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Pricing-inventory model with discrete demand and delivery orders

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Abstract

This paper aims to develop an inventory model considering discrete demand, coordinated pricing, and multiple delivery policies in a single-buyer single-supplier production-inventory system. The shortage is not allowed and the planning horizon is considered to be infinite. The main objective of the framework is to equip the decision-maker with optimal order, pricing, and shipment quantities to maximize the total profit of the system. The results obtained from the numerical example reveal that the proposed approach with an average selling price equal to about 94% of the classical model, has resulted in an average profit increase of about 16% and an average order increase of about 34% compared to the classical approach.

Keywords: *discrete demand, pricing, multiple discrete deliveries, optimization*

1. Introduction

Integrated production inventory planning as an operations research and management science problem has received a considerable amount of attention [34]. It strongly depends on materials handling, which significantly affects the costs and the capability of revenue generation. It also regulates material flow within and between the various organizations along with the integrated system [44]. Besides, in today's globalized competitive business environment, manufacturers are looking for supply and distribution of materials, components, and finished products esall over the world. If they do not achieve reliable delivery, it will be switched from one supplier to another [18]. Therefore, the importance of integrated production-inventory management is further declared [61]. Managing the production-inventory system is strongly related to determining the volume and order points. Therefore, several models developed address this issue in the relative literature.

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The first inventory model, named economic order quantity (EOQ), was introduced in 1913; to develop the replenishment policy, the economic production quantity (EPQ) model was introduced. However, so many assumptions addressed in these models make them far from real-world situations [41]. Recent research papers try to remove a number of these assumptions to obtain a more realistic model. For example, although many companies face discrete demand that directly depends on the selling price, the models assume demand is continuous and constant. Just in time (JIT) is the other inventory system that is not only capable of reducing inventory cost but ensuring the reliability of the supplier's production system, using a small-batch delivery strategy [56]. As an integrated system, the JIT has better performance than other ones [6] in terms of linking manufacturer and retailer in a mutually rewarding, long-term partnership to provide a cost-effective inventory system [37]. This paper tries to develop an original approach to obtain an integrated production-inventory system through the JIT concept. It considers discrete demand, coordinated pricing, and multiple delivery policies in a single-buyer single-vendor production-inventory system. The vendor and the buyer share their costs to achieve a globally optimal solution along a two-stage supply chain. The objective of the system is to optimally determine how many items should be ordered and how many items should be delivered per shipment. According to the authors' knowledge, no inventory model in the relative literature addresses price-dependent demand, discrete demand, and multiple deliveries simultaneously, in a two-stage supply chain. The shortage of the final product is not allowed and the planning horizon is considered to be infinite. The contributions of the paper are briefly as follows:

- formulating an integrated production-inventory system through single set-up multiple delivery (SSMD) strategies and the JIT inventory philosophy is the main contribution of the paper;
- proposing an optimal framework that addresses the discrete and periodic retailer's demand while considering the discrete and periodic delivery is the other main contribution of the research;
- the pricing decisions and price-dependent demand rates included in the developed framework to solve the resulting general integrated production-inventory planning problem through an innovative solution approach.

The rest of the paper is organized as follows. A review of the relative literature is provided in Section 2. In Section 3, the model for the integrated single-vendor single-buyer inventory model is formulated. Section 4 examines the effectiveness of the model through numerical experimentation. And, finally, Section 5 provides conclusions and opportunities for future research.

2. Literature review

The concept of the integrated production-inventory system was popularized in 1977 by Grubbström and Lundquist [23], and since then many variations have been developed. Kim and Ha [32] introduced an SSMD model for a vendor-buyer system. It considers frequent and small-lot deliveries that may be integrated with the JIT inventory system to minimize the cost function of the system. Using a non-periodic JIT system, Rau and OuYang [43] developed a similar model with a finite time horizon and a linear trend for the demand. A multi-stage production-inventory system using the JIT delivery policy and Kanban is proposed by Wang and Sarker [55] and further solved by the branch and bound method. More advanced Internet-of-things-based JIT-oriented inventory systems are also reviewed in [19] for manufacturing logistics systems. Zanoni and Zavanella [64] studied an integrated made-to-order production-inventory system in a steel production industry considering finite capacity. Law and Wee [33] investigated this system from the perspectives of both the manufacturer and the retailer. Chung and Wee [9] included pricing policy, imperfect production, inspection planning, warranty period, stock-level-dependent demand, partial backorder, and inflation in the same problem. Taking into account discrete and periodic delivery policy in an EPQ model, Pasandideh and Niaki [42] formulated a model and proposed a genetic algorithm to solve it. Yan, Banerjee [59] developed an integrated manufacturer-retailer system considering discrete delivery lot quantity and an exact cost function. The same modeling approach was introduced by Sarkar [49] and Chang [8] to find the system's minimum cost using the algebraic approach. They were also used by Cárdenas-Barrón [5] to develop the derivation of EOQ/EPQ inventory models with two back-order costs and have been previously introduced by Sphicas [52]. Wee and Widyadana [33] integrated a single-vendor single-buyer production-inventory model considering multiple deliveries and lost sales to lessen the inventory cost. Jha and Shanker [27] formulated an integrated single-vendor multi-buyer system using a batch production policy. Cao and Hu [4] claimed that addressing discrete delivery in the model formulation may obtain a better optimal solution with lower cost regarding the step-wise characteristics of a multi-delivery strategy. Comparing the single set-up single delivery (SSSD), SSMD, and multiple set-up multiple-delivery (MSMD), Kim and Banerjee [31] further claimed that the SSMD is the best delivery policy whenever the set-up cost is relatively high. Hoque and Goyal [25] developed an optimal policy for the same system by introducing successive batches of a lot transferred to the buyer in a finite number of unequal and equal sizes. Giri and Sharma [14, 15] also developed the same approach by examining unequal shipments, using renewal theory. Sadeghi, Makui [48] addressed a multi-level assembly system with random lead time with periodic interval demand and random lead time. Maddah and Noueihed [36] studied an EOQ model considering random demand. AlDurgam, Adegbola [1] introduced an integrated single-vendor single-manufacturer production-inventory model considering stochastic demand in an SSMD system.

Feng and Chan [11] studied the pricing decisions and price-dependent demand rates in the integrated production-inventory planning problems. They claimed that price is a major factor in demand, based on marketing and economic theory. Considering a price-sensitive demand, Weng [57] introduced a single-vendor single-buyer model. Khan and Shaikh [30] formulated a mathematical model of economic order quantity by considering price as a decision variable and proving their optimality. They assumed that demand is dependent on price and also, shortages are considered and these depend on the customer waiting time. And Khan, Shaikh [29] also considered the EOQ model with full and partial payment, assuming shortages are allowed and the demand function is considered as price and stock-dependent. Khan, Shaikh [28] formulated a mathematical model for a single deteriorating item with demand dependent on the frequency of advertisement and the selling price of the product. And also used the advanced payment policy and assumed that shortages were allowed. Using a multi-replenishment scenario in a finite period system, Datta and Paul [10] analyzed an inventory system where the demand rate is influenced by both displayed stock level and selling price. Considering a quantity discount pricing strategy, Yang [60] developed an optimal pricing and ordering policy for a deteriorating item with price-sensitive demand.

Golpîra [16] considers an agile manufacturing setting for single-product under a vendor-managed inventory (VMI) strategy to seize a new market opportunity by using bilevel programming. Although

Table 1. Summary of the features of surveyed publications

Authors	Vendor–buyer model	Finite production rate	Infinite horizon	Price -dependent demand	Discrete delivery orders	Discrete demand
Harris [24]			✓			
Goyal [21]	✓		✓			✓
Datta, Paul [10]				✓		
Weng [57]			✓	✓		
Goyal, Nebebe [22]	✓	✓	✓		✓	
Ouyang, Wu [40]		✓	✓			
Yang [60]			✓	✓		
You [62]			✓	✓		
Mukhopadhyay, Mukherjee [38]			✓	✓		
You, Hsieh [63]				✓		
Pasandideh, Niaki [42]	✓	✓	✓		✓	
Widyadanaa, Wee [58]	✓	✓	✓		✓	
Wee, Widyadana [56]	✓	✓	✓		✓	
Alfares, Ghaithan [2]		✓	✓	✓		
AlDurgam, Adegbola [1]	✓	✓	✓		✓	
Fu, Chen [12]	✓	✓	✓		✓	
Omar, Zulkipli [39]	✓	✓	✓		✓	
Chan, Fang [7]	✓	✓	✓			
Sadeghi [46]		✓	✓		✓	✓
Liu, Li [35]	✓	✓			✓	
Sadeghi [45]		✓	✓		✓	✓
Ben-Daya, As' ad [3]	✓	✓	✓		✓	
Sadeghi, Golpîra [47]	✓	✓	✓		✓	✓
This paper	✓	✓	✓	✓	✓	✓

dealing with inventory in the construction industry is different [20], Golpîra [17] proposes the first mathematical framework that successfully captures the outcomes and role of the VMI strategy for the problem of construction supply chain integration. Hsiao and Yu [26] consider the same strategy for deteriorating items using the SSSD, and SSMD policies. They assume that the shortage is not allowed and that both researchers and industry with constructive management insight in inventory management decisions, minimizing the overall cost of item deterioration and overall carbon footprint. Sarkar and Debnath [50] assume both the supplier and the manufacturer follow through with an SSMD policy for shipment in a two-stage supply chain. Sarkar and Chung [51] proposed a model to obtain the optimal flexible production rate with the reduced total cost of the supply chain also through the SSMD policy. You [62] investigated the problem of jointly determining the order size and optimal prices for a perishable inventory system under the condition that demand is time- and price-dependent. Mukhopadhyay and Mukherjee [38] introduced an inventory replenishment policy for deteriorating items addressing a price-dependent demand. You and Hsieh [63] developed a continuous inventory model to find the strategy for an enterprise that sells a seasonal item over a finite planning time. The model aims at maximizing the expected profit by determining the optimal ordering quantity and price-setting strategy. Srivastava and Gupta [53] studied an EPQ model for a single product with price-dependent demand under a markdown policy. Zhang, Bai [65] introduced the problem of simultaneously determining the price and inventory control strategies for deteriorating items. In the model, the rate of deterioration is reduced through effective preservation technology investment, and the demand rate is a function of the selling price.

Teksan and Geunes [54] developed a generalized EOQ model for an end item. They consider that the quantity of input components available for production is a non-decreasing function of the price offered by the manufacturer to its vendor, with a price-dependent demand. Alfares and Ghaithan [2] simultaneously considered the variability of the demand rate, the unit holding cost, and the unit purchase cost to model an inventory system. It includes a selling price-dependent demand rate, a storage time-dependent holding cost, and an order size-dependent purchase cost based on an all-unit quantity discount. For obtaining more clarity, the critical features of some more relative publications surveyed in this section are summarized in Table 1.

3. Problem description and notations

3.1. Problem definition

Taking into consideration the discrete step-wise lot delivery policy, a single-supplier single-buyer system is addressed in this paper. The demand is considered to be handled by the buyer as a multi-period and discrete schema. Such price-dependent discrete demand $D(v)$ is also captured as a function of the selling price v per period. The time between two consecutive demands is denoted as t_s. This is because the lower the selling price, the higher the annual demand is. In each cycle time T, the buyer orders q units of the product, and the supplier sends the products to the buyer through a step-wise strategy.

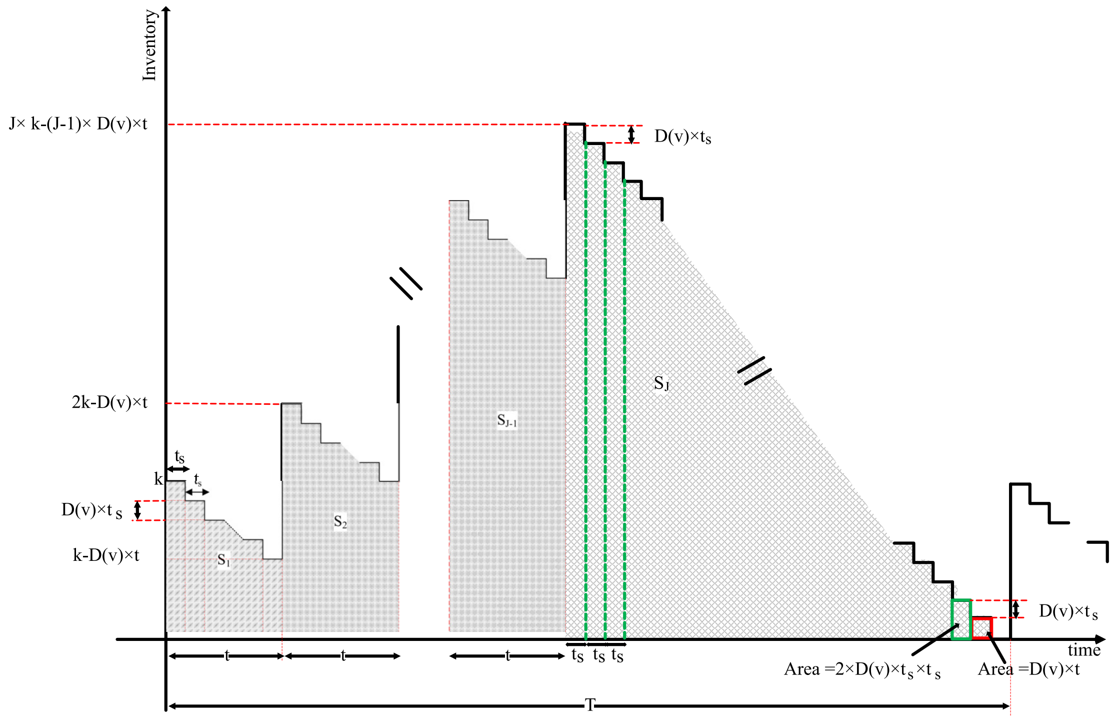

Figure 1. The proposed inventory policy

As shown in Figure 1, according to the SSMD strategy taken by the supplier, k units of the products are supplied with production rate P in each of the J steps during the interval time t. In other words, the buyer requests q unit of product in each cycle time. Given the perishability of the products, it is better not to deliver all the order quantities in one shipment. Delivering the products at the m stage makes it possible to decrease the amount delivered. This is more acceptable for the buyer due to decreasing the chance of perishability of the deliverables. The supplier delivers the issued order at the m stage and sent k units of a product at every stage. So, the amount of each order is equal to Jk. The buyer faces C as the unit selling price from the supplier and also the fixed order cost A as well as the holding cost h and the batch transportation cost B. The objective of the model is to determine the optimal order quantity, the optimal number of deliveries, and the selling price handled by the buyer. To do so, some assumptions are considered in the model formulation:

- The model is designed for a single-supplier single-buyer system.
- The strategy of delivery follows the SSMD context.
- The production rate is constant through the known production time cycle that is greater than that of the annual demand.
- The demand of the buyer is discrete, periodic, and price-dependent.
- The production rate is constant.
- The shortage is not allowed.
- Transportation time between the vendor and the buyer is considered.

3.2. Notations

The following notations will be used through the model formulation provided in Section 4:

Parameters

p	– supplier's production rate
$D(v)$	– buyer's price-dependent discrete demand, unit/year
T	– duration of a cycle, year
t	– interval time between two sequential shipments, year
t_s	– the time between two consecutive demands, year
J	– number of shipments in each cycle
h	– holding cost per unit, \$
B	– unit transportation cost per shipment, \$
C	– unit purchasing cost, \$
A	– fixed ordering cost per cycle, \$

Decision variables

v	– selling price, \$
k	– shipment quantity
q	– retailer order quantity per cycle

4. Problem formulation and solution approach

In the following subsections, a mathematical model is formulated to support the proposed framework. A novel heuristic-based solution approach is clearly defined to obtain a globally optimal solution, which is clearly discussed and further investigated using a numerical example in Section 5.

4.1. Problem formulation

As aforementioned, the system considered in this paper includes a buyer and a supplier, which should be optimized from the perspective of the buyer. Given the fixed ordering cost A, if the number of shipments at each cycle sets to J, the total order in each cycle is given as Jk. With the unit purchasing cost C, the total purchasing cost can be calculated as CJk. The total transportation cost can be calculated as JB paid for m shipment with the unit transportation cost B. After the order delivery, the holding cost $H(q,\, v,\, k)$ is paid by the buyer during each order cycle time with duration T. According to Figure 1, the total inventory holding cost is subdivided into J terms corresponding to J shipment that occurred during the ordering cycle.

Each term is devoted to the inventory holding cost paid during the period t as the interval time between two sequential shipments. Holding cost in each period can be calculated as $h\,S_j,\ j \in J = \{1,\, \ldots,\, J\}$ where parameter h is the unit inventory holding cost and $S_j,\ j \in J = \{1,\, \ldots,\, J\}$ are provided through equations (1)– (4)

$$\begin{aligned} S_1 &= \left(k + (k - D\,(v)\,t_s) + (k - 2D\,(v)\,t_s) + \cdots + \left(k - \left(\frac{t}{t_s} - 1\right) D\,(v)\,t_s\right)\right) t_s \\ \Rightarrow S_1 &= (2K - t\,D\,(v) + t_s\,D\,(v))\ \frac{t}{2} \end{aligned} \tag{1}$$

$$\begin{aligned} S_2 = \Big(&\big(2k - D(v)\,t\big) + \big(2k - D(v)\,t - D(v)\,t_s\big) + \big(2k - D(v)\,t - 2D(v)\,t_s\big) + \cdots + \\ &\left(2k - D(v)t - \left(\frac{t}{t_s} - 1\right) D(v)t_s\right) t_s \Rightarrow S_2 = (4K - 3\,D(v)\,t + D(v)\,t_s)\ \frac{t}{2} \end{aligned} \tag{2}$$

$$S_j = \big(2\,(jk) - (2j-1)\,D(v)\,t + D(v)t_s\big)\,\frac{t}{2}, \quad j = 1,\, \ldots,\, J-1 \tag{3}$$

$$\begin{aligned} S_J &= \big(D(v)\,t_s\big)t_s + 2\,(D(v)t_s)\,t_s + \cdots + (Jk - (J-1)D(v)tt_s) \\ \Rightarrow S_J &= (D(v)\,t_s)\ t_s\,\left(1 + \cdots + \frac{(J\,k - (J-1)\,D(v)\,t)}{D(v)\,t_s}\right) \end{aligned} \tag{4}$$

Then, one calculates the sum of the inventory over the ordering cycle T

$$\begin{aligned} S &= \sum_{j=1}^{J} S_j = \frac{1}{2}\,(J-1)\,t\,(kJ + D(v)\,(t - Jt + t_s)) + S_J \\ \Rightarrow S &= \frac{kJ\big(kJ + D(v)\,(t - Jt + t_s)\,\big)}{2D(v)} \end{aligned} \tag{5}$$

Therefore, the holding cost of each cycle is

$$H(q,\, v,\, k) = hS \tag{6}$$

Since the annual cycle is calculated as $T = \dfrac{Q}{D(v)} = \dfrac{Jk}{D(v)}$, the total annual cost $T(q, v, k)$, can be obtained through equation

$$\begin{aligned} T(q, v, k) &= \frac{D(v)}{Jk}\big(A + CJk + JB) + H(q, v, k)\big) \\ \Rightarrow T(q, v, k) &= CD(v) + \frac{BD(v)}{k} + \frac{AD(v)}{kJ} + \frac{h}{2}(kJ + D(v)(t - Jt + t_s) \end{aligned} \tag{7}$$

Now, it is time to calculate the annual income of the system to make the capability to calculate the total profit. Given the selling price C, the total income $I(q, v, k)$ can be provided through equation

$$I(q, v, k) = vD(v) \tag{8}$$

Then, the total annual profit can be provided through the equation

$$\begin{aligned} \Pi(q, v, k) &= I(q, v, k) - T(q, v, k) \\ &= vD(v) - \left(C\,D(v) + \frac{B\,D(v)}{k} + \frac{A\,D(v)}{kJ} + \frac{h}{2}\left((kJ) + D(v)\,(t - (Jt) + t_s)\right)\right) \end{aligned} \tag{9}$$

According to Figure 2, during the interval time t, the supplier provides k units of product with the production rate P and, then, sends them to the buyer, which means $k = Pt$.

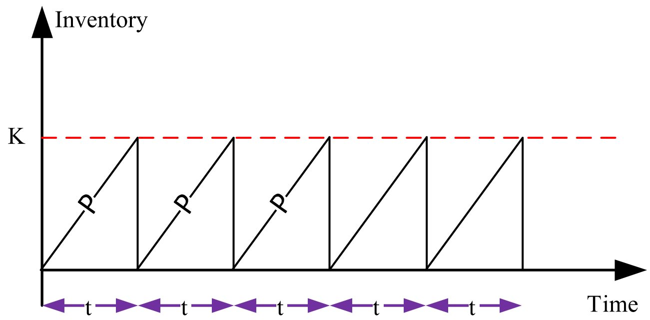

Figure 2. The supplier inventory level

Given $k = Pt$, $D(v) = a - bv$, and $q = Jk$, the total annual profit $\Pi(q, v, k)$ can be further summarized as follows:

$$\Pi(q, v, k) = (v - C)\,(a - bv) - (a - bv)\left(\frac{B}{k} + \frac{A}{q}\right) - \frac{h}{2}\left(q + (a - bv)\left(\frac{k}{P} - \frac{q}{P} + t_s\right)\right) \tag{10}$$

Then the final model can be formulated as

$$\max \Pi(q, v, k), \quad q, k \geq 0 \,\&\, \text{integer}, \quad v \geq 0 \tag{11}$$

4.2. Solution approach

As one can see in equation (11), the model is finally formulated as the non-constrained maximization problem, which can be solved through a simple derivative approach. Due to the maximization type of the problem, at first, it is necessary to define if the objective function is concave. To do so, the following procedure, begun by Theorem 1, should be completely followed.

Theorem 1. A function, i.e., the total annual profit in this paper, is concave if $\mathbf{X}^t\mathbf{A}\mathbf{X} < 0$ in which $\mathbf{A}$ denotes the Hessian matrix and $\mathbf{X}$ is a decision vector.

Proof. Vector $\mathbf{X}$ in the introduced model is defined as $\mathbf{X} = \begin{bmatrix} q & v & k \end{bmatrix}^t$, hence the relative Hessian matrix is defined as

$$\mathbf{A} = \begin{bmatrix} \frac{\partial^2 TC}{\partial q^2} & \frac{\partial^2 TC}{\partial q \partial k} & \frac{\partial^2 TC}{\partial q \partial v} \\ \frac{\partial^2 TC}{\partial k \partial q} & \frac{\partial^2 TC}{\partial k^2} & \frac{\partial^2 TC}{\partial k \partial v} \\ \frac{\partial^2 TC}{\partial v \partial q} & \frac{\partial^2 TC}{\partial v \partial k} & \frac{\partial^2 TC}{\partial v^2} \end{bmatrix} = \begin{bmatrix} -\frac{2A(a-bv)}{q^3} & 0 & -b\left(\frac{h}{2P}+\frac{A}{q^2}\right) \\ 0 & -\frac{2B(a-bv)}{K^3} & \frac{1}{2}b\left(-\frac{2B}{k^2}+\frac{h}{P}\right) \\ -b\left(\frac{h}{2P}+\frac{A}{q^2}\right) & \frac{1}{2}b\left(-\frac{2B}{k^2}+\frac{h}{P}\right) & -2b \end{bmatrix} \tag{12}$$

To test the availability of the theorem, the term $\mathbf{X}^t\mathbf{A}\mathbf{X}$ is calculated as equation (13), which is negative. Therefore, the annual profit outlined in equation (11) is strictly concave.

$$x^T A\, x = -\left(\frac{2a\,(Ak+Bq)}{kq} + \frac{bv\,(h\,(q-k)+2Pv)}{P}\right) \tag{13}$$

□

Due to the non-convexity of $\Pi(q,\, v,\, k)$, the global optimal solution can be provided by the first derivative set equal to zero. To do so, equations (14)–(16) are simply provided.

$$\frac{\partial\psi(q,\, v,\, k)}{\partial q} = 0 \Rightarrow \frac{A\,(a-bv)}{q^2} - \frac{h\,(-a+P+bv)}{2P} = 0 \Rightarrow q^*(v) = \frac{\sqrt{2AP(a-bv)}}{\sqrt{h\,\left(P-(a-bv)\right)}} \tag{14}$$

$$\frac{\partial\psi(q,k,v)}{\partial k} = 0 \Rightarrow \frac{1}{2}\left(\frac{2B}{k^2}-\frac{h}{P}\right)(a-bv) = 0 \Rightarrow k^* = \sqrt{\frac{2BP}{h}} \tag{15}$$

$$\begin{gathered} \frac{\partial\psi(q,\, v,\, k)}{\partial v} = 0 \Rightarrow a+bc+\frac{bB}{k}+\frac{Ab}{q}-2bv+\frac{bh\,(k-q+Pt_s)}{2P} = 0 \\ \Rightarrow v^*(q) = \frac{2AbkP+bhk^2q+2bBPq+2akPq+2bckPq-bhkq^2+bhkPqt_s}{4bkPq} \end{gathered} \tag{16}$$

Corresponding to equations (14)–(16) and the three unknown decision variables, the optimal values for the retailer's order quantity per cycle, shipping quantity, and selling price, denoted respectively by q^*, k^*, and v^*, are calculated. Although the optimal solution is obtained, there is no guarantee for q^* and k^* to be an integer. However, they are restricted to be integer-valued through equation (11). To obtain integer values for these decision variables, given the rounded values of k^*, i.e., $\mathrm{round}\left(\sqrt{2BP/h}\right)$, equations (14) and (16) are both recalculated, simultaneously. If the value of order quantity is going to be an integer, then the constraints of equation (11) are satisfied; otherwise, it will be rounded and replaced in equation (16), and the relative value of v^* is achieved. According to García–Laguna, San–José [13], the upper- and lower-bound of q^*, can be provided respectively through equations

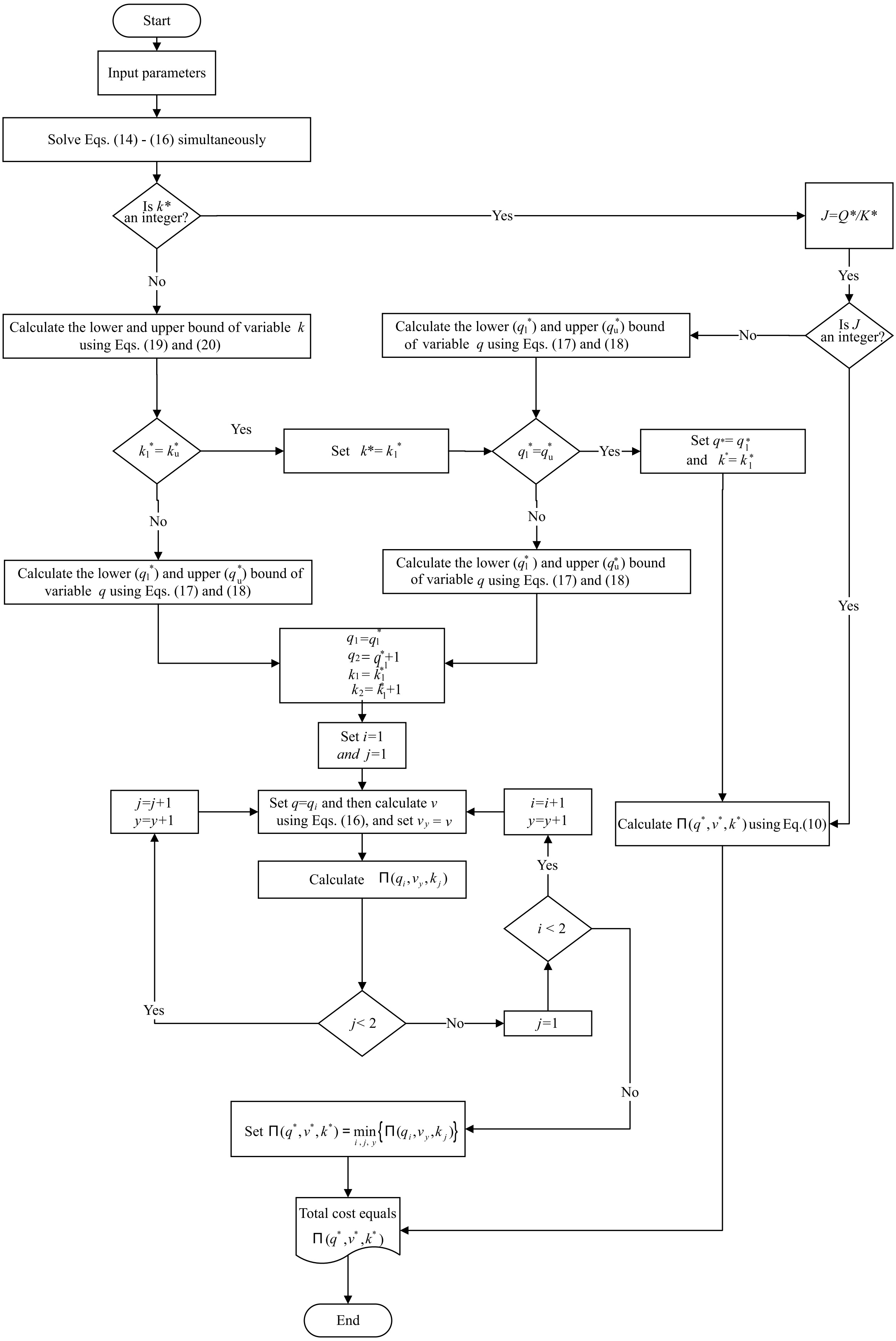

Figure 3. The proposed solution approach

$$q_l{}^* = \left\lceil -0.5 + \sqrt{0.25 + \frac{2AP(a-bv)}{h\ (P-(a-bv))}} \right\rceil \tag{17}$$

$$q_u{}^* = \left\lfloor 0.5 + \sqrt{0.25 + \frac{2AP(a-bv)}{h\ (P-(a-bv))}} \right\rfloor \tag{18}$$

where $\lceil . \rceil = [.] + 1$ and $\lfloor . \rfloor = [.]$ are the ceiling and floor functions, respectively.

Hereafter, if $-0.5 + \sqrt{0.25 + \frac{2AP(a-bv)}{h\ (P-(a-bv))}}$ is not an integer, then the optimal solution is obtained as $q_l{}^* = q_u{}^*$; otherwise, two optimal solutions, i.e., $q_l{}^*$ and $q_u{}^* = q_l{}^* + 1$, are obtained. The same procedure is also done to obtain optimal value/s for k. Given equations (19) and (20)), if $-0.5 + \sqrt{0.25 + (2BP/h)}$ is not an integer, then the optimal solution is obtained as $k_l{}^* = k_u{}^*$; otherwise, there are two optimal solutions $k_l{}^*$ and $k_u{}^* = k_l{}^* + 1$

$$k_l{}^* = \left\lceil -0.5 + \sqrt{0.25 + \frac{2BP}{h}} \right\rceil \tag{19}$$

$$k_u{}^* = \left\lfloor 0.5 + \sqrt{0.25 + \frac{2BP}{h}} \right\rfloor \tag{20}$$

Here, $J = \frac{q}{k}$ depends on the values obtained for q and k. Hence, J takes the integer value, whenever q is an integer multiple of k. The optimal value of q, then, takes the value of $q_1{}^* = \left[\frac{q^*}{k^*}\right] k^*$, $J = \left[\frac{q^*}{k^*}\right]$ or $q_2{}^* = \left[\frac{q^*}{k^*} + 1\right] k^*$, $J = \left[\frac{q^*}{k^*} + 1\right]$. If $\Pi(q_1^*, v, k) \geq \Pi(q_2^*, v, k)$ then $q_1{}^*$ is the optimal solution; otherwise, q_2^* is the final optimal solution. To obtain more clarity about the proposed solution approach, it is completely represented in Figure 3.

5. Numerical example and sensitivity analysis

5.1. Problem data

In this section, a numerical example is designed in which the supplier production rate sets to 100 units per year, and the retailer's demand is defined as $D(v) = 100 - 0.3v$ units per year. The time that exists between two consecutive demands is set to 0.01 years and the purchasing cost is also set to 40$. The retailer's product order is received one time per cycle with 1000$ as its fixed ordering cost. The time of the order delivery is not significant, corresponding to the certain lead time, addressed in this paper. The annual holding and the transportation costs are set to 20$ per unit and 20$ per order, respectively.

5.2. Numerical results

Using equations (19) and (20) the lower and upper bound of variable are achieved as

$$k_l^* = \left\lceil -0.5 + \sqrt{0.25 + (2\,20\,100)/20} \right\rceil = 14, \quad k_u{}^* = \left\lfloor 0.5 + \sqrt{0.25 + (2\,20\,100)/20} \right\rfloor = 14$$

So, the optimal value of shipment quantity is obtained as 32. Afterward, upper and lower optimal values of the order quantity are defined through equations (16)–(18). The values are obtained as

$$q_u{}^* = \left\lfloor 0.5 + \sqrt{0.25 + \frac{2\,1000\,100\,(100 - 0.30\,v)}{20\ (100 - (100 - 0.30\,v))}} \right\rfloor$$

$$q_l{}^* = \left\lceil -0.5 + \sqrt{0.25 + \frac{2\,1000\,100\,(100 - 0.30\,v)}{20\ (100 - (100 - 0.30\,v))}} \right\rceil$$

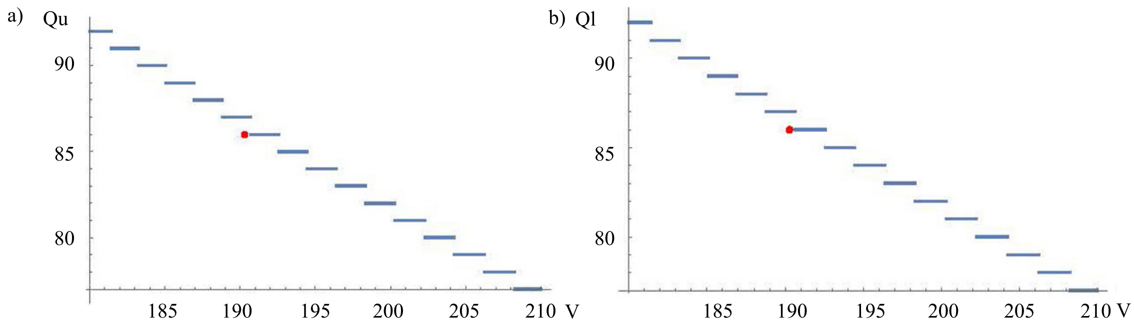

Figure 4. The number of and relative to various values of are outlined in Figure 4 (a) and Figure 4 (b)

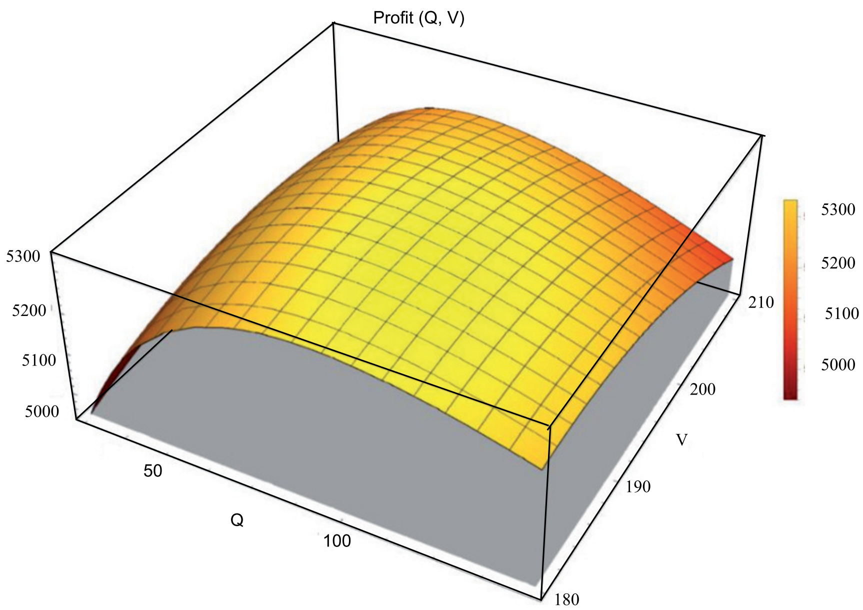

Figure 5. Relationship between the objective function, selling price, and the order quantity

Due to the dependency of the optimal order quantities to the value of v, Figures 4a and 4b outline the number of q_u and q_l relative to various values of v, respectively. As one can see, those figures are similar, hence the optimal value of v is obtained as 109.224, and, the optimal value of v can be, then, calculated as 46 units. The point $(v^{*},\, q^{*}) = (189.515,\, 87.114)$ that is colored red, outlines the obtained optimal solution as well. Figure 4 further shows the relationship between the objective function, selling price, and order quantity. The number of shipments in each cycle, i.e., J can be, then, calculated as $J = 84/15 = 5.6$. Accordingly, the value of J can be calculated as $J = 5$ or $J = 6$. Therefore, the optimal value of q can be provided as $q_1^{*} = (q^{*}/k^{*})\, k^{*} = 6 \times 14 = 84$ or $q_2^{*} = (q^{*}/k^{*} + 1)\; k^{*} = 7 \times 14 = 98$. The total profit for $q_1{}^{*}$ and $q_2{}^{*}$ take the values of 5333.37 and 5327.53, respectively. Therefore, the optimal order quantity is 84 units per cycle and the optimal number of shipments per cycle is 2. Figure 6 further shows the sensitivity of the objective function to the order quantity. The optimal value is outlined as the red point in the figure.

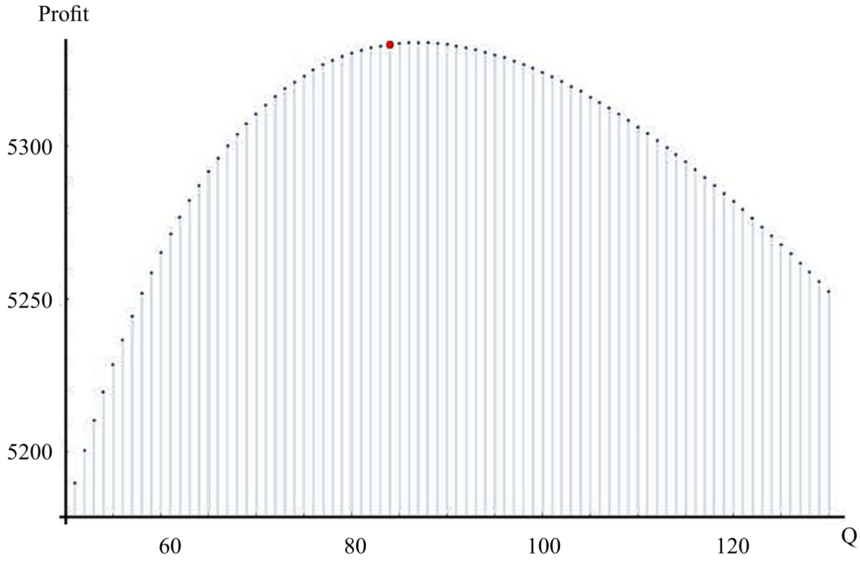

Figure 6. Sensitivities of profit function relative to the order quantity

5.3. Analysis of results

The results obtained from the proposed model, the proposed SSMD model, and the classic model, the traditional SSSD model, are summarized in Table 2. As shown in the table, three parameters have to be specified before the results computations: 1) fixed order cost, 2) holding cost, and 3) transportation cost. So, the sensitivity of the results to these parameters has been prepared to provide more confidence in the model. In doing so, a parameter can be changed, while the others are set to fixed values. In the SSSD model, the manufacturer provides the retailer with all the quantity it ordered; multi-delivery ordering is not allowed. The annual cost of the order, considering the transportation cost, can be written as equation (21) in which B is the transportation cost paid per delivery.

$$\Pi(q,\, v,\, k) = (v - C)\,(a - bv) - (a - bv)\left(\frac{B}{Q} - \frac{A}{Q}\right) - \frac{1}{2}h\left(Q + (a - bv)\left(\frac{Q}{P} - \frac{Q}{P} + t_s\right)\right) \quad (21)$$

As shown in the table, the proposed approach, with an average selling price equal to about 94% of the average selling price of the classical approach, resulted in an average profit increase of about 16% and an average relative percent difference (RPD) factor increase of about 14%.

Table 2. Results of the sensitivity analysis

Cost variability	Parameters				Proposed model				Classic model			RPD
	O	h	B	k	q	J	v	P	Q	v	P	[%]
Fixed order	500	20	20	14	56	4	189.795	5620.96	47	199.736	4884.49	13.10
	500	20	20	14	56	4	189.795	5620.96	47	199.736	4884.49	13.10
	600	20	20	14	70	5	188.917	5556.85	52	199.936	4818.46	13.29
	700	20	20	14	70	5	189.631	5495.11	56	200.217	4756.01	13.45
	800	20	20	14	84	6	188.693	5436.26	59	200.596	4695.73	13.62
	900	20	20	14	84	6	189.288	5384.71	62	200.925	4639.61	13.84
	1000	20	20	14	84	6	189.883	5333.37	66	201.042	4590.27	13.93
	1100	20	20	14	98	7	188.843	5283.22	69	201.288	4540.82	14.05
	1200	20	20	14	98	7	189.353	5239.07	72	201.500	4494.01	14.22
	1300	20	20	14	98	7	189.864	5195.07	74	202.021	4446.32	14.41
	1400	20	20	14	98	7	190.374	5151.23	77	202.008	4403.94	14.51
	1500	20	20	14	112	8	189.227	5109.96	80	202.142	4363.38	14.61
Holding	1000	10	20	20	180	9	185.969	5614.83	92	199.768	4886.91	12.96
	1000	12	20	18	108	6	189.181	5585.89	85	200.059	4818.61	13.74
	1000	14	20	17	102	6	189.217	5516.87	79	200.301	4756.3	13.79
	1000	16	20	16	96	6	189.34	5452.22	74	200.543	4697.95	13.83
	1000	18	20	15	90	6	189.559	5391.33	69	200.898	4640.73	13.92
	1000	20	20	14	84	6	189.883	5333.37	66	201.042	4590.27	13.93
	1000	22	20	13	78	6	190.326	5277.32	63	201.248	4540.95	13.95
	1000	24	20	13	78	6	190.006	5226.8	60	201.52	4492.43	14.05
	1000	26	20	12	72	6	190.609	5175.03	59	201.721	4488.52	13.27
	1000	28	20	12	72	6	190.314	5128.32	55	202.048	4400.77	14.19
	1000	30	20	12	72	6	190.019	5081.66	53	202.276	4357.84	14.24
Transportation	1000	20	10	10	90	9	188.772	5369.37	65	196.209	4990.93	7.05
	1000	20	12	11	88	8	189.094	5361.5	65	197.209	4908.95	8.44
	1000	20	14	12	84	7	189.652	5353.26	65	198.209	4827.58	9.82
	1000	20	16	13	91	7	188.927	5346	66	199.042	4750.21	11.14
	1000	20	18	13	91	7	189.003	5339.33	66	200.042	4669.94	12.54
	1000	20	20	14	84	6	189.883	5333.37	66	201.042	4590.27	13.93
	1000	20	22	15	90	6	189.256	5327.52	66	202.042	4511.19	15.32
	1000	20	24	15	90	6	189.322	5321.76	66	203.042	4432.72	16.71
	1000	20	26	16	80	5	190.579	5313.63	66	204.042	4354.84	18.04
	1000	20	28	17	85	5	190.023	5311.39	66	205.042	4277.57	19.46
	1000	20	30	17	85	5	190.081	5306.34	66	206.042	4200.89	20.83

[1] Symbols used: O – fixed order cost, h – holding cost, B – transportation cost, k – optimal shipment quantity, q – optimal order quantity, J – number of shipments, v – optimal selling price, P – optimal profit, RPD – relative percent difference.

The RPD is the factor that has been defined to compare the optimal profits obtained corresponding to the proposed model with those of the classic model (see Table 2). It, traditionally, measures the variation in a set of data calculated

$$\text{RPD} = \frac{\text{Optimal profit (proposed model)} - \text{Optimal profit (classic model)}}{\text{Optimal profit (proposed model)}} \times 100\% \qquad (22)$$

These results are quite justified given the average order increase of about 34% in the proposed approach compared to the classical approach. However, based on Figure 7, the decrease in profit due to the increase in the fixed costs is almost the same in both models and shows an average difference of about 2%. This is even though the number of shipments in the proposed model is more than in the classic model.

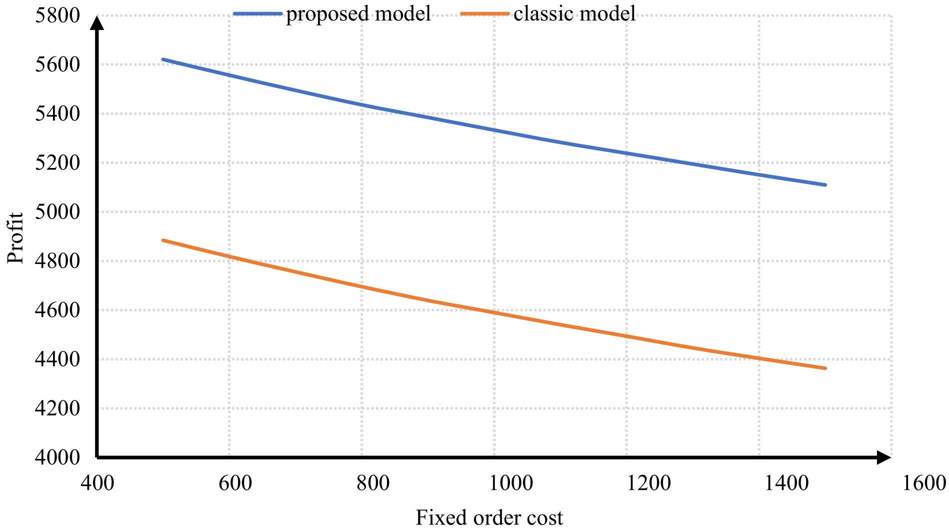

Figure 7. Optimal profit against the various fixed order costs

In such a case, in the classical model, the logical behavior of the buyer would be to increase the selling price to partially offset the increase in the fixed order cost, as shown in Figure 8. At the same time, despite the changes in the selling price resulting from the proposed approach, this approach overall has much better relative stability than the classical approach. This can be seen due to the negligible slope (0.0004) of the trend-line for the proposed model results compared to that of the classical model.

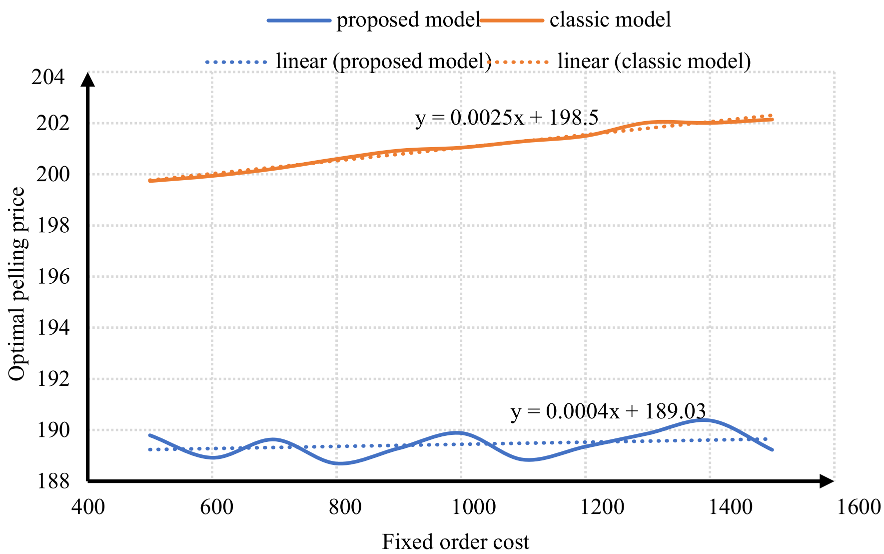

Figure 8. Optimal selling price against the various fixed order costs

In the proposed model, since the amount of shipment quantity (k) is remaining fixed against the various values of the fixed order cost, the increasing number of shipments (J) results in very little change in the holding costs. At the same time, due to the need to provide a balance between the fixed order and the holding costs, the proposed model with a lower holding cost, in a greater fixed order cost, faces a greater order quantity, as shown in Figure 9. In other words, given $q = Jk$, the increasing number of shipments increases the order quantity. Increasing the fixed order cost may increase the buyer's order quantity due to the direct relationship between the system's total cost and the number of orders a year. Accordingly, a negligible change in the total cost of the system makes the retailer not see the need to increase the selling price. Since the demand and the selling price are reversely related, unlike the classical model, the stability of the price makes the demand unchanged. Therefore, unlike the classical model with the decreasing demand, in the proposed model the demand remains fixed, hence the optimal order quantity may increase more sharply with the slope of 0.047 compared with that of the classical model (0.32).

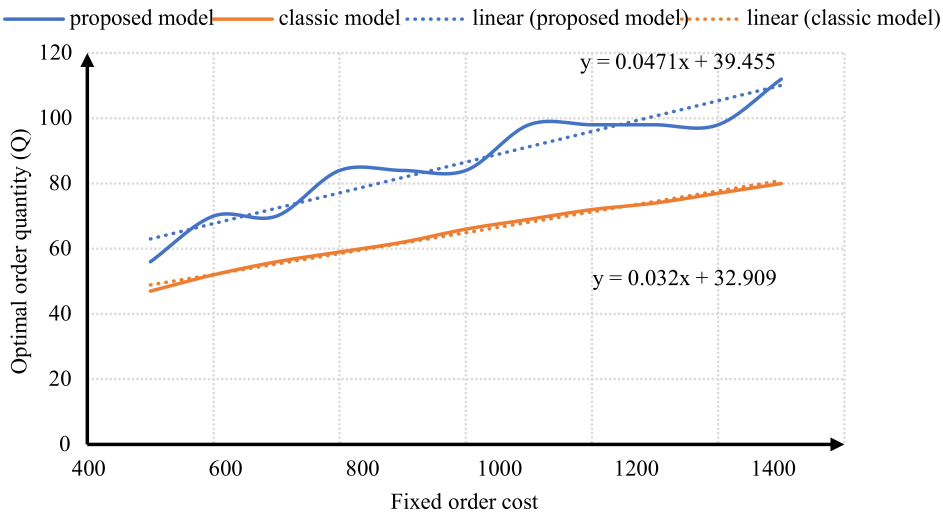

Figure 9. Optimal order quantity against the various fixed order costs

In the following, the effect of changes in the unit holding cost on the optimal results of the model is also examined. As shown in the 11 middle rows of Table 2 and further outlined in Figure 10, unlike the effect of the increase in the fixed order cost, shown in Figure 8, any increase in the unit holding cost has reduced the order quantity. This is logical because increasing the unit holding cost increases the total holding cost of the system. And decreasing the order quantity, in both models, may moderate the increasing intensity of the holding costs to some extent.

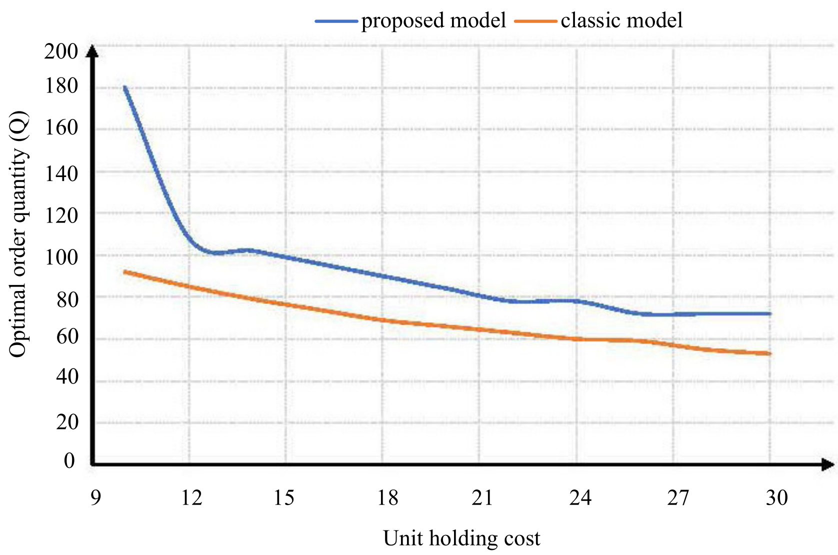

Figure 10. Optimal order quantity against the various unit holding costs

Decreasing the amount of the order quantity may increase the number of orders a year. Therefore, the total cost of the system may increase; hence, decreasing the profitability, as shown in Figure 11. To compensate for some of this decline in profits, the logical solution is to increase the price of the product, as declared in Figure 12. However, the dependence of demand on the selling price moderates the trend of increasing the price of the product. Because excessive price increases make a large decrease in demand and this in turn reduces the total profit.

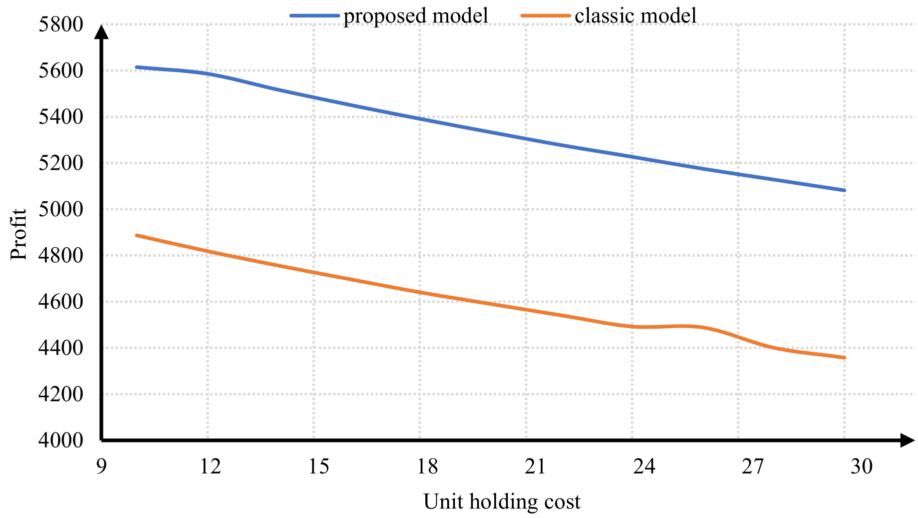

Figure 11. Optimal profit against various unit holding costs

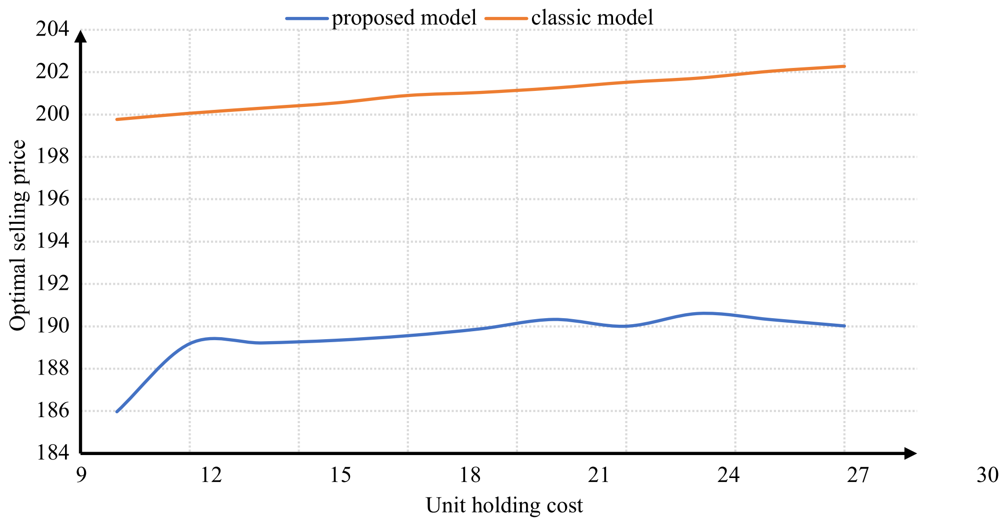

Figure 12. Optimal selling price against various unit holding costs

To further analyze the sensitivity of the results to the transportation cost, given the fixed number of shipments per cycle that is equal to one, the corresponding costs are added to the fixed ordering cost. Therefore, the transportation cost of each order for the classical model behaves similarly to the fixed ordering cost. In the proposed model, on the other hand, the transportation cost per cycle is equal to the product cost of each order in the number of shipments. Therefore, to maintain a balance in transportation costs, the number of shipments decreases with increasing transportation costs. At the same time, by reducing the number of orders per cycle, the amount of each receipt increases to maintain a balance between supply and demand. Therefore, increasing the transportation cost increases the system's costs and, as a result, the total profit decreases, as shown in Figure 13. However, as can be seen in the figure, the reduction rate in the proposed model is less than the classic model due to its flexibility in the quantity and number of shipments.

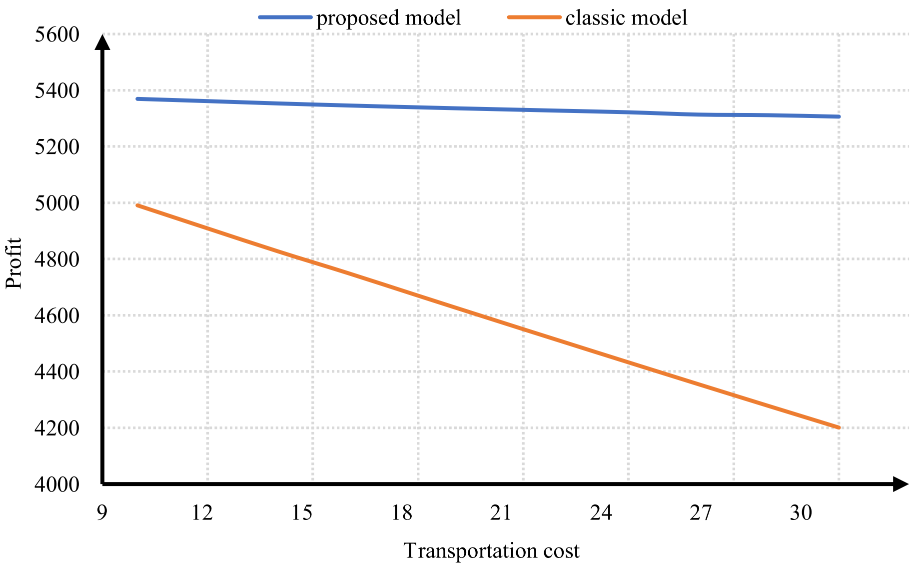

Figure 13. Optimal profit against various transportation costs

In this regard, reducing the profit increases the buyer's selling price to offset part of the system's costs. However, as shown in Figure 14, since increasing the selling price reduces demand, these price changes in the proposed model have been made very cautiously compared to the classic model. This is in full compliance with the slight slope of the profit change of the proposed model, outlined in Figure 13.

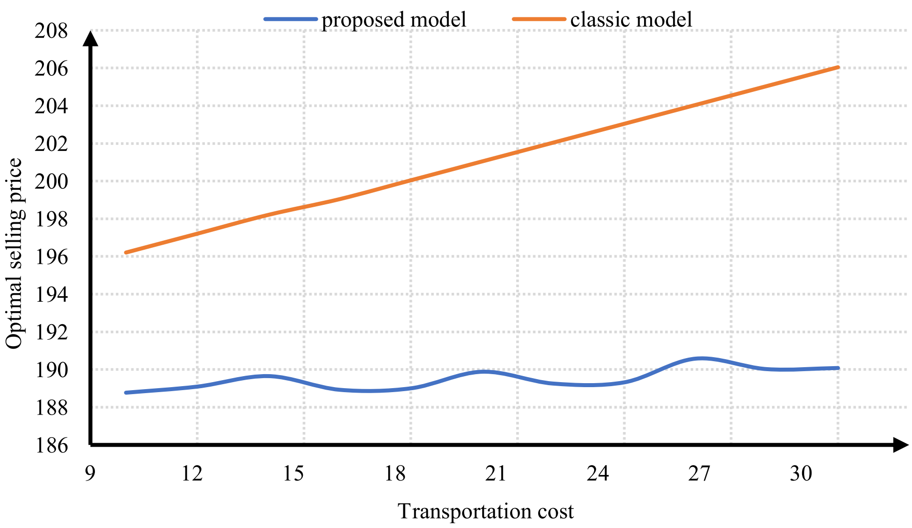

Figure 14. Optimal selling price against various transportation costs

6. Conclusion

This study investigates an optimal integrated production-inventory system from the perspectives of both the supplier and the buyer. A step-wise delivery strategy, which is more useful for such perishable products as alcohol, naphthalene, and food is considered to capture a large amount of fixed order and inventory costs. The demand is periodic and depends on the selling price through an infinite planning horizon. To simply solve the model, a solution approach is introduced to keep the reliability and correctness of the calculation process. A numerical example and a well-designed sensitivity analysis clearly validate the model and the obtained results. The results show the superiority of the model in providing results that are nearer to real-life situations. The numerical results show that the proposed approach, with an average selling price equal to about 94% of the average selling price of the classical model, has resulted in an average profit increase of about 16% and an average RPD factor increase of about 14%. It also results in an average order increase of about 34%

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