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INVESTORS' PREFERENCES AND PAYOFFS FROM STRUCTURED PRODUCTS

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Abstract: The attractiveness of structured products is mainly due to the fact that the diversity of their structures corresponds to the specific preferences of different groups of investors. To determine the appropriate investment product for an investor it is necessary to identify investor preferences, attitude toward risk. The paper contains an analysis of investors preferences in relation to the payoffs from the typical structured products. Investors' preferences are based both on the rational choice theory and the prospect theory.

Keywords: structured financial products, utility function, prospect theory

INTRODUCTION

Structured financial products have gained great popularity since the last decade of the 20th century. At the peak of the last global bull market of 2007, the estimated value of the investment in structured products exceeded 1000 billion EUR. The total volume of sales of these products in Poland in 2011 amounted to over 10 billion PLN. The attractiveness of structured products is mainly due to the fact that the diversity of their structures corresponds to the specific preferences of different investors. To determine the appropriate investment product for an investor it is necessary to identify the payoff from the product and the investor preferences, attitude toward risk and loss. The aim of this paper is to show that different investors' preferences may determine the choices of particular structured products and also indicate whether to invest in a given structured product or in an asset underlying this product.

After outlining theoretical background referring to the structured products, both the rational choice theory and the prospect theory, example structured

products are presented, expected utilities and certainty equivalents of their payoffs are then computed and finally concluding remarks are presented.

THEORETICAL BACKGROUND

Structured products are synthetic investment instruments[1]. They are tailor-made investments created to meet specific needs that cannot be met from the standardized financial instruments available in the markets generally. They are composed of several elements or component parts, each providing a specific exposure or protection for the investor. In general a structured product is constructed by combining a bond (typically a zero-coupon bond) and financial options on the underlying asset (single or many). There may be a broad scope of these underlying assets, the same as list of assets that options may be written for - particular stock, stock indexes, commodities, exchange rates etc. In most cases, these underlying assets and the derivatives markets which offer exposure to these assets are not easily accessible to individual investors.

The combination of a bond and a financial option results in the capital protection secured by the bond and possible return outcomes provided by the option. The investment is structured in the sense that the investor knows the package of underlying assets included in the product and the method of calculating gains and risk from investment in the specific product. This enables investors to establish a payoff from a structured product which shows a risk profile of such product based on different forms of capital protection: full, partial or conditional. The typical product has a payoff based on the performance of an underlying asset. The investor may benefit from its good performance and simultaneously may receive a minimum guaranteed level of the capital protection in the case of poor underlying returns. The cost of the guarantee is generally covered by modifying the payoff usually capping it at some level or reducing participation in underlying returns.

Structured products fall into three broad categories[2]:

- Capital Protection
- Yield Enhancement
- Participation

It is generally considered that capital protection products carry the least risk of the three and participation products carry the most risk.

Capital protected products offer full capital protection of the initial investment and others offer partial or conditional capital protection. Conditional capital protection is commonly linked to the performance of the underlying asset.

[1] For a broad review of structured products see Blumke A. (2011) Jak inwestować w produkty strukturyzowane, Wolters Kluwer Polska, Warszawa

[2] The terms used may depend on the issuer.

If specific conditions are met, for example if the price of the underlying asset falls below an agreed threshold during the investment period, the capital protection disappears and the investor may incur a loss at maturity.

Yield enhancement products offer no protection for the initial investment. The purpose is to generate a fixed return that is higher than a bond. Yield enhancement products may be desirable to investors when the market for the underlying is rather stable. The yield potential can be above this market, however the capital may be at risk

Participation products are very closely tied to the underlying assets. They offer leveraged upside potential or downside protection with no or only partial and conditional capital protection. Usually no coupon is paid on these instruments and a return at maturity is calculated by multiplying the performance of the underlying asset by a fixed percentage, called the participation rate.

Structured products can have features such as a barrier or multiple barriers. A barrier causes a certain feature of an instrument to come into effect once a predetermined condition is met. This is usually based on the price movement of the underlying asset.

Structured products have a strong consumer appeal since they combine upside potential with downside protection in a single instrument. As there is a large variety of structured products offered by financial institutions, virtually every investor may find the one which is suitable to her or his needs and preferences.

Utility functions enable to measure investor's preferences for wealth and the amount of risk they are willing to undertake in the hope of attaining greater wealth. In general a utility function widely used in economics as representative for a rational investor is a twice-differentiable function of wealth $U(W)$ which has the properties of non-satiation $(U'(W) > 0)$ and risk aversion $(U''(W) < 0)$. The principle of expected utility maximization states that a rational investor, when faced with a choice among a set of competing feasible investment alternatives, acts to select an investment which maximizes his expected utility of wealth.

In the paper four basic and popular utility functions are considered and a function which reflects behavioral finance theory which has an S-shape around a point of reference (the S-shaped utility function).

The important property of a utility function is an assumption about how the investor's preferences change with a change in wealth. The Arrow-Pratt measures of absolute (ARA) and relative (RRA) risk aversion may be applied to examine this behaviour.

$$ARA(W) = \frac{-U'(W)}{U(W)} \tag{1}$$

$$RRA(W) = W \cdot \frac{-U'(W)}{U(W)} = W \cdot ARA(W) \tag{2}$$

The list of considered utility functions with their properties is shown in Table 1.

Table 1. Functional forms of utility functions and their properties

Name	Form	Type of ARA	Type of RRA
Logarithmic	$\ln(W)$	Decreasing (DARA)	Constant (CRRA)
Power	W^{α}	Decreasing (DARA)	Constant (CRRA)
Exponential	$-e^{-\alpha \cdot W}$	Constant (CARA)	Decreasing (DRRA)
Quadratic	$W-\alpha \cdot W^2$	Increasing (IARA)	Increasing (IRRA)

Source: own elaboration

The applications of utility functions with CRRA are frequent supported by the classical studies by Friend and Blume (1975) and Pindyck (1988). Some studies also recognize the potential of DRRA utility function [Ogaki and Zhang 2001].

A number of studies indicate that behavioral factors such as loss aversion affect the investment decision of the individual investor. Prospect theory (PT) presented by Kahneman and Tversky (1979) and further extended into cumulative prospect theory (CPT) in 1992 [Tversky, Kahneman 1992] examines how people make decisions involving risk. Many later studies (see for example [Fischer 2007], [Doebeli and Vanini 2010], [Hens and Bachmann 2010]) show that the behavioral approach is useful in analyzing investor preferences with respect to investment in structured products.

The prospect theory assumes that the investor does not focus on absolute levels of final wealth W but rather view the gains and losses measured as from a certain reference point ΔW. The utility of gains and losses are evaluated separately. In the region of gains, the utility function (originally in PT called a value function) is concave whereas in the region of losses it is convex[3]. In other words, investors are risk averse over gains and risk seeking over losses. Moreover, an investor's value function is steeper for losses than for gains pointing that keeping the magnitude equal, losses hurt more than gains please.

The functional form of value function in CPT proposed by Tversky and Kahneman (1992) and used later in this paper is expressed as[4]:

$$v(\Delta W)=\begin{cases}(\Delta W)^{\alpha} & \text{if } \Delta W \geq 0\\ -\lambda \cdot (-\Delta W)^{\beta} & \text{if } \Delta W < 0\end{cases} \tag{3}$$

[3] This kind of function is also called the S-shaped utility function

[4] For the review of other functional forms for the value function as well as for decision weights described in the next paragraph proposed by other researchers see Glimcher P., Camerer C., Fehr E., Poldrack R. (2009) Neuroeconomics: Decision Making and the Brain, Elsevier Academic Press, London, 145-170

The parameters obtained by Tversky and Kahneman (1992) in their study were the following: $\alpha = 0{,}88,\ \beta = 0{,}88, \lambda = 2{,}25$ and λ is a coefficient of loss aversion.

In CPT, to obtain the value of a prospect (reported as CPT utility in the next section of the paper), the value of an outcome $v(\Delta W)$ is weighted not by its probability (as is the case in the utility theory) but by the decision weight instead. These weights reflect the fact that moderate to high probabilities are underweighted (which intensifies risk aversion for gains and risk-seeking for losses in this range) and low probabilities are overweighted (which reverses the attitude towards risk in this range of probabilities and leads to risk seeking for gains and risk aversion for losses).

The functional forms of decision weights with their originally estimated parameters proposed by Tversky and Kahneman (1992) and employed in the paper are as follows:

$$w^{+}(p) = \frac{p^{\gamma}}{(p^{\gamma} + (1-p)^{\gamma})^{\frac{1}{\gamma}}} \qquad w^{-}(p) = \frac{p^{\delta}}{(p^{\delta} + (1-p)^{\delta})^{\frac{1}{\delta}}} \qquad (4)$$

$$\gamma = 0{,}61,\ \delta = 0{,}69$$

DATA ANALYSIS AND RESULTS

Two structured products issued on Polish financial market and based on the Warsaw Stock Exchange Index WIG20 were analyzed. One of them is an example of a capital protected product with a barrier while the second is representative for the class of participation products.

Zyskuj z WIG20 - Strategia 100 (Profit with WIG20 – strategy 100) product

This product is a kind of a capital guaranteed product with a knock-out barrier. This structured product is an example of a Shark note. It is built using a zero-coupon bond plus an up-and-out call option. Such option has a barrier embedded, which, if breached, causes the option to "die" All participation accumulated once the barrier is breached is lost. Zyskuj z WIG20 product, like most Shark notes includes a rebate (6,5% in this case), which is a return for the investor if the barrier has been breached. The short description of this product is presented in Table 2 below[5].

[5] Full termsheet for this product is available at http://www.analizy.pl/fundusze/produkty-strukturyzowane/produkt/PSALR009/Zyskuj-z-WIG20--strategia-100.html . Accessed: 20/06/2013

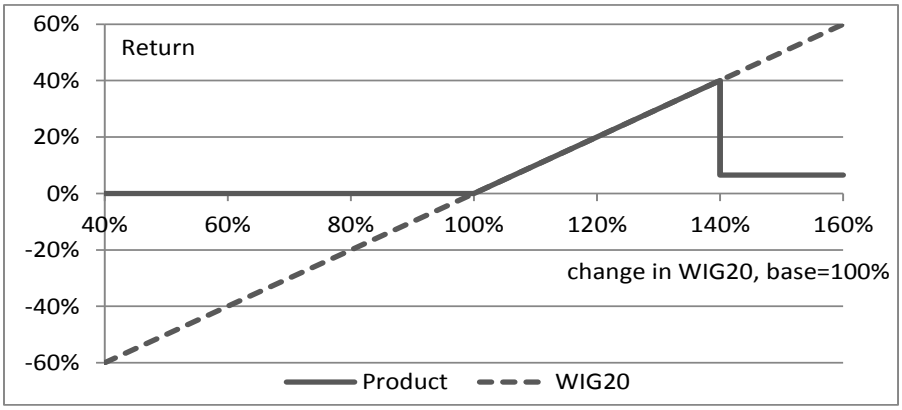
Table 2. Basic parameters for Zyskuj z WIG20 – Strategia 100 product

Issuer	Alior Bank SA
Currency	PLN
Unit certificate issue price	100 PLN
Subscription period	26/07/2010-20/08/2010
Initial fixing date	27/08/2010
Final fixing date	27/08/2012
Redemption date	03/09/2012
Underlying asset	WIG20 index
Two-year investment offering 100% capital protection on the Redemption Date. The investor participates in the 100% growth of WIG20 index (underlying asset) until the WIG20 reaches 140% (barrier) at any time of the investment compared to its value at the initial fixing date. If the barrier is reached the investor receives a coupon of 6,5%..	

Source: product termsheet

The payoff from Zyskuj z WIG20 – Strategia 100 product is illustrated in Figure 1.

Figure 1. Payoff from Zyskuj z WIG20 – Strategia 100 product



Source: own elaboration based on product termsheet

WIG20 Twin Win product

This product is an example from the class of participation products and attractive looking investment profiting both the rise of WIG20 index (unlimited) and fall (capped). The short description of this product is presented in Table 3[6].

[6] Full termsheet for this product is available at http://www.gpw.pl/info_produkty_strukturyzowane?isin=AT0000A10550&ph_tresc_glowna_start=show. Accessed: 20/06/2013

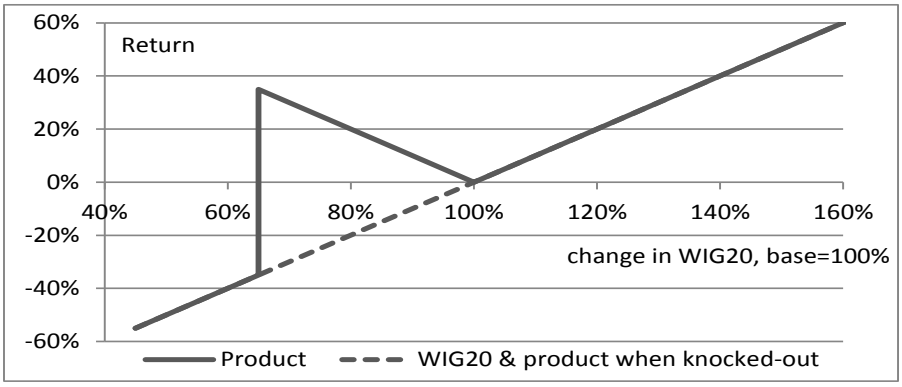
Table 3. Basic parameters for WIG20 Twin Win product

Issuer	Raiffeisen Centrobank AG
Currency	PLN
Unit certificate issue price	1000 PLN
Subscription period	15/04/2013-24/04/2013
Initial fixing date	26/04/2013
Final fixing date	27/10/2015
Redemption date	30/10/2015
Underlying asset	WIG20 index
Two and a half-year investment offering 100% participation in the increase of WIG20 index. The decrease of WIG20 is turned to profit 1:1 until the WIG20 falls 35% (barrier) at any time of the investment compared to its value at the initial fixing date. If the barrier is reached the product return tracks exactly movement in the WIG20 index.	

Source: product termsheet

The payoff from WIG20 Twin Win product is illustrated in Figure 2.

Figure 2. Payoff from WIG20 Twin Win product



Source: own elaboration based on product termsheet

The structure of such product is constructed by means of a long zero strike call option, and long two down-and-out put options, where the strike is set at-the-money. If the barrier is knocked-out the Twin-Win transforms itself in a certificate tracking the WIG20 index.

The simulations of returns for two described products were carried out. Note, that Zyskuj z WIG20 product has already expired[7] whereas WIG20 Twin Win is in course of investment period. The starting period of simulations was 02/01/2000 for both products. For Zyskuj z WIG20 there were 2170 simulations including 500 overlapping observations each (two-year investment period, the last simulation ended just before final subscription day 20/08/2010). For WIG20 Twin Win there

[7] Final value of one certificate paid at redemption date was 100 PLN (return 0%)

were 2715 simulations, each lasting for two and a half-year investment period, the last one ended before final subscription day 24/04/2013. Details about simulation returns are presented in Tables 4 and 5.

Table 4. Distribution of returns for WIG20 index and Zyskuj z WIG20 product

Range of return	Average return in the range for:		Probability
	WIG20	Product	
$R < 0\%$	-31,1%	0	0,438
$R \in \langle 0\%; 40\%)$	20,5%	20,5%	0,149
$R \geq 40\%$	65,6%	6,5%	0,413

Source: own calculations

Table 5. Distribution of returns for WIG20 index and WIG20 Twin Win product

Range of return	Average return in the range for:		Probability for:	
	WIG20	Product	WIG20	Product
$R \leq -35\%$	-42,8%	-41,9%	0,175	0,172
$R \in (-35\%; 0\%\rangle$	-17,9%	-25,5%	0,232	0,132
$R \geq 0\%$	57,1%	49,9%	0,593	0,696

Source: own calculations

Based on the distributions of returns, expected utilities and certainty equivalents were computed for the investments in WIG20 and both products[8]. The results are shown in Tables 6 and 7. For exponential and quadratic utility functions results for two different levels of risk aversion are reported whereas for CPT utility two different coefficients of loss aversion were examined.

Generally the results show that for the given sample, investments in both products as well as directly in WIG20 (asset underlying) could be considered attractive except for few cases with high level of risk or loss aversion. There is also not much difference in results regarding different functional forms of utility function and absolute and relative risk aversion. However there is a noticeable difference comparing investments in WIG20 and in particular product. Volatility of returns from Zyskaj z WIG20 is so low that most investors prefer direct investment in much riskyWIG20 index. The capital protection seems insufficient reward for potential profit lost. On the contrary WIG20 Twin Win is preferred in all cases to the direct investment in WIG20, but according to strong loss aversion this preference is weakest for investors exhibiting behavioral biases (for higher loss aversion both investments in WIG20 and WIG20 Twin are for them unattractive).

[8] For the purpose of calculations, final levels of wealth reflecting distributions of returns were scaled for quadratic and exponential utility functions. For CPT utility gains and loss were considered taking 100 (initial investment) as a reference point. Certainty equivalents in all cases are comparable to the initial investment of 100.

Table 6. Expected utilities and certainty equivalents for WIG20 index and Zyskuj z WIG20 product

Functional form	Expected utility for:		Certainty equivalent for:	
	WIG20	Product	WIG20	Product
Logarithmic	4,678	4,658	107,57	105,46
Power $\alpha = 0{,}5$	10,586	10,274	112,06	105,56
Exponential $\alpha = 0{,}5$	-0,572	-0,590	111,58	105,55
Exponential $\alpha = 2$	-0,139	-0,122	98,67	105,12
Quadratic $\alpha = 0{,}5$	0,109	0,100	115,06	105,38
Quadratic $\alpha = 2$	0,085	0,083	108,88	105,19
CPT $\lambda = 1{,}25$	5,355	4,336	106,73	105,30
CPT $\lambda = 2{,}25$	-3,196	4,336	98,51	105,30

Source: own calculations

Table 7. Expected utilities and certainty equivalents for WIG20 index and WIG20 Twin Win product

Functional form	Expected utility for:		Certainty equivalent for:	
	WIG20	Product	WIG20	Product
Logarithmic	4,729	4,755	113,24	116,12
Power $\alpha = 0{,}5$	10,858	10,972	117,19	120,38
Exponential $\alpha = 0{,}5$	-0,556	-0,548	117,48	120,08
Exponential $\alpha = 2$	-0,126	-0,118	103,37	106,60
Quadratic $\alpha = 0{,}5$	0,114	0,116	120,51	122,48
Quadratic $\alpha = 2$	0,089	0,090	114,69	117,57
CPT $\lambda = 1{,}25$	5,912	6,660	107,53	108,63
CPT $\lambda = 2{,}25$	-2,571	-1,276	98,84	99,48

Source: own calculations

CONCLUDING REMARKS

Structured financial products have gained more and more popularity in recent years, however due to their variety and complexity there is much to be done for the analysis of their expected utility for investors with different preferences. The results presented in this paper are only little contribution in this field. Many popular structured products use behavioral factors, like loss aversion. Hens and Rieger (2008) show that the currently most popular products cannot be explained even within the framework of prospect theory, but only when taking into account probability mis-estimation. This suggests the possible extensions of the studies.

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