QUANTITATIVE METHODS IN ECONOMICS
Vol. XV, No. 2, 2014, pp. 112 – 124

THE RATING SCALE MODEL IN THE CONSTRUCTION OF FUZZY TOPSIS METHOD

Bartłomiej Jefmański
Department of Econometrics and Computer Science
Wrocław University of Economics
e-mail: bartlomiej.jefmanski@ue.wroc.pl

Abstract: Fuzzy TOPSIS method enables linear ordering of objects characterized by linguistic variables, which values constitute expressions emerging from natural language. Crucial, however, often neglected phase of this method is a selection of the way of introducing linguistic expressions by fuzzy numbers. Therefore, in this article one suggested a modification of fuzzy TOPSIS method using Rating Scale Model (RSM) to establish triangular fuzzy numbers. A suggested method enables establishing the rank of objects on the basis of objective criteria and subjective weights expressed in the form of triangular fuzzy numbers. Usability of the suggested method was confirmed by an empirical example, concerning linear ordering of selected smartphones models.

Keywords: IRT models, rating scale model, TOPSIS, fuzzy number

INTRODUCTION

TOPSIS method belongs to the linear ordering group allowing for synthetic assessment of multidimensional objects. In an original version of the method suggested by Hwang and Yoon [1981], synthetic measure is evaluated on the basis of Euclidean distance from positive-ideal solution and negative-ideal solution. Fuzzy modification of TOPSIS method suggested by Chen [2000], enables for a synthetic assessment of multidimensional objects with the implication of linguistic variables and triangular fuzzy numbers. However, this method does not suggest the way of expressing linguistic variables by triangular fuzzy numbers. The way of estimating fuzzy numbers' parameters plays an important role in a final linear ordering of objects according to synthetic criterion. Therefore, the purpose of this article is to present a proposition of fuzzy TOPSIS method modification, which is based on the implementation of the Rating Scale Model in estimating the parameters of triangular fuzzy numbers. A suggested method assumes that criteria's values are shown on a metric scale, while criteria's weights are introduced in the form of linguistic values. Usability of a suggested method was presented on the basis of empirical example.

FUZZY TOPSIS METHOD

Let's assume that a certain set of objects $A = \{A_i | i = 1,...,n\}$ and a set of criteria $C = \{C_j | j = 1,...,m\}$, where $\tilde{X} = \{\tilde{x}_{ij} | i = 1,...,n; j = 1,...,m\}$ stand for a set of fuzzy evaluation criterion and $\tilde{W} = \{\tilde{w}_j | j = 1,...,m\}$ a set of fuzzy weights. Linear ordering of objects with the above outlined assumptions is possible among others through the application of fuzzy TOPSIS method. An example of applying this method can be found among others in studies of: Uyun and Riadi [2011], Yayla and in. [2012], Madi and Tap [2011], Matin et al. [2011], Chang and Tseng [2008], Erdoğan et al. [2013], Ataei [2013].

Application of fuzzy TOPSIS method requires the accomplishment of the following steps [Chen 2000]:

Step 1. Calculation of normalized fuzzy evaluation criteria:

$$\tilde{z}_{ij}(x) = \frac{\tilde{x}_{ij}}{\sqrt{\sum_{i=1}^{n} \tilde{x}_{ij}^2}}, \quad i = 1,...,n; \; j = 1,...,m. \tag{1}$$

Step 2. Calculation of weighted normalized fuzzy evaluation criteria:

$$\tilde{v}_{ij}(x) = \tilde{w}_j \tilde{z}_{ij}(x). \tag{2}$$

Step 3. Appointing positive-ideal solution A^+ and negative-ideal solution A^- development:

$$\tilde{A}^+ = \{\tilde{v}_1^+(x), \tilde{v}_2^+(x), ..., \tilde{v}_j^+(x), \tilde{v}_m^+(x)\}$$
$$\{(\max_i \tilde{v}_{ij}(x) | j \in J_1), (\min_i \tilde{v}_{ij}(x) | j \in J_2) | i = 1,...,n\}, \tag{3}$$

$$\tilde{A}^- = \{\tilde{v}_1^-(x), \tilde{v}_2^-(x), ..., \tilde{v}_j^-(x), \tilde{v}_m^-(x)\}$$
$$\{(\min_i \tilde{v}_{ij}(x) | j \in J_1), (\max_i \tilde{v}_{ij}(x) | j \in J_2) | i = 1,...,n\}, \tag{4}$$

where J_1 and J_2 are respectively the benefit criterion and the cost criterion.

Step 4. Calculation for each object a distance from positive-ideal solution d_i^+ and negative-ideal solution d_i^- (in an original work it is an Euclidean distance).

Step 5. Calculation of a synthetic measure:

$$C_i^+ = \frac{d_i^-}{d_i^+ + d_i^-}, \ i = (1,...,n). \tag{5}$$

Measure values (5) are normalized in an interval $<0;1>$. The smaller the distance of an object is from a positive-ideal solution and the bigger from a negative-ideal solution, the closer the value of a synthetic measure is to cohesion.

Step 6. Establishing the objects ranking. The best object owns the biggest value of a synthetic measure.

FUZZY TOPSIS METHOD BASED ON THE RATING SCALE MODEL

RSM model

RSM model is one of the best known IRT (*Item Response Model*) models. It was suggested by Georg Rasch [1960] and then extended by David Andrich [1978]. This model enables to estimate the choice probability by a respondent of a certain category, in the assessment of a selected item scale. Probability depends on the level of "difficulty" of item scale, "the ability" of the individual and threshold for a certain category. In accordance with RSM model, the choice probability by the n-th respondent of the category x on i-th item scale is expressed by the equation [Andrich 1978]:

$$\pi_{nix} = \frac{\exp \sum_{j=0}^{x} \left[\beta_n - \left(\delta_i + \tau_j\right)\right]}{\sum_{k=0}^{m} \exp \sum_{j=0}^{k} \left[\beta_n - \left(\delta_i + \tau_j\right)\right]} \tag{6}$$

where: β_n – the level of the n-th respondent's ability to give a correct answer for i-th item scale,

δ_i – the level of difficulty of i-th item scale,

τ_j – threshold for j-th category within i-th item scale.

A presented model allows to convert measure results from ordinal scale into interval scale. However, it does not find an application and answer in the analysis of "extreme" patterns on the item scale (e.g. when a respondent chooses extreme categories like "definitely unimportant" or "definitely important" within all items scale). An advantage of this model is a fact that parameters concerning a respondent and item scale are expressed by a common measure unit (described as logit) on the same continuum. It is also necessary to emphasise that its application requires to accept the assumption about one-dimensionality scale (all items scale measure only one latent variable) and local independence of item scale (an answer for certain item scale is independent from the answer for other suggestions).

The most important parameters of RSM model, from the view of conversion of verbal categories to the form of triangular fuzzy numbers, are threshold values for these categories. Threshold values are appointed on continuum of latent variable in the point of characteristic curves intersection of characteristic and adjacent with each other categories. Therefore, a threshold value constitutes a point in which a choice probability of a respondent one of the two adjacent categories is the same and comes to 50%. A detailed characteristics of threshold values in IRT models together with a graphical presentation includes Linacre study [2010].

Characteristics of fuzzy RSM-TOPSIS method

Let's assume that a measurement of criteria's values takes place in a metrical measure scale. Variables weights are assigned in a direct way (by experts, respondents etc.) through linguistic values (e.g. very important, unimportant etc.). In such a case, linear ordering of objects is possible through application of fuzzy RSM-TOPSIS method. It constitutes the hybrid linking fuzzy TOPSIS method and RSM model. This method assumes that, criterion weights expressed in the form of linguistic values are transformed to the form of triangular fuzzy numbers with the application of RSM model. The accomplishment of this method takes place in five distinguished stages outlined below:

Stage 1. Selection of criteria for objects' assessment;

Stage 2. Normalization of criteria's assessments (there is a need to apply normalization formula appropriate for metric scales);

Stage 3. Assessment of the criteria's importance by linguistic expressions;

Stage 4. Conversion of linguistic values to the form of fuzzy numbers.

A suggested approach assumes at this stage the use of RSM model procedure. Support for triangular fuzzy numbers for ordered linguistic values are determine in accordance with threshold values, which are assigned to certain categories. Table 1 illustrates formulas needed to establish triangular fuzzy numbers parameters for each of the categories distinguished within j-th criterion. An example concerns rating scale with five ordered verbal criteria, which are properly appointed as follows: definitely unimportant (DUI), unimportant (UI), medium important (MI), important (I), definitely important (DI).

Table 1. Formulas for estimating triangular fuzzy numbers parameters for linguistic values

Category	Fuzzy number parameters		
	a	b	c
DUI	-4	-4	τ_{i1}
UI	τ_{i1}	$\frac{\tau_{i1}+\tau_{i2}}{2}$	τ_{i2}
MI	τ_{i2}	$\frac{\tau_{i2}+\tau_{i3}}{2}$	τ_{i3}
I	τ_{i3}	$\frac{\tau_{i3}+\tau_{i4}}{2}$	τ_{i4}
DI	τ_{i4}	4	4

Source: own study

In the case of parameters b and c of a fuzzy number assigned to the most beneficial category it is determined on level 4. A characteristic feature of RSM model is a fact that, threshold values for particular categories can differ within criteria of objects' assessment. It means that, suggested in this article approach requires estimation of fuzzy numbers parameters separately for each criterion.

Stage 5. Averaging the assessment of significance through calculating arithmetic mean of fuzzy numbers in accordance with an equation:

$$\tilde{w}_j = \sum_{i=1}^{n} \tilde{w}_{ij} \tag{7}$$

where: $\tilde{w}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ - weight j-th criterion assigned by i-th respondent.

Stage 6. Rating normalized weighted criteria's assessments

Normalized weighted assessment i-th object according to j-th criterion is estimated in the following pattern:

$$\tilde{v}_{ij}(x) = \tilde{w}_j z_{ij}(x) \tag{8}$$

According to the principles of arithmetic of fuzzy numbers described among others in the study of Iron [1998] equation's result (8) is also a fuzzy number.

Stage 7. Determining a positive-ideal solution and negative-ideal solution of development.

According to the fact that, the obtained results in stage 4 have triangular form of fuzzy numbers, there is a problem of determining criteria's values for positive-ideal solution and a negative-ideal solution. Therefore, there is an issue of comparing fuzzy numbers and then a choice of maximal and minimal fuzzy number for each of these criterion. In order to do that, defuzzification of normalized weighted assessments is suggested, to show for each of criterion the best and worst value. Next, they will constitute a positive-ideal solution's coordinates, depending on the fact if a criterion influences benefit criterion or cost criterion on synthetic criterion.

Formulas of defuzzification fuzzy numbers were included among others in the study of Opricovic and Tzeng [2003].

Stage 8. Calculation the distance of assessed objects from a positive-ideal solution and negative-ideal solution.

It is suggested to apply Euclidean distance in estimating the distance of i -th object from positive-ideal solution and negative-ideal solution. This distance for two triangular fuzzy numbers $\tilde{A}$ and $\tilde{B}$ is expressed by an equation:

$$d\left(\tilde{A},\tilde{B}\right)=\sqrt{\frac{1}{3}\left(a_1-b_1\right)^2+\left(a_2-b_2\right)^2+\left(a_3-b_3\right)^2} \quad (9)$$

Stage 9. Determining the ranking of objects on the basis of an equation:

$$C_i^+=\frac{d_i^-}{d_i^++d_i^-} \quad (10)$$

The higher the value of synthetic measure is, the higher object's position in a ranking is.

EMPIRICAL EXAMPLE

Fuzzy RSM-TOPSIS method was applied to linear ordering of 10 selected smartphones, available on Polish market. Selected models according to report[1] of skąpiec.pl, were the most wanted ones in January 2014 r. The models were characterized by six criteria:

x_1 – screen size (inches),

x_{2a} – screen resolution horizontally (px),

x_{2b} – screen resolution vertically (px),

x_3 – resolution of built-in digital camera (Mpx),

x_4 – quantity of built-in memory (GB),

x_5 – RAM memory (GB),

x_6 – maximal time of conversations (h).

The importance of particulary criteria in using smartphones was assessed on the basis of survey research results (internet survey), which was conducted among smartphones' users in June 2014 r. This attempt was of purposeful character and its numerical amount came to 47 respondents.

Criteria were normalized in accordance with a formula of linear scale transformation [Shih et al. 2007]. Normalized criteria's values were distinguished in Table 2.

[1] Special report: Telephones, Servis Skąpiec.pl, January 2014.

Table 2. Normalized data matrix

Model	x_1	x_{2a}	x_{2b}	x_3	x_4	x_5	x_6
Samsung Galaxy S4 I9505	0,11	0,15	0,15	0,16	0,14	0,16	0,14
Samsung Galaxy S3 i9300	0,11	0,10	0,10	0,10	0,14	0,08	0,17
myPhone Next	0,10	0,08	0,08	0,10	0,03	0,08	0,04
Samsung Galaxy S III mini I8190	0,09	0,07	0,06	0,06	0,07	0,08	0,11
Samsung Galaxy S DUOS S7562	0,09	0,07	0,06	0,06	0,02	0,06	0,10
Samsung Galaxy Note III N9005	0,13	0,15	0,15	0,16	0,28	0,23	0,17
Sony Xperia Z	0,11	0,15	0,15	0,16	0,14	0,16	0,11
Goclever Quantum 4	0,09	0,07	0,06	0,02	0,03	0,04	0,04
Apple iPhone 5 16 GB	0,09	0,09	0,09	0,10	0,14	0,08	0,06
Sony Xperia J	0,09	0,07	0,07	0,06	0,02	0,04	0,06

Source: own calculations

The importance of particulary criteria, respondents assessed through rating scale, which points constituted linguistic value: “definitely unimportant”, “unimportant”, “medium important”, “important”, “definitely important.” None of the respondents assessed criterion as “definitely unimportant” or “unimportant”, therefore in a further analysis three other linguistic values were used.

On the basis of the results of assessments‘ importance and in accordance with RSM model, characteristic curves were estimated for all criteria. Graphical picture of curves was distinguished in Figure 1.

In accordance with a suggested in this article method a basis of conversion of the results of assessments’ importance to the form of fuzzy numbers are the points of intersection of particulary characteristic curves. These points’ values for each of criteria were distinguished in Table 3.

In accordance with formulas included in Table 1, the conversion results is an expression of linguistic values through triangular fuzzy numbers. Estimating within each criterion an arithmetic mean from fuzzy numbers allowed to obtain averaged weights for each of criterion. Parameters of triangular fuzzy numbers being criteria’s weights were distinguished in table 3.

Table 3. Threshold values and weights for criteria

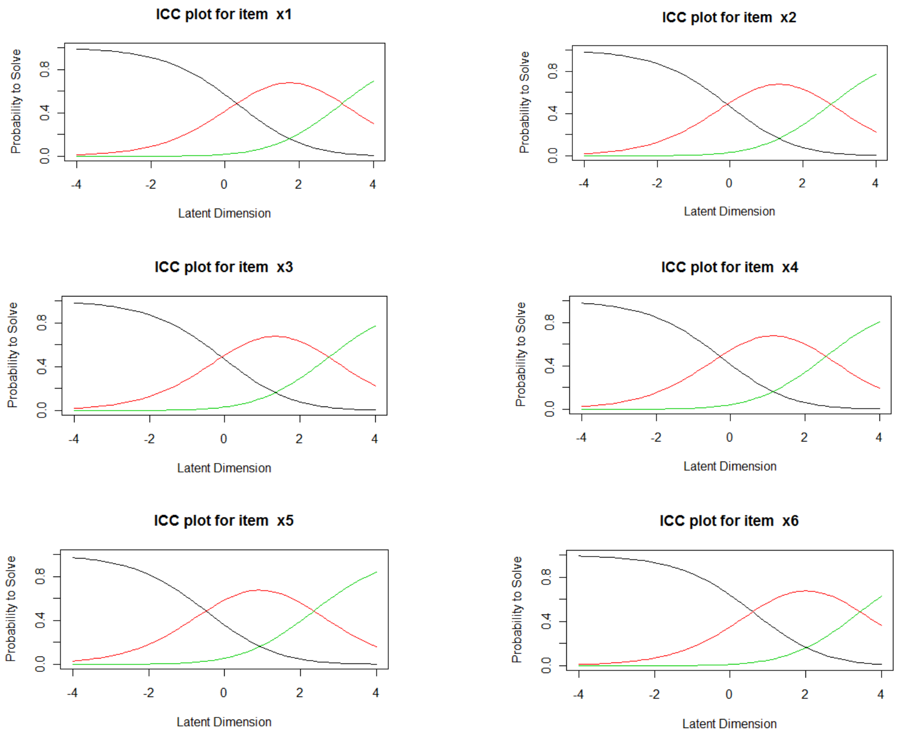
Criteria	Thresholds		Weights		
	τ_1	τ_2	a	b	c
x_1	0,32	3,16	1,06	2,21	3,30
x_{2a}	-0,08	2,77	0,99	2,25	3,17
x_{2b}	-0,08	2,77	0,99	2,25	3,17
x_3	-0,28	2,57	0,96	2,18	3,13

Criteria	Thresholds		Weights		
	τ_1	τ_2	a	b	c
x_4	-0,49	2,36	0,97	2,36	3,17
x_5	0,60	3,44	0,92	2,84	3,15
x_6	0,32	3,16	0,83	1,59	3,08

Source: own calculations with the application of eRm package of R programme

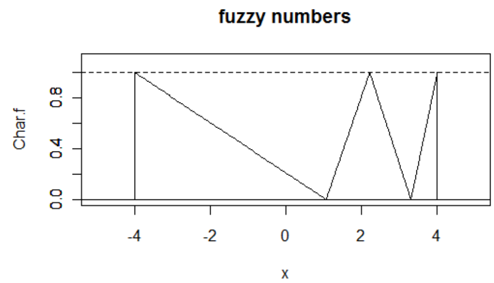
A graphical form of the obtained average assessments' importance in the forms of triangular fuzzy numbers was presented on the basis of criterion x_1 (see figure 2).

Figure 1. Characteristic curves for criteria



Source: own calculation with the application of eRm package of R programme

Figure 2. Average assessment's importance for criterion x_1



Source: own study with the application of fuzzyOP package of R programme

Normalized weighted assessments of criteria were estimated in accordance with arithmetic principles for fuzzy numbers and equation 8. The results in the form of parameters of triangular fuzzy numbers are introduced in table 4.

Establishing positive-ideal solution and negative-ideal solution requires recommendation among normalized weighted assessments maximal and minimal value. In order to do that through the defuzzification method presented in the studies of Ding and Liang [2005] and Wysocki [2010] 10 fuzzy numbers were compared within each criterion:

$$q_i = \left(a_i + 4b_i + c_i\right)/6 \tag{11}$$

Because all criteria influence in a benefit way on synthetic criterion, it was assumed in ideal-point solution for each criterion maximal weighted assessments, however in a negative-ideal solution minimal assessments. Values for fuzzy ideal-point solution and negative-ideal solution were distinguished in table 5.

The distance of particulary objects from a positive-ideal solution and negative-ideal solution was estimated in accordance with an equation (9). The distance of objects together with the values of synthetic measure (10) was introduced in table 6.

In a ranking gained on the basis of a suggested method, there are objects very close to a positive-ideal solution as well as the ones of a very low value of a synthetic criterion. The highest value of a synthetic measure was obtained for Samsung Galaxy Note III N9005 model. There are also on a high place in a ranking, other models like Galaxy S4 I9505 and Sony Xperia Z. The lowest value of a synthetic measure gained Goclever Quantum 4 and Sony Xperia J models.

Table 4. Values of normalized weighted criteria's assessments

No.	Model	Criteria																				
		x_1			x_{2a}			x_{2b}			x_3			x_4			x_5			x_6		
		a	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*	*a*	*b*	*c*
1	Samsung Galaxy S4 I9505	0,12	0,25	0,37	0,34	0,34	0,48	0,15	0,35	0,49	0,16	0,35	0,51	0,13	0,33	0,44	0,14	0,44	0,49	0,11	0,21	0,42
2	Samsung Galaxy S3 i9300	0,11	0,24	0,35	0,23	0,23	0,32	0,10	0,23	0,33	0,10	0,22	0,31	0,13	0,33	0,44	0,07	0,22	0,25	0,14	0,27	0,52
3	myPhone Next	0,11	0,22	0,33	0,17	0,17	0,24	0,08	0,17	0,25	0,10	0,22	0,31	0,03	0,08	0,11	0,07	0,22	0,25	0,03	0,06	0,12
4	Samsung Galaxy S III mini I8190	0,09	0,20	0,29	0,15	0,15	0,22	0,06	0,15	0,20	0,06	0,14	0,20	0,07	0,16	0,22	0,07	0,22	0,25	0,09	0,18	0,35
5	Samsung Galaxy S DUOS S7562	0,09	0,20	0,29	0,15	0,15	0,22	0,06	0,15	0,20	0,06	0,14	0,20	0,02	0,04	0,05	0,06	0,17	0,19	0,09	0,16	0,32
6	Samsung Galaxy Note III N9005	0,13	0,28	0,42	0,34	0,34	0,48	0,15	0,35	0,49	0,16	0,35	0,51	0,27	0,65	0,88	0,22	0,67	0,74	0,14	0,27	0,51
7	Sony Xperia Z	0,12	0,25	0,37	0,34	0,34	0,48	0,15	0,35	0,49	0,16	0,36	0,51	0,13	0,33	0,44	0,14	0,44	0,49	0,09	0,18	0,34
8	Goclever Quantum 4	0,09	0,20	0,29	0,15	0,15	0,22	0,06	0,15	0,20	0,02	0,05	0,08	0,03	0,08	0,11	0,04	0,11	0,13	0,03	0,06	0,12
9	Apple iPhone 5 16 GB	0,09	0,20	0,29	0,20	0,20	0,29	0,09	0,21	0,29	0,10	0,22	0,31	0,13	0,33	0,44	0,07	0,22	0,25	0,05	0,10	0,20
10	Sony Xperia J	0,09	0,20	0,29	0,15	0,15	0,22	0,07	0,16	0,22	0,06	0,14	0,20	0,02	0,04	0,05	0,04	0,11	0,13	0,05	0,09	0,18

Source: own calculations

Table 5. Criteria's values for positive-ideal solution and negative-ideal solution

Criteria	Positive-ideal solution			Negative-ideal solution		
	a	b	c	a	b	c
x_1	0,12	0,25	0,37	0,09	0,20	0,29
x_{2a}	0,34	0,34	0,48	0,15	0,15	0,22
x_{2b}	0,15	0,35	0,49	0,06	0,15	0,20
x_3	0,16	0,35	0,51	0,02	0,05	0,08
x_4	0,27	0,65	0,88	0,02	0,04	0,05
x_5	0,22	0,67	0,74	0,04	0,11	0,13
x_6	0,14	0,27	0,52	0,03	0,06	0,12

Source: own calculations

Table 6. Objects' distance from a positive-ideal solution and negative-ideal solution together with the values of synthetic values

No.	Model	d^-	d^+	C_i	Ranking place
1	Samsung Galaxy S4 I9505	2,71	1,03	0,73	2
2	Samsung Galaxy S3 i9300	1,80	1,94	0,48	4
3	myPhone Next	0,67	3,07	0,18	7
4	Samsung Galaxy S III mini I8190	0,78	2,96	0,21	6
5	Samsung Galaxy S DUOS S7562	0,47	3,29	0,12	8
6	Samsung Galaxy Note III N9005	3,79	0,08	0,98	1
7	Sony Xperia Z	2,63	1,12	0,70	3
8	Goclever Quantum 4	0,07	3,67	0,02	10
9	Apple iPhone 5 16 GB	1,25	2,49	0,33	5
10	Sony Xperia J	0,23	3,51	0,06	9

Source: own calculations

SUMMARY

TOPSIS method belongs to the group of most often applied methods in a linear ordering of multidimensional objects. Its fuzzy modification enables to conduct analyses in the fuzzy information conditions, when objects' assessments and/or criteria's weights are introduced in the form of linguistic values. It allows respondents to formulate assessment in a more natural way than through numbers, but at the same time it causes that this description is less precise and subjective. A theory of fuzzy sets seems to be helpful and allows for among others to express out of vague and ambiguous terms thanks to fuzzy numbers. Taking into consideration a fact that the results of a linear ordering of objects can depend on parameters describing fuzzy numbers at their estimation there is a need to apply appropriate methods.

This article presents a suggestion of fuzzy TOPSIS method modification based on RSM model. It allows for conversion of the assessments of criteria's importance expressed by linguistic values to the form of triangular fuzzy numbers. As it results from the algorithm of a suggested in this article method, it can be sensitive for several parameters subjectively chosen by a researcher: normalization formula of variable, distance measure between fuzzy numbers and the way of comparing fuzzy numbers by defuzzification methods. It is also worth emphasizing that, RSM model is one of many IRT models, which can be applied in a fuzzy TOPSIS method. Therefore, in further research for this method, it is planned to conduct a conversion of linguistic values to the form of triangular fuzzy numbers, also thanks to such models like: Partial Credit Model, Generalised Partial Credit Model and Graded Response Model.

REFERENCES

Andrich D. (1978) A rating formulation for ordered response categories, "Psychometrika", vol. 43, pp. 561-573.

Ataei E. (2013) Application of TOPSIS and Fuzzy TOPSIS Methods for Plant Layout Design, "World Applied Sciences Journal", vol. 24, iss. 7, pp. 908-913.

Chang S.-H., Tseng H.-E. (2008) Fuzzy Topsis Decision Method for Configuration Management, "International Journal of Industrial Engineering", vol. 15, iss. 3, pp. 304-313.

Chen C.-T. (2000) Extensions of the TOPSIS for group decision-making under fuzzy environment, "Fuzzy Sets and Systems", no. 114, pp. 1-9.

Ding J.F., Liang G.S. (2005) Using fuzzy MCDM to select partners of strategic alliances for linear shipping, "Information Sciences", vol 1-3, pp. 197-225.

Erdoğan M., Bilişik Ö.N., Kaya İ., Baraçh H. (2013) A customer satisfaction model based on fuzzy TOPSIS and SERVQUAL methods, "Lecture Notes in Management Science", vol. 5, pp. 74-83.

Hwang C.L., Yoon K. (1981) Multiple Attributes Decision Making Methods and Applications, Springer, Berlin Heidelberg.

Iron A. (1998) Fuzzy rules and fuzzy functions: A combination of logic and arithmetic operations for fuzzy numbers, "Fuzzy Sets and Systems", vol. 99, iss. 1, pp. 49-56.

Linacre J.M. (2010) Transitional categories and usefully disordered thresholds, "Online Educational Research Journal", pp. 1–10.

Madi E.N., A.O.M Tap (2011) Fuzzy TOPSIS Method in the Selection of Investment Boards by Incorporating Operational Risks, "Proceedings of the World Congress on Engineering", vol. 1, pp. 291-295.

Opricovic S., Tzeng G. (2003) Defuzzification within a multicriteria decision model, "International Journal of Uncertainty Fuzziness and Knowledge-Based Systems", vol. 11, no. 5, pp. 635-652.

Special report: Telephones, Skąpiec.pl, January 2014.

Rasch G. (1960) Probabilistic Models for Some Intelligence and Attainment Tests, Danish Institute for Educational Research, Copenhagen (Expanded edition, University of Chicago Press, 1980).
Shih H.-S., Shyur H.-J., Lee E.S. (2007) An extension of TOPSIS for group decision making, “Mathematical and Computer Modelling”, vol. 45, no. 7, pp. 801-813.
Uyun S., Riadi I. (2011) A Fuzzy Topsis Multiple-Attribute Decision Making for Scholarship Selection, “Telkomnika”, vol.9, no.1, pp. 37-46.
Wysocki F. (2010) Metody taksonomiczne w rozpoznawaniu typów ekonomicznych rolnictwa i obszarów wiejskich, Wydawnictwo Uniwersytetu Przyrodniczego w Poznaniu, Poznań.
Yayla A.Y., Yildiz A., Özbek A. (2012) Fuzzy TOPSIS Method in Supplier Selection and Application in the Garment Industry, “FIBRES & TEXTILES in Eastern Europe”, vol. 20, no. 4, pp. 20-23.