Studia Philosophiae Christianae
UKSW
49(2013)2

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CAUSATION AND INTENSIONALITY IN ARISTOTELIAN LOGIC

Abstract: We want to show that Aristotle's general conception of syllogism includes as its essential part the logical concept of necessity, which can be understood in a causal way. This logical conception of causality is more general then the conception of the causality in the Aristotelian theory of proof ("demonstrative syllogism"), which contains the causal account of knowledge and science outside formal logic. Aristotle's syllogistic is described in a purely intensional way, without recourse to a set-theoretical formal semantics. It is shown that the conclusion of a syllogism is justified by the accumulation of logical causes applied during the reasoning process. It is also indicated that logical principles as well as the logical concept of causality have a fundamental ontological role in Aristotle's "first philosophy".

Keywords: Aristotle, syllogism, proof, necessity, cause, intension

1. INTRODUCTION

It is a well know fact in the history of Aristotelian logic that Aristotelian proof (*apodeixis*, "demonstrative syllogism"), as described in Aristotle's *Posterior Analytics*[1], is a sort of deductive causal reasoning,

[1] See Aristotle, *Analytica Priora et Posteriora*, eds. D. Ross, L. Minio-Paluello, Oxford University Press, Oxford 1964; Aristotle, *Posterior Analytics*, transl. by J. Barnes, 2nd. ed., Oxford University Press 2002; Aristotle, *Posterior Analytics*, transl. by G.R.G. Mure, The Internet Classics Archive, 1994–2000, http://classics.mit.edu//Aristotle/posterior.html. (Hereafter cited as *An.Post.*)

where, starting from causes, we derive their effect: "the premisses (…) must be true, primary, immediate, better known than and prior to the conclusion, and they must be causes of the conclusion".[2] Let us recall that in a proof, according to Aristotle, predication is essential, i.e. holds of something in itself (and not accidentally): "scientific demonstrations are concerned with what holds of things in themselves (...) and proceed from such items"[3], in distinction to the syllogism about the accidental. More precisely, in the premisses "the middle term must hold of the third term, and the first of the middle, because of itself".[4] Aristotle emphasizes that the conclusion is not necessary and not about a cause (reason, *to dioti*) if it does not hold of things in themselves.[5] Aristotle also speaks of a syllogism where premisses are not causes, having in mind syllogisms that are not proofs.[6]

However syllogism as a whole also possesses a sort of necessity independently of whether each premiss in itself and the conclusion in itself hold of necessity. "A syllogism is an argument (*logos*) in which, certain things being posited, something other than what was laid down results by necessity because these things are so".[7]

We will argue that, according to Aristotle's account of syllogism, a sort of causality is contained in each syllogism (demonstrative or not). Namely, that something (conclusion) *results* (*symbainei*) *by necessity* from *something else* (premisses) because of this "something

[2] "(…) ex alēthōn (…) kai prōtōn kai amesōn kai gnōrimōterōn kai proterōn kai aitiōn tou symperasmatos." Aristotle, *An.Post*. A 2, 71b 20–22.

[3] "(…) peri tōn kath' hauta hyparchontōn (…) kai ek tōn toioutōn." Aristotle, *An.Post.* A 6, 75a 29–31. On "in themselves" see A 4, 73a 34 ff.

[4] "(…) di' hauto (…) dei kai to meson tō tritō kai to prōton tō mesō hyparchein." Aristotle, *An.Post*. A 6, 75a 35–37.

[5] Aristotle, *An. Post*. A 6, 75a 31–35, 18–20.

[6] Aristotle, *An.Post*. A 2, 71b 20–26, 29–31. We understand that Aristotle has here in mind *real* causes, not causes in an abstract, merely formal sense.

[7] "Syllogisms de esti logos en hō tethentōn tinōn heteron ti tōn keimenōn ex anagkēs symbainei tō tauta einai." Aristotle, *Prior Analytics, Book 1*, transl. by G. Striker, Oxford University Press 2010, A 1, 24b 18–20; Aristotle, *Prior Analytics*, transl. by A. J. Jenkins, The Internet Classics Archive 1994–2000, http://classics.mit.edu//Aristotle/prior.html. (Hereafter cited as *An.Pr.*). See also Aristotle, *Analytica Priora et Posteriora*, op. cit.

else" seems to say that the conclusion of a syllogism is effectuated by the premisses of the syllogism.[8] Moreover, a syllogism should contain a full account of the causation of the conclusion by the premisses. This is indicated by Aristotle's interpretation of the meaning of his expression "because these things are so" (*tō tauta einai*) as "resulting through these" (*to dia tauta symbainein*), and further, as "no term is required from outside for the necessity to come about" *(to mēdenos exōthen horou prosdein pros to genesthai to anagkaion)*.[9]

Although we will focus on the concept of cause, it seems that other concepts of Aristotle's "first philosophy", too, like substance, accident, relation, time, space, are also, in a way, formally preconceived within Aristotle's theory of syllogism. In a sense (with all differences to Kant), we can even speak of something like Aristotelian "metaphysical deduction of categories" from their corresponding logical forms. In case of Aristotelian causality it would mean that there is one and the same necessity which yields a conclusion from its premisses in any valid syllogism, and which yields an effect from its causes in the world given outside our reasoning.[10] In addition, the interrelationship of Aristotelian logic with the concept of causality may also serve as an introductory and partial test for Gödel's idea about the "fundamental" role of the concept of causality in logic (and in philosophy in general).[11]

2. ARISTOTLE'S INTENSIONAL SEMANTICS

Instead of a proof-theoretical or extensional model-theoretical account of Aristotle's syllogistic, we want to emulate what can be ex-

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[8] Cf. also the expression for a causation in Aristotle, *An.Post.* B 11, 94a 21–22: "tinōn ontōn anagkē tout' einai."

[9] Aristotle, *An.Pr*. A 1, 24b 20–22.

[10] For Kant's formulation of the idea of the "metaphysical deduction of categories" see in I. Kant, *Kritik der reinen Vernunft*, *2. Aufl.*, W. de Gruyter, Berlin 1968 (*Kant's Werke*, 3), B 105. According to Kant, the same "function" that gives a unity to representations in a judgment, also gives a unity to representations in an intuition.

[11] See K. Gödel, *Collected Works*, eds. S. Feferman et al., vol. 3, Oxford University Press, Oxford 1995, 433–434.

plicitly found in Aristotle as a semantic account of logic. To that end we connect Aristotle's syllogistic (as described in *Prior* and *Posterior Analytics)* with the foundations of logic as explicated especially in *Metaphysics* Γ.[12] We restrict ourselves to Aristotle's assertoric (non-modal) logic of quantified propositions "all B are A" (*a*), "no B are A" (*e*), "some B are A" (*i*) and "not all B are A" (*o*) and of corresponding syllogisms inasmuch they can be reduced to the first figure syllogistic moods.[13] As object language we will use (1) the vocabulary with *A*, *B*, *C* etc. as basic terms, (2) four operators *a*, *e*, *i* and *o*, (3) compound terms (not-A, AB), and (4) sentences of the form A*a*B, A*e*B, A*i*B and A*o*B (where A and B are terms). Non-quantified sentences ('Horse is animal') are not part of the object language, but have a significant role in the semantic metalanguage.

The distinction extensional/intensional is crucial for Aristotle's causal account of proofs. For example, although "being near" and "not twinkling" are convertible (*antistrephonta*) and extensionally equivalent terms, "being near" is the cause (middle term in a proof proper, i.e. causal proof, *apodeixis tou dioti*, *demonstratio propter quid*) of "not twinkling" of planets, while "not twinkling" is not the cause of "being near" of planets.[14] At the same time, "not twinkling" is more evident than "being near" and thus more apt for the evidential inference ("proof of the fact", *apodeixis tou hoti*, *demonstratio quia*) that planets are near.

Primitives (Belongs to, Attribute, Subject). Instead of set and membership as primitive (metatheoretical) notions, we encounter in Aristotle *belonging* (*hyparchein*, attribution), *attribute* (hyparchon) and *subject* (hypokeimenon) of attribution as primitives, by means of which he intensionally defines the meaning of all his logical notions. Aristotle

[12] Aristotle, *Metaphysica*, ed. W. Jaeger, Oxford University Press, Oxford 1973.

[13] For Hilbert-style formalization of Aristotle's syllogistic, see J. Łukasiewicz, *Aristotle's Syllogistic: from the Standpoint of Modern Formal Logic*, 2nd ed., Oxford University Press, Oxford 1957. For natural deduction and corresponding extensional semantics, see J. Corcoran, *Completeness of an ancient logic*, Journal of Symbolic Logic 37(1972), 696–702 and Idem, *Aristotle's natural deduction system*, in: *Ancient Logic and its Modern Interpretations*, ed. J. Corcoran, Reidel, Dordrecht 1974, 85–131.

[14] See Aristotle, *An.Post.* A 13 and A 6, 75a 31–37.

says that A belongs to B, which is in ordinary language usually expressed by saying that B is A. Here, A is an attribute and B the subject of attribution. (Notice that, unlike Corcoran, we did no exclude the case that in quantified sentences (A*a*B etc.) A is the same as B[15]. If A belongs to B, we also say, like Aristotle, that B *is an* A and that B *can be taken* as an A.

Attribution of A to B taken semantically, as the *meaning* associated to the expression that B is A, is predication:

Definition (Predication):

A is predicated of B if and only if A *is expressed* to belong to B. We say that A is the predicate (*katēgoroumenon*) and B the subject of the predication (*to kath' ou katēgoreitai*, what is predicated of).

We can distinguish predication to subjects (1) *as of themselves* (essentially, and therefore necessarily; for instance, horse is in itself an animal) and (2) *accidentally*, and thus at this or that moment of time, in this or that case, in this or that respect (for instance, horse is accidentally black).[16]

Following and generalizing Aristotle's analysis of demonstrative science, we define an intensional *frame* of reasoning (inferring):

Definition (Intensional Frame):

An intensional frame is an interpreted structure consisting of the following items:

(1) "about what" (*peri ho*, domain) we are reasoning, i.e. what is assumed to be: genus (*genos*, general subject) ,

(2) "from what" (*ex hōn*) we are inferring: the first principles,

(3) "what" (*ha*) do we infer: meaning of the basic terms – basic attributes (properties, *pathē*).[17]

Ad (1). Genus is a *whole* (*holon*, not a set) comprising species (*eidē*) as its parts (merē). It is a whole in the distributive sense of One (each of the Many is One), not in the collective sense (One consisting of

[15] See J. Corcoran, *Aristotle's natural deduction system*, op. cit., 99; Idem, *Completeness of an ancient logic*, op. cit., 696.

[16] See e.g. Aristotle, *An.Post.* A 4, 73a 34–73b 16.

[17] Cf. Aristotle, *An.Post.* A 10, 76b 11–22.

many only as taken together).[18] We take that species of species is also a species of genus, and that there are lowest species (*species infimae*). Lowest species belong to (i.e. are predicated of) individuals ("primary substances"), which do not belong to anything. In this sense we say that individuals are included by the genus in the domain, although not as parts of the genus itself.

Ad (2). There are two fundamental principles of "belonging", i.e. of predication: the principle of non-contradiction (NC) and the principle of excluded middle (EM), which also determine the negation (the meaning of 'not'):

NC It is impossible that A belongs and does not belong to B at the same time and in the same respect.

EM Necessarily, A belongs to B, or A does not belong to B.

EM follows from NC if we assume a contradictory sense of negation. That is, according to NC, at least one part of contradiction should be denied, which means that either it is not so that A belongs to B, or it is not so that A does not belong to B. If we understand negation in the contradictory sense, EM follows.[19]

According to these principles it follows for an arbitrary attribute and an arbitrary subject that the attribute belongs or does not belong to the subject and not both.

Ad (3). The third element of an intensional frame are *basic attributes* that belong to the genus. For example, if a species of stone is blue, stone is blue (non-quantified), and therefore "blue" is an attribute in the model with the domain "stone".

Example (Arithmetical frame):

Genus: number (units). Principles: arithmetical axioms (e.g. implicit principle "take equals from equals and equals remain" as applied to numbers). Attributes: even, odd, square, cube, commensurable, equal, greater, lesser.[20]

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[18] See Aristotle, *Metaph.* Δ 26, 1023b 28–33.

[19] Cf. Aristotle, *Metaph.* Γ 4, 1008a 2–7.

[20] See Aristotle, *Metaph.* Γ 2, 1004b 10–13; *An. Post.* A 10, 76b 6–8.

Subject A can be quantified so that we can speak of "all", "some", or "none" of A. In a quantified categorical sentence (*a*, *e*, *i*, *o*) the predicate is predicated (or not predicated) to all or to some parts of the subject of predication.

To get an intensional model based on an intensional frame we should add an *interpretation*.

Definition (Interpretation):

An interpretation is a non-ambiguous determination of the meaning of each basic term. The meaning of a basic term is an attribute of the frame. The determination of meaning is based on a natural language and its ordinary or scientific use. The meaning of a basic term, say, '*A*' (representing a term of the natural language) will be denoted simply by '*A*', i.e. '*A*' means *A*, since the meaning of terms, as determined by natural language, is understood in metalanguage.

Definition (Truth of a quantified categorical sentence):

(1) A*a*B (A belongs to each B) is true iff no B can be taken to which A is not predicated,

(2) A*e*B (A belongs to no B) is true iff no B can be taken (if any) to which A is predicated,

(3) A*i*B (A belongs to some B) is true iff AeB is not true, i.e. a B can be taken to which A is predicated,

(4) A*o*B (A does not belong to some B) is true iff AaB is not true, i.e. a B can be taken, if any, to which A is not predicated).[21]

Remark. Aristotle explains the truth of A*a*B also by B being *in* A as in a whole, and this is further explained by (1) of the definition above.[22]

Definition (Compound attribute):

(a) Not-A is the whole of attributes X such that A does not belong to X,

(b) AB is the whole of attributes X such that A belongs to X and B belongs to X.

Notice that, if there is no (part of) B to be taken, A cannot be predicated. I.e., if there is no subject of predication, there is no predication

[21] See Aristotle, *An.Pr.* A 1, 24a 18–19, 24b 28-30, cf. also *An.Post*. A 4, 73a 28–29.

[22] See Aristotle, *An.Pr.* A 1, 24b 26–30.

either. This is a special case of (2). It denies condition (1), which requires that there are B that can be taken, in order to confirm that the predication relation obtains between A and B (i.e. that it is not so that non-predication of A obtains). In addition, notice that according to case (4) A*o*B is true also if there is no B.[23]

We have already mentioned essential predication – where predication holds of a subject as of itself, which is a special intensional feature of Aristotelian semantics. In distinction to set theory, it is not so that if B and C belong to the same things, then B = C. If A essentially belongs to each B (and only accidentally to each C), and if B essentially belongs to each D (and C only accidentally to each D), it follows that A essentially belongs to D because of B, not because of C, which means that B is the *cause* (essentially, not only logically) of A*a*D. Whereas in syllogisms in general an essential as well as an accidental predication may be indistinguishably included, in proofs proper (*demonstratio propter quid*) only essential predication is allowed. Note that the essential predication implies the essential (not merely accidental) existence of things to which predication refers. (See the remark on "convertibles" in the second paragraph of section 2 above).

On the other hand, an accidental predication (for example, according to a semantic convention) in general syllogistic allows merely accidental existence of objects to which predication refers (subject), as well as merely accidental truth. An example is the proposition that hornstage (*tragelaphos*) is horned, where we assume that hornstage exists merely inasmuch as the copula 'is' ('is horned') is used to describe what is meant by 'tragelaphos'. Another example is the sentence 'Homer is a poet', according to which Homer exists inasmuch as he is a poet, although he is in fact dead (i.e. according to Aristotle, non-existing, non-substance) at the time at which the sentence is spoken.[24]

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[23] On A*o*B, see T. Parsons, *The traditional square of opposition*, in: *The Stanford Encyclopedia of Philosophy* (Fall 2012 Edition), ed. E. Zalta, http://plato.stanford.edu/archives/fall2012/entries/square, and G. Boger, *Aristotle's underlying logic*, in: *Handbook of the History of Logic*, eds. D.M. Gabbay, J. Woods, vol. 1, Elsevier, Amsterdam etc. 2004, 101–246.

[24] Aristotle, *De Interpretatione* 11, 21a 25–28, in Aristotle, *Categoriae et Liber de Interpretatione*, ed. L. Minio-Paluello, Oxford University Press, Oxord 1949.

The definition of the truth of categorical propositions is closely connected with the principle of *ecthesis* (exposition), which is explicitly used in some places of *Prior Analytics*, and is implicitly present in the justification of all logical rules of *Prior Analytics* mentioned above. Ecthesis: If there is an A, then C may be taken as an A, where C did not occur in the previous reasoning.

By means of ecthesis and the definition of the truth of quantified categorical sentences the whole assertoric "working logic" of quantified sentences (with the first figure inferences, *e*-, *a*- and *i*- conversion, and indirect inference as rules) can be derived form the principles of non-contradiction and of excluded middle as they are stated in *Metaphysics* Γ. Aristotle himself indicates how this can be done in the case of conversion rules,[25] and refers to the definition of truth of quantified categorical sentences[26] when dealing with syllogisms of the first figure, too.[27] Besides, ecthesis is mentioned, for example, in the justifications of the third figure modi *Darapti*, *Disamis*, *Datisi* and *Bocardo.*[28] We note that in formalizations, ecthesis is not usually conceived of as a foundational means for the justification of "working logic" (including all figures of assertoric syllogism) but as an alternative ("extra-systematic", Corcoran[29]) or systematic (Żarnecka-Biały) means for the reduction of all figures to the first one. However, let us also mention that, in distinction, the conception of the foundational role of ecthesis has some similarities to the "Buridan project" of the reduction of valid syllogisms to "expository syllogism" (with a singular middle term).[30]

[25] Aristotle, *An.Pr.* A 2, 25a 14–17.

[26] Aristotle, *An.Pr.* A 1, 24b 26–30.

[27] Aristotle, *An.Pr.* A 4, 25b 39-40, 26a 24, 27.

[28] See J. Łukasiewicz, op. cit., 59ff; E. Żarnecka-Biały, *Aristotle's proofs by ecthesis*, Bulletin of the Section of Logic 22(1993)1, 40–44.

[29] J. Łukasiewicz, op. cit., 128 ftn. 20.

[30] See F. Rombout, *Buridan project: how to reduce all valid syllogisms to the third figure*, preprint no. 0613835, http://www.academia.edu/1286302/Buridan_on_Expository_Syllogism_How_to_reduce_all_valid_syllogisms_to_the_3rd_figure.

Definition (Valid inference):
We say that the inference from $p_1, \ldots, p_n$ to q is valid if and only if the truth of q can be justified in each intensional model where $p_1, \ldots, p_n$ are true.

Here is the proof of the intensional validity of the conversion of A*e*B to B*e*A on the grounds of the ecthesis and NC, where we follow Aristotle's line of thought (from *An. Pr.* A 2) but elaborate upon it on the explicit level of truth conditions:

1. No B can be taken to which A is predicated	assumption, truth of AeB
2. Let an A can be taken to which B is predicated	added assumption
3. C is a B, to which A is predicated	ecthesis
4. Impossibly: A is predicated to C and A is not predicated to C	NC
5. It is not so that no B can be taken to which A is predicated	from 3, 4; ecthesis (3) discharged
6. No A can be taken to which B is predicated	ass. 2 cannot be added (since it contradicts 1)

In the justification of line 6 it is said that that assumption 2 itself contradicts 1 because lines 3–6 are nothing else but an analysis of 2 by ecthesis. As Aristotle indicates, *e* conversion is used in proving conversions of the propositions *a* and *i*.[31]

As an example of the foundational use of ecthesis, NC and EM in syllogism, we prove the intensional validity of syllogism Barbara from NC and EM:

1. No M can be taken to which P is not predicated	major, truth of PaM
2. No S can be taken to which M is not predicated	minor, truth of MaS
3. A is taken as an arbitrary S	ecthesis

[31] Aristotle, *An.Pr.* A 2, 25a 17–22.

4. It is not so that M is not predicated to A	from 2, 3
5. Impossibly: M is not predicated to A and it is not so that M is not predicated to A	NC
6. M is predicated to A or M is not predicated to A	EM
7. M is predicated to A (i.e. A is an M)	from 4, 5, 6
8. It is not so that P is not predicated to A	from 1, 7
9. no S can be taken to which P is not predicated	since A is an arbitrary S, ecthesis (3) discharged

Similarly, the other first figure modes can be proved from NC and EM by ecthesis. In the indirect proof, if sentence *p* contradicts the premisses, the contradictory of *p* must be true if the premisses are, on the grounds of NC and EM.

3. SYLLOGISM AND CAUSATION

Aristotle's definition of a syllogism from the beginning of *Prior Analytics*, quoted at the beginning of this paper, explicitly contains the concept of necessity, which we interpret as a logical (formal) causality since, according to Aristotle, in a valid syllogism the conclusion results *because* the premisses hold. Not only in demonstrative syllogisms, for which Aristotle explicitly states that premisses are causes of the conclusion, but even in the case where premisses express only accidental states of affairs there are still logical necessity and logical causality yielding the conclusion.[32]

Hence, in accordance with Aristotle, the whole syllogism can be conceived of as being in the scope of a causal necessity. In this sense

[32] Cf. "[aitia] hōs to symbebēkos." Aristotle, *Metaph.* Δ 2, 1013b 34–35.

it is justified to represent syllogisms as axioms or theorems, like in Łukasiewicz's interpretation of Aristotle's syllogistic, and prefix them with the necessity operator to express the necessity of the inference itself, for example:

$\Box$(if P*a*M and M*a*S, then P*a*S).

On the ground of modal reasoning it follows that

if $\Box$ P*a*M and $\Box$ M*a*S, then $\Box$ P*a*S,

from which we obtain a form of demonstrative syllogism: $\Box$ P*a*M, $\Box$ M*a*S $\vdash$ $\Box$ P*a*S.[33] This would, in a causal interpretation, have the following meaning: if P*a*M holds for some cause and if M*aS* holds for some cause, then, consequently, P*a*S holds for some cause, too.

General features of Aristotle's syllogism, especially its structural rules, additionally confirm that this syllogism is conceived (at least formally) in a causal way. We give a few examples. It is obvious that syllogistic reasoning, like causality (taken in an ordinary sense), does not include (a) reflexivity ($p \vdash p$) nor (b) monotonicity (if $\Gamma \vdash p$ then $\Gamma, \Delta \vdash p$)[34], but (c) includes transitivity ($\Gamma \vdash p$ & $\Delta, p \vdash q \Rightarrow \Gamma, \Delta \vdash q$).

(a) Non-reflexivity is related to the impossibility of self-causation: self-causation implies that one and the same thing or event should exist before it exists, or exist independently of its own existence, to cause its own existence, which is impossible.[35] We can only improperly say that a thing causes itself, for example in the way that one of its properties causes some of its other properties. Correspondingly, according to Aristotle's definition of the syllogism, and according to the form of singular syllogistic moods, it is obvious that the conclusion should be a *new* proposition, at least formally: subject and the predicate are di-

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[33] J. Barnes, *Introduction*, in: Aristotle, *Posterior Analytics*, transl. by J. Barnes, 2nd ed., Oxford University Press, Oxford 2002, xvii–xviii.

[34] In distinction, for instance, to a formalized sense that can be given to the concept of causality by means of justification logic, where causation could be monotonic. See S. Kovač, *Modal collapse in Gödel's ontological proof*, in: *Ontological Proofs Today*, ed. M. Szatkowski, Ontos, Frankfurt 2013.

[35] Cf. "to gar aition proteron hou aition." Aristotle, *An.Post.* B 16, 98b 17.

rectly connected for the first time only in the conclusion. It may happen that they are already materially connected in premisses, but in this case one of them occurs formally as the middle term, like in the following example: A*a*A, A*a*B / A*a*B, where A of the conclusion is predicate, and A of the minor premiss is middle term. What is applied in this example is not the reiteration rule, but the syllogistic mood *Barbara*, containing the formal causation in the inference (AaB in one form is a cause of A*a*B in another form). Also, for example, during the reduction to the first figure, we may invoke an already assumed premiss, but not as a syllogistic conclusion. Non-reflexivity is especially obvious in proofs (demonstrative syllogisms) where premisses should be "prior" and "better known" than the conclusion, and therefore the conclusion cannot be only a reiterated premiss. It follows also that nothing can be concluded from only one premiss[36]: an "immediate consequence" (by conversion) is only an analysis of one and the same proposition, that is, only an analysis of one and the same fact, without its causal relationship.

(b) Regarding non-monotonicity, we compare it with the addition of redundant or even irrelevant factors to the factors that already suffice to cause an event. We would not include these redundant factors as parts of the cause. Correspondingly, the addition to an already valid syllogism of new, redundant, premisses, which are not used in the syllogism, does not yield a (valid) syllogism.[37]

(c) Transitivity, which we easily recognize in Aristotelian polysyllogism, and which can be reduced to sorites by means of omitting intermediate conclusions, also uncover a causal sense of syllogistic reasoning, since causality in general is often conceived of as transitive. We do not enter here into the discussion of counterexamples to transitivity adduced in contemporary literature. We merely note that in such coun-

[36] See Aristotle, *An.Post.* B 11, 94a 24–25.

[37] Notice the distinction to a stronger proposition, rejected by Corcoran, that no deduction contains redundant premisses (and that in judging deductions an independence proof of premisses would be required). See J. Corcoran, *Aristotle's demonstrative logic*, History and Philosophy of Logic 30(2009)1–20, 4, incl. ftn. 9.

terexamples "cause" is usually taken elliptically or not each time in the same sense.[38]

(d) Additionally to the already mentioned features there is an interesting and disputed feature of Aristotelian logic, the rejection of non-p $\models$ p (*An. Pr.* B 4, 57b 3–17), which in some cases can be valid in classical propositional logic. Aristotle analyzes the relation of truth of the premisses of a syllogism and the truth of the conclusion as a special case of a consequence relation, which we denote as $\models$ where p $\models$ q means: if p is true then necessarily q is true, and where p expresses the middle term and q is a conclusion.[39] Aside from Łukasiewicz[40] and Patzig[41], who argue that non-p $\not\models$ p is an error on Aristotle's side, and aside from connexive logics, which formally incorporate non-p $\not\models$ p,[42] Aristotle's non-p $\not\models$ p can be seen as a further confirmation of a formal causal sense required for Aristotelian consequence relation in each syllogism: absence of fact p cannot be as such a cause of p.

Further, on the basis of this rejection, Aristotle argues that q could impossibly be a consequence both of p and of non-p:[43]

non-p $\models$ q

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[38] For the contemporary discussion about properties of causation (including backward causation, transitivity, as well as preemption cases and the counter-factual account of causality), see, for example, two articles by N. Hall, *Causation and the price of transitivity*, in: *Causation and Counterfactuals*, eds. J. Collins, N. Hall, L.A. Paul, MIT Press, Cambridge (Mass.) etc. 2004, 181–203; Idem, *Two concepts of causation*, in: *Causation and Counterfactuals*, eds. J. Collins et al., MIT Press, Cambridge (Mass.) etc. 2004, 225–276.

[39] Cf. the following Aristotle's example: "Why is the moon eclipsed? Because the light leaves it when the earth screens it", where the second sentence is meant to give the cause as the middle term. Aristotle, *An.Post.* A 2, 17–19.

[40] J. Łukasiewicz, op. cit., 49–50.

[41] G. Patzig, *Die aristotelische Syllogistik*, 3. ed., Vandenhoeck & Ruprecht, Göttingen 1969, 205.

[42] See S. McCall, *Connexive implication*, Journal of Symbolic Logic 31(1966), 415–433; H. Wansing, *Connexive modal logic*, in: *Advances in Modal Logic*, vol. 5, eds. R. Schmidt et al., College Publications, London 2005, 367–383.

[43] See Aristotle, *An.Pr.* B 4, 57b 15–17. Also G. Patzig, op. cit., 205–206.

$$\frac{\text{non-q} \models \text{p} \qquad \text{p} \models \text{q}}{\text{non-q} \models \text{q}}$$

Patzig gives an example which he means should confirm that non-p $\models$ q and p $\models$ q make a good sense in ordinary discourse: a situation may occur where a patient could die (q) in case (s)he undergoes a surgery (p) as well as in the case (s)he does not undergo a surgery (non-p). However, this example shows precisely that, in this case, surgery stands in no causal connection with the death of the patient, and that because of this we cannot have q here as a consequence either from non-p or from p.

4. CAUSES OF A SYLLOGISM

Let us take a closer look at the causal structure of a syllogism. On the ground of Aristotle's distinction of four types of causes, we propose the following causal structure of a syllogism: (1) the premisses of a syllogism are the material cause of the conclusion[44], (2) the figure (*schēma*), i.e. the position of the middle term is the formal cause of the conclusion (*eidos*, *paradeigma*), (3) ecthesis is the efficient cause that first, after premisses and figure are given, sets the syllogistic reasoning in motion (see the example of *Barbara* above), (4) the conclusion is the final cause (*ergon*) of the premisses (*organa*).[45] Notice that the mood of a syllogism is determined by the material and formal cause taken together as a whole (*synolon*), since the premisses (material cause) contain the quality and quantity (of propositions), which together with the form (figure) give the mood of a syllogism. The principles of non-contradiction (NC) and excluded middle (EM) should also be counted to the premisses, although they are in syllogisms, in practice, used al-

[44] Explicitly confirmed by Aristotle, e.g. *Metaph.* Δ 2, 1013b 20–21.

[45] For the different meanings of 'cause' we have applied, see Aristotle, *Metaph.* Δ 2. For the middle as the cause of a syllogism cf. B. Despot, *Logički fragmenti*, CKD, Zagreb 1977, 38–41.

most always only implicitly.[46] At the same time, they formally restrict (simplify) the behaviour of terms in such a way (as in the example above) that it becomes possible to come to a conclusion from a given (valid) figure in a certain syllogistic mood. Hence, NC and EM should in a foundational analysis count to the material as well as to the formal causes of a syllogism. We remark that the inclusion of the foundational principles NC and EM in Aristotle's "first philosophy" confirms his commitment to the ontological meaning of logical "forms" of reasoning. Although ecthesis seems to have in Aristotle only auxiliary, didactic role, it in fact enables the application of NC and EM to the defined truth conditions (essence) of premisses. Note that essence (*ti esti*) is a sort of formal cause for Aristotle.

Let us come back to the example of *Barbara* and find what are the causes needed for the conclusion and the whole syllogism to be finally established (we extend the analysis of *Barbara* given above with the indications of causes and their accumulation as it will be explained below). The essence of proposition p is denoted by 'Ess(p)' and the ecthesis of A from B by 'ecth(B,A)'.

1. P*a*M	given proposition
2. M*a*S	given proposition
3. no M can be taken to which P is not predicated	Ess(P*aM*)
4. no S can be taken to which M is not predicated	Ess(M*aS*)
5. A is taken as an arbitrary S	ecth(S,A)
6. It is not so that M is not predicated to A	by Ess(MaS), ecth(S,A)
7. Impossibly: M is not predicated to A and it is not so that M is not predicated to A	NC, principle

[46] For exceptions see Aristotle, *An.Post.* A 12.

8. M is predicated to A or M is not predicated to A	EM, principle
9. M is predicated to A (i.e. A is an M)	by (NC, EM), (Ess(MaS), ecth(S,A))
10. It is not so that P is not predicated to A	by PaM, ((NC, EM), (Ess(MaS), ecth(S,A)))
11. no S can be taken to which P is not predicated	by ecth(S,A), (Ess(PaM), ((NC, EM), (Ess(MaS), ecth(S,A))))
12. PaS	11 is Ess(PaS)

We start from the causal sense of the premisses, first simply as something given ("material cause"), which, as a whole, reveal a form, i.e., syllogistic figure: P*a*M, M*a*S (with the middle position of the middle term). With the premisses given, we have also their truth conditions (essences) satisfied (lines 3, 4). Now it is possible to enter into the essence of the premisses by ecthesis ("take an instance of a term) and to activate this essential causal structure (line 5) in order to arrive at the conclusion (final cause), i.e. to causally establish the connection of S and P. The reasoning process begins by the application of the ecthesis of an S to the essence of M*a*S. The effect is the proposition that it is not so that M does not belong to A (line 6). Then the first logical principles are invoked (lines 7, 8), which results in the proposition that M belongs to A (line 9). All this activated causality, together with the essence of P*a*M (line 10), gives the proposition that it is not so that P does not belong to A (line 10). With the ecthesis discharged the generality results and the essence of P*a*S is established (line 11). The resulting proposition P*a*S simplifies the structure given initially by the two premisses.

Hence the cause of the conclusion in *Barbara* is indicated in the causal justification of line 11 by causal accumulation ecth(S,A), (Ess(P*a*M), ((NC, EM), (Ess(M*a*S), ecth(S,A)))). The cause of the whole syllogism as such is the "resulting" itself of the cause of the conclusion from the given premisses, which can be denoted as (P*a*M, P*a*A); c, where c is short for the complex expression given above for the cause of the conclusion, and where everything before ";" has a hypothetical sense.[47]

5. A FINAL REMARK

We have already mentioned that Aristotle included NC and EM in his "first philosophy", not restricting what would be later called logic, only to his analytics. Even more, NC and EM, as necessary in themselves, can, in a way, be conceived of as incorporated in the "unmoved mover", God of Aristotle's *Metaphysics* Λ. This can be seen from Aristotle's notion of God as the "simple", "actual" substance, where the simple (*to haploun*) is for Aristotle the necessary in itself: "the necessary in the primary and strict sense is the simple; for this does not admit of being in more than one way, so that it cannot be in this way and another".[48]

Since God is essentially simple, and as such cannot be "in more than one way", any contradiction in Him is impossible and any indeterminateness in Him should be excluded by His own essence. Since God is, according to Aristotle, the unmoved end (*telos*), which attracts all other being and thinking, it follows that God is also the last end (final cause) of whole logic and reasoning.[49]

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[47] On a formalized logic of justifications as proofs, in natural deduction format, see S. Artemov, E. Bonelli, *The intensional lambda calculus*, in: *Logical Foundations of Computer Science*, eds. S. Artemov, A. Nerode, Springer, Berlin 2007, 12–25; G. Primiero, *Intuitionistic logic of proofs with dependent proof terms*, Preprint Series of the Isaac Newton Institute for Mathematical Sciences, vol. NI12026-SAS, 2012, http://logica.ugent.be/centrum/preprints/dependent_evidence_newnot.pdf.

[48] Aristotle, *Metaph*, Γ 5, 1015b 11–13.

[49] An analogous result can be obtained also with respect to the Christian conception of the omnipotent God; see K. Świętorzecka, *Some remarks on formal description of God's omnipotence*, Logic and Logical Philosophy 20(2011), 307–315, as well as E. Nieznański, *Elements of modal theodicy*, Bulletin of the Section of Logic 37(2008), 253-264. In formal system TW by Świętorzecka, theorem $\neg W(\perp)$, 'God does not want a contradiction', holds. $\neg Cb\ (p \wedge \neg p)$, with the same meaning, is a theorem of a system by Nieznański, op. cit.). Since A/WA (A being a formula) is a rule of TW, it follows that God wants non-contradiction, $\perp$, as well as all logical axioms and theorems. Even more, according to the alternative system (section 3B in K. Świętorzecka, op. cit.), God does not want any lack of His omnipotence (obviously with respect to logic, too).

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