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CONTENT ANALYSIS OF THE *DEMONSTRATION OF THE EXISTENCE OF GOD* PROPOSED BY LEIBNIZ IN 1666

INTRODUCTION

The final part of Leibniz's early work *De arte combinatoria* from 1666 includes, at the end, the *Demonstration of the existence of God.*[1] The demonstration is "mathematical" in character and is built according to the Euclidean formula. It starts with premises consisting of definitions, postulates, axioms and one premise called an "observation." This is followed by a deductive argument in which particular verses, according to Leibniz's indications, lead to the final concluding statement confirming the existence of God. However, the initial definition of God consists in presenting Him as disembodied substance of infinite power. I have subjected the content of the demonstration to an attempt at formalisation which, on the one hand showed the lack of precision of the demonstration in its clarification, whilst on the other, by the necessity of searching for the missing deductive links it helped to clarify the views of young Leibniz both on the concept of substance which runs through the proof proposed by him, as well as in the methodology he used to construct his demonstration. Leibniz's demonstration was translated into English by Leroy E. Loemker.[2] This text, along with my logical analysis of the proof is available in English in

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[1] Gottfried Wilhelm LEIBNIZ, *Sämtliche Schriften und Briefe*, Reihe 6, Bd. 1 (Berlin: Akademie-Verlag, 1971), 169–170.

[2] Gottfried Wilhelm LEIBNIZ, *Philosophical papers and letters*, trans. and ed. Leroy E. Loemker (Dordrecht: D. Reidel, 1969), 73–74.

Studies in Logic, Grammar and Rhetoric (1982), and in Polish in my book entitled *Leibniz. Wczesne pojęcie substancji* [*Leibniz. The early concept of substance*] (1992).[3] I refer to them in the content analysis of the proof presented below and which in the translation into English repeats, with some slight modifications, the content of the fifth chapter of the aforementioned book.

1. THE INFLUENCE OF ERHARD WEIGEL ON THE MATHEMATICAL CHARACTER OF LEIBNIZ'S *DEMONSTRATION OF THE EXISTENCE OF GOD*

An attempt to formalize Leibniz's demonstration of 1666 was flawed by an error of contradiction.[4] However, even this defective record gives rise to various observations that otherwise would have escaped the researcher's attention. The mere necessity of penetrating into the deeper structure of the premises of the proof confronts us with multifarious questions, including that of the ontological status of the said premises. At the beginning it is worth mentioning that the *Demonstratio*... owes its logical construction to the theoretical influences of Leibniz's teacher, Erhard Weigel. These influences were forgotten for a long time, and it was not until the last few decades of the twentieth century that Weigel's name appeared again in the scholarly literature concerned with Leibniz as well as general German cultural history of the seventeenth century.[5] In Weigel we find not only some of

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[3] Krystyna [KRAUZE-]BŁACHOWICZ, "Leibniz's 'Demonstratio Existentiae Dei ad mathematicam certitudinem exacta.' A logical Analysis," *Studies in Logic, Grammar and Rhetoric* 2 (1982): 43-55; EADEM, *Leibniz. Wczesne pojęcie substancji* (Białystok: Zakład Teorii Poznania. Filia Uniwersytetu Warszawskiego, 1992), 47–55.

[4] Cf. K. KRAUZE-BŁACHOWICZ, *Leibniz. Wczesne pojęcie substancji*, 54, footnote 10.

[5] The issue of Weigel's influence on young Leibniz is discussed by the author of Leibniz's extensive biography, Eric J. AITON, *Leibniz: A Biography* (Bristol & Boston: A. Hilger Ltd., 1985), 14–16; in Polish literature Erhard Weigel and his influence on Leibniz, particularly in the area of pedagogy, is discussed only briefly by Waldemar VOISÉ, "Erhard Weigel (1625-1699) czyli u progu Wieku Oświecenia (z okazji 300 rocznicy jego książki *Idea Matheseos Universae*)," *Kwartalnik Historii Nauki i Techniki* 15 (1970), 3: 527–544. The extent to which the sources for Leibniz, and in particular Weigel, have been forgotten, was reported in the early 1980s by Konrad Moll, who with some satisfaction pointed out the errors committed by scholars when they came in contact with this not quite well-known figure, cf. Konrad MOLL, *Der Junge Leibniz*, t. I–II Stuttgart-Bad Cannstatt), II, 38. After my publication of the text presented here, i.e. after the year 1992 the question of importance of Weigel was addressed by Christia MERCER, "The Vitality and Importance of Early Modern Aristotelianism," in *The Rise of Modern Philosophy*, ed. Tom Sorell (Oxford: Clarendon Press, 1993), 33–67, and EADEM, "The Young Leibniz and his Teachers," in *The Young*

Leibniz's ideas, but also some of the formal constructions introduced by the latter into his philosophy. *Demonstration of the existence of God* was constructed according to the theoretical guidelines of Weigel's *Analysis Aristotelica ex Euclide restituta* (1658) and it is not an isolated phenomenon in Leibniz's work.[6] Beginning with Weigel's *Analysis* ... up until modern times Euclid's *Elements* have been interpreted in the categories of Aristotle's *Analytics*, although to this day the exact relationship between them is not known. This resulted from the fact that *Analytics* are the only fully preserved ancient exposition of lecture on the deductive theory of the demonstration. Of the well-known ancient texts only *Elements* use the deductive method of the lecture in practice.[7] In the case of Weigel's theory and its practical implementation in Leibniz's writings, things are quite different. Their relationship is undoubted. Thus, it is obvious that when interpreting Leibniz's demonstration, it will often be necessary to resort to the categories used by Leibniz's teacher.

2. A FEW COMMENTS ON REAL AND NOMINAL DEFINITIONS

The placement of the symbol of equation between the definitions of our formalisation was in the first place determined not only by their structure, but also to a large extent by Leibniz's labelling them as defining statements. Although we do not have the information regarding Leibniz's position on this type of premises until 1670,[8] there is no reason to suppose that his concepts would move away definitions beyond the circle of equivalent sentences. Indeed, this way they would depart both from the existing tradition and later opinions of Leibniz himself.[9] Therefore, the decision on the equiva-

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Leibniz and his Philosophy 1646-76, ed. Stuart Brown (Dordrecht, Boston, Leiden: Kluwer Academic Press, 1999), 19–40.

[6] In 1690, Leibniz formulated the *Demonstratio contra atomos* (Demonstration against the existence of atoms), identical in form to the above-presented *Demonstration of the existence of God*, and—as if the over twenty-year interval between them had not occurred—it constituted a continuation of its content. This demonstration repeats Leibniz's position regarding the *continuum.*

[7] Cf. Krystyna KRAUZE-BŁACHOWICZ, "'Archai'. Problem rozumienia pierwszych zasad w systemie Euklidesa," *Idea. Studia nad strukturą i rozwojem pojęć filozoficznych* 1 (1986): 8–17; EADEM, "Early Greek Mathematics—a Branch of Dialectic?," *Studies in Logic, Grammar and Rhetoric* 5 (1986): 5–20.

[8] Hans BURKHARDT, *Logic und Semiotik in der Philosophie von Leibniz* (München: Philosophia Verlag, 1980), 208.

[9] Ibidem, 211.

lent character of the first three grounds of the demonstration also concerns the definition of substance, which is the most interesting for us.[10] This sentence, had it occurred in a different context that would not imply the defining process, could take an alternative form of $\forall x\ [\mathrm{Mv}(x) \vee \exists y\ \mathrm{M}(x, y) \rightarrow \mathrm{Sb}(x)]$.[11] The formalisation of the proof also eliminated the difference in the formulation that occurs between definition 1. and 3., and definition 2. As we know, definition 2. was expressed, as it is called today, in "metalanguage stylization," unlike "object stylization" used in the remaining ones. This situation provokes a question whether such stylization was relevant to Leibniz and, according to him, what was the character of the introduced definitions.

The contemporary knowledge of definitions departs from the older division of definitions into real and nominal ones. It treats all definitions as linguistic phenomena. However, it introduces the said "stylizations" referring directly to the previous tradition.[12] Definitions in objective stylizations—as theorems concerning non-linguistic subjects—give a clear description of the subject, and thus correspond to "real definitions," as understood by Kazimierz Ajdukiewicz.[13] A "metalinguistically" stylised definition would correspond to a nominal definition, constituting, in a way, an explanation of the meaning of certain expressions in a certain language. The "traditional" classification of definitions reaches back to Aristotle's *Analytics.*[14] The names "real" and "nominal" originate from scholasticism ("*definitiones reales et nominales*") and, with the beginning of modern science in the 17th century, they are put to use in various theories of definitions, however also in different meanings.[15] We also find them in Leibniz's classification of definitions. Modern reconstruction of his theory of definition is based on Leibniz's late texts (after 1675). Indeed, his writings from the years 1670–1675 contain merely references (discussed below). However, there is a lack of Leibniz's comments from before 1670.[16]

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[10] "Definition 2: I call substance whatever moves or is moved." G.W. LEIBNIZ, *Philosophical papers and letters*, 73; cf. K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 46.

[11] In my formalisation this definition took the following form: $\forall x\ [\mathrm{Sb}(x) \equiv \mathrm{Mv}(x) \vee \exists y\ \mathrm{M}(x,y)]$, K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 49.

[12] Cf. Tadeusz PAWŁOWSKI, *Tworzenie pojęć w naukach humanistycznych* (Warszawa: PWN, 1986), 20; Witold MARCISZEWSKI, *Metody analizy tekstu naukowego* (Warszawa: PWN, 1977), 131.

[13] Kazimierz AJDUKIEWICZ, "O definicji," in *Język* i *poznanie,* t. II: *Wybór pism z lat 1945–1963* (Warszawa: PWN, 1985), 227.

[14] ARISTOTELES, *Analytica Posteriora,* II, 10, 93b-94a.

[15] H. BURKHARDT, *Logik und Semiotik*, 206.

[16] Ibidem, 208.

3. THE LATE-SCHOLASTIC THEORY OF DEFINITION BY ERHARD WEIGEL

As we have said at the beginning, *Demonstratio*… is a model example of the implementation of the Weigel's directive. Let us therefore first reach to to the theory of definitions proposed in *Analysis Aristotelica ex Euclide restituta*, of which young Leibniz was undoubtedly aware in 1666. Weigel holds on to the late-scholastic classification.[17] He divides definitions into "conceptual," i.e., such "thanks to which a notion is conceived in the mind" ("*notionales, quae notionem in intellectu gignunt*"), and "real ones" (*reales*).[18] Conceptual definitions are further divided into "nominal" (*nominales*) and "essential" (*essentiales*).[19]

Nominal definitions establish signs for the terms used in proofs. They are used both when a complex term requires replacing it with a simple one, and when it is necessary to develop a simple, incomprehensible term. Hence, *definitum*[20] is a verbal symbol generally chosen in accordance with the linguistic tradition. However, a formally nominal definition depends only on the speaker's will.[21]

In turn, essential, or logical (*logicae*), definitions are to distinguish the essence of a thing being defined from the essences of other things by pointing to the distinctive features. This happens *per genus et differentiam specificam.* The definition that arises this way does not give us true knowledge of the essence of things, only "conceptually" allows to uniquely distinguish it from other.[22]

Conceptual definitions belong to the principle of truth that is referred to as "suppositive" (*suppositivae*) and treated hypothetically. They include definitions and hypotheses (in the sense of Aristotle's *Posterior Analytics*).[23] They explain the concepts in the proof thesis and open the proof ("*ab iis … inchoetur demonstratio*").[24]

This hypotheticity distinguishes them from other principles of the proof, called *principia perfectiva*—"performative," because thanks to them the proof

[17] As presented by Fonseca, cf. H. BURKHARDT, *Logik und Semiotik*, 207.
[18] Erhard WEIGEL, *Analysis Aristotelica ex Euclide restituta* (Jena: Grosium, 1658), 56.
[19] Ibidem, 64.
[20] This is how Weigel and Leibniz refer to *definiendum*.
[21] E. WEIGEL, *Analysis Aristotelica*, 66.
[22] Ibidem, 64.
[23] ARISTOTELES, *Analytica Posteriora*, I, 2, 71b-72b; 10, 76a-77a.
[24] E. WEIGEL, *Analysis Aristotelica*, 51.

proper is "achieved" or "fulfilled." They are axioms and postulates ("*axiomata et postulata*")[25] .

Weigel does not include real definitions within suppositive ones, as these are the definitions which according to Aristotle give the source of existence of a defined object. Like in Aristotle's writings, this definition is closely linked to the theory of proof. A real definition is, according to Weigel, either a ready definition of a certain proof (*definitio conclusiva*), or the proof is presented in the definiens (the definiens depicts the definitum cause—*causa*, hence the name *definitio causalis*—the causal definition).[26]

4. LEIBNIZ'S VIEWS ON DEFINITION

Leibniz's first mention of definition comes from 1670 from the introduction to the editing of the work of Nizolius.[27] The definition—according to this text—gives the meaning of a given expression ("*definitio enim nihil aliud est, quam significatio verbis expressa, sive brevius significatio significata*").[28] Thus, the definition is exactly equivalent to the nominal definition in Weigel's sense.

Only later Leibniz's writings allow to classify definitions as nominal or real. A nominal definition includes in the definiens a list of attributes or constituents of a thing so that it is distinguishable from others. This is for Leibniz, equivalent to the fact that definitum is made up by a "combination" of simpler terms in the definiens, which in turn can be expressed in even simpler terms until a combination of original concepts is achieved.[29] Definitions *per genus et differentiam specificam*—referred to by Weigel as logical or essential—are practically no different from Leibniz's nominal definitions. They are treated as a combination of two terms formulating the concept of

[25] Ibidem, pp. 96-97.

[26] Ibidem, p. 76.

[27] In 1670 Leibniz prepared a new edition of the work of an Italian humanist Marius Nizolius, entitled *Anti-Barbarus, seu de veris principiis et vera ratione philosophandi contra pseudophilosophos*, published in Parma in 1533.

[28] Gottfried Wilhelm LEIBNIZ, *Die Philosophischen Schriften*, ed. Carl Imannuel Gerhardt, Bd. IV (Berlin: Weidmann, 1880), 140; cf. *Opuscules et fragments inédits de Leibniz*, ed. Louis Couturat (Paris: F. Alcan, 1903), 528.

[29] Gottfried Wilhelm LEIBNIZ, *Die Philosophischen Schriften*, ed. Carl Imannuel Gerhardt, Bd. VII (Berlin: Weidmann, 1890), 293; cf. Raili KAUPPI, *Über die Leibnizsche Logik* (Helsinki: Societas Philosophica, 1960), 103.

the definitum. There is no fundamental distinction between *genus* and *differentia* because they can, according to Leibniz, be exchanged between themselves, once expressing the first one, then expressing the other in the form of an adjective.[30] Thus, as we can see, nominal definitions according to Leibniz are not only the nominal definitions of the scholastics and their continuator Weigel, but also any definitions that by following Weigel's lead we could place under a common name of conceptual definitions.

A real definition, according to Leibniz, requires that the combination of concepts in definiens be possible. In this sense, a real definition is such a nominal definition as to which we are certain that the terms occurring in its definiens are compatible (*compatibilia*).[31] A "pure" real definition is a nominal definition, for which the said compatibility of constituent terms is experimentally proven. In other words: "pure real definition" = "nominal definition" + "statement of existence of its object." The other types of real definitions that Leibniz distinguishes are causal definitions.[32]

5. DEFINING SUBSTANCE WITH THE USE OF A REAL DEFINITION

The first determination that will be concerned with Definitions 1.–3. of Leibniz's proof may be referred to as a negative determination. Indeed, none of the definitions seems real either in terms of scholastic terminology or in the categories proposed in Leibniz's mature works. In Weigel's sense these are suppositive principles—providing successive explanations of the meanings of concepts combining the notion of God that occurs in the thesis of the proof. They are therefore nominal in every sense. The existence of defined objects is not guaranteed by the definitions themselves, but is proved in the course of the demonstration on the basis of an empirical existential premise—"observation." Thus, upon the completion of the proof these definitions are converted into real definitions which are conclusive in Weigel's sense, however not earlier. If we apply here the terminology of mature Leibniz (admitting to committing an obvoius anachronism) they become pure real definitions. For "observation" is a premise based on experience, therefore

[30] H. BURKHARDT, *Logik und Semiotik*, 211; G.W. LEIBNIZ, *Die Philosophischen Schriften*, ed. C.I. Gerhardt, Bd. VII, 292.

[31] G.W. LEIBNIZ, *Die Philosophischen Schriften*, ed. C.I. Gerhardt, Bd. VII, 293.

[32] H. BURKHARDT, *Logik und Semiotik*, 215.

definiens is real by experience. Thus, within the framework of the proof we receive not only the thesis of the existence of God, but also its real definition, and moreover the REAL DEFINITION OF SUBSTANCE.

Therefore, is Definition 2. different from the other ones in terms of content or just stylistically? If we continue to follow Weigel's classification, we can say that Definitions 1. and 3. are essential definitions. Definition 2. is far from the ideal of *genus proximum et differentia specifica*, unless we treat motion as something that distinguishes" substances from everything else. However, this does not affect the wording of Definition 2. It would even be in some way contradictory to Leibniz's subsequent growing tendency to a certain standardisation of definitions, called conceptual by Weigel's and nominal by Leibniz. In the light of the above considerations, the occurrence of the expression *autem voco* in the definition appears to be stylistically justified information—in this central place for all three definitions—that all of them constitute a sequence of nominal explanations. Definition 3. refers to the concept of *virtus infinita*—infinite power, defined as the original capacity to move the infinite. Definition 3. is used in point (4) of the proof and in the remaining analogous points. A moving body is—in accordance with Axiom 4.—indefinitely divisible. Leibniz's reasoning is as follows: a body requires a mover according to Axiom 5. The mover is either incorporeal or corporeal. If it is incorporeal and moves the body that is infinitely divisible, it means that it possesses the power capable of moving the infinite. According to Leibniz, it is the infinite power (*virtus infinita*). In the case when the mover is a body, there is no need to decide whether it has infinite power. Leibniz leaves us with three interpretative possibilities:

1) either he omits the fact that he defined infinite power as an ORIGINAL capacity, thus everything that moves something infinite (here: infinitely divisible) has infinite power;

2) or anything that moves something infinite, has the original capacity to move the infinite—therefore the body that is the mover possesses the original capacity to do so, even assuming that it is the capability stimulated or modified by an external action[33];

3) or, finally—Leibniz makes the entimematic assumption that if the infinite is moved by something incorporeal, then the ability to do this is original, but if it is a body, the ability is secondary. This way he refers to further

[33] As can be seen in mature writings, according to which the bodies have their internal *vis activa*, only modified from the outside, this is not an improbable assumption.

content of Definition 3., where the primary and secondary causes (*causae primae et secundae*) are distinguished: in the incorporeal substance one would find the primary cause, in the corporeal one, the secondary one.

For p. 1)—2) as the formalization of Definition 3. the following suffices:

$$\forall x \; \{\mathrm{Vi}(x) \equiv \exists y \; [\mathrm{M}(x, y) \wedge \mathrm{If}(y)]\}.$$ [34]

In the process of formalising the proof I applied this simple version of Definition 3., which theoretically corresponds to situations 1) and 2). In all variants: 1)–3) God can be any incorporeal substance, if only it puts the body into motion.

The adoption of any of the variants causes that having assured the existence of God (any incorporeal motor is—by virtue of point (4) of the proof—God), we cannot be certain that it is the only one. Or vice versa, that aside from the only God, there are no other incorporeal substances capable of putting the corporeal substance in motion.

6. WEIGEL'S CLASSIFICATION OF THE REMAINING PRINCIPLES

Besides the "suppositive" (according to Weigel's terminology) definitions, the proof containes perfect premises—*perfectiva*: axioms and postulates. Axioms are self-evident statements that express real beings.[35] A statement, according to Weigel, is a complex being (*esse complex*). Axioms are statements from which it is directly (*nude*) understood that something exists out of necessity. They are divided into rational and experimental. The former are axioms *par excellence* (κατ' ἐξοχήν), the other are the so-called "observations" (*observationes*) based on experience manifested in sensory or intellectual perception.[36] A postulate, in turn, first gives existence only to terms (a term is a simple being—*esse simplex*), by means of which it strives

[34] Cf. K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 49.

[35] "Axiomata demonstrativum effatum realiter esse faciunt, hoc est necessitant." E. WEIGEL, *Analysis Aristotelica*, 97.

[36] "Sunt autem Axiomata generali definitione nihil aliud quam Effata Propositioni extranea, quibus aliquid necessario esse nude saltem intelligitur. Haec in se considerata commodissime distinguuntur in rationalia et eperimentalia, quorum illa κατ' ἐξοχήν et quasi per Antonomasiam dicuntur Axiomata; haec Observationis vocabulo ... indigitantur." E. WEIGEL, *Analysis Aristotelica*, 105; "Experientia distingui potest vel ratione instrumenti, quo fit perceptio, in sensualem et intellectualem, vel ratione rei perceptae in realem et impositivam." Ibidem, 109–110.

to give reality to the whole sentence.[37] In another place, Weigel says that since postulates are self evident statements thanks to which something may exist, it exists nominally (nominaliter). [38]

7. THE POSTULATE: THE CONCEPT OF *TOTUM* AND ITS SET-THEORETIC ASPECT.

The "postulate" in Leibniz's proof introduces the concept of *totum*—the whole, used in point (14) of the proof and in line 86. of the formalisation.[39] The formal record of the postulate is slightly closer to the set-theoretic record of the axiom of comprehension. At a certain stage the whole can be presented, as I have done it, as a set, and so its description will remain within the framework of the basic terms of set calculus and predicate calculus of the first order. This, however, is not a fully successful attempt because Leibniz makes no clear distinction between what is now called mereological set and a set in the sense of set theory. *Totum*, once constructed by the postulate, ceases to be abstract—it is treated by Leibniz as any individual. The elements making up such an individual-aggregate are its parts. The transition from a *totum*—understood abstractly to a concrete *totum*[40]—is undoubtedly a weak point in the formalisation. This is not an entirely uncorrigible element from the point of view of today's formal calculi and if one takes into account additional assumptions.[41] However, it is an eloquent testimony to the fact that Leibniz is still far from the consistent understanding of a set as an abstract in Cantor's sense. The fact that many of Leibniz's calculi can be presented—though in an incomplete form—as Boole's algebra[42] and the

[37] "Postulata vero effatorum demonstrantium terminis prius largiuntur esse simplex, ąuibus mediantibus demum ad ipsius effati complexum et realem essendi actum quasi necessitandum concurrunt." Ibidem, 97.

[38] "Postulata demonstrativa nihil aliud sunt, quam effata per se nota, quibus aliąuid fieri posse atque ita saltem nominaliter esse (licet quoad rem saepius etiam actu sit) postulatur concedi quippe cum hoc nulla demonstratione indigeat." Ibidem, 98.

[39] K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 52: v. 86; cf. EADEM, *Leibniz. Wczesne pojęcie substancji*, 53: v. 81.

[40] K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 52: v. 85.–127., cf. EADEM, *Leibniz. Wczesne pojęcie substancji*, 53: v. 83a.–120.

[41] An example of a solution to the problem was shown by Witold Marciszewski in "On Leibniz' Anticipation of the Comprehension Axiom in the Light of the Formalization of 'Demonstratio Existentiae Dei'," *Studies in Logic, Grammar and Rhetoric* 2 (1982): 29–42.

[42] Nicholas Rescher attempted such an interpretation. He claims that Leibniz's calculi form a system that is essentially the Boolean algebra, except that it is deprived of the zero element.

efforts aimed at reconciliation of Leśniewski's calculi with Boole's algebra,[43] and, above all, the consistently concrete approach of Leibniz to the examples of sets presented by him rather encourage an attempt at formulating a "mereological" explanation of Leibniz's thought.

8. THE ROLE OF AXIOMS AND "OBSERVATIONS" IN THE PROCESS OF MAKING SUBSTANCE DEFINITION REAL

Axiom 1 guarantees not only the existence of a mover for everything that is moving, but also that the mover is DIFFERENT from what is moving. The consequence of the axiom is that every substance that is moving is moved by ANOTHER SUBSTANCE (point (6) and the subsequent points of the proof). This is because, according to Axiom 1., it is moved by something else. This "thing" moves it, hence according to Definition 2 is a substance. Since the mover is external, the predicate "M" ("moves") is introduced with an additional assumption about the mover's externality.[44] This assumption immediately finds its expression in Axiom 2, where each moving body is again understood as a body moving "something else."

Axiom 3 states the following: "If all its parts are moved, the whole is moved." The existence of God is guaranteed in Leibniz's proof by the movement of the EXISTING WHOLE of a chain of moving bodies. Hence, in the sentence formulation of Axiom 3., we introduce in the preceding one a premise which is not expressed *explicite* in Leibniz's text—the assumption of an existence of at least one part of the whole. This assumption will be fulfilled in the course of the proof as a consequence of the empirical "observa-

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Nicholas RESCHER, "Leibniz's Interpretation of his Logical Calculi," *Journal of Symbolic Logic* 19 (1954): 1–20. This interpretation is difficult to maintain in the light of M. Dummett's criticism, cf. Michael DUMMETT, "N. Rescher's *Leibniz's Interpretation*…Review," *Journal of Symbolic Logic* 21 (1956): 197–199; cf. next footnote.

[43] Andrzej Grzegorczyk, on the other hand, presented Leśniewski's mereology in the form of the Boolean algebra without the zero element (Andrzej GRZEGORCZYK, "The Systems of Leśniewski in Relation to Contemporary Logical Research," *Studia Logica*," *Studia Logica* 3 (1955): 91–95), subjected to criticism by contemporary experts on Leśniewski (cf. RICKEY, V. Frederick. "A Survey of Leśniewski's Logic." *Studia Logica* 36 (1977): 422) . Both interpretations, although as it was found not entirely free of errors, point out some similarities between the discussed theories.

[44] Therefore, besides the explanation that $M(x, y)$ means "x moves y," we add the formula "$x \neq y \wedge \neg P(x, y)$" saying that "$x$ is different from y and external in relation to it." Cf. K. [KRAUZE-] BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 48; cf. EADEM, *Leibniz. Wczesne pojęcie substancji*, 50, where this externality is expressed in lines 5a and 5b of the formalisation.

tion" of existence of a certain moving object.[45] The thus true line 112 of the analysis "$\exists x$ P(x, c)"[46] along with line 89: "$\forall x$ [P(x, c) $\rightarrow$ Mv(x)]" based on Axiom 3. will result in line 115: "Mv(c)"—"The whole is moved," from which follows the existence of its divine mover.[47] Axiom 3., along with lines 90-115[48] of the formalisation are therefore important for showing the "reality" of the definition of God based on one empirical premise of the proof, namely the "observation." The aggregate-whole has this property (as expressed by Axiom 3.) that if all the parts are moved, then the whole is moved as well. In developing of the proof, we have also applied an implicit premise that what moves the whole also moves the parts. It can be treated as a result of a more general assumption that the motion of the whole implies the motion of a part. Did Leibniz make such an assumption? Taking into account the syntax of the sentence, it is impossible to completely exclude such a possibility, i.e. that in Axiom 3. Leibniz included a sentence of equivalence, although if he did so—it was rather unfortunate from the linguistic point of view. The implicit premise is essential if at the same time we wish to precisely convey the meaning of the expression: "since we have already included all bodies, back to infinity" (p. (18) of Leibniz's proof), that is we have to guarantee that every body that moves the whole "c" becomes a part of it. However, this unfortunately is a premise leading to the contradiction of the formalisation.[49]

As for Axiom 4.: "Every body whatsoever has an infinite number of parts; or, as is commonly said, the *continuum* is infinitely divisible.[50] To Leibniz "to have an infinite number of parts" means the same as "to be infinite." The limitation of the body does not preclude its infinity. The consequence of this in the proof is the fact that even a body subjected to the action of the divine *virtus* requires that it be *infinita*—infinite. It is worth noting, however, that Leibniz uses this axiom in a limited manner. The moving whole "c" is certainly corporeal—and thus, in the concept of Axiom 4.—infinite. Nevertheless, in point (19) of the proof Leibniz does not refer to the thus understood infinity but to a potentially infinite number of elements—*partes* of his set.

[45] K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 52–53: vv. 90–112; cf. EADEM, *Leibniz. Wczesne pojęcie substancji*, 53–54: vv. 86–108.

[46] *P*(x,c)—*x* is a part of *c*, *c*—a whole (*totum*).

[47] Cf. K. KRAUZE-BŁACHOWICZ, *Leibniz. Wczesne pojęcie substancji*, vv. 105, 85, 108.

[48] Cf. ibidem, vv. 86–108.

[49] Cf. K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 53, v. 119; cf. EADEM, *Leibniz. Wczesne pojęcie substancji*, 54, v. 112, footnote 10 ("implicit premise" is introduced in proof formalisation as Axiom 3.).

[50] EADEM, "Leibniz's *Demonstratio Existentiae Dei*," 47.

As for the "Observation," it is certainly an experimental axiom, as Weigel understood it (see above). As has already been said, it is relevant for making the thesis of the proof real, and, at the same time, of the definitions occurring in it. It therefore makes the definition of substance a real definition.

9. "MATHEMATICAL CERTAINTY" AS THE CAUSE FOR THE EMERGENCE OF *A NEW CONCEPT* OF SUBSTANCE (ARISTOTELIAN TRADITION IN CONFRONTATION WITH THE RESULTS OF THE PROOF ANALYSIS)

Let us summarise the results of our analysis of Leibniz's proof, referring to the problems included in Aristotle's *Physics*. In his considerations of the moving factor Aristotle assumes that the motion factor may be outside but also within the moving object.[51] Only his reflections on the first factor of motion challenge this thesis in relation to the first mover. Indeed, it is only the tacitly adopted assumption that if the whole moves, the parts of the moving thing move as well, that excludes the possibility of a motionless mover being an internal factor of movement. And that is because as part of that thing it would have to move.[52] Leibniz, in turn, immediately recognises the distinctiveness of the factor of movement and its location outside the moving object.

Aristotle tries to prove that the first factor of motion is the only one.[53] Leibniz, assuming the presupposition of the *oneness* of God, is unaware that his proof creates the possibility of the opposite conclusion: the existence of an infinite number of first movers. It is also worth mentioning that Leibniz consciously makes use of Aristotelian notion of a *continuum*. Probably, just as Aristotle, he silently assumes that if the whole moves, then the parts move as well, the consequence of which is expressed by the implicit premise that the factor moving the whole puts in motion also its parts.[54] To finish with these comparisons, let us speak of the most important aspect. Aristotle elaborately demonstrates the state of immobility of the first mover. Leibniz leaves us guessing. However, this is obvious in the light of both Axioms 1. and 2. According to these axioms, a moving substance requires a mover.

[51] ARISTOTELES, *Physica*, VIII, 255b-256a.

[52] Ibidem, VII, 241b-242a; VIII, 254b-256a; cf. K. KRAUZE-BŁACHOWICZ, *Leibniz. Wczesne pojęcie substancji*, chapter III, § 4–5, 39–42.

[53] ARISTOTELES, *Physica*, VIII, 258b.

[54] Cf. above, footnote 48.

God, therefore, cannot be in motion as this would entail calling to life another causal force, to some extent higher than God. In this case, Leibniz's God would have been reduced to the role of *causa secunda in virtute primae operans*. This is unacceptable in the light of the definition of God (Definition 1.) and the definition of infinite power (Definition 3.). We can therefore assume that Leibniz was guided by the presupposition of rest of the divine being. The proof that he makes is based on the Thomistic argument "from motion." Motion will therefore constitute the most important predicate for objects which truths are expressed in the lines of the proof. The first empirical premise is to say that a certain object is moving. Leibniz chooses the simplest option, limiting predicates in the definition of substance to "moving" and "being moved." The two terms are required because, as we have just shown, we have an assumption in the proof that God is still, with which Definition 2. should not stand in contradiction: "I call substance whatever moves or is moved."[55] Thus, from among the predicates concerned with movement, the first one, i.e. "moving" is vested in God. The "movement" itself is not enough for the definition chain that considers Definition 2. to lead to such a transformation as to obtain the concept of a motionless God. The content of the definition of substance is therefore chosen in such a way as to meet the deductive requirements of the proof.

General considerations of nature contained in the beginning of book II of *Physics* allow for a reconstruction of Aristotle's definition of substance based on the concept of motion.[56] Aristotle did not feel the need to preserve the strict rules of defining (even his own) in this place because it was not an intended definition. The reconstructed definition is a partial one and can be expressed as follows: Everything that possesses the principle of motion is a substance.[57] Again in this place Leibniz differs from Aristotle. In the proof of a clearly stated structure, his definition found a place among equivalence definitions. Thus, was movement to be not just a sufficient condition but also a necessary one for the establishment of a substance?

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[55] K. [KRAUZE-]BŁACHOWICZ, "Leibniz's *Demonstratio Existentiae Dei*," 46.

[56] ARISTOTELES, *Physica*, II, 192b: "All things therefore which have this kind of principle [namely—motion], have a nature; Moreover, all this is a substance, because it is a certain substrate and nature is always present in the substrate" (translation from Greek K. K.-B.). Cf. K. KRAUZE-BŁACHOWICZ, *Leibniz*. *Wczesne pojęcie substancji*, chapter III, § 3, 39.

[57] ARISTOTELES, *Physica*, II, 192b allows the following reconstruction of Aristotle's reasoning: Everything that possesses the principle of movement has a nature. Having a nature is a determinant of something being a substance. And that is because nature requires a substrate for itself, a substrate in turn is a substance. Hence, EVERYTHING THAT POSSESSES THE PRINCIPLE OF MOVEMENT IS A SUBSTANCE. Cf. K. KRAUZE-BŁACHOWICZ, *Leibniz*. *Wczesne pojęcie substancji*, chapter III, § 3, 39.

It is also worth mentioning here, using what we have established above, that in the light of Weigel's principles, which served Leibniz as guidance, the definition of a substance included in the proof as a nominal definition, due to the realness of the premise referred to as an "Observation," with the completion of the proof also turns into a real definition. As we can see, all that we have said above is not headed towards the negation of the view of the influence of traditional Aristotelianism on the young Leibniz. However, in view of the conclusions towards which the examination of the consequences of the proof leads, it is clear that this is modified Aristotelianism for the purposes of the deductive requirements formulated by Erhard Weigel—the teacher, who was the first true authority for the young philosopher both in mathematics and when it came to the application of mathematical accuracy in other sciences. The construction of a philosophical proof which is to be, as Leibniz himself defines, *ad mathematicam certitudinem exacta*, imposes certain limitations on the content introduced in the premises. Leibniz, adapting himself to the first paradigm known to him—the geometric paradigm—had to contemplate the notion of substance which he should have defined for the purposes of the proof. Hence, according to all the principles taught by learned Weigel, he constructed the definition of a substance that takes into account not only the factor of movement but also of the moving (in the definition itself the postulate of an existence of a divine substance is not an immediate one), and, additionally, the appearance of these factors becomes a necessary precondition. In the process of the proof the proposition is obtained which in conjunction with the "Observation" not only guarantees the existence of God, but also the existence of a defined (Def. 2.) substance.

Thus, the nature of the notion of a substance evolving from the proof is inseparable from the geometric pattern used in the proof. It is possible that Leibniz, while adapting the Aristotelian tradition to his mathematical pattern, did not reach for the writings of the Stagirite himself. He went in the direction of reconciling not so much Aristotle himself as one of his interpretations, which Leibniz and some of his masters regarded as the proper teaching of Aristotle himself,[58] with newer physical concepts. He began with Gassendi and Hobbes, making the first steps towards the future concept of a substance embedded in physical concepts: motion, and much later—energy.[59]

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[58] What it meant to be an Aristotelian in Leibniz's times is explained by Christia Mercer in the article "The Vitality and Importance of Early Modern Aristotelianism"; see above footnote 5.

[59] Whether and to what extent, one might seek the beginnings of later metaphysical concepts in youthful writings of Leibniz can still be debated; see Maria Rosa ANTOGNAZZA, *Leibniz*: *An In-*

APPENDIX

Gottfried Wilhelm LEIBNIZ, *Philosophical papers and letters*. Translated and edited with an Introduction by Leroy E. Loemker (Dordrecht: D. Reidel, 1969), 73–74.

I. DEMONSTRATION OF THE EXISTENCE OF GOD
[G, IV, 32–33]

HYPOTHESES [PRAECOGNITA]:

1. Definition 1. *God is* an incorporeal substance of infinite power [*virtus*].
2. Definition 2. I call *substance* whatever moves or is moved.
3. Definition 3. *Infinite power* is an original capacity [*potentia*] to move the infinite. For power is the same as original capacity; hence we say that secondary causes operate by virtue [*virtus*] of the primary.
4. Postulate. Any number of things whatever may be taken simultaneously and yet be treated as one whole. If anyone makes bold to deny this, I will prove it. The concept of *parts* is this: given a plurality of beings all of which are understood to have something in common; then, since it is inconvenient or impossible to enumerate all of them every time, one name is thought of which takes the place of all the parts in our reasoning, to make the expression shorter. This is called the *whole.* But in any number of given things whatever, even infinite, we can understand what is true of all, since we can enumerate them all individually, at least in an infinite time. It is therefore permissible to use one name in our reasoning in place of all, and this will itself be a *whole.2*
5. Axiom 1. If anything is moved, there is a mover.
6. Axiom 2. Every moving body is being moved.
7. Axiom 3. If all its parts are moved, the whole is moved.
8. Axiom 4. Every body whatsoever has an infinite number of parts; or, as is commonly said, the continuum is infinitely divisible.
9. Observation. There is a moving body.

PROOF:

1. Body *A* is in motion, by hypothesis No. 9.
2. Therefore there is something which moves it, by No. 5,
3. and this is either incorporeal
4. because it is of infinite power, by No. 3;
5. since *A,* which it moves, has infinite parts, by No. 8;

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tellectual Biography (New York: Cambridge University Press, 2009); Christia MERCER, *Leibniz's Metaphysics: Its Origins and Development* (New York: Cambridge University Press, 2001); Daniel GARBER, *Leibniz: Body, Substance, Monad* (Oxford: Oxford University Press, 2010).

6. and is a substance, by No. 2.
7. It is therefore God, by No. 1 Q.E.D.
8. Or it is a body,
9. which we may call *B*.
10. This is also moved, by No. 6,
11. and what we have demonstrated about body *A* again applies, so that
12. either we must sometime arrive at an incorporeal power, as we showed in the case of *A,* in steps 1–7 of the proof, and therefore at God;
13. or in the infinite whole there exist bodies which move each other continuously.
14. All these taken together as one whole can be called C, by No. 4.
15. And since all the parts of C are moved, by step 13,
16. C itself is moved, by No. 7,
17. and by some other being, by No. 5,
18. namely, by an incorporeal being, since we have already included all bodies, back to infinity, in C, by step 14. But we need something other than C, by 17 and 19,
19. which must have infinite power, by step No. 3, since C, which is moved by it, is infinite, by steps 13 and 14;
20. and which is a substance, by No. 2,
21. and therefore God, by No. 1.

Therefore, God exists. Q.E.D.

Symbol	Usage	Informal Reading
Mv	Mv(x)	x is moved
M	M(x, y)	x moves y
Sb	Sb(x)	x is a substance
Vi	Vi(x)	x has infinite power
If	If(x)	x is infinite
Ic	Ic(x)	x is incorporeal
P	P(x, y)	x is part of y

Table 1. Interpretation of formal symbols used.

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ANALIZA TREŚCIOWA „DOWODU NA ISTNIENIE BOGA" ZAPROPONOWANEGO PRZEZ LEIBNIZA W 1666 ROKU

Streszczenie

Młodzieńcze dzieło Leibniza *De arte combinatoria* z 1666 r. opatrzone zostało na końcu „Dowodem na istnienie Boga". Dowód ma charakter „matematyczny" i jest zbudowany zgodnie z wzorcem Euklidesowym. Dowód ten w 1982 r. został poddany przeze mnie próbie formalizacji logicznej. W niniejszym tekście, wykorzystując wyniki tamtej analizy, staram się na podstawie treści zawartych w wierszach dowodu pokazać, jakim pojęciem substancji posługiwał się Leibniz na użytek dowodu. Zostaje ponadto zrekonstruowana Leibnizjańska koncepcja całości i części oraz jego metoda definicyjna Leibniza.

CONTENT ANALYSIS OF THE *DEMONSTRATION OF THE EXISTENCE OF GOD* PROPOSED BY LEIBNIZ IN 1666

Summary

Leibniz's juvenile work *De arte combinatoria* of 1666 included the "Proof for the Existence of God." This proof bears a mathematical character and is constructed in line with Euclid's pattern. I attempted to logically formalize it in 1982. In this text, on the basis of then analysis and the contents of the proof, I seek to show what concept of substance Leibniz used on behalf of the proof. Besides, Leibnizian conception of the whole and part as well as Leibniz's definitional method have been reconstructed here.

Translated by Jan Kłos

Słowa kluczowe: Leibniz; substancja; fizyka; całość; część; definicja.
Key words: Leibniz; substance; physics; whole; part; definition.

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