QUANTITATIVE METHODS IN ECONOMICS
Vol. XIV, No. 1, 2013, pp. 27 – 36

PRODUCTIVITY CHANGES OVER TIME – THEORETICAL AND METHODOLOGICAL FRAMEWORK

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Abstract: The TFPC Malmquist index is one of the tools that does not require knowledge of the price level and gives information on factors affecting productivity changes over time. The DEA-based approach allows decomposing of the TFPC indices into: technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). A panel data from the companies of a key food processing sector, namely the meat processing, was used in the paper. The sample consisted of above 200 objects. The results indicated which of the decomposed indices (ΔT, ΔTE, ΔSE) had the greatest impact on productivity changes in the analyzed sector.

Keywords: productivity growth, the Malmquist index, food processing sector

INTRODUCTION

The purpose of the paper was to assess the productivity as a measure of efficiency-based relations in a selected sector of food processing. The accomplishment of this goal funds a strong economic background. The theoretical background of productivity and efficiency was presented within the framework of the paper. The assessment of productivity was conducted by using advanced computing techniques, namely the total factor productivity index – the TFPC[1] Malmquist index which bases on deterministic approach. A decomposition of the resulting index was conducted in the paper. Its components are a subject of evaluation, namely the change in manufacturing techniques (technical progress), the change in technical efficiency and scale efficiency change. The decomposition

[1] TFPC, *total factor productivity change*.

of the index extends the interpretation of the sources of the changes in efficiency-based relations over time.

The measuring of TFP (*total factor productivity*) indices is very commonly used in literature. Two main approaches, referred to as the axiomatic and the economic approach, can be distinguished in the literature of TFP indices [Diewert 1992]. The axiomatic approach postulates a number of properties that any index number should satisfied. By contrast, the economic approach is based on the economic theory and its behavioral assumptions. An example of the economic approach is the Malmquist index [Caves et al. 1982b]. One advantage of economic approach to TFP measurement is the availability of decompositions that gives information on the underlying sources of productivity growth [Kuosmanen and Sipilainen 2009]. In the paper the economic approach that bases on the application of the decomposed TFPC Malmquist index was applied.

ECONOMIC BACKGROUND

A firm operating on a competitive market seeks possibilities for maximization of expected profit by increasing production, especially by non-decreasing returns to scale[2]. Researchers confirm that the output growth in the agri-food sector is determined by a growth of demand for agri-food products occurring at a specific time[3]. The low growth rate of demand for agri-food products can limit the growth in the agri-food sector, and consequently, the production and processing growth inducing technical change. Therefore, the low growth rate of demand for agri-food products must determine the change of efficiency-based relations treated as a main growth factor in the sector. Thus, the authors believe that not the increase of input factors but the efficiency of its use is the main factor of firms' ability for long-term and effective growth and performance.

In the paper, it was assumed – based on W. Rembisz and A. Bezat-Jarzębowska – that changes in production efficiency are the endogenous condition of a company [Rembisz and Bezat-Jarzębowska 2013]. It is uncontested that the efficiency is a base source of profitability improving [Rembisz 2008]. Thus, it is important to understand which factors lead to improving of efficiency. The issue refers to both the allocation and distribution. Firstly, the aspect of allocation is described by the changes in production techniques (technical relations) – thus the productivity of each production factors. Secondly, the aspect of distribution refers to remuneration of each production factor. This should result – based on

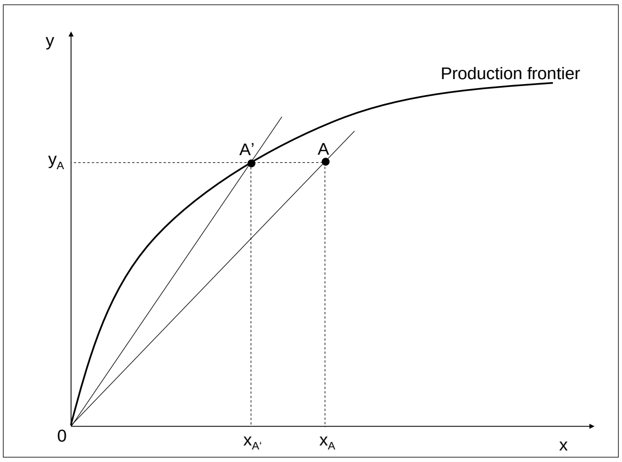
[2] Autors assume that in competitive market equilibrium the price is fixed for processor in agri-food sector.

[3] For more details see: Figiel Sz., Rembisz W. (2009) Przesłanki wzrostu produkcji w sektorze rolno-spożywczym – ujęcie analityczne i empiryczne, Multi-annual Programme 2005-2009, No. 169, Wyd. IERiGŻ, Warszawa.

neoclassical assumption – from the productivity of each factor, in particular from its marginal productivity (which is reflected by the scale of economic activity).

The concepts of productivity and efficiency are often used interchangeably; however, they do not mean exactly the same things [Coelli et al. 2005]. Some theoretical considerations on it were presented in the paper, nevertheless to illustrate the difference between them, a simple manufacturing process was shown, in which a single input (*x*) is used for obtaining a single output (*y*). Figure 1 illustrates the production frontier with one input and one output. Productivity of object A is calculated as output/input quotient (y_A/x_A), hence it is the value for the tangent of the angle between the x-axis and the 0A curve. The highest productivity to be achieved by object A (after reducing number of inputs for a given level of outputs) was determined by point A' ($y_A/x_{A'}$) [Coelli et al. 2005]. Further, it can be stated that the highest value of productivity is achieved by an object lying at the point of tangency of the production frontier and a curve drawn from the origin of the coordinate system [Jarzębowski 2011].

Figure 1. Efficiency and productivity - a comparison



Source: own work based on Coelli et al. 2005, p. 5

The efficiency of A is calculated as the ratio of minimum input ($x_{A'}$) needed to achieve a given level of output and actual input used to achieve this output (x_A). The highest efficiency (equal to 1) is obtained by objects lying on the production frontier. Therefore, efficient objects not necessarily achieve the highest level of productivity, since not each of them lies on the tangent to the production frontier. The result of the above evidence is the fact that productivity is not a synonymous term with efficiency.

There are various measures of productivity, depending on criterion (quantitative or valuable), used in order to express inputs and outputs. The decision on the choice of one from all the forms depends on the subject of the study, the purpose of its analysis and practical reasons (such as the availability of data) [Coelli et al. 2005].

For analysis of productivity change, indices of value are used. They are applied when changes in prices and volumes are measured over time; they allow also comparing productivity during the given time in a number of companies, industries, regions or countries. A price index may include consumer prices, prices of inputs and outputs, prices of import and export, etc., while a value index can measure changes in the volume of the outputs produced or inputs used in companies and industries taking into account changes over time or comparing companies among themselves.

Indices of value play an important role in economic sciences. The indices of Laspeyres and Paasche are one of the most important contributions, dated for the late XIX century. These indices are still used by statistical offices around the world. Irving Fisher's book "The Making of Index Numbers", published in 1922, showed application of many statistical formulas for determining value indices. And the Tornquist's index (1936) is formula which plays the main role in measuring of efficiency.

Traditional productivity measures can be applied in the analysis of companies' performance in one period. In contrary to the classical measures, the Malmquist index allows to analyze productivity change of companies or industries over time. The second problem connected with the productivity measurement is the fact that, one needs to know price levels. The most common examples of measurement using prices of production factors are the indices of: Laspeyres, Paasche and Fischer. Aggregation of the values expressed in other measurement units than prices is difficult. However, it is possible to express changes in analyzed inputs and outputs in a given period compared to the previous one.

The DEA-based Malmquist index is one of the tools that does not require knowledge of the price level and gives information on factors affecting productivity changes over time [Coelli et al. 2005].

METHODOLOGICAL BACKGROUND

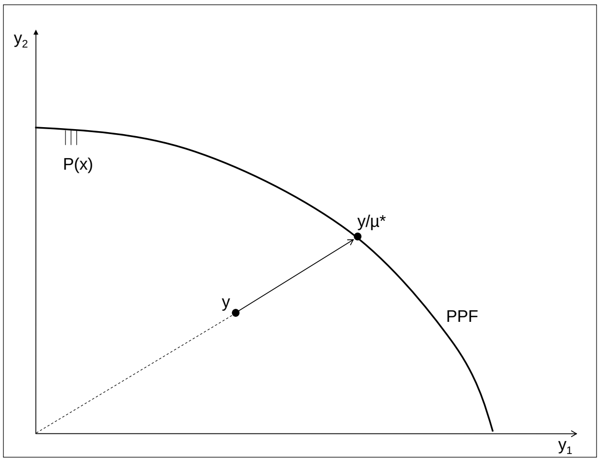
The productivity is almost always defined as the output-input ratio. However, it is quite common among researchers to analyse aggregated multi-outputs and multi-inputs cases. This is formally defined as *total factor productivity* (TFP) of a firm. In order to measure the changes in productivity a productivity change index has to be built. The *total factor productivity change index*is simply calculated as a ratio of the TFP of firm (i) in period (t) relative to the TFP of this firm (h) in period ($t+1$). This simple concept is used in case of the Mamlquist TFPC index which application was made in the paper.

The Malmquist productivity index was introduced as a theoretical index by D.W. Caves, L.R. Christensen and W.E. Diewert. [Caves at al. 1982a] and popularized as an empirical index by R. Färe, S. Grosskopf, B. Lindgren, P. Roos

[Färe et al. 1994]. The big advantage of the Malmquist TFPC index is that the interpreting of its value is relative simple and it is not necessary to know the factor prices. Further advantage is that the Malmquist TFPC index allows indicating the factors that affect productivity and its changes. The index can be decomposed into three different factors, namely: technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). It is important not to miss any of these factors because the results could be interpreted incorrectly.

When calculating the TFPC Malmquist index one bases on the distance function. A distance function (defined as $D_O(\boldsymbol{x},\boldsymbol{y}) = min\{\mu: \boldsymbol{x}/\mu \epsilon P(\boldsymbol{x})\}$, where $\boldsymbol{x}$ – input vector, $\boldsymbol{y}$ – output vector, P – output set) takes an output-expanding approach to the measurement of the distance from a producer to the production possibility frontier (PPF, see Figure 1) [Krumbhakar and Lovell 2004, p. 28]. This function gives the minimum value μ by which an output vector can be deflated and still remain producible with a given input vector. As it is shown in Figure 2, the output vector $\boldsymbol{y}$ is producible with input $\boldsymbol{x}$, but so is the radially expanded output vector (y/μ^*), and so $D_O(\boldsymbol{x},\boldsymbol{y}) = \mu^* < 1$. An object for which $D_O(\boldsymbol{x},\boldsymbol{y}) = 1$[4], is an efficient one, and if the value is smaller than one, an object is inefficient [Bezat 2012].

Figure 2. An output distance function



Source: own work based on Krumbhakar and Lovell 2004, p. 31

Application of distance functions (which are incorporated in the DEA method)allows measuring of the total productivity over time and allows making comparisons across firms and for a given firm over time.

[4] The relation is true for objects lying on the production possibility frontier.

The output-oriented Malmquist productivity index[5] using the period (*t*) benchmark technology is defined as [Färe et al. 1994]:

$$M_O^t(y_t, y_{t+1}, x_t, x_{t+1}) = \frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} \tag{1}$$

where (x^τ, y^τ) ($\tau = t, t+1$) is an observed τ –period input-output vector, the "*O*" denoted the output-orientation.
The period (*t+1*) Malmquist index using the output orientation is defined as:

$$M_O^{t+1}(y_t, y_{t+1}, x_t, x_{t+1}) = \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^t, y^t)} \tag{2}$$

The TFPC Malmquist index can be estimated by using the output-oriented Malmquist index between period (*t*) and period (*t+1*). It is defined as the geometric mean of two Malmquist indices between two time periods ($M_O^{t,t+1}$) [Cantner et al. 2007]. Thus, it's defined as[6]:

$$M_O^{t,t+1} = [M_O^t \times M_O^{t+1}]^{1/2} = \left[\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} \times \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^t, y^t)}\right]^{1/2} \tag{3}$$

It is possible that observed productivity improvement (change) reflected in the Malmquist TFPC index could be the result of improvement of the underlying production technology (technical change) and/or of technical efficiency change (technical efficiency change). In this case the above Malmquist index (3) can be decomposed into ΔT[7] and ΔOTE[8] as follows [Cantner et al. 2007]:

$$M_O^{t,t+1} = \left[\frac{D_O^t(x^{t+1}, y^{t+1})}{D_O^{t+1}(x^{t+1}, y^{t+1})} \times \frac{D_O^t(x^t, y^t)}{D_O^{t+1}(x^t, y^t)}\right]^{1/2} \times \frac{D_O^{t+1}(x^{t+1}, y^{t+1})}{D_O^t(x^t, y^t)} = \Delta T \times \Delta OTE \tag{4}$$

Under the CRS (*Constant Returns to Scale*) assumption, in case of index (4), there will be no scale efficiency change. However the productivity of some firms can increase by changing the scale of its operation what leads to operating an optimal scale of production by the company. Following T.J. Coelli, P. Rao, Ch.J. O`Donnell and G.E. Battese (2005) we have the decomposition of the ΔOTE [Coelli et al. 2005]:

[5] This section could be written using input distance functions to define an input-oriented Malmquist productivity index, and nothing in the way of decomposing would change.

[6] We can have $M_O^{t,t+1} > 1$, $M_O^{t,t+1} = 1$ or $M_O^{t,t+1} < 1$ according as productivity growth, stagnation or decline occurs between period (*t*) and (*t+1*).

[7] The ΔT is a geometric mean of the shift in production frontier in time (*t*) and (*t+1*) at inputs levels x^t and x^{t+1}(graphically – shift of production possibilities frontier).

[8] The ΔOTE measures the change in the technical efficiency between periods (*t*) and (*t+1*) (graphically – shift towards or away from production possibilities frontier).

$$\Delta OTE = \frac{D_{O^*}^{t+1}(x^{t+1}, y^{t+1})}{D_{O^*}^{t}(x^{t}, y^{t})} \times \left[\frac{\frac{D_O^{t+1}(x^{t+1},y^{t+1})}{D_O^{t+1}(x^{t+1},y^{t+1})}}{\frac{D_O^{t}(x^{t},y^{t})}{D_O^{t}(x^{t},y^{t})}}\right] =$$

$$=\Delta TE \times \left[\frac{SE^{t+1}(x^{t+1},y^{t+1})}{SE^{t}\ (x^{t},y^{t})}\right] = \Delta TE \times \Delta SE \quad (5)$$

where the ΔTE measures technical efficiency change on the best practice technologies (production frontier), denoted as "O^*", and the ΔSE measures the change in scale efficiency from period (*t*) to period (*t+1*)[9]. The ΔSE is measured by using the output distance of observed input-output vectors from the CRS frontier relative to the VRS (*Variable Returns to Scale*) frontier. The product of this tree changes is equal to the TFPC Malmquist index in time period (*t*) to (*t+1*). Thus, the TFPC Malmquist index can be written as:

$$\text{TFPC} = \Delta T \times \Delta TE \times \Delta SE \quad (6)$$

If a data on a cross-section of companies in periods (t) and (t+1)is available then it is possible to use for example the DEA to calculate the TFPC index [Coelli et al. 2005]. The application of the DEA-based TFPC Mamlquist index was conducted in the next part of the paper.

APPLICATION OF THE DEA-BASED TFPC MALMQUIST INDEX

The productivity change assessment was carried out on the basis of data from the *Monitor Polski B*. The sample covered 223 companies from meat processing sector from across Poland from period 2006-2011 (the balanced panel). The selection of a specific sector was made because of different production's technologies in different food processing sectors. The data was reported as revenue/expenditure denominated in PLN in constant prices. The TFPC Malmquist index was calculated for a single output and two inputs. The inputs and the output are identified in table 1. The variables were selected to reflect the cost sources and production possibilities on the input side and the revenue sources on the output side.

Table 1. Inputs and outputs used to assess the efficiency scores

Inputs	Outputs
X1 – operational costs	Y –revenue on sales
X2 – value of assets	

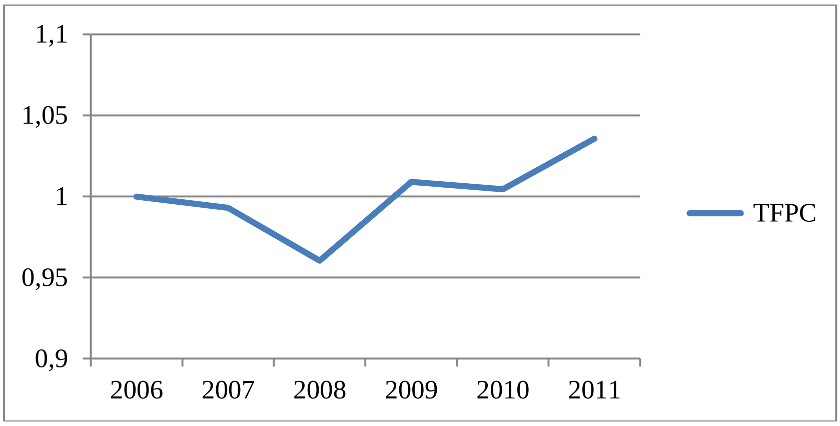
Source: own work based on Coelli et al. 2005, p. 5

Figure 3 shows the changes in productivity growth in the period 2006–2011. The significant progress (increase in productivity) can be observed in the period

[9]Graphically movements around the frontier surface to capture economies of scale.

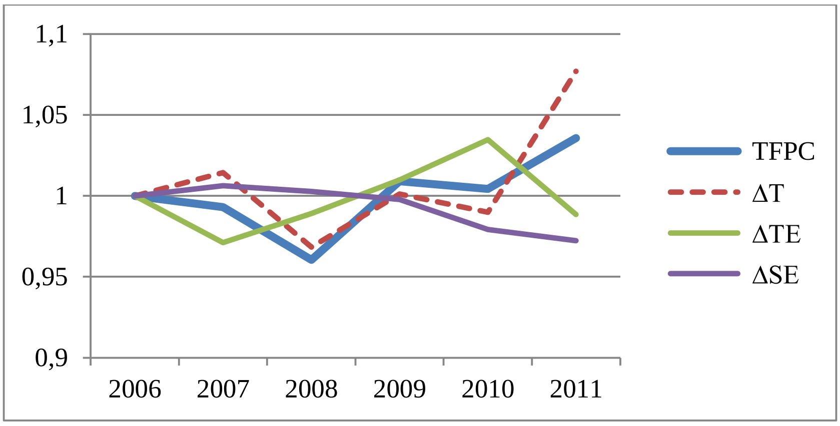
2008–2011. If the decomposition was not be made in the paper, the Figure 3 would be the only information regarding changes in the productivity in the Polish meet sector. The decomposition of the index allows researchers to analyze different sources of the productivity change (see Figure 3).

Figure 3. The changes of the mean value of the TFPC in the meat processing sector in period 2006–2011 (previous year = 100)



Source: own calculations

Figure 4. The changes of the mean value of the TFPC, the mean value of the ΔT, the ΔTE and the ΔSE in the meat processing sector in period 2006–2011 (previous year = 100)



Source: own calculations

Figure 4 reports the cumulative indices of each component of the proposed decomposition, namely technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). The results suggest that the technical change was the main driver of TFPC change. An increase in the ΔT was observed in the period 2008–2011. It indicates the shift upwards of production frontier. The scale efficiency has slightly decreased in the sample. Other growth factor was the technical change which has not changed much over the analyzed years.

CONCLUSIONS

Within the framework of the paper the theoretical background for relationships between productivity and efficiency was presented. Some theoretical and analytical implications for modeling of productivity changes are detected in the study as well.

The analysis of productivity changes is of main interest of economic researcher. Among tools which can be used to measure productivity change one can find the TFPC Malmquist index. The use of the DEA-based Malmquist index allows including a number of inputs which might influence the productivity. The decomposition of TFPC index allows detecting some sources of productivity changes which might be: technical change (ΔT), technical efficiency change (ΔTE) and scale efficiency change (ΔSE). The decomposing of the productivity growth provides valuable information to managers and planners.

The application of the TFPC Malmquist index in the analyzed sample of companies allows concluding that the technical change (ΔT) was the main driver of the productivity changes in the meat processing sector in the period 2006–2011.

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ACNOWLEDGEMENTS

The research conducted within this paper was granted by the Ministry of Science and Higher Education from the funds for science in 2013 as a scientific project „The increase in productivity as a measure of changes in efficiency-based relations over time"(„Wzrost produktywności jako miernik zmian relacji efektywnościowych w czasie").