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ON THE CONCEPT OF TRUTH IN AN INTENDED MODEL OF THE LOGIC OF BELIEFS*

Abstract. The article is all about – firstly – to build a logic of beliefs on the way *a posteriori*, i.e. observation developed on the principle how, people usually think, what kind of propositions seem about reality and what actually is described by the truth of the judgements under the influences of beliefs. In this situation, we have to depart from customary practice *a priori* semantics of possible worlds to semantics for models intended. Secondly, we find that in practice, human judgments accept indirectly a *logic of thinking at all*, which leads us – thirdly – to define this logical system as extension of the intuitionistic logic. And fourth, and finally, our logic of beliefs, empirically generated, proved to be the logic at most of hypothetical and positing beliefs, because the propositions issued on the basis of intuitionistic logic are not assertive judgments.

Keywords: logic of beliefs, intuitionistic logic, semantics of intended models, concept of truth

1. INTRODUCTION

Following Aristotle we repeat:

1) "to say that what is, is, and what isn't, is not, that's true"[1],

and following Thomas Aquinas:

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[1] Aristotle, *Metaphysics*, transl. W.D. Ross, http://classics.mit.edu/Aristotle/metaphysics.html (6.05.2013), Γ7, 1011 b 26–27.

2) “Veritas est adequatio intellectus et rei”[2].

Alfred Tarski was teaching that:

3) “*x* is a true sentence if and only if *p*”, where ‘x’ marks the sentence ‘p’ [3].

Ludwik Borkowski gave a defintion:

4) “A sentence is true if and only if it describes a certain state of affairs (states the existence of a certain state of affairs) and this state of affairs exists (occurs)”[4].

Classically comprehended *truth* once is treated as a sentence-forming metalinguistic operator “... is true”, other time as a property of sentences (propositions), and yet another – as “an intentional identity” (*adequatio*) in relation of sentences (*intellectus*) to facts (*res*) . However, in the Max Black’s[5] so-called *philosophical theory of truth*, the truth is an existential attribute of the situation itself.

5) “The situation is true, when it occurs, (when it is a fact)”.

From the very beginning, this classical understanding of the truth – according to the intention of faithfulness to the facts – the contemporary logical semantics attempted to assimilate. For this faithfulness, however, the need of explaining the truth of *de dicto* modal sentences turned out to be an obstacle. Insofar in 1933, Tarski didn’t refrain yet from the determinations in the kind: a true sentence is a sentence which expresses, that things are such and such, and things are so just, or “the snow is falling down” is a true sentence if and only if the snow is falling down[6], soon afterwards the logicians inserted into semantics some

[2] This formula is probably derived from the Arabic philosophers, as indicated in *Quaestiones disputatae de veritatae* I, transl. R.W. Mulligan, Chicago 1952, question 1.

[3] „Zdanie ‘p’ jest prawdziwe wtedy i tylko wtedy, gdy p”. A. Tarski, *Pojęcie prawdy w językach nauk dedukcyjnych*, Warszawa 1933, 5.

[4] „Zdanie jest prawdziwe wtedy i tylko wtedy, gdy opisuje pewien stan rzeczy (stwierdza istnienie pewnego stanu rzeczy) i ten stan rzeczy istnieje (zachodzi)”. L. Borkowski, *Pewna wersja definicji klasycznego pojęcia prawdy*, Roczniki Filozoficzne 28(1980)1, 119.

[5] M. Black, *The Semantic Definition of Truth*, Analysis 8(1948)4, 49–63.

[6] A. Tarski, *Pojęcie prawdy w językach nauk dedukcyjnych*, op.cit., 4–5.

new concepts: firstly, the relative truth, the truth in the model, and then the truth in possible worlds.

Kripke's idea of possible worlds – put in the set-theoretic codes and paraphrases – has been widely adopted as a godsend for the real world. And nothing in this proceeding would be unfortunate – because 'sets' are codes more readable than the traditional 'concepts' and the new paraphrases seem to solve some of the problems of ambiguity of modal sentences – if not for the fact that this way a Kantian revolution has been made in the semantics, according to a prescription: "we can know a priori of things only what we ourselves put into them"[7].

In a number of studies on this subject Marian Przełęcki proposes – without denying the value of *the relative truth in possible models* – a return to *absolute truth* and the *intended models* in certain circumstances, in which, even without avoiding modern paraphrases and codes, we would be faithful to the traditional principle of concepts and propositions creation *cum fundamento in re*[8]. In his book "The Logic of empirical theories", Przełęcki defines what he means by the concept of *absolute truth in the intended model*: "If L is to be an interpreted, meaningful language, we must define, with respect to it, an 'absolute' concept of truth : say what it is to mean for a sentence of L to be simply 'true'. This may be realized by selecting from among all possible interpretations of the language L interpretation of the actual or intended: to say that from all models of the language L (all fragments of reality which L can speak about) its proper, or intended, model (that fragment of reality which L does speak about)"[9].

[7] I. Kant, *Critique of Pure Reason*, transl. and ed. by P. Guyer, A.W. Wood, Cambridge 1998, 111.

[8] M. Przełęcki, *Interpretacja systemów aksjomatycznych*, Studia Filozoficzne 4(1960)6, 97–98; Idem, *Z semantyki pojęć otwartych*, Studia Logica 12(1964)15, 202; Idem, *Problem interpretacji języka empirycznego w ujęciu teoriomodelowym*, Studia Filozoficzne 16(1972)1, 135; Idem, *Logika teorii empirycznych*, Warszawa 1988, 23–39.

[9] Idem, *The Logic of empirical theories*, London 1969, 17.

2. LOGIC OF BELIEFS

In this paper we address the issue of overcoming the recalcitrant material to create *a posteriori* a logical theory, as a pure opportunity to build a formal language on the basis of *intended models*. We want to create *ex nihilo* a logic of beliefs. That is to say – otherwise than it is in the habit – not to take as a starting point any possible worlds, what just means here *ex nihilo*, but looking at how a particular person actually thinks, to develop, or rather reveal, the *a priori* laws and rules of the logic of beliefs.

Let us consider propositional expressions: α, β, γ,..., that is, the indicative sentences and propositional functions of the any language. Let $\underline{\alpha}$, $\underline{\beta}$, $\underline{\gamma}$,... be non-linguistic correlates of the formulas α, β, γ,..., respectively. Let Φ marks the set of all formulas of the chosen language. Then a set of all correlates, K, we determine, the following:

6) $K = \{\underline{\alpha}: \alpha \in \Phi\}$

Since some formulas may be inconsistent, their correlates are impossible, whereas the ones denoted through consistent formulas, which means they are possible (sign "$\Diamond$"), will be called *situations*. The set of all situations, associated with a set Φ, will be denoted by "S" and defined as:

7) $S = \{\underline{\alpha} \in K: \Diamond\underline{\alpha}\}$

Some situations may occur in the real world, others – do not. These α situations that occur, or exist, $E(\alpha)$, we call facts, and their set will be signified by the letter F:

8) $F = \{\underline{\alpha} \in S: E(\underline{\alpha})\}$

The following relationship takes place:

9) $F \subseteq S \subseteq K$

We can take shortcuts for phrases: 'α is true in the set of situations S' and 'α is false in a set of situations S', respectively:

10) $V(\alpha, S) = 1$, $V(\alpha, S) = 0$

and specify them:

11) $V(\alpha, S) = 1$ iff $\underline{\alpha} \in F$

12) $V(\alpha, S) = 0$ iff $\underline{\alpha} \notin F$)

What these words now mean: 'I think α' – symbolically – '$\mu(\alpha)$'? Probably in both cases:

13) $\mu(\alpha)$ iff $\mu(V(\alpha, S) = 1)$

14) $\mu(\alpha)$ iff $\mu(\underline{\alpha} \in F)$

However, we have to notice immediately that the phrase "I think" is an elliptical sentence, because it contains two implicit individual constants: the "I" (marked by "c_0") and "now" (marked by "t_0"), i.e., its complete form is the expression: 'I am thinking now that α':

15) $\mu(c_0, t_0, \alpha)$

This means, however, that we can also say generally that any person (personal variable "c") at any time (temporal variable "t") may think that α:

16) $\mu(c, t, \alpha)$

Let us consider two types of inferences (recognitions): conditional (rule): $\alpha, \beta, \ldots \vdash \gamma$ (the sign $\vdash$ denotes operators: *hence*, *so*, *ergo*) and unconditional (claim): $\vdash \alpha$. There are various grounds for the recognition of the unconditional. It used to enumerate: the experience (external and introspective), linguistic convention, intuition, epistemic authority and recognition based on conditional inference from *the empty set of assumptions*.

We attempt to establish the rules and theorems of thinking. Our source is an introspection. The results we are going to list step by step in the records of elliptical forms in order to determine what happens to the rational reasoning, when I am thinking *now*.

The generalization of these findings to any person and time (any of the variables c and t) is treated as a legitimate operation which is called here Melsen's principle about so-called species structure of the individual being. Structure: individual as a species means that each determination, the form of the matter, whatever might be the nature of such a determination, or form, is carried out in such a way that, in principle, it is not limited to one particular individual thing, material or event[10].

[10] Andrew G. van Melsen, *Philosophy of Nature*, Pittsburgh 1961, 177. Guessing the characteristics of quality in things and events unit was already known to Aristotle, and the induction of elimination in general can not function without a guess.

Immediately we must state that our thinking is inherently incomplete, but consistent:

17) $\exists\alpha\ [\sim\mu(\alpha) \wedge \sim\mu(\sim\alpha)]$

18) $\forall\alpha\ [\mu(\alpha) \rightarrow \sim\mu(\sim\alpha)]$

Because there are sentences and situations, about which I think – for instance during a dream – quite nothing, neither that they are, nor that there are their denials. It does not happen yet (rejecting abnormality) that I thought α, and I thought that $\sim\alpha$ at the same time.

19) Here's a set of rules, the obviousness of which is rather impossible to deny:

($\wedge$1): $\mu(\alpha), \mu(\beta) \vdash \mu(\alpha\wedge\beta)$

On this basis that I think α, and I think β, I also think that $\alpha \wedge \beta$.

($\wedge$2): $\mu(\alpha\wedge\beta) \vdash \mu(\alpha)$

($\wedge$3): $\mu(\alpha\wedge\beta) \vdash \mu(\beta)$

($\vee$1): $\vdash\mu(\alpha) \vdash\ \vdash\mu(\alpha\vee\beta)$

($\vee$2): $\vdash\mu(\beta) \vdash\ \vdash\mu(\alpha\vee\beta)$

Based on the affirmation that I think that α or on this, that I affirm that I think, that β, I affirm, that I think, that α or β.

($\vee$3): $[\mu(\alpha\vee\beta), \mu(\alpha)\vdash\mu(\gamma), \mu(\beta)\vdash\mu(\gamma)] \vdash \mu(\gamma)$

I think γ when thinking that $\alpha \vee \beta$, I conclude that μ (γ) both that μ (α) and from the fact that μ (β).

($\rightarrow$1): $\vdash(\alpha\rightarrow\beta) \vdash \mu(\alpha\rightarrow\beta)$

Since I consider that if α, β is, I think hence also that $\alpha \rightarrow \beta$.

($\rightarrow$2): $\mu(\alpha), \mu(\alpha\rightarrow\beta) \vdash \mu(\beta)$

The consistency rule in thinking ($\rightarrow$ 2) states the conditional recognition $\mu(\beta)$ on the basis of the facts $\mu(\alpha)$, $\mu(\alpha \rightarrow \beta)$.

($\sim$1): $\mu(\alpha) \vdash \sim\mu(\sim\alpha)$[11], hence $\mu(\sim\alpha)\vdash\sim\mu(\alpha)$, hence $\mu(\sim\sim\alpha)\vdash\sim\mu(\sim\alpha)$ etc.

($\sim$2): $\mu[\alpha\rightarrow (\alpha\wedge\sim\alpha)] \vdash \sim\mu(\alpha)$

I do not think that α when I think that α implies a contradiction α and $\sim\alpha$.

(Q): $\mu(\alpha\wedge\sim\alpha) \vdash \mu(\beta)$

[11] The theorem of the consistency of thinking 18) is based directly on the rule ($\sim$1) and vice versa.

The rule *quodlibet*, (Q), states that from thinking a contradiction, thinking just anything follows.

On the basis of the foregoing introspective analysis of thinking we discovered the rules that bear a striking similarities to the rules of the propositional intuitionistic logic outlined in Gentzen's natural deduction. Just compare, for example, the rules of the logic in the presentation of Dirk van Dalen[12]. We therefore suggest the possibility of opposing ontological to epistemic reality on a basis that the first – referring to the existence of "the world outside of us" – is to be described by the classical logic LK, and the second – referring to the phenomena of thinking – a theory of the $T\mu$ which is built on the basis of the intuitionistic logic LI.

Arend Heyting axiomatized the Logic LI in the 1930s. It was based on one rule and eleven primary theses (axioms)[13]:

The rule of detachment RO: $\vdash\alpha, \vdash\alpha\rightarrow\beta \vdash \vdash\beta$[14]

The axioms of *LI*:

[1]. $\vdash ((\alpha\rightarrow\beta) \rightarrow ((\beta\rightarrow\gamma) \rightarrow (\alpha\rightarrow\gamma)))$

[2]. $\vdash (\alpha \rightarrow (\alpha \vee \beta))$

[3]. $\vdash (\beta \rightarrow (\alpha \vee \beta))$

[12] Dirk van Dalen, *Intuitionistic Logic*, in: *The Blackwell Guide to Philospphical Logic*, ed. L. Goble, Oxford 2001, 228.

[13] These axioms are presented in a version given in H. Rasiowa, R. Sikorski, *The Mathematics of Metamathematics*, Warsaw 1963, 379.

[14] The rule of detachment is here given as an admissible rule, that is the rule can be applied only to found formulas. On the basis of our *a posteriori (intuitive) logic* is also *valid* the rule of detachment: $\alpha, \alpha \rightarrow \beta \vdash \beta$ that is, one which is based on intuitionistic logic theorem $\vdash ((\alpha \wedge (\alpha \rightarrow \beta) \rightarrow \beta$). Proof of this thesis is the number 2.23 in the book J. Dopp, *Logiques construites par une méthode naturelle de déduction*, Louvain – Paris 1962. The theorem that each valid rule is also admissible one (though not vice versa), has a proof in the article: K. Świętorzecka, *O stosowalności niektórych modalnych reguł inferencji w rozumowaniach pozalogicznych*, Filozofia Nauki 10(2002)1, 117.

[4]. $\vdash ((\alpha\rightarrow\gamma) \rightarrow ((\beta\rightarrow\gamma) \rightarrow ((\alpha\vee\beta) \rightarrow \gamma)))$

[5]. $\vdash ((\alpha\wedge\beta) \rightarrow\alpha)$

[6]. $\vdash ((\alpha\wedge\beta) \rightarrow\beta)$

[7]. $\vdash ((\gamma\rightarrow\alpha) \rightarrow ((\gamma\rightarrow\beta) \rightarrow (\gamma \rightarrow (\alpha\wedge\beta))))$

[8]. $\vdash ((\alpha\rightarrow(\beta\rightarrow\gamma)) \rightarrow ((\alpha\wedge\beta) \rightarrow \gamma))$

[9]. $\vdash (((\alpha\wedge\beta)\rightarrow\gamma) \rightarrow (\alpha\rightarrow(\beta\rightarrow\gamma))$

[10]. $\vdash ((\alpha\wedge{\sim}\alpha) \rightarrow \beta)$

[11]. $\vdash ((\alpha\rightarrow (\alpha{\sim}\alpha) \rightarrow {\sim}\alpha)$

At this point, let us recall what has already been mentioned above. The characters of inference are ambiguous, because their meaning depends on the context of use for a particular system of deduction. In our discussion we would have to use a triple marking for double inference, because we are dealing with the system L μ, the logic of LI, and, in the end we believe in the possibility of creating a system that is our ultimate goal – the logic of beliefs LP. Instead of multiplying the symbols it is better to take a different approach: we denote uniformly different inferences, indicating their single sense by means of the same type of numbering: n) for L μ, [n] for the LI and (n) for LP, or writing about thesis α, to which the system belongs: $\alpha \in L\mu$, $\alpha \in LI$, $\alpha \in LP$.

Because, firstly, we believe that there is no thoughtless acceptance and, secondly, that the rules described under no. 19) are "valid rules"[15], so for translating theses from the logic of LI into theorems of L μ, we will use the rules:

20) $\vdash\alpha$ iff $\vdash\mu(\alpha)$

21) If $\vdash\mu(\alpha)$, then $\mu(\alpha)$

[15] See footnote 14.

22) If $\vdash (\alpha\to\beta)$, then $(\alpha\vdash\beta)$

It proves now that, based on the LI-axioms: [1] – [11] theorems of the theory L μ are:

23) $\vdash (\mu(\alpha\to\beta) \to (\mu(\beta\to\gamma) \to\mu(\alpha\to\gamma)))$

24) $\vdash (\mu(\alpha) \to \mu(\alpha \vee \beta))$

25) $\vdash (\mu(\beta) \to \mu(\alpha \vee \beta))$

26) $\vdash (\mu(\alpha\to\gamma) \to (\mu(\beta\to\gamma) \to (\mu(\alpha\vee\beta) \to \mu(\gamma)))$

27) $\vdash (\mu(\alpha\wedge\beta) \to\mu(\alpha))$

28) $\vdash (\mu(\alpha\wedge\beta) \to\mu(\beta))$

29) $\vdash (\mu(\gamma\to\alpha) \to (\mu(\gamma\to\beta) \to (\mu(\gamma)\to \mu(\alpha\wedge\beta))))$

30) $\vdash (\mu(\alpha\to(\beta\to\gamma)) \to (\mu(\alpha\wedge\beta) \to\mu(\gamma)))$

31) $\vdash ((\mu(\alpha\wedge\beta)\to\mu(\gamma)) \to \mu(\alpha\to(\beta\to\gamma)))$

32) $\vdash (\mu(\alpha\wedge\sim\alpha) \to \mu(\beta))$

33) $\vdash ((\mu(\alpha)\to\mu(\alpha\wedge\sim\alpha)) \to \mu(\sim\alpha))$

And for intuitively established rules belonging to the system L μ, under the no. 19), we may find a proper justification in logic LI.
All rules listed there are "valid":

($\wedge$1), because '$\alpha\to(\beta\to (\alpha\wedge\beta))$'$\in$*LI*. Proof of this thesis one can see in Dopp[16].

($\wedge$2), for [5]; ($\wedge$3), for [6]; ($\vee$1), for [2]; ($\vee$2), for [3]; ($\vee$3), for [4]; ($\to$1), for 20) and 21); ($\to$2), for '$\alpha\to((\alpha\to\beta)\to\beta)$'$\in$*LI*. Proof in Dopp under no. 1.6.

[16] J. Dopp, op.cit., thesis 2.24.

Rule (~ 1) is just a consideration of thinking normal, or in accordance with the assertion 18).

(~ 2) is valid on the basis of [11] and rule *quodlibet* (Q) – [10].
How to build a logic of beliefs LP? First, we realize that it is the logic in which analysed beliefs can be:
– fixed (strong, assertive, certain) what we write in the form of 'A (α)' and read: "I believe firmly that α";
– assumed (hypothetical, views), what we write in the form of 'P (α)' and read: "I suppose that α";
– admitted (allowed, suppositional), what we write in the form of 'D (α)' and read: "I admit that α".
Using the definitions, we specify the meaning of these new epistemic operators[17]:

Df.A: $\vdash [A(\alpha) \leftrightarrow \mu(\alpha) \wedge \mu(\sim\exists t\, \mu(t,\sim\alpha))]$[18]

I believe firmly that α iff (if and only if) I think α and at the same time I think that I never think that $\sim\alpha$.

Let us note that the presence of quantifiers in the argument of the operator "μ" is deductively irrelevant because this is a non-extensional operator, but it is important for the explication of the meaning of the operator "A".

Df.P: $\vdash [P(\alpha) \leftrightarrow \mu(\alpha)]$

I suppose α when I think α. From the assumptions we do not expect – but also do not rule out – assertive beliefs.

Df.D: $\vdash [D(\alpha) \leftrightarrow \sim\mu(\sim\alpha)]$

I admit that α when I do not think that $\sim\alpha$.

On the basis of the adopted definitions we can immediately infer several theorems in logic of beliefs:

(1) $\vdash [A(\alpha) \rightarrow P(\alpha)]$, because Df.A, Df.P

[17] Definitions are equivalences type $\alpha \leftrightarrow \beta$ what in intuitionistic logic is understood as a conjunction of two implications: $(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$, rules applied here are: $\vdash(\alpha\leftrightarrow\beta) \vdash \vdash(\alpha\rightarrow\beta)$; $\vdash(\alpha\leftrightarrow\beta) \vdash \vdash(\beta\rightarrow\alpha)$; $\vdash(\alpha\rightarrow\beta), \vdash (\beta\rightarrow\alpha) \vdash \vdash(\alpha\leftrightarrow\beta)$.

[18] Specifically, with the same default constant personal c_o: $\vdash [A(t_o,\alpha) \leftrightarrow \mu(t_o, \alpha) \wedge \mu(t_o, \sim\exists t\, \mu(t,\sim\alpha))]$.

(2) $\vdash [\, P(\alpha) \rightarrow D(\alpha)]$, because Df.P, Df.D, (~1)

(3) $\vdash [A(\alpha) \rightarrow D(\alpha)]$, because Df.A, Df.D

(4) $\vdash [D(\alpha) \leftrightarrow \sim P(\sim\alpha)]$, because Df.D i Df.P

(5) $\vdash [\, P(\sim\alpha) \leftrightarrow \sim D(\alpha)]$, because (4): $\alpha/\sim\alpha$

(6) $\vdash [A(\alpha) \rightarrow\sim P\sim\alpha)]$, because (3) and (4)

(7) $\vdash [P(\sim\alpha) \rightarrow\sim A(\alpha)]$, because (6)

(8) $\vdash [A(A(\alpha)) \quad P(A(\alpha))]$, because (1): $\alpha/A\alpha$

The next theorems we get from those already listed, on the basis of the tautologies: *LI:* [1] i $\vdash ((\alpha\rightarrow\beta) \rightarrow (\sim\beta \rightarrow \sim\alpha))$[19].
We note also in our system L μ that every affirmation α is at once the acceptance of two other facts:

34) If $\vdash\alpha$ then $\vdash\mu(\alpha)$

35) If $\vdash\alpha$ then $\vdash\mu(\sim\exists t\ \mu(t,\sim\alpha))$

but at the same time we reject ($\dashv$) relationships that:

36) $\dashv [\alpha\rightarrow\mu(\alpha)]$

37) $\dashv [\alpha \rightarrow\mu(\sim\exists t\ \mu(t,\sim\alpha))]$

38) $\dashv [\mu(\alpha)\rightarrow\alpha]$

39) $\dashv [\mu(\sim\exists t\ \mu(t,\sim\alpha))\rightarrow\alpha]$

because there are cases – and what's more numerous – that α is, but there is no thinking about it, and vice versa: I think α, when it does not occur. Based on the findings of 34) – 39) we can draw conclusions that:

40) $(\vdash\alpha)\vdash (\vdash\mu(\alpha))$

[19] J. Dopp, op.cit., thesis 3.86.

41) $(\vdash\alpha)\vdash(\vdash\mu(\sim\exists t\ \mu(t,\sim\alpha))$

and hence – on the basis of ($\wedge$1) and Df.A – we recognize the so called "permissible"[20] Gödel's rule (RG):

42) RG: $(\vdash\alpha)\vdash(\vdash A(\alpha))$

and we reject the implication:

43) $\dashv [A(\alpha)\rightarrow\alpha]$

Let us call the set of all established propositional expressions a basis of the deductive system – as in our case LI is such a basis of L μ. Then, for each thesis α belonging to this basis, we have the right to join – on the basis of the rule RG – a theorem Aα, as proved. For example, '$\alpha \rightarrow \alpha$' $\in$ LI, so 'A ($\alpha \rightarrow \alpha$)' $\in$ μ L, and $\vdash [A(\alpha \vee \sim \alpha) \rightarrow (\alpha \vee \sim \alpha)]$, because '$\alpha \vee \sim \alpha$' $\notin$ LI[21].

In conclusion we mark:

I. The set of all theorems of the intuitionistic logic, the scope of the LI, is the basis of the logic LP;
II. The logic of beliefs was intuitively generated on the basis of the theory of L μ, consistent in its nature with the logic of LI;
III. One rule of inference, RO, is enough to create axiomatic systems LI and L μ;
IV. To construct a logic of beliefs in addition to the axioms and the rule RO, the rule RG and definitions of epistemic operators are necessary.
V. For the final results of our research we should consider the issue of our ability to properly define operator "A" in logic LI, without quantifier "$\exists$" in the definiens and without predicate "μ" taken from everyday language.
VI. We present, therefore, *a priori*, what we detected *a posteriori*.

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[20] Compare footnote 14. Since the RG rule is only admissible in *L*μ and is not valid, then by 40) we derive the rule $\vdash A(\alpha) \vdash \vdash A(A(\alpha))$, while the formula: $\vdash [A(\alpha) \rightarrow A(A(\alpha))]$ is not a thesis.

[21] See J. Dopp, op.cit., thesis 2.1.

The proposed logic of beliefs LP is an essential extension of the intuitionistic logic LI. The original term is “P”, which, together with its argument, we write as ‘P α' and we read: "I suppose α".
Firstly, the axioms of the logic-LI are axioms for the logic-LP: [1] – [11]. Then we take a proper axiom introducing the prime term of the logic-LP:

01. $P\alpha \leftrightarrow \alpha$

and the definition of the term: 'D α' – "I admit that α":

Df.D: $(D\alpha \leftrightarrow \sim P \sim \alpha)$

We get out of here immediately theorems of non-contradiction, incompleteness, and separation of the *attribute* of supposition towards the implication[22].

02. $(P\alpha \rightarrow \sim P \sim \alpha)$ based on: Dopp 1.1 $p \rightarrow p$, 3.2 $p \rightarrow \sim\sim p$

03. $\sim(P\alpha \wedge \sim P\alpha)$, because Dopp 3.26 $(p \wedge \sim p)$

04. $(P(\alpha \rightarrow \beta) \rightarrow (P\alpha \rightarrow P\beta))$, because Dopp 1.1 $p \rightarrow p$, (1)

The primary rules of the system are:

1. The rule of substitution
2. The rule of detachment: RO: $\alpha, \alpha \rightarrow \beta \vdash \beta$

3. SOME METATHEORETICAL REMARKS

We accept as in the metatheory of LP – as in the L μ – that:

$K = \{\alpha: \alpha \in \Phi\}$,

which means, that correlates are extra-linguistic counterparts to formulas. But beyond the language, there are two fields in which a projection of language occurs: one field introspective – totality of all propositions,

[22] One could even try to apply Lukasiewicz’s procedure for rejecting (for example, one can use axiom: $\dashv p \vee \sim p$ (Dopp 2.1)), and trivalent Heyting matrix.

(which somebody thinks actually or potentially), and the second – the outer one also in relation to the thinking and thoughts. The projection of the language LP is done, of course, into the first field.

$$S = \{\underline{\alpha} \in K: \Diamond \underline{\alpha}\}$$

In the present case, the situations, are the judgments which are thought (not necessarily given). Judgments have contents. When the content is incomprehensible, unknown, unresolved, etc. we have only judgment thought. ("I suppose Aristotelian *goat-deer* is a mammal". "I guess you're right").

$$F = \{\underline{\alpha} \in S: E(\underline{\alpha})\}$$

Now, the fact about a judgment is that "it is", "occurs", that it is a proposition given (by the user of a language). The logical value of truth is in this case determined by the equivalence:

$$V(\alpha, S) = 1 \text{ iff } \underline{\alpha} \in F.$$

In the logic LP, and in fact in its metatheory, another substantial equivalence is valid now:

$$\underline{\alpha} \in F \text{ iff } \underline{P\alpha} \in F.$$

Given the axiom 01. and the definition Df.D, we see that the theorems of the LP are:

05. $P\alpha \rightarrow D\alpha$, because $p \rightarrow \sim\sim p$, and

06. $D\alpha \leftrightarrow \sim\sim\alpha$

At first glance, it would seem that the equivalency: $A\alpha \leftrightarrow P\alpha \wedge \sim D \sim \alpha$ could be the definition of assertion (certainty, belief), because it seems to be clear that I believe firmly that there is life on Mars when I suppose it is there and I do not admit (as anything possible) that it is not there. But the implication $p \rightarrow \sim\sim\sim\sim p$ is a logical tautology in intuitionistic logic, hence our $A\alpha$ would be equivalent to $P\alpha$[23].

[23] $A\alpha \leftrightarrow \alpha \wedge \sim\sim\sim\sim\alpha \leftrightarrow \alpha \leftrightarrow P\alpha$.

If on the other hand we would accept axiomatically that $A\alpha \rightarrow P\alpha$, although it would sound reasonably, then it would guide us directly to the thesis: $A\alpha \rightarrow \alpha$, which would not have been reasonable, because it would be inconsistent with what we'd agreed earlier *a posteriori* that – if you do not aspire to the attribute of infallibility – things are not always such, as we strongly have them in mind[24].

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[24] It is also not especially helpful reading the operator "A" as "I know" or "I believe" because from the fact that I know that p, does not follow that p.