	0,32	0,42	0,43
12	Jędrzejowski	0,33	0,39	0,33
13	Kazimierski	0,25	0,38	0,31

Source: own authoring

There are several powiats in every class – table 5.

Table 5. Class districts by synthetic indicator

Class number	Number of powiat	Change for 3 years
I	2,4,5,8,12,13	↗↘
II	3,6,7,9	↘↘
III	1	↘↗
IV	11	↗↗
V	10	=↘

Source: own authoring

In the first class after an increase of the synthetic indicator we mention a decrease. But the indicator decreases in the second year in the second and the third classes. Whereas in the third class after this decrease we observe an increase, the probability that the synthetic indicator in the first class will increase in the fourth year equals $\frac{1}{5}$, because there are four powiats in the second class and one powiat in the third class. Accordingly, the probability of decrease is $\frac{4}{5}$. All another probabilities of increase and decrease are introduced in the following table 6.

Table 6. Probabilities of increase and decrease of the synthetic indicator

Class number	P(↗)	P(↘)
I	$1/5$	$4/5$
II	$1/5$	$4/5$
III	$1/7$	$6/7$
IV	$1/7$	$6/7$
V	$1/5$	$4/5$

Source: own authoring

However the synthetic indicator yet can be constant the next year. Whereas in this classification we can observe it only in the fifth class, moreover it is only during one change, the probability that indicators will be constant is $\frac{1}{26}$. Then for full analysis we need to make the matrix of transition probabilities. In this case it must be the matrix with five columns and five strings [Dziekański 2016, p. 56-63].

Let us consider the first class. In the matrix of transition probabilities the first value is $\frac{1}{26}$, the second –$\left(1-\frac{1}{26}\right)*\frac{4}{5}=\frac{10}{13}$, the third - $\left(1-\frac{1}{26}\right)*\frac{1}{5}=\frac{5}{26}$. The fourth and the fifth value in this matrix is zero, because in the third year in all regions from the first class the synthetic indicator decrease. So this indicator cannot be changed like the synthetic indicators in the fourth and the fifth classes, because we do not observe a decrease of the synthetic indicator in any of them. The matrix of transition probabilities for this classification is as follows – table 7.

Table 7. The matrix of transition probabilities

$$\left|\begin{matrix} \frac{1}{26} & \frac{10}{13} & \frac{5}{26} & 0 & 0 \\ 0 & \frac{1}{26} & \frac{5}{26} & 0 & \frac{10}{13} \\ \frac{75}{91} & 0 & \frac{1}{26} & \frac{25}{182} & 0 \\ \frac{6}{7} & 0 & 0 & \frac{1}{7} & 0 \\ 0 & \frac{10}{13} & \frac{5}{26} & 0 & \frac{1}{26} \end{matrix}\right|$$

Source: own authoring

We can write this matrix of transition probabilities with decimal fractions – table 8.

Table 8. Matrix of transition probabilities with decimal fractions

0,04	0,76	0,2	0	0
0	0,04	0,2	0	0,76
0,82	0	0,04	0,14	0
0,86	0	0	0,14	0
0	0,76	0,2	0	0,04

Source: own authoring

Consequently, the synthetic indicator from the first class will change like the synthetic indicator from the second class the next year with the probability 0,76. It means that the value of the synthetic indicators from the first class will decrease. The synthetic indicator from the second class will be in the fifth class next year, but it must decrease, too. And so on.

Butwe can be in situation when we donot know in which class we are. For instance, consider these probabilities for our situation. Supposing, we are in the first class with probability 0,3 in the second – 0,2 and in the third, fourth and fifth with probabilities 0,2, 0,1 and 0,2 consequently. So, let us consider the new matrix of transition probabilities – table 9

Table 9. Matrix of transition probabilities

0,04· 0,3	0,76· 0,3	0,2· 0,3	0	0
0	0,04· 0,2	0,2· 0,2	0	0,76· 0,2
0,82· 0,2	0	0,04· 0,2	0,14· 0,2	0
0,86· 0,1	0	0	0,14· 0,1	0
0	0,76· 0,2	0,2· 0,2	0	0,04· 0,2

Source: own authoring

After calculation, we obtain the following results – table 10.

Table 10. Matrix of transition probabilities

0,012	0,228	0,06	0	0
0	0,008	0,04	0	0,152
0,164	0	0,008	0,028	0
0,086	0	0	0,014	0
0	0,152	0,04	0	0,008

Source: own authoring

Consequently, the exploring of the evolution of the synthetic indicators allows to make statistical model of the state of the economic situation in the country. We can do it for every country, if only we now the value of the synthetic indicators for the sequent three years.

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