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Objectives and Content of the Mathematics E-learning Course Preparing Students for the School-leaving Exam in Mathematics

Abstract

Information and communication technologies (ICT) can help solve the problems connected with forming mathematical competencies in students and are used in the process of teaching mathematics. The first part of the paper presents the theoretical background of the subject matter, including: the description of mathematical competencies and their identification at secondary school in Poland, Niemierko's taxonomy, the programmed learning theory, and the structure of the system of education/learning in the e-learning environment. It expresses the preconditions, expected results, concepts, objectives, hypotheses, and research methods. The practical part describes the structure of "Mathematics with Moodle," a system for individual learning based on the original authorial *MatLearn* module, and its graphic representation. A didactic tool is proposed – an e-learning course preparing students for the school-leaving exam in mathematics and improving students' mathematical competencies. Its aim is to increase the level of competencies, especially those which have not been mastered yet. In order to construct study activities in the course, the programmed learning principles and Niemierko's taxonomy were used.

K e y w o r d s: information and communication technologies (ICT), key competencies, mathematical competencies, e-learning, Niemierko's taxonomy, programmed learning, *MatLearn* module, Mathematics with Moodle

Introduction

Mathematical competence and basic competencies in science and technology are ranked third in the list of "eight key competences for lifelong learning" ("Key competences for lifelong learning…," 2007, p. 3). Key competencies were defined, developed, and accepted in the document "Recommendation of the European Parliament and of the Council of 18 December 2006 on key competences for lifelong learning." The document says: "Competences are defined here as a combination of knowledge, skills and attitudes appropriate to the context. Key competences are those which all individuals need for personal fulfilment and development, active citizenship, social inclusion and employment" ("Recommendation…," 2006).

"Recommendation" states eight key competencies:

1) communication in the mother tongue;
2) communication in foreign languages;
3) **mathematical competence and basic competences in science and technology**;
4) digital competence;
5) learning to learn;
6) social and civic competences;
7) sense of initiative and entrepreneurship; and
8) cultural awareness and expression ("Recommendation…," 2006).

Mathematical competence can be found in the currently applicable new mathematics core curriculum in accordance with the Regulation of the Minister of National Education (in 2012) on the core curriculum for pre-school child development and general education in specific types of schools (Journal of Laws 2012, item 977, p. 245).

An analysis of the Programme for International Student Assessment (PISA) findings carried out since 2003 shows the results in mathematics in Poland – the highest number of students is at the average level. It can also be noticed that the abovementioned results have remained at the same level for the long time, and the level of mathematical competence of students has not increased. With reference to Polish graduation exams in mathematics, which are obligatory, a visible decrease of certain mathematical competencies has been noticed. In order to solve this problem and increase the level of certain skills and knowledge – in particular those least mastered by students – we try to improve the process of teaching mathematics by developing a course for individual learning of mathematics supported by ICT.

After graduating from the primary school, a student continues general education at the third and the fourth stage of education. The third stage of education in Poland is executed in middle schools, whereas the fourth stage of education is executed in secondary schools. Although they are executed in two different types of schools, the third and the fourth stage of general education form a coherent whole and constitute a basis of education, making it possible to gain varied professional qualifications and to improve or modify them at a later stage, opening the process of lifetime education.

In accordance with the Regulation, an important aim of the school at the third and the fourth stage of education is to prepare students for living in the information society. Teachers should create favourable conditions for students to gain skills in searching, ordering, and using information from different sources with the application of ICT from different subjects. The Regulation also contains teaching content, that is, detailed requirements concerning mathematics at the fourth stage of education.

The main purpose of the research is to develop and assess the system for learning mathematics with the application of ICT, the element of which is a didactic tool – an e-learning course which contains a teaching module developing mathematical competence of students. It is enriched with elements of programmed learning and applies the principle based on the gradual increase of the difficulty level.

Mathematical competencies are also very important in the context of STEM education and the future career of young people, whose choices are influenced, on the one hand, by appropriately directed education at the lower stages of education, and, on the other hand, by the demand for specialists in the scope of applied studies: science, technology, engineering, and mathematics.

STEM Education: Background

As Larson (2017) stresses, "STEM education is a focus of many policy makers, business and industry leaders, philanthropic foundations, and education leaders because the data indicate there will be accelerated growth in the number of STEM jobs the economy will generate over the next decade, particularly compared to other professions (see, for example, STEM 101: Intro to tomorrow's jobs). Additional data indicate beginning salaries and salary growth for STEM majors will outpace those for other majors and careers."

Escuder and Furner (2012) note that "The President's Council of Advisors on Science and Technology (PCAST) released an executive report in November 2010 where specific recommendations to the administration are given to ensure that the United States is a leader in Science, Technology, Engineering, and Mathematics (STEM) education in the coming decades. One of the recommendations is to recruit and train 100,000 new STEM middle and high school teachers over the next decade that are able to prepare and inspire students and have strong majors in STEM fields and strong content-specific pedagogical preparation. PCAST regards teachers as the most important factor in ensuring excellence in STEM education. Despite the ongoing efforts to promote the use of technology in education (e.g., National Council of Teachers of Mathematics [NCTM], 2000; National Educational Technology Standards for Teachers [NETS.T], 2008), teachers' ineffective use of technology has been reported in the literature. One reason frequently cited is that teachers are not trained in utilizing technology in the classroom within context" (Escuder & Furner, 2012, p. 76).

According to Kelley and Knowles (2016), "The global urgency to improve STEM education may be driven by environmental and social impacts of the twenty-first century which in turn jeopardizes global security and economic stability. The complexity of these global factors reach beyond just helping students achieve high scores in math and science assessments. Friedman (2005) helped illustrate the complexity of a global society, and educators must help students prepare for this global shift. In response to these challenges, the USA experienced massive STEM educational reforms in the last two decades. In practice, STEM educators lack cohesive understanding of STEM education. Therefore, they could benefit from a STEM education conceptual framework."

Other researchers, Schmidt and Fulton (2016) observe in their study that "The need to prepare students with 21st-century skills through STEM-related teaching is strong, especially at the elementary level. However, most teacher education preparation programmes do not focus on STEM education." Schmidt and Fulton's (2016) "findings suggest that while inquiry-based STEM units can be implemented in existing programmes, creating and testing these prototypes requires significant effort to meet PSTs' learning needs, and that iterating designs is essential to successful implementation" (Schmidt & Fulton, 2016, p. 302).

Returning to Kelley and Knowles (2016), "The process of integrating science, technology, engineering, and mathematics in authentic contexts can be as complex as the global challenges that demand a new generation of STEM experts. Educational researchers indicate that teachers struggle to make connections across the STEM disciplines. Consequently, students are often disinterested in science and math when they learn in an isolated and disjoined manner missing connections to crosscutting concepts and real-world applications." The aim of their article is to "operationalize STEM education key concepts and blend learning theories to build an integrated STEM education framework to assist in further researching integrated STEM education framework" (2016). The authors describe one example: "Teachers and Researchers Advancing Integrated Lessons in STEM (TRAILS)" (2016) (Smyrnova-Trybulska, Morze, Kommers, Zuziak, & Gladun, 2016).

Methodological Bases and Content of the E-learning Course

The development of the authors' programme "Matematyka z Moodle" ["Mathematics with Moodle"] is based on the ADDIE model, whose name is an acronym for the English words: analysis, design, development, implementation, and evaluation (Clark & Mayer, 2002). The ADDIE model consists of the analysis phase, the assumptions and conditions, the course design, the course development component, implementation, and evaluation. Constructing a good e-course, which runs under the ADDIE model, is an ongoing process. After the evaluation stage, there is the next stage of analysis, which starts the next phase of work on the course, and which is aimed at creation of a bug-free, efficient, and user friendly product.

Analysis

The training goals and expected results of our proposed system were defined (Heba, 2014). Then, surveys intended for 500 students and 500 teachers in secondary schools were performed. The detailed results are described in Heba (2010). Based on them, the computer software was designed. It was designed to support:

- teaching mathematics as well as technology connected with e-learning, and
- extending knowledge and mathematical skills that were selected.

An analysis of mathematical competence of secondary school pupils in Poland was done. The results are presented in Heba (2014). Math tasks for the e-learning course were chosen according to Niemierko's taxonomy of educational goals: A, B, C, and D (Niemierko, 1999).

Design

The system contains the description of objectives and tasks for mastering mathematical competence for secondary schools in the examination requirement standards. The schedule, organisational structure, duration, and pace of the proposed learning system were outlined.

The acting subject asks questions about effectiveness of own practice, observes own actions, assigns a specific meaning to them, makes own conclusions, which become the source of initiatives, and their effects generate next questions and ideas addressed to the action. This strategy has all signs of a "never-ending story" with participation of the researcher of own practice in the leading role. It is illustrated by the figures below, containing the charts of the procedure of action research. The results of cognition and action are noticed and assessed in their course, but are also confronted with the processes and structures in the wide context, in which the social and educational practice runs. Therefore, it may be stated that this procedure of disclosure of mechanisms governing the functioning of persons and communities is a specific type of *learning through experiencing* (Czerepaniak-Walczak, 2010, p. 231).

It should be emphasised that the models of the process of research on action – despite differences between each other in details – have some common features, namely: sequencing of thinking and acting, and cyclicality of sequences, that is, the steps of the action in every element and of the elements in the process. In fact, each of these models is a detailed model of the process prepared by Kurt Lewin (Czerepaniak-Walczak, 2010).

“The three stages of the change process determined by Kurt Lewin (unfreeze–change–freeze) consist of the following phases:

- facing an unknown problem or difficulty, which raises dissatisfaction of the person or of the group, and, as a result, produces a need for a change of project – realisation of the problem (unfreeze);
- implementation of the project, diagnosing the changed situation, checking the effects of new behaviours and actions, application of new measures, organisational changes, etc. (action–change); and
- evaluation of change implementation; if the evaluation results are satisfactory, they are introduced into practice on a permanent basis (freeze), and if they generate new problems, not known so far, new projects are formulated (new unfreeze); therefore, a new cycle of action and research starts.

Such a structure of the process of action research illustrates its cyclicality. The first element in the cycle is planning the application of the change factors. These factors may be: actions of the persons engaged in the process of teaching, educating, and learning; the used didactic measures; management of the space in which the educational interactions take place; the forms of time organisation, etc.” (Czerepaniak-Walczak, 2010, p. 327).

The integrated process of cognition and transformation of own practice is called – apart from action research – “methodological research,” “practice-based research,” “practice-oriented research,” or “participating research.” Each of these names contains information that it is a cognitive procedure used for improvement of own practice in interaction with a reflection on it through a disclosure of the mechanisms governing it and its personal interpretation. It allows for dealing with a specific practical problem experienced in a natural situation. It is a combination of a research critical and constructive strategy with an empirical and analytical strategy.

This procedure originates from the Cartesian coup, reversing the order of contemplation and action in the process of knowledge creation. According to Hannah Arendt (2000, p. 314), the certainty of knowledge could have been obtained only after fulfilling two conditions: “firstly, that the knowledge relates to something done on one’s own [...], and secondly, that this knowledge – due to its nature – can be checked solely through further doing of something” (Czerepaniak-Walczak, 2010, p. 322).

Knowledge creation has become an element of the process of transformation of reality. In this situation, both the knowledge of the subject on the object and the object itself are subject to a change. Human action has acquired a new dimension. It has become the object of cognition but also its driving force, while the knowing subject has become the cognised object for themselves as well as the author of the change in the cognition object. The examples of such an approach to complementarity of cognition and action, learning and knowledge expansion are found in philosophy, psychology, and pedagogy of John Dewey, who – apart from Lewin – is considered to be the precursor of action research (Czerepaniak-Walczak, 2010).

The charts and diagrams representing the manner of training content were developed. A prototype lesson in the e-learning course with the application of the *MatLearn* module was created, and it was tested in the experimental group in order to verify the hypothesis of a potential increase of mathematical competence in a particular topic. The methods of and conditions for assessing were specified. Methods of system evaluation and collecting data for analysis and the reporting were established.

An analysis of objectives, scope of teaching, and activities of a teacher in the course is followed by the formation of the educational content in the e-learning environment. It is assumed that the programme corresponds to mathematics in terms of a subject matter and is constructed according to the following rules:

- introduction: educational objectives, abstracts, contents, references, definitions of terms, a forum, a registration questionnaire;
- thematic modules: a pre-test (diagnostic test), study materials, a block of tasks, verification and control of information, creative tasks, interactive communication – a teacher with students and among students; and
- summary: an exam test, a final questionnaire.

Our “Matematyka z Moodle” system contains:

- documents: a new core curriculum in mathematics, Polish educational programme *Matematyka z plusem*, a guide for graduation examination by the Central Examination Board;
- an e-learning course preparing for the graduation exam in mathematics with the authors’ *MatLearn* module; and
- a user manual for the students.

After evaluating the system, necessary adjustments were done, and a methodological guide for the teachers is being prepared. It will include numerous scenarios of mathematics lessons with the use of our e-learning course “Matematyka z Moodle.”

A course preparing for the graduation exam contains the following parts:

- introduction to the course;
- eight e-learning units, each of which will contain a maximum of five lessons;
- sets of tasks for the graduation exams from previous years; and
- end of the course.

The e-learning course contains a module forming mathematical competence. The module is controlled by the *if–then–else* condition. It verifies whether the condition placed after *if* is fulfilled. If it is, the block of instructions following *then* is realised. If this condition is not fulfilled, the block of instructions following *else* is realised.

Development

The final product – the e-learning course is developed. Study materials, tasks, *GeoGebra* applets, and tests are prepared. The *MatLearn* module is based on a number of theoretical resources (Bertrand, 1998): behaviourism (Skinner, 1974), constructivism (Piaget, 1977, 1985; Weicker, 2005), as well as Niemierko's taxonomy of educational objectives (Hudecová, 2003; Niemierko, 1999) and Skinner's programmed learning elements as described in the work by Kapounová and Pavlíček (2003):

- the principle of small steps;
- the principle of active response;
- the principle of immediate confirmation;
- the principle of self-pacing; and
- the principle of self-evaluation.

Nowadays, computers at school may be used as teaching machines for the realisation of programmed learning in combination with hypertext and multimedia in e-learning.

Implementation

Study materials are stored on the Moodle platform, and information on how to operate the course is given to students and teachers in the form of specific training sessions. An administrator assists with technical problems, and the teachers help the students with their problems in mathematics. The training/experiment is managed according to a schedule planned in the design stage. During the pilot stage, the functionality of the *MatLearn* module was tested and contents of Niemierko's categories, mathematics tasks, timings of tests, and understanding of tasks were verified.

Evaluation

At the final stage the operation, form and content of the course was evaluated. The formal evaluation of the e-learning course follows mainly Kirkpatrick and Kirkpatrick's model of training effectiveness (J. Kirkpatrick & K. W. Kirkpatrick, 2009).

Teaching Mathematics Using *GeoGebra*

Escuder and Furner (2012) note: "The most powerful feature of *GeoGebra* is the connection it makes between Geometry, Algebra, Calculus, and Statistics. *GeoGebra* is a dynamic geometry system in which you work with points, vectors, segments, lines, and conic sections. *GeoGebra* is also a dynamic algebraic system, where equations and coordinates can be entered directly. Functions can be defined algebraically and then changed dynamically afterwards. *GeoGebra* has a simple CAS in the background, which has the ability to deal with variables for numbers, vectors and points, find derivatives and integrals of functions and offers commands like Root or Extremum" (pp. 77–78). As the authors stress further, "These two views are characteristic of *GeoGebra*: an expression in the algebra window corresponds to an object in the geometry window and vice versa. The spreadsheet view has been added recently making it possible to enter data in the spreadsheet and view graphs in the geometry window while maintaining its dynamic characteristic. Although *GeoGebra* has been primarily intended for mathematics instruction in secondary schools, it certainly has uses in Higher Education and even now being brought down to the elementary levels as well" (Escuder & Furner, 2012, pp. 77–78). Figure 1 shows the example of the *GeoGebra* Applets.

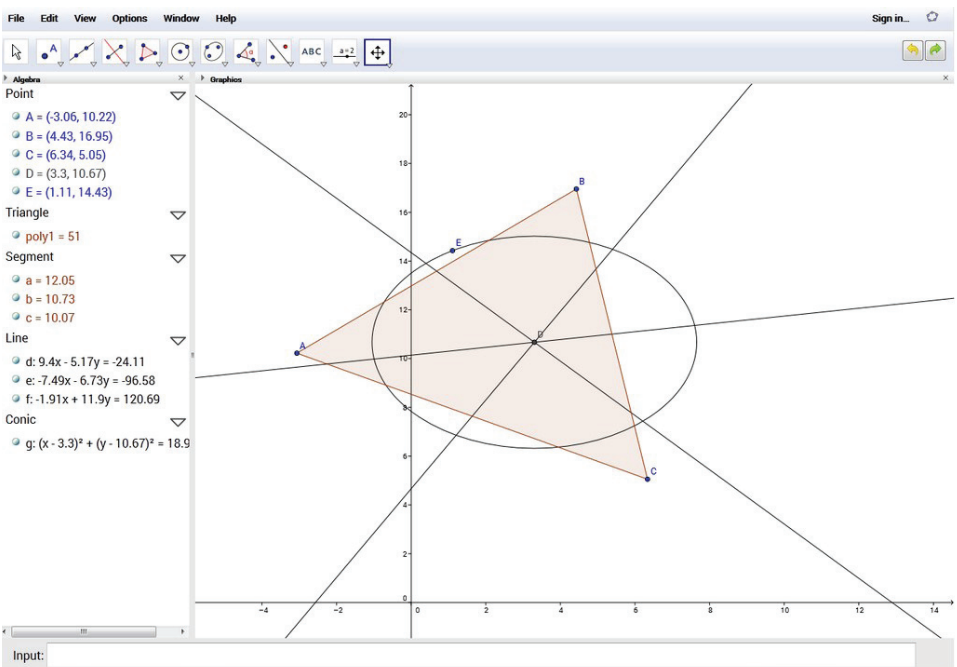

Figure 1. GeoGebra Applets.

S o u r c e: CopyScreen of the example in GeoGebra Applets.

Teaching Mathematics Using an E-learning Course

Teaching with the use of the platform is carried out in accordance with a predetermined schedule and a scheme of the e-learning course. At the beginning, a student analyses papers concerning the school-leaving exam, which can be found on the platform's main page. Afterwards, one proceeds to the actual course, fills out a public questionnaire, and then is redirected to other parts which can be found in the introductory part of the course: operating instructions concerning the platform and course, literature, information on the computer programmes used during the teaching. Next, one is redirected to the introductory test – a pre-test consisting of 30 closed school-leaving exam tasks. A student continues to the next part regardless of his or her test results. After one finishes the pre-test, the system redirects one to the individual e-learning units. Each unit consists of a maximum of 5 classes.

Each class is a multimedia programme which consists of 4 modules at 4 levels of the categories of taxonomic aims by Niemierko (A, B, C, D), and of the course structure, which consists of the following:

- knowledge (of the course contents);
- e-tasks (exercises, tasks with the use of the *GeoGebra* programme);
- Test I, Test II, Test III (parts that determine the level of mastering of the learned skills), which are a part of the e-tasks; and
- a help-teacher module.

All the information, instructions, and directions are forwarded in a written form. Within the course, a student moves by using hypertext or a button, which can be preset in the Moodle system.

The first part of each class determines its objectives, introductory requirements, and problematic issues that will be solved. The knowledge is passed in an interactive manner. The individual modules are divided into screens on which students – through the tasks – acquire knowledge and learn to form mathematical competencies. They can also use the *GeoGebra* programme.

The second part of the e-tasks consists of tasks which consolidate and form the acquired knowledge and skills. Each task contains samples of solution, some of which come with the *GeoGebra* applets. After a student completes this part (at each level), he or she proceeds to the third part.

The third part of the class is the module of tests (Test I, Test II, and Test III). The tests provide both students and teachers with feedback on students' ability to master knowledge and skills at a particular level of a particular curriculum. In this part, a student can no longer use the help or return to the already solved tasks.

At the end, there is a help-teacher module, which makes it possible for a student to become acquainted (on his or her own, or with a teacher's help) with the solution of each level's test, to compare it with his or her own solution, and to consolidate the knowledge and skills in the tasks that he or she solved incorrectly.

At the conclusion of the e-learning unit, a student solves homework – a set of open tasks which is evaluated by a teacher and sent back to a student (and thus providing feedback) via the Moodle system.

Students' working with the e-learning units is being evaluated all the time. Thanks to automatically generated reports, a teacher has a permanent access to information on how his or her students have dealt with individual mathematical competencies.

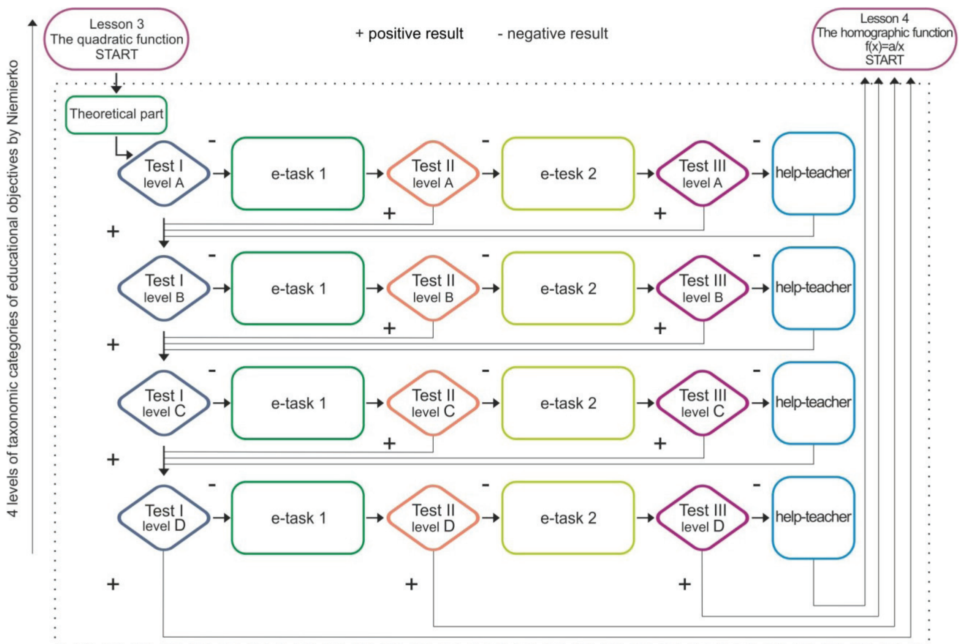

Figure 2. The structure of the *MatLearn* module.

S o u r c e: Heba, Kapounova, & Smyrnova-Trybulska, 2014b.

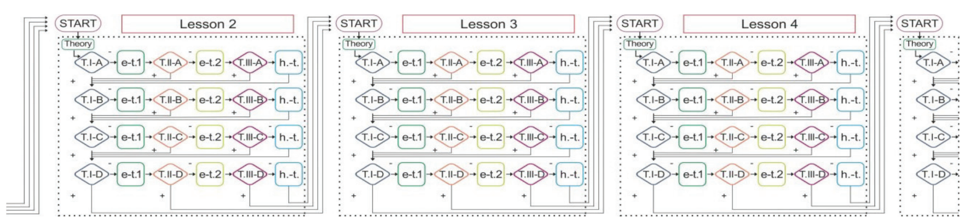

Figure 3. The structure of the e-learning units.

S o u r c e: Heba, 2014.

Teachers can use parts of the course in regular mathematics classes and also in tutoring classes. In the course, students can work independently at home and can choose their own pace, time, and environment for studying.

The development of students' mathematical competencies is measured twice: by a pre-test taken before the start of the course and by a test taken at the end of the course. The aim of it is the evaluation of the studying progress of the students who are preparing for their school-leaving exam. Before the start of the course, a student tries to solve 5 sets of school-leaving exam tasks from previous years. To compare their results, a student uses the help-teacher module, which will offer him or her correct results.

Conclusion

The mathematics authorial course, which contains elements of programmed learning, was proposed and tested. It is based on Niemierko's taxonomy of educational aims and works on the principle based on the gradual increasing of the difficulty level of the solved tasks. The proposed system is realised as a Moodle platform e-learning course. It was tested for three areas of mathematics: functions and their qualities, analytic geometry, and planimetry and stereometry. The mathematical experiment – which took place in the experimental and control groups – proved that students who used the "Matematyka z Moodle" system for their preparation for the school-leaving exam improved their competencies in the taught areas of mathematics. More information about it can be found in Heba, Kapounova, and Smyrnova-Trybulska (2014a, 2014b, 2014c).

Conclusions obtained in the course of pedagogical experiment have brought concepts of further development. The plans are:

- completing a methodological guide for teachers after evaluating the proposed learning system by mathematics and ICT examiners, and an examiner from the Central Examination Board;
- conducting long-term studies of a complete e-learning course (8 thematic modules) at upper secondary school in a larger group of students from several voivodships;
- developing a system for individual mathematics instructional systems – "Moodle Math" – throughout the core and extended learning cycles;
- conducting an analysis of the learning path of the pupil on the Moodle platform;
- adapting the MatLearn course to video conferencing capabilities (Skype, Clickmeeting, etc.);
- adapting the MatLearn course to mobile applications;
- adapting the MatLearn course to other free platforms such as eFront used in Polish schools;
- adapting the projected system to other subjects in Polish post-gymnasium schools; and

- conducting similar studies for other subjects and other types of post-gymnasiums.

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Cele i zawartość matematycznego kursu e-learningowego przygotowującego uczniów do egzaminu maturalnego z matematyki

Streszczenie

Technologie informatyczne (technologie ICT) mogą pomóc w rozwiązaniu problemu związanego z kształtowaniem kompetencji matematycznych u uczniów i są wykorzystywane w procesie nauczania matematyki. Pierwsza część niniejszego artykułu przedstawia teoretyczne tło tej tematyki, m.in. opis kompetencji matematycznych i ich identyfikację w szkole ponadgimnazjalnej w Polsce, taksonomię Niemierki, teorię nauczania programowanego i strukturę systemu edukacji/nauczania w środowisku e-learningowym. Określa ona warunki wstępne, oczekiwane wyniki, koncepcje, cele, hipotezy i metody badawcze. Część praktyczna opisuje strukturę systemu nauczania indywidualnego, „Matematykę z Moodle", opartą na autorskim module *MatLearn*, oraz jego formę graficzną. Zaproponowane zostało narzędzie dydaktyczne – kurs e-learningowy przygotowujący uczniów do egzaminu maturalnego z matematyki i podnoszący kompetencje matematyczne uczniów. Jego celem jest podwyższenie poziomu kompetencji, w szczególności tych, które nie zostały jeszcze opanowane. W celu zbudowania czynności edukacyjnych w ramach kursu wykorzystano zasady nauczania programowanego i taksonomię Niemierki.

Słowa kluczowe: technologie teleinformatyczne (ICT), kluczowe kompetencje, kompetencje matematyczne, e-learning, taksonomia Niemierki, programowane nauczanie, moduł MatLearn, Matematyka z Moodle

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Цели и содержание электронного учебного курса Математика для подготовки учащихся к выпускным экзаменам по математике в школе

Анотация

Информационные и коммуникационные технологии (ИКТ) могут помочь решить проблемы в процессе обучения математике. Первая часть статьи представляет теоретические основы, в частности: описание математических компетенций и их идентификацию в старшей школе в Польше, таксономию Немерко, теорию программированного обучения и структуру системы образования / обучения в электронной среде обучения. Описаны предпосылки, ожидаемые результаты, концепции, цели, гипотезы и методы исследования. Практическая часть описывает структуру системы для индивидуального обучения математики с Moodle на основе оригинального авторского модуля MatLearn и его графическое представление. Был предложен дидактический инструмент - электронный курс подготовки учащихся к выпускным экзаменам по математике и совершенствование математических компетенций. Его цель состоит в том, чтобы повысить уровень компетенций, особенно тех, которые еще не были освоены. Для построения исследовательской деятельности в процессе, были использованы запрограммированные принципы обучения и таксономия Немерко.

К л ю ч е в ы е с л о в а: информационные и коммуникационные технологии (ИКТ), ключевые компетенции, математические компетенции, электронное обучение, таксономия Немерко, программированное обучение, модуль MatLearn, Математика с Moodle

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Objetivos y contenido del curso e-learning de matemáticas de preparación de los estudiantes para el examen final de la escuela de matemáticas

R e s u m e n

Las tecnologías de la información y la comunicación (TIC) pueden ayudar a resolver problemas y por ello se utilizan en el proceso de enseñanza de las matemáticas. La primera parte del trabajo presenta los antecedentes teóricos de estos aspectos, entre otros: la descripción de las competencias matemáticas y su identificación en la escuela superior de Polonia, la taxonomía de Niemierko, la teoría del aprendizaje programado y la estructura del sistema de educación / aprendizaje en entornos de e-learning. Expresa las condiciones previas, resultados esperados, conceptos, objetivos, hipótesis y métodos de investigación. La parte práctica describe la estructura del sistema para el aprendizaje individual de las matemáticas en Moodle basado en el módulo MatLearn original del propio autor y su representación gráfica. Se propone una herramienta didáctica - un curso e-learning de preparación de los estudiantes para el examen de graduación de matemáticas y de mejora de las competencias matemáticas de los estudiantes. Su objetivo es aumentar el nivel en las competencias, especialmente en aquellas que aún no han sido dominadas. Para elaborar las actividades de estudio del curso, se han utilizado los principios de aprendizaje programado y la taxonomía de Niemierko.

P a l a b r a s c l a v e: tecnologías de la información y la comunicación (TIC), competencias clave, competencias matemáticas, e-learning, taxonomía de Niemierko, aprendizaje programado, módulo MatLearn, matemáticas con Moodle