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ESTIMATION OF POPULATION MEAN USING MULTI-AUXILIARY CHARACTERS WITH SUBSAMPLING THE NONRESPONDENTS

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ABSTRACT

The aim of this paper is to suggest a class of two phase sampling estimators for population mean using multi-auxiliary characters in presence of non-response on study character. The expressions for bias and mean square error are obtained. The condition for minimum mean square error of the proposed class of estimators has been given. The optimum values of the size of first phase sample, second phase sample and the sub sampling fraction of non-responding group have been determined for the fixed cost and for the specified precision. A comparative study of the proposed class of estimators has been carried out with an empirical study.

Key words: Population mean; Bias; Mean square error; Multi-auxiliary characters.

1. Introduction

Hansen and Hurwitz (1946) suggested that the effect of non-response while conducting a sample survey can be reduced by using the method of subsampling from non-responding units. Further, the improvement in reducing the effect of non-response was considered by El-Badry (1956) and Foradori (1961). Rao (1986, 90) and Khare and Srivastava (1996, 97, 2000) proposed some estimators for population mean by using the auxiliary character with known population mean in presence of non-response while Khare and Sinha (2009) proposed some classes of estimators for population mean by using multi-auxiliary characters in presence of non-response.

But sometimes it has been observed that the population means of the auxiliary characters are not known due to the change in scenario. In such case we draw a

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first phase sample to observe the auxiliary characters which is used in estimating the population mean of the auxiliary characters. Khare (1992) proposed two phase sampling regression estimators for population mean in presence of non-response. Further, Khare and Srivastava (1993, 1995) proposed two phase sampling ratio and product estimators and made a comparative study for two phase sampling ratios, product and regression type estimators in presence of non-response while Khare and Sinha (2002) proposed general classes of two phase sampling estimators for population mean using an auxiliary character in presence of non-response.

In this paper, we have suggested a class of two phase sampling estimators (T) for population mean using multi-auxiliary characters with unknown population means in presence of non-response on study character only. The expressions for bias, mean square error and the condition for attaining the minimum mean square error of the proposed class of estimators have been obtained. The optimum values of the size of the first phase sample (n'), second phase sample (n) and the subsampling fraction $(1/k)$ of non-responding group have been determined for the fixed cost and for the specified precision. The merits of the suggested class of estimators have been judged through an empirical study.

2. The suggested class of estimator

In most of the situations it usually happens that the list of the units is available but the population means $\bar{X}_1, \bar{X}_2, \ldots\ldots, \bar{X}_p$ of the auxiliary characters $x_1, x_2, \ldots\ldots, x_p$ respectively are not known. In such situations, we select a larger sample of size n' from N in the first phase by using simple random sampling without replacement (SRSWOR) method of sampling and collect information regarding the auxiliary characters and estimate $\bar{X}_j$ $(j = 1,2,\ldots,p)$ based on n' units by $\bar{x}_j'$ $(j = 1,2,\ldots,p)$. Again, select a second phase sample of size n $(< n')$ from selected n' first phase units by using SRSWOR method of sampling and observe the study character y. We observe that n_1 units are responding and n_2 units are not responding in the sample of size n for the study character y. Further, from n_2 non-responding units, we select a subsample of size r $\left(r = \frac{n_2}{k}, k > 1\right)$ using SRSWOR method of sampling by making extra effort and observe the study character y.

Now, we have information on $(n_1 + r)$ selected units for the study character y. Using Hansen and Hurwitz (1946) technique, the estimator for $\bar{Y}$ based on $(n_1 + r)$ units is given by

$$\bar{y}^* = \frac{n_1}{n}\bar{y}_1' + \frac{n_2}{n}\bar{y}_2'', \tag{2.1}$$

where $\bar{y}_1'$ and $\bar{y}_2''$ are the sample means of the character y based on n_1 and r units respectively.

The estimator $\bar{y}^*$ is unbiased and has the variance given by

$$V(\bar{y}^*) = \frac{1-f}{n} S_0^2 + \frac{W_2(k-1)}{n} S_{0(2)}^2 , \qquad (2.2)$$

where S_0^2 and $S_{0(2)}^2$ are the population mean square of y for the entire population and for the non-responding part of the population respectively and $W_i = N_i/N$, $(i = 1,2)$, $f = {}^n/_N$.

Let $\bar{x}_j'$ and $\bar{x}_j$ denote the sample means of the auxiliary characters $x_j (j = 1,2, \ldots, p)$ based on n' and n units respectively. Let $Y_l, X_{1l}, X_{2l}, \ldots X_{pl}$ be the $l^{th} (l = 1,2, \ldots, N)$ unit of the characters $y, x_1, x_2, \ldots, x_p$ respectively in the population of size N and are assumed to be non-negative.

In case when population means $\bar{X}_1, \bar{X}_2, \ldots\ldots, \bar{X}_p$ of the auxiliary characters $x_1, x_2, \ldots\ldots, x_p$ respectively are not known then they are estimated by $\bar{x}_1', \bar{x}_2', \ldots, \bar{x}_p'$ which are based on larger first phase of size n'. In such situation, when we have incomplete information on the study character but complete information on the auxiliary characters from the sample of size n, we propose some generalized two phase sampling estimators with suitably chosen constants v_{1j}, v_{2j}, v_{3j} and β_{1j} for population mean $\bar{Y}$

$$T_{01}^{**} = v \, exp\left[\sum_{j=1}^{p} v_{1j} \log z_j\right], \qquad (2.3)$$

$$T_{02}^{**} = v \sum_{j=1}^{p} W_j \, z_j^{(v_{2j}/W_j)}, \qquad \sum_{j=1}^{p} W_j = 1 \qquad (2.4)$$

and

$$T_{03}^{**} = \sum_{j=1}^{p} \left[W_j \, z_j^{(v_{3j}/W_j)}\right] \left[v + \beta_{1j}(z_j - 1)\right], \qquad (2.5)$$

which are the members of our suggested class of two phase sampling estimators T given by

$$T = f(v, \boldsymbol{z}') \qquad (2.6)$$

such that

$$f(\bar{Y}, \boldsymbol{e}') = \bar{Y} , \quad f_1(\bar{Y}, \boldsymbol{e}') = \left(\frac{\partial}{\partial v} f(v, \boldsymbol{z}')\right)_{(\bar{Y}, \boldsymbol{e}')} = 1 \qquad (2.7)$$

where $\boldsymbol{z}$ and $\boldsymbol{e}$ denote the column vectors $(z_1, z_2, \ldots, z_p)'$ and $(1,1, \ldots, 1)'$ respectively and $v = \bar{y}^*$, $z_j = \frac{\bar{x}_j}{\bar{x}_j'}$, $(j = 1,2, \ldots, p)$.

The function $f(v, \boldsymbol{z}')$ also satisfies the following conditions:

(i) For any sampling design, whatever be the sample chosen, $(v, \boldsymbol{z}')$ assumes values in a bounded closed convex subset D, of $p + 1$ dimensional real space containing the point $(\bar{Y}, \boldsymbol{e}')$.

(ii) In D, the function $f(v, \boldsymbol{z}')$ and its first and second derivatives exist and are continuous and bounded.

Here, $f_1(v, \boldsymbol{z}')$ and $f_2(v, \boldsymbol{z}')$ denote the first partial derivatives of $f(v, \boldsymbol{z}')$ with respect to v and $\boldsymbol{z}'$ respectively. The second partial derivative of $f(v, \boldsymbol{z}')$ with respect to $\boldsymbol{z}'$ is denoted by $f_{22}(v, \boldsymbol{z}')$ and the first partial derivative of $f_2(v, \boldsymbol{z}')$ with respect to v is denoted by $f_{12}(v, \boldsymbol{z}')$.

On account of the regularity conditions imposed on $f(v,\boldsymbol{z}')$, it may be seen that the bias and mean square error of the estimator T will always exist.

3. Bias and mean square error (mse)

Expand $f(v,\boldsymbol{z}')$ about the point $(\bar{Y},\boldsymbol{e}')$ by using Taylor's series up to second order partial derivatives and using (2.7), the expressions for bias and mean square error of T up to the terms of order n^{-1} for any sampling design are as follows:

$$B(T) = E(v-\bar{Y})(\boldsymbol{z}-\boldsymbol{e})'\,f_{12}(v^*,\boldsymbol{z}'^*) + \frac{1}{2}E(\boldsymbol{z}-\boldsymbol{e})'f_{22}(v^*,\boldsymbol{z}'^*)(\boldsymbol{z}-\boldsymbol{e}) \tag{3.1}$$

and $$M(T) = V(\bar{y}^*) + 2E(v-\bar{Y})(\boldsymbol{z}-\boldsymbol{e})'\,f_2(\bar{Y},\boldsymbol{e}') + E\left(f_2(\bar{Y},\boldsymbol{e}')\right)'(\boldsymbol{z}-\boldsymbol{e})(\boldsymbol{z}-\boldsymbol{e})'f_2(\bar{Y},\boldsymbol{e}') \tag{3.2}$$

where $v^* = \bar{Y} + \emptyset(v-\bar{Y})$, $\boldsymbol{z}^* = \boldsymbol{e} + \boldsymbol{\emptyset_1}(\boldsymbol{z}-\boldsymbol{e})'$ and $\emptyset_1 = diag\left[\emptyset_{11},\emptyset_{12},\dots,\emptyset_{1p}\right]_{p\times p}$.

such that $0 < \emptyset,\emptyset_{1j} < 1, \quad \forall j = 1,2,\dots,p$

The optimum value of $f_2(\bar{Y},\boldsymbol{e}')$ for which the $M(T)$ will be minimum is given by

$$f_2(\bar{Y},\boldsymbol{e}') = -(E(\boldsymbol{z}-\boldsymbol{e})(\boldsymbol{z}-\boldsymbol{e})')^{-1}\,E(v-\bar{Y})(\boldsymbol{z}-\boldsymbol{e}) \tag{3.3}$$

and the minimum value of $M(T)$ is given by

$$M(T)_{min.} = V(\bar{y}^*) - E(v-\bar{Y})(\boldsymbol{z}-\boldsymbol{e})'\,(E(\boldsymbol{z}-\boldsymbol{e})(\boldsymbol{z}-\boldsymbol{e})')^{-1}\,E(v-\bar{Y})(\boldsymbol{z}-\boldsymbol{e}) \tag{3.4}$$

Considering simple random sampling without replacement (SRSWOR) method, let $\boldsymbol{A}_0 = \left[a_{0jj'}\right]$ be a $p\times p$ positive definite matrix and $\boldsymbol{b} = (b_1,b_2,\dots,b_3)'$ is a column vector such that

$$a_{0jj'} = \rho_{jj'}\,C_jC_{j'}, \qquad j \neq j' = 1,2,\dots,p$$

and $$b_j = \rho_jC_j, \qquad j = 1,2,\dots,p,$$

where $C_j^2 = \frac{S_j^2}{\bar{X}_j^2}$, $S_j^2 = \frac{1}{N-1}\sum_{l=1}^{N}\left(X_{jl}-\bar{X}_j\right)^2$ and $\rho_{jj'}$ is the correlation coefficient between the auxiliary characters $(x_j,x_{j'})$ while ρ_j is the correlation coefficient between y and x_j for the entire group of the population.

Now, the expressions for bias and mean square error of T in the case of simple random sampling without replacement (SRSWOR) method of sampling are given by

$$B(T) = \gamma\left((S_0\boldsymbol{b}')f_{12}(v^*,\boldsymbol{z}'^*) + \frac{1}{2}\,trace\boldsymbol{A}_0f_{22}(v^*,\boldsymbol{z}'^*)\right) \tag{3.5}$$

and $$M(T) = V(\bar{y}^*) + \gamma\left(\left(f_2(\bar{Y},\boldsymbol{e}')\right)'.\boldsymbol{A}_0.f_2(\bar{Y},\boldsymbol{e}') + 2S_0\boldsymbol{b}'f_2(\bar{Y},\boldsymbol{e}')\right) \tag{3.6}$$

wher $\gamma = \frac{1}{n} - \frac{1}{n'}$ and $S_0^2 = \frac{1}{N-1}\sum_{l=1}^{N}(Y_l-\bar{Y})^2$.

The mean square error $M(T)$ will attain minimum value if

$$f_2(\bar{Y}, \boldsymbol{e}') = -S_0 \boldsymbol{A}_0^{-1} \boldsymbol{b} \tag{3.7}$$

and the minimum value of $M(T)$ is given by

$$M(T)_{min.} = V(\bar{y}^*) - \gamma S_0^2 \boldsymbol{b}' \boldsymbol{A}_0^{-1} \boldsymbol{b} \tag{3.8}$$

The suggested class of two phase sampling estimators has a wider class of estimators and all the members of this class attain minimum value of mean square error given in (3.8) if the condition (3.7) is applied. Now, using the conditions (3.7) in case of SRSWOR method of sampling, the constants involved in T_{01}^{**}, T_{02}^{**} and T_{03}^{**} can be evaluated. The condition (3.7) is sometimes obtained in the form of constants along with some parameters and sometimes in the form of some conditions between parameters. The later one is difficult to realize in practice and rarely used. In former case, the value of constants in the form of parameters can be computed on the basis of past data. Reddy (1978) showed that such values are stable over time and region. However, if no guess value is available from the past data then one can estimate it on the basis of sample observations without any loss in efficiency. Srivastava and Jhajj (1983) showed that up to the terms of order (n^{-1}), the efficiency of such type of estimators does not decrease if we replace the optimum values of the constants by their estimates based on sample values.

4. Determination of n', n and k for fixed cost $C \leq C_0$

Let C_0 be the total cost (fixed) of the survey apart from overhead cost. The cost function C′ for the cost incurred on the survey apart from overhead expenses can be expressed by

$$C' = C_1' n' + C_1 n + C_2 n_1 + C_3 \frac{n_2}{k} \tag{4.1}$$

Since C' will vary from sample to sample, so we consider the expected cost C to be incurred in the survey apart from overhead expenses, which is given by

$$C = E(C') = C_1' n' + n \left[C_1 + C_2 W_1 + C_3 \frac{W_3}{k} \right], \tag{4.2}$$

where

C_1': The cost per unit of identifying and observing auxiliary characters,
C_1: The cost per unit of mailing questionnaire/visiting the unit at the second phase,
C_2: The cost per unit of collecting and processing data for the study character y obtained from n_1 responding units and
C_3: The cost per unit of obtaining and processing data for the study character y (after extra efforts) from the subsampled units.

The $M(T)$ can be expressed in terms of the notations V_{01}, V_{11}, V_{21} which is given as

$$M(T) = \frac{1}{n}V_{01} + \frac{1}{n'}V_{11} + \frac{k}{n}V_{21} + terms\ independent\ of\ n, n'\ and\ k \quad (4.3)$$

where V_{01}, V_{11} and V_{21} are the coefficients of the terms of $\frac{1}{n}$, $\frac{1}{n'}$ and $\frac{k}{n}$ respectively in the expressions of $M(T)$.

Now, for minimizing the $M(T)$ for the fixed cost $C \leq C_0$ and to obtain the optimum values of n', n and k, we define a function ψ given as

$$\psi = M(T) + \lambda\left\{C_1'n' + n\left(C_1 + C_2W_1 + C_3\frac{W_2}{k}\right) - C_0\right\} \quad (4.4)$$

where λ is Lagrange's multiplier.

Now, differentiating ψ with respect to n', n and k and equating to zero, we have

$$n' = \sqrt{\frac{V_{11}}{\lambda C_1'}} \quad (4.5)$$

$$n = \sqrt{\frac{V_{01} + kV_{21}}{\lambda\left(C_1 + C_2W_1 + C_3\frac{W_2}{k}\right)}} \quad (4.6)$$

and

$$k_{opt.} = \sqrt{\frac{C_3W_2V_{01}}{(C_1 + C_2W_1)\,V_{21}}} \quad (4.7)$$

Using the value of k from (4.7) and putting the values of n' and n from (4.5) and (4.6) in (4.2), we have

$$\sqrt{\lambda} = \frac{1}{C_0}\left[\sqrt{C_1'V_{11}} + \sqrt{\left(V_{01} + k_{opt.}V_{21}\right)\left(C_1 + C_2W_1 + C_3\frac{W_2}{k_{opt.}}\right)}\right]. \quad (4.8)$$

It has also been observed that the determinant of the matrix of second order derivative of ψ with respect to n', n and k is positive for the optimum values of n', n and k, which shows that the solutions for n', n given by (4.5), (4.6) and the optimum value of k under the condition $C \leq C_0$ minimize the variance of T. It is also important to note here that the subsampling fraction $1/k_{opt.}$ will decrease as $\sqrt{C_3/(C_1 + C_2W_1)}$ increases.

The minimum value of $M(T)$ can be obtained by putting the optimum values of n', n and k in the expression (4.3). Hence, we have

$$M(T)_{min.} = \frac{1}{C_0}\left[\sqrt{C_1'V_{11}} + \sqrt{\left(V_{01} + k_{opt.}V_{21}\right)\left(C_1 + C_2W_1 + C_3\frac{W_2}{k_{opt.}}\right)}\right]^2 - \frac{S_0^2}{N}. \quad (4.9)$$

In case of $\bar{y}^*$, the expected total cost is given by

$$C = E(C') = n\left(C_1 + C_2W_1 + C_3\frac{W_2}{k}\right). \tag{4.10}$$

For fixed cost C_0, the expression for $M(\bar{y}^*)_{min.}$ is given by

$$M(\bar{y}^*)_{min.} = \frac{1}{C_0}\left[\sqrt{V_0(C_1 + C_2W_1)} + \sqrt{(C_3W_2V_2)}\right]^2 - \frac{S_0^2}{N} \tag{4.11}$$

where V_0 and V_2 are the coefficients of the terms of $\frac{1}{n}$ and $\frac{k}{n}$ respectively in the expression (2.2).

5. Determination of n', n and k for the specified variance $V = V_0''$

Let V_0'' be the variance of the estimator T fixed in advance and we have

$$V_0'' = \frac{1}{n}V_{01} + \frac{1}{n'}V_{11} + \frac{k}{n}V_{21} - \frac{S_0^2}{N}. \tag{5.1}$$

For minimizing the average total cost C for the specified variance of the estimator T (i.e. $M(T) = V_0''$), we define a function ψ' which is given as

$$\psi' = C_1'n' + n\left(C_1 + C_2W_1 + C_3\frac{W_2}{k}\right) - \mu(M(T) - V_0'') \tag{5.2}$$

where μ is Lagrange's multiplier.

Now, for obtaining the optimum values of n', n and k, differentiate ψ' with respect to n', n and k and equating to zero, we have

$$n' = \sqrt{\frac{\mu V_{11}}{C_1'}} \tag{5.3}$$

$$n = \sqrt{\frac{\mu(V_{01} + k\,V_{21})}{\left(C_1 + C_2W_1 + C_3\frac{W_2}{k}\right)}} \tag{5.4}$$

and

$$k_{opt.} = \sqrt{\frac{V_{01}C_3W_2}{V_{21}(C_1 + C_2W_1)}}. \tag{5.5}$$

Again, by putting the values of n' and n from (5.3) and (5.4) utilizing the optimum value of k in (5.1), we get

$$\sqrt{\mu} = \frac{\left[\sqrt{C_1'V_{11}} + \sqrt{(V_{01} + k_{opt.}V_{21})\left(C_1 + C_2W_1 + C_3\frac{W_2}{k_{opt.}}\right)}\right]}{\left[V_0'' + \frac{S_0^2}{N}\right]}. \tag{5.6}$$

The minimum expected total cost incurred in attaining the specified variance V_0'' by the estimator T is then given by

$$C(T)_{min.} = \frac{\left[\sqrt{C_1' V_{11}} + \sqrt{\left(V_{01} + k_{opt.} V_{21}\right)\left(C_1 + C_2 W_1 + C_3 \frac{W_2}{k_{opt.}}\right)}\right]^2}{\left[V_0'' + \frac{S_0^2}{N}\right]}. \quad (5.7)$$

6. An empirical study

96 village wise population of rural area under Police-station -Singur, District – Hooghly, West Bengal has been taken under the study from District Census Handbook 1981. The first 25% villages (i.e. 24 villages) have been considered as non-response group of the population. Here, we have taken the number of agricultural labours in the village as the study character (y) while the number of literate persons (x_1), the area (in hectares) of the village (x_2) and the number of cultivators in the village (x_3) are used as auxiliary characters. The values of the parameters of the population under study are given below:

$\bar{Y}$= 137.9271	$\bar{X}_1$= 955.8750	$\bar{X}_2$= 144.8720	$\bar{X}_3$= 185.2188
S_0 = 182.5012	C_1 = 1.1066	C_2 = 0.8115	C_3 = 1.0529
$S_{0(2)}$= 148.6390	ρ_1 = 0.705	ρ_2= 0.773	ρ_3= 0.786
ρ_{12}= 0.772	ρ_{13} = 0.770	ρ_{23} = 0.819	

In this problem, we have considered the estimator

$$T_{01}^{**} = v\, exp\left[\sum_{j=1}^{p} v_{1j}\, log\, z_j\right]$$

as a member of the proposed class of estimator T to study the relative efficiency with respect to $\bar{y}^*$ and other relevant estimators. The optimum values of the constant v_{1j} involved in T_{01}^{**} can be calculated by using the condition (3.7), which are as follows:

$$T_{01}^{**} \quad \begin{cases} T_{01}^{**}(p=1) : v_{11} = -0.8430 \\ T_{01}^{**}(p=2) : v_{11} = -0.3205,\ v_{12} = -0.9232 \\ T_{01}^{**}(p=3) : v_{11} = -0.1530,\ v_{12} = -0.5507,\ v_{13} = -0.5163 \end{cases}$$

The mean square error and relative efficiency (R. E.) of T_{01}^{**} with respect to $\bar{y}^*$ using one, two and three auxiliary characters for different values of the subsampling fraction $(1/k)$ in case of fixed n and n' are given in Table 6.1. The relative efficiency of the proposed class of estimators with respect to $\bar{y}^*$ for fixed cost (C_0) and their expected cost for specified precision (V_0'') are given in Table 6.2 and Table 6.3 respectively.

Table 6.1. Relative efficiency R. E. $(\cdot)$ in % with respect to $\bar{y}^*$ (for fixed $n' = 60$ and $n = 40$ for different values of k)

Estimators	Auxiliary character(s)	$1/k$		
		1/4	1/3	1/2
$\bar{y}^*$	–	100.00 (899.8656)*	100.00 (761.7809)	100.00 (623.6962)
T_{01}^{**}	x_1	118.10 (761.9478)	122.11 (623.8631)	128.39 (485.7784)
T_{01}^{**}	x_1, x_2	123.95 (726.0187)	129.57 (587.9340)	138.65 (449.8493)
T_{01}^{**}	x_1, x_2, x_3	126.24 (712.8124)	132.55 (574.7277)	142.84 (436.6430)

*Figures in parenthesis give $M(\cdot)$.

On comparing the $R.E.(.)$, it is clear from Table 6.1 that the estimator T_{01}^{**} is more efficient than $\bar{y}^*$ in case of one auxiliary character for the different values of k. The mean square error of T_{01}^{**} decreases as the subsampling fraction $(1/k)$ increases. It has also been observed that the relative efficiency of T_{01}^{**} with respect to $\bar{y}^*$ is increasing by increasing the subsampling fraction $(1/k)$ and the number of auxiliary characters used. It means that all the numbers of the suggested class of two phase sampling estimators T in presence of non-response attain minimum mean square error for every number of auxiliary characters used if the condition (3.7) is applied.

Table 6.2. Relative efficiency R. E. $(\cdot)$ with respect to $\bar{y}^*$ (for fixed cost $C_0 =$ Rs. 200.00)

Estimators	Auxiliary character(s)	$C_1' = Rs.0.70$ $C_1 = Rs.2.00$ $C_2 = Rs.5.00$ $C_3 = Rs.25.00$				
		$k_{opt.}$	$n'_{opt.}$ $(approx.)$	$n_{opt.}$ $(approx.)$	$M(.)$	$R.E.(.)$ in %
$\bar{y}^*$	–	2.3383	-	24	1367.0545	100.00
T_{01}^{**}	x_1	1.4864	56	16	1151.9815	118.67
T_{01}^{**}	x_1, x_2	1.1664	68	14	933.4605	146.45
T_{01}^{**}	x_1, x_2, x_3	1.0240	74	13	835.0246	163.71

From the Table 6.2, we observe that in case of one auxiliary character $M(T_{01}^{**})$ is less than $M(\bar{y}^*)$ and the relative efficiency of T_{01}^{**} with respect to $\bar{y}^*$ increases by increasing the number of the auxiliary characters for fixed cost.

Table 6.3. Expected cost for different estimators for specified precision $V_0'' =$ 800.00

Estimators	Auxiliary character(s)	$C_1' = Rs.0.70$ $C_1 = Rs.2.00$ $C_2 = Rs.5.00$ $C_3 = Rs.25.00$			
		$k_{opt.}$	$n'_{opt.}$ $(approx.)$	$n_{opt.}$ $(approx.)$	$Expected\ cost$ $(in\ Rs.)$
$\bar{y}^*$	–	2.3383	-	36	298.88
T_{01}^{**}	x_1	1.4864	73	21	261.38
T_{01}^{**}	x_1, x_2	1.1664	76	15	223.72
T_{01}^{**}	x_1, x_2, x_3	1.0240	76	13	206.11

Further, for the specified precision, the Table 6.3 shows that the expected cost incurred in T_{01}^{**} is less than the cost incurred for $\bar{y}^*$ and the expected cost for attaining the specified precision of T_{01}^{**} decreases by increasing the number of auxiliary characters.

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