Oksana Mandrazhy
PhD, associate professor of The Chair of Physics and Higher Mathematics, Kharkiv National Agrarian University named after V.V.Dokuchayev, Ukraine
e-mail: OksanaMandraji23@list.ru

Intellectual Property and Copyright Protection as Essential Knowledge for Students

Wiedza Studentów na Temat Ochrony Praw Własności Intelektualnej i Praw Autorskich

Abstract:
The article is dedicated to the revelation of important of study of the basis of law and on a few concrete examples based on mathematical material it has been shown the importance the validity of the legal questions raised.

Keywords:
Students' research activity, intellectual property, copyright protection, the rule of law.

Streszczenie:
Artykuł poświęcony jest znaczeniu nauki prawa własności intelektualnej i praw autorskich w nauczaniu i działalności badawczej studentów. Dla podkreślenia tematu ukazano problem na kilku konkretnych przykładach opartych na materiale matematycznym, co ukazuje znaczenie ważności podnoszonych kwestii prawnych.

Słowa kluczowe:
działalność badawcza studentów, własność intelektualna, ochrona praw autorskich, praworządność.

Problem definition.

The affiliation of certain reasoning to some people, the authorship of sayings, the right to be the founder of assumptions, statements, findings, and inventions – all these issues are likely to have been of interest since ancient times. After all, the history has never revealed to us the name of the one who invented the wheel, but, at the same time, it has recorded a lot of quite unusual situations when people were solving the issues of primacy on the basis of ethics and morality and on the basis of the rule of law. Unfortunately, this is not always taken into account. Nowadays, the problem of intellectual property and copyright protection is quite a pressing issue. The purpose of this article is to draw attention not as much to the lack of references in students' essays, reviews and similar school projects, but mostly to students' understanding of those issues in relation to foreign intellectual property and copyright, and especially with regard to their own copyright and intellectual property.

Presentation of the material.

In school, when students are faced with writing a paper or performing a survey, their teachers, as a rule, among other things always pay attention to the reference list format as well as the mandatory use of links and references. The educators are trying to communicate the idea that borrowing information from the Internet and presenting it as your own achievement is wrong because such an action is similar to theft. Any material we have used should contain a reference to the source. It is important that students understand the difference between authorship, on the one hand, and facts, ideas and principles, on the other hand, as copyright does not extend to facts and ideas. No one can own the right to a fact. Copyright protects only the words this fact is described with.

Students are also introduced to the idea that while writing their school projects, they can use some excerpts of someone else's text (again it is mandatory to use a reference) with informative and scientific purposes or for the purpose of writing critical comments. However, this does not mean that it is necessary to rewrite the entire article. The amount of the citation is defined based on the purpose for which the selected text, thought, or reasoning is used. It should also be accompanied by their explanations, comments or personal understanding of the issue.

Very often in a classroom setting, teachers use parts of texts, assignments, and articles for educational purposes and not in violation of any legal rights or moral laws.

At the same time, we rarely teach students what can be attributed to those developments which are considered intellectual property and should be protected by copyright. For example, if a teacher wrote an article and in it gave the examples of his or her students' research results without their consent or without indicating the students' names, it's definitely a violation of the students' copyright. Even if that article was posted on an educational website, we can't claim that nobody's rights were violated. In this situation, the teacher was bound to know the opinion of the students regarding the use of their findings in his or her article and if I may say so, the teacher had to get their permission. I am sure, such a permission is often viewed as a formality. I think the students are already in awe of the fact that their ideas will get the fame. Nevertheless, the step mentioned above must be made because, without asking the author's permission, we are free to use only the articles printed in newspapers and magazines on current economic, political, social and religious issues. Indeed, such articles we can post without the author's consent, but, as usual, with the obligatory indication of the source.

Some people claim that using the results of students' school projects without their permission is not shameful, especially if it does not bring you monetary profit. However, we must remember that the use of the creative achievements of another person without his/her permission, whoever the author may be or even if it is not about money is still a violation of the law. Overall, the situation described above is a very sensitive issue. In my opinion, we should not, perhaps, forget that regardless of legality, it's moral character and reputation that are not less important.

The cooperation between teachers and students in the classroom, the use of innovative teaching methods can often lead to some very interesting lessons as well as interesting finding. During the class organized in the form of a research, the students can become so passionate that, at some point, they begin to feel they are pioneers, and with shining eyes they perceive the results as something new, hitherto unknown. This might only be a feeling, but for them it seems real. For those students, the results obtained in the research is their own discovery. And even if those materials do not bring anything new to humanity, they are very important for those who received them independently, as a result of their intellectual activity. And sometimes, even though those facts are not new, the thinking, the approaches, and the reasoning leading to obtaining those results may be of some interest. In such moments, it is important to emphasize and explain to the students what can be considered intellectual property and what cannot. Here are a few concrete examples based on mathematical material which I am going to use to support the validity of the legal questions raised.

Exploring geometric fractals, the members of the "Advanced Math Study" club at Small Academy of Sciences in Ukraine were very surprised at seeing fractal's properties. Indeed, if $n \to \infty$, the length of the Sierpinski carpet is $L_n \to \infty$, and its area is $S_n \to 0$. n – the step of the iteration, and the Sierpinski carpet can be constructed in the following way: you should first take a square with sides equal to units of measurement; after that each side of the square is divided into three equal parts, and accordingly, the square is divided into nine equal squares with a side equal to $\frac{1}{3}$. A central square is cut out of the resulting figure. After that, the same thing is done with each of the eight remaining squares, etc. (Fig. 1).

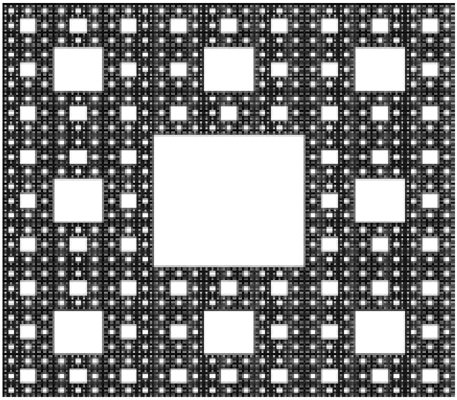

Fig. 1

The surprise was a stimulus to conduct a further research and to get a proof of properties of the considered fractal.

Suppose the side of the Sierpinski carpet is equal to a, then the length of curve that limits it is $L_0 = 4a$. After the first step, when the square in the middle with the side equal to $\frac{1}{3}a$, is cut out, the length of the curve L_1, which limits the area remained, is $L_1 = 4a + 4 \cdot \frac{a}{3} = 4a(1 + \frac{1}{3})$. Reasoning similarly, on the second stage, we have $L_2 = 4a + 4 \cdot \frac{a}{3} + 8 \cdot 4\frac{a}{9} = 4a(1 + \frac{1}{3} + \frac{8}{9})$, on the third stage, we have $L_3 = 4a + 4 \cdot \frac{a}{3} + 8 \cdot 4 \cdot \frac{a}{9} + 8^2 \cdot 4 \cdot \frac{a}{27} = 4a(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3})$. If a closer look is taken, a certain consistent pattern can be noticed, so we can surmise that $L_n = 4a + 4 \cdot \frac{a}{3} + 8 \cdot 4 \cdot \frac{a}{9} + 8^2 \cdot 4 \cdot \frac{a}{27} + \cdots + 8^{n-1} \cdot 4 \cdot \frac{a}{3^n} = 4a \cdot \left(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3} + \cdots + \frac{8^{n-1}}{3^n}\right)$. The correctness of this assumption can be easily proved by the method of mathematical induction on the basis of belonging of the figure to self-similar figures. Thus, starting with $n \geq 2$, we have: if $n = 2,\ L_2 = 4a\left(1 + \frac{1}{3} + \frac{8}{9}\right)$, so the formula is true. Suppose $n = k$, then $L_k = 4a \cdot \left(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3} + \cdots + \frac{8^{k-1}}{3^k}\right)$ is also the correct equality, and we will prove the correctness of the derived formula for $n = k + 1$, i.e. we need to get $L_{k+1} = 4a \cdot \left(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3} + \cdots + \frac{8^{k-1}}{3^k} + \frac{8^k}{3^{k+1}}\right)$. In fact, in order to get the length of the Sierpinski carpet at the stage $k + 1$, the length of all the squares which were added at the next stage should be added to the length of step k. Starting with the second stage, the quantity of squares which are formed is 8 in an appropriate degree, and as a result we have $L = 4a \cdot \left(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3} + \cdots + \frac{8^{k-1}}{3^k}\right) + 4a \cdot \frac{8^k}{3^{k+1}} = 4a \cdot \left(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3} + \cdots + \frac{8^{k-1}}{3^k} + \frac{8^k}{3^{k+1}}\right)$, quod erat demonstrandum.

Let us consider the derived formula again:

$$L_0 = 4a$$

$$L_1 = 4a + \frac{4a}{3} = 4a(1 + \frac{1}{3})$$

$$L_2 = 4a + \frac{4a}{3} + 8 \cdot 4\frac{a}{9} = 4a(1 + \frac{1}{3} + \frac{8}{9})$$

$$L_3 = 4a + \frac{4a}{3} + \frac{32a}{9} + 8^2 \cdot 4 \cdot \frac{a}{27} = 4a(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3})$$

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$$L_n = 4a + 4 \cdot \frac{a}{3} + 8 \cdot 4 \cdot \frac{a}{9} + 8^2 \cdot 4 \cdot \frac{a}{27} + \cdots + 8^{n-1} \cdot 4 \cdot \frac{a}{3^n} = 4a\left(1 + \frac{1}{3} + \frac{8}{3^2} + \frac{8^2}{3^3} + \cdots + \frac{8^{n-1}}{3^n}\right)$$

In the brackets, starting from the second summand, we have a sum of geometric progression, where $b_1 = \frac{1}{3}$ and $q = \frac{8}{3}$, which can be found by the formula: $S_n = \frac{b_1(1-q^n)}{1-q}$.

$$S_n = \frac{\frac{1}{3}\left(1 - \left(\frac{8}{3}\right)^n\right)}{1 - \frac{8}{3}} = \frac{1 - \left(\frac{8}{3}\right)^n}{-5}$$

If n tends to infinity, we have

$$L_n = 4a \cdot \left(1 - \frac{1-\left(\frac{8}{3}\right)^n}{5}\right) = 4a \cdot \frac{4-\left(\frac{8}{3}\right)^n}{5}$$

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} 4a \cdot \frac{4-\left(\frac{8}{3}\right)^n}{5} = 4a \lim_{n\to\infty} \frac{4-\left(\frac{8}{3}\right)^n}{5} = 4a \lim_{n\to\infty} \frac{\frac{4}{\left(\frac{8}{3}\right)^n} - 1}{\frac{5}{\left(\frac{8}{3}\right)^n}} = \infty$$

Thus, together with the students, we have derived a general formula of the length of curve, which limits the area of the Sierpinski carpet, and have determined that with unrestricted growth of steps/stages (when $n \to \infty$), the length of this curve also increases unrestrictedly $L_n \to \infty$.

Now let us consider how to calculate the area of the Sierpinski carpet, how it changes at every step of recursion and what it tends to if $n \to \infty$. According to the considerations given above, we can write:

$$S_0 = a^2$$

$$S_1 = a^2 - \frac{a^2}{9} = \frac{8a^2}{9}$$

$$S_2 = \frac{8a^2}{9} - \frac{8a^2}{81} = \frac{8}{9}a^2$$

$$S_3 = \frac{64a^2}{81} - 8^2\frac{1}{27^2}a^2 = \frac{(9\cdot 64 - 64)a^2}{9^3} = \frac{8^3a^2}{9^3}$$

Having taken a closer look, we can admit that $S_n = \frac{8^n a^2}{9^n}$, and the correctness of the formula can be proved by the method of mathematical induction, based on the fact that the Sierpinski carpet is a self-similar figure. So, if $n = 1$, then $S_1 = \frac{8a^2}{9}$, which is correct. Suppose that $S_k = \frac{8^k a^2}{9^k}$ if $n = k$, and let us prove the correctness of the formula for $n = k + 1$:

$$S_{k+1} = S_k - \frac{8^k a^2}{\left(3^{k+1}\right)^2} = \frac{8^k a^2}{9^k} - \frac{8^k a^2}{9^k \cdot 9} = \frac{9 \cdot 8^k a^2 - 8^k a^2}{9^k \cdot 9} = \frac{8^k a^2 (9-1)}{9^{k+1}} = \frac{8^{k+1} a^2}{9^{k+1}}$$

Thus, with the use of the method of mathematical induction, it has been proved that the area of the Sierpinski carpet at every step can be found by the formula: $S_n = \frac{8^n a^2}{9^n}$.

If $n \to \infty$, then $\lim_{n \to \infty} S_n = \lim_{n \to \infty} a^2 \cdot \left(\frac{8}{9}\right)^n = a^2 \lim_{n \to \infty} \left(\frac{8}{9}\right)^n = 0$.

As a result, we have found an amazing thing: if $n \to \infty$, then $L_n \to \infty$, and $S_n \to 0$.

The study of the basic properties of the Sierpinski carpet gave the opportunity to the students to derive a general formula calculating the length and area. For mathematicians, the conclusion and the proof of these properties is not a complicated task and it is important to explain to students that this is the reason why it is not given in the information materials, but this does not mean that no one has ever done that before. Of course, dealing with these issues, it is very interesting to know how these results were obtained by other researchers, and, therefore, we decided to publish these thoughts (Mandrazhy 2016, p. 37).

It was a similar situation when a member of the club "Advanced Math Study", Mikhaylichenko Igor, and I were discussing the question of finding the area of polygons with vertices at the nodes of a triangular lattice, similar to how to calculate the area of polygons with vertices at the nodes of squared paper by Pick's Theorem. A simple and elegant formula of mathematician George Alexander Peak fascinated Igor and he wanted to pay more attention to it to find an unusual application or an interesting development under new conditions. The formula of Peak is the following: we consider only those polygons, all vertices of which lie at the nodes of a square lattice, i.e. the points where the lines of the specified net intersect. The area of these polygons can be found:

$$S = B + \frac{\Gamma}{2} - 1,$$

B is the number of nodes inside the polygon, and Γ is the number of nodes on the boundary.

While working with a squared paper and admiring the way that in half a minute we were able to find the area of the intricate polygon, we also became curious whether it is possible to deduce a similar formula for polygons that are located on triangle paper. From that moment on, a fascinating research began. The derivation was based on an empirical method that led us to the assumption that to find the area of a polygon drawn on a triangular lattice can be presented in the following way:

$$S = \Gamma + 2B - 2 \qquad (1)$$

B is the number of nodes inside the polygon, and Γ is the number of nodes on the boundary.

Obtaining the formula is a really pleasant moment, but now it is important to think how to prove it. First, we considered the parallelograms all sides of which lie on the given straight lattice (Fig. 2, left). Their area in triangular units is very easily counted based on the number of triangles of which they are composed. Suppose we are given a parallelogram with length of sides equal to m and n, then the number of triangles that make up the length of m will be 1 less than the number m.

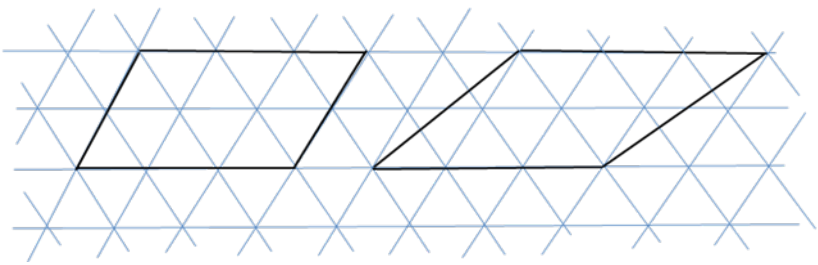

Fig. 2

Let us agree to call parallelograms located along a single straight line of a simple lattice as a row. Then with the side of m except for ($m - 1$) triangles, this number is supplemented by the same number of them. On the n side, the number of such rows will be $(n - 1)$, because the row at the side m is already considered. Thus, we get $S = 2(m - 1)(n - 1)$. Now let us calculate the area using the formula which we are proving. So, if we have a parallelogram with sides m and n, the number of nodes on the boundary Γ is equal to $m + m$ plus the number of nodes on the other two sides, but without the nodes of the vertices that are included in m, that is $\Gamma = 2m + 2(n - 2) = 2m + 2n - 4$. The number of nodes inside of the parallelogram $B = (m - 2)(n - 2) = mn - 2m - 2n + 4$.

Using the formula (1), we will receive:

$$S = \Gamma + 2B - 2 = 2m + 2n - 4 + 2(mn - 2m - 2n + 4) - 2 =$$

$$= 2m + 2n - 4 + 2mn - 4m - 4n + 8 - 2 = 2mn - 2m - 2n + 2.$$

On the other hand, calculating the area of triangle units, we received:

$$S = 2(m - 1)(n - 1) = 2(mn - m - n + 1) = 2mn - 2m - 2n + 2.$$

For these parallelograms, the formula is proved. If a parallelogram has two opposite sides that are not lying on the straight of the lattice (Fig. 2, right), we can always easily obtain a parallelogram of the same area, but with all sides being parts of the lines of the grid by following the step shown in Figure 3. If all the sides of a parallelogram do not lie on the grid, the described action should be performed twice: first for one pair of opposite sides, then for the other.

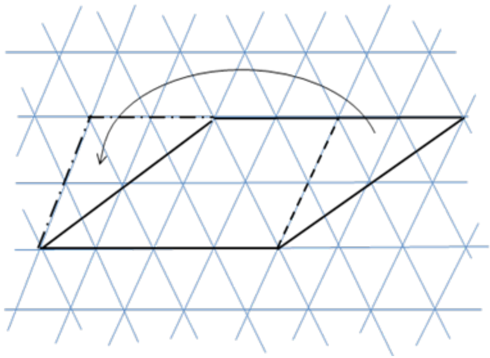

Fig. 3

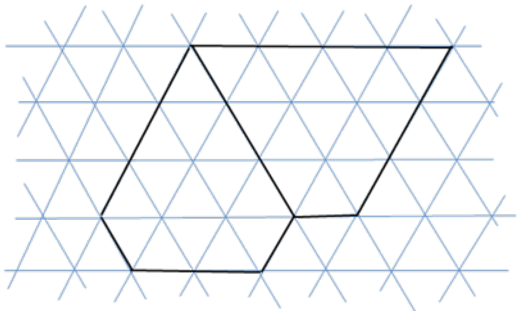

Fig. 4

Next, for the proof we use the fact that the sum of the areas of two polygons is equal to the area of their merging, and note that of two polygons having a common side, we can always make one, if we remove this side. Let us prove that by this procedure the number of $\Gamma + 2B - 2$ for the resulting polygon will be the sum of the corresponding numbers for the original polygons. Suppose we have two polygons M_1 and $\measuredangle_2$ (Fig. 4), for which the number of nodes on the edge are respectively equal to Γ_1 and Γ_2, and the number of nodes inside are B_1 and B_2. Then, according to the property for the area, $S_{1,2} = S_1 + S_2$, where $S_{1,2}$ is the area of a polygon $M_{1,2}$, merging the areas M_1 and M_2. Let us find the areas of all three polygons.

$S_1 = \Gamma_1 + 2B_1 - 2$, $S_2 = \Gamma_2 + 2B_2 - 2$, thence

$$S_{1,2} = \Gamma_1 + \Gamma_2 + 2(B_1 + B_2) - 4.$$

Since the merged polygons have a common side, they also have common nodes. If we name the number of shared nodes O we will find the corresponding values $\Gamma_{1,2}$ and $B_{1,2}$. The number of boundary nodes when merging the polygons can be added, keeping in mind that the shared nodes will be counted twice and besides now they will become internal, except for the two node vertices. Thus, $\Gamma_{1,2} = \Gamma_1 + \Gamma_2 - 2O + 2$. The number of nodes inside will be enriched by the nodes, which we already mentioned, that is $B_{1,2} = B_1 + B_2 + (O - 2)$. So we have $S_{1,2} = \Gamma_1 + \Gamma_2 - 2O + 2 + 2(B_1 + B_2 + (O - 2)) - 2 = (\Gamma_1 + \Gamma_2) + 2(B_1 + B_2) - 4$.

As you can see, the formulae matched and that's what we wanted to prove. Now we have to conclude that the number $\Gamma + 2B - 2$ for any triangle obtained by dividing a parallelogram with a diagonal will be equal to half of the corresponding sum for a parallelogram and, therefore, because we are talking about parallelograms, it will mean the area of a triangle. But any polygon with vertices in the nodes of a triangular grid can be obtained by combining and then deleting a few of such triangles. So our statement is true for any polygon with vertices in the nodes of a triangular lattice. Note that the formula for area for a triangular lattice differs from the formula of area of square for a factor of 2. Indeed, from a square with the area of 1 we can easily get a rhombus with an acute angle 30° which is divided into two equilateral triangles by a smaller diagonal, thus, the triangular lattice has the area of 2.

Igor and I have not found the sources where the deduced formula will be described, however, this is not a reason to say that such reasoning hasn't been done before. It was therefore decided to write an article in which we shared the progress of the research and its results (Mandrazhy, Mikhaylichenko 2014, p. 37).

Another example for the reasoning is presented by one of the participants of the "Advanced Math Study club", Kyrylova Amiliya. During math classes when studying the topic of "Geometric progression", Amiliya was faced with the fact that the textbook gives a standard formula nth term of a geometric progression $b_n = b_1 \cdot q^{n-1}$ and in the section «Examples how to solve main tasks» it all comes down to using only this formula. For example, in the same textbook, we find a task where you have to calculate b_{12}, if we know two consecutive members of a geometric progression, b_3 and b_4. It is clear that to find b_{12}, a student can first find $q = \frac{b_4}{b_3}$, and then, using the formula nth member of a geometric progression for b_3 or b_4 find b_1 so that then, knowing b_1 and q, again according to the formula of the nth member, the student can calculate b_{12}. Of course, the way of thinking we describe is not the only possible approach to finding a solution, but we were trying to come to the result using only the formulae from the corresponding textbook section. Therefore, it becomes interesting to solve a task where we are asked to find b_{12}, if we know b_2 and b_4. In the section «Let's solve it together», we see a similar task that is recommended to solve using a system approach. That is, it is known that there are two members of a geometric sequence $b_4 = 2$ and $b_7 = -54$, we must find b_1 and q. It is proposed, using the formula of nth member, to write the following: $b_4 = b_1 \cdot q^3$, and $b_7 = b_1 \cdot q^6$, to make a system $\begin{cases} 2 = b_1 \cdot q^3 \\ -54 = b_1 \cdot q^6 \end{cases}$ and solve it using term dividing, that is dividing the second equation into the first one. As a result of the division, they get $q^3 = -27$, and $q = 3$, then they substitute the value found in the first equation and find $b_1 = -2/27$. As we can see, the authors while finding the solution are trying to rely only on the formulae from the text of the corresponding textbook section and it is good. But still, if we can solve the task by reasoning, then this method, in our opinion, is not only necessary, but also very important to be shown. Because the analyzed task could have led to a very interesting research. As we have described, knowing two consecutive members of a geometric sequence b_3 and b_4 and understanding the definition of geometric progression, a student easily finds the denominator $q = \frac{b_4}{b_3}$. But on the condition that b_2 and b_4 are known, a student will have to think and come to the conclusion that the system b_4 to b_2 gives q^2. Thus, if b_4/b_2, we get q^2 (or in case of b_7/b_4 we will get q^3). The described approach could be generalized by assuming that q can always be easily found for any two members of a geometric progression (not necessarily the first one). Suppose we are given b_x and b_y and that $y \neq x$ (for clarity, let us assume that $y > x$):

$$q^{y-x} = \frac{b_y}{b_x}, \quad b_x \neq 0.$$

Of course, the formula remains valid provided $b_y = b_x$.

For example, let us take $b_5 = 27$ and $b_3 = 3$ then

$$q^{5-3} = \frac{27}{3}$$

it turns out $q^2 = 9$,

$q = 3$.

For the proof, we will use the basic formulae from the textbook section and we will write:

$b_y = b_1 \cdot q^{y-1}$, a $b_x = b_1 \cdot q^{x-1}$, then

$$\frac{b_y}{b_x} = \frac{b_1 \cdot q^{y-1}}{b_1 \cdot q^{x-1}} = \frac{q^{y-1}}{q^{x-1}} = q^{y-1-(x-1)} = q^{y-1-x+1} = q^{y-x}$$

I wonder if we can use the formula provided $b_y = b_x = 0$? And is it possible to rewrite the proved equation using arithmetical root? All these are questions for further research. Generalizing all that can be said based on the described formula, it is easy to get a useful result, in which initially to calculate any member of a geometric progression, instead of the standard school formula we can use the formula for which knowing the first member is not necessary: we only have to know any member of the progression b_x and q:

$b_n = b_x \cdot q^{n-x}$.

Yes, in the textbooks we have not found these formulae, but mathematicians understand that the very essence of progression is in it. For Amiliya, the derived formula was something new and unexplored, and in this situation it is very important to explain to the student that this approach contains the arguments themselves while solving such problems, but it is not shown in a generalized form, however, in no means, does it diminish the importance of the study (Mandrazhy, Kyrylova 2016, p. 52).

While working with the students in our “Advanced Math Study” club, my students and I face such situations quite often. For example, during a 10-grade math class, we were solving the following problem: from point M, which belongs to the angle BAC, we drew lines perpendicular to AB and AC. The lines lengths are $\sqrt{7}$cm and $2\sqrt{7}$ cm. We need to find the length of the segment AM, if the angle BAC is equal 60°.

The task was solved in different ways, one of which was as following: we can consider the triangle BCM (Fig. 5), in which $\angle BMC = 120°$ (for example, of the quadrilateral $ABMC$) and using the law of cosines, we can find BC: $BC^2 = 7 + 28 - 2\sqrt{7} \cdot 2\sqrt{7} \cdot cos120°$. Thence $BC = 7$ cm.

Further, if you look closely at the quadrilateral $ABMC$, we can easily see that the points A, B, M and C are the points of the same circle ($\angle ABM + \angle MCA = 180°$).

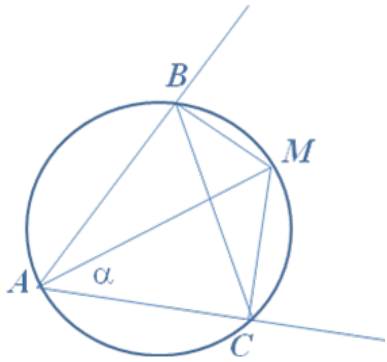

Fig. 5

Thus, we have: the triangle ABC is inscribed in a circle, $\angle A = 60°$ by condition, we found $BC = 7$ cm, the sought one for AM, the diameter of the built circle (the right angles are subtended by AM). The result we get from the law of sines is: $\frac{7}{sin60°} = 2R$, $2R = \frac{14}{\sqrt{3}}$, $2R = AM = \frac{14}{\sqrt{3}}$ cm.

In the previous solution, we used an additional construction. We built a circle. This figure prompted the members of our club to get to the following idea. This angle $60°$ corresponds to the chord $BC = 7$ cm, and the angle $90°$ corresponds to the diameter AM. Is it possible to set up a correspondence? After all, the circles of different diameter are correspondent to the different chord lengths that subtend the arc $60°$. So how do we make such a correspondence? Does it even exist? The first stage of All-Ukrainian contest and defense of scientific research projects of students-members of Minor Academy of Sciences of Ukraine, where we took the problem to solve from, takes place in the fall, so the tenth-grade students at this point only know some basic information from calculus, which can help them to hold their own, although minor but independent, study. We can't help mentioning this wonderful atmosphere of this search, when everyone is thinking hard, suggesting, doubting, and giving argumentation. In collaboration, students receive the answer to the question as well as one more way of solving the problem. So, let us find what can connect the chord, the corresponding inscribed angle and the diameter and the ventral angle which corresponding to the inscribed one. And what is the most convenient way to arrange a central angle of $120°$, so that we could use it to establish the required correspondence? Finally, what was suggested can be seen in Fig. 6.

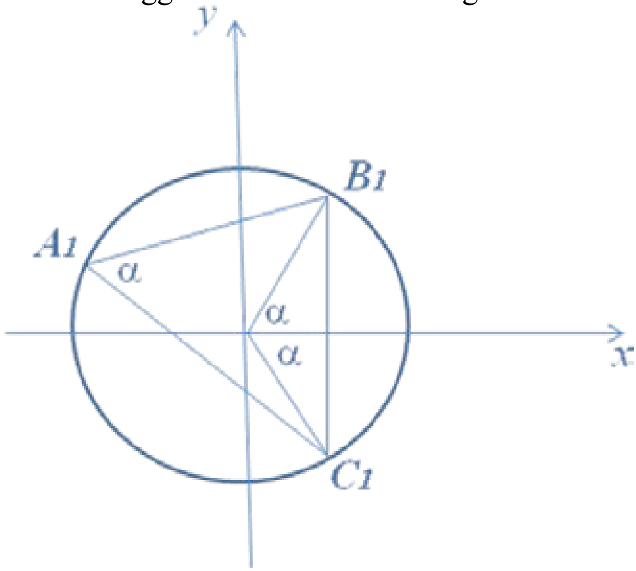

Fig. 6

The chord BC corresponds to the inscribed angle of $60°$ or the corresponding central angle of $120° = 60° + 60°$. For the purpose of derivation of a formula, let us consider a unit circle, i.e., $d = 2$, in which the chord B_1C_1 also corresponds to the inscribed angle of $60°$ and is perpendicular to the axis OX. The corresponding central angle is $B_1OC_1 = 120°$. $B_1C_1 = \sqrt{3}$ as the lengths of $\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$. In the task $BC = 7$, we will make a proportion (the value of $sin\alpha$ is not dependent on d because the triangles for α at the vertex O are similar): $d = B_1C_1 \cdot \frac{2}{\sqrt{3}}$. Thus, $AM = 7 \cdot \frac{2}{\sqrt{3}} = \frac{14}{\sqrt{3}}$.

The problem is solved, but we can also find other formulae for the angles of, for example, $30°$ or $45°$. Actually, we can create an entire table for angles that are often found in school tasks. The students were so enthusiastic that they began to feel as if they were creating additional information on trigonometry. But in fact, they just mastered the basics of trigonometry, and did it on their own. It is very important to be objective, to show students that in their reasoning was the history of trigonometry, but it does not diminish the relevance of their search and research.

Conclusions.

School research is an important activity for students, which, as we hope, will help them form good habits not only to participate in real discoveries in the future, but also to demonstrate a high-quality performance of their work. The research is a good soil to show the students some examples of legal aspects relating to intellectual property and copyright. Let us be respectful to one another without violating anybody's rights.

References:

1. **MANDRAZHY O.**, 2016 *Amazing Properties of Fractal Figures*, International Journal of Pure Mathematical Sciences, Switzerland, 2016, Vol. 16, pp. 37–43. – https://www.scipress.com/IJPMS.16.37
2. **MANDRAZHY O., MIKHAYLICHENKO I.**, *Simple and elegant Formula*, European science review, May-June, Vienna, 2014, № 3, pp. 37–40.
3. **МАНДРАЖИ О., КИРИЛЛОВА А.**, *Загадки геометрической прогрессии*, The scientific heritage, Budapest, Hungary, 2016, No 6 (6), P.2, p. 52–54. – http://tsh-journal.com/wp-content/uploads/2016/12/VOL-2-No-6-6-2016.pdf