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THE DEVELOPMENT OF ALTERNATIVE STRATEGIES FOR THE ISSUERS OF UNIT-LINKED LIFE INSURANCE CONTRACTS WITH INVESTMENT GUARANTEES

(Summary)

One of the consequences of the financial markets' development, and implicitly of insurance markets, has been the emergence of unit-linked life insurance products in the last two decades. Unit-linked contracts are life insurance policies with investment components. Due to the financial instability caused by the Global Crisis and the amplification of market competitiveness, insurers from international markets have started to incorporate guarantees in unit-linked products. So a unit-linked life insurance policy with an asset value guarantee is an insurance policy whose benefit payable on death or at maturity consists of the greater of some guaranteed amount and the value of the units from the investment funds. One of the most challenging issues concerns the pricing of minimum death benefit and maturity benefit guarantees and the establishment of proper reserves for these guarantees. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for pricing and assessing unit-linked insurance contracts with incorporated investment guarantees. Therefore the model provides a theoretical framework for valuing these investment guarantees. Our findings reveal that the investment policy is optimal in that it provides the Romanian insurance company with a hedging strategy against the investment risk connected with these benefit guarantees.

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1. Introduction

One of the most interesting life insurance products which have emerged in recent years has been the unit-linked contract[1]. The payoff in such contracts depends on two factors: the value of some underlying financial instrument(s) and some insurance-type event in the life of the owner of the contract (death, retirement, survival to a certain date etc.)[2].

Due to the financial instability caused by the Global Crisis and the amplification of market competitiveness, insurers from international markets have started to incorporate guarantees in unit-linked products. So a unit-linked life insurance policy with an asset value guarantee is an insurance policy whose benefit payable on death or at maturity consists of the greater of some guaranteed amount and the value of the units from the investment funds. One of the most challenging issues concerns the pricing of minimum death benefit and maturity benefit guarantees and the establishment of proper reserves for these guarantees. Although the Romanian legislation authorizes the Romanian insurers to offer unit-linked contracts without investment guarantees, this research provides a proposal of a theoretical and empirical basis for pricing and assessing unit-linked insurance contracts with incorporated investment guarantees. Therefore the model, proposed by the authors Boyle and Schwartz, provides a theoretical framework for valuing these investment guarantees. The aim of this study is to determine the equilibrium values of guarantees on single premium contracts and regular premium contracts. This study contributes to the existing literature regarding the issue of appropriate pricing of life insurance contracts and hedging of the risks involved, with an exclusive focus on the unit-linked life insurance contracts with investment guarantees.

The structure of this paper is as follows. Section 2 discusses some previous research on the issue. Section 3 presents some legislative changes regarding the unit-linked life insurance contracts with investment guarantees. Section 4 outlines

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1 **P.P. Boyle**, **E.S. Schwartz**, *Equilibrium Prices of Guarantees Under Equity-Linked Contracts*, The Journal of Risk and Insurance 1997/64(4), pp. 1–9.

2 **Y. Romanyuk**, *Imperfect hedging and risk management of equity-linked life insurance contracts*, Library and Archives Canada 2006, pp. 5–17.

the methodology. The empirical results regarding the computation of the equilibrium prices of guarantees on single premium contracts and regular premium contracts are presented in Section 5. Section 6 provides a summary of the main findings and some concluding remarks.

2. Literature review

In this section the authors intend to present the main studies oriented to research issues related to life insurance unit-linked products.

There is an extensive number of studies on the pricing, hedging and risk management of unit-linked contracts: The first articles elegantly described some of the option elements of life insurance products and demonstrated how the option pricing theory of Black and Scholes could be applied to value these innovative products[3]. Other studies determine premiums for a large variety of equity-linked contracts, including those with payoffs where the contract policyholder chooses the larger of the values of two risky assets (and possibly a guaranteed amount) at maturity of the contract[4]. One study examines the pricing, hedging and accounting of equity-indexed annuities[5]. Another article discusses the modelling and risk management for equity-linked life insurance; the focus of his research is on stochastic modelling of embedded guarantees that depend on equity performance[6]. Some authors have developed the theory to measure and manage risks that are contingent on demographic experience as well as on financial variables[7]. Another study focuses on the risk analysis and hedging of variable annuities in incomplete markets[8].

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[3] **P.P. Boyle**, **E.S. Schwartz**, *Equilibrium Prices of Guarantees…*, pp. 1–9; **M. Brennan**, **E. Schwartz**, *Alternative Investment Strategies for the Issuers of Equity Linked Life Insurance Policies with an Asset Value Guarantee*, Journal of Business, 1979/52/1, pp. 63–93.

[4] **S. Ekern**, **S.-A. Persson**, *Exotic Unit-Linked Life Insurance Contracts*, The Geneva Papers on Risk and Insurance Theory 1996/21(1), pp. 35–63.

[5] **S.E.I. Tiong**, *Equity indexed annuities in the Black–Scholes environment*, Bell&Howell Information and Learning, 2000, p. 29.

[6] **M. Hardy**, *Investment Guarantees. Modelling and Risk Management for Equity-Linked Life Insurance*, John Wiley&Sons, New Jersey 2003, pp. 22–196.

[7] **M. Dickson**, **M.R. Hardy**, **H.R. Waters**, *Actuarial Mathematics for Life Contingent Risks*, Cambridge University Press, 2009, p. 495

[8] **G. Argesanu**, *Risk analysis and hedging in incomplete market*, ProQuest Information and Learning Company 2004, p. 27.

3. Regulatory changes

This section proposes some legislative changes in the Romanian legislation regarding unit-linked life insurance market that may authorize the Romanian insurers to offer unit-linked contracts with and without investment guarantees.

In the table below the authors have summarized antithetical elements of the current and proposed legislation on unit-linked life insurance contracts.

TABLE 1: *Regulatory changes regarding Romanian unit-linked contracts*

Current legislation regulating unit-linked life insurance market	**The proposed legislation to regulate unit-linked life insurance market**
According to the Romanian legislation which regulates the unit-linked life insurance market, unit-linked life insurance contracts pass most of the investment risk to the policyholder and involve no investment risk for the insurer. In general the European Union requirements and the national requirements are common for all life insurance products offered by an insurance company and also there is not a very clear distinction between the legislation regarding unit-linked life insurance products and the one regarding traditional life insurance products. The rest of the information regarding the insurance company and the insurance commitment is common to all insurance contracts.	A beneficial normative change in the Romanian legislation which regulates unit-linked life insurance market may be the authorization and consent given to insurance companies to offer to policyholders a wider range of life insurance products, respectively unit-linked contracts with and without investment guarantees. The design of unit-linked products should ensure that they must offer above-market risk-adjusted returns compared with those available on portfolios of bonds and index funds, deposits, currencies, etc. Policyholders should fully understand how these types of financial products will generate the expected financial result and the method of computation the returns generated by the investments in their chosen investment funds. A key regulatory issue is whether unit-linked insurance products are suitable investments.

S o u r c e: authors' processing based on the: Directive 2002/83/EC of the European Parliament and of the Council of 5 November 2002 concerning life assurance, art. 25, al. 1–2, an. 3.

4. Methodology

The economists' interest is to attempt to calculate equilibrium prices for financial contracts of which insurance contracts are a special case. In order to determine equilibrium prices aspects regarding transactions costs, sales costs and institutional constraints are ignored.

It is possible to form a portfolio of an option and a share so that the rate of return on the combined portfolio is non-stochastic or riskless[9].

In order to determine the equilibrium price of guarantees it is convenient to use the following notation:

- S = represents the price of a non-dividend-paying common stock at time *t*;
- r = represents the instantaneous risk free interest rate;
- W = the equilibrium price of a call option on the reference portfolio;
- T = time to maturity (years);
- E = exercise price of the option;
- μ = expected instantaneous rate of return on the reference portfolio;
- σ^2 = the instantaneous variance of the rate of return on the reference portfolio.

The authors applied the following formula to compute equilibrium prices for investment guarantees:

$$P(S,T) = -SN(-d_1) + Ee^{-rT}N(-d_2) \tag{1}$$

5. Empirical results

The Option Valuation Model provides a theoretical and empirical framework for pricing and evaluating investment guarantees on unit-linked contracts.

In this section, the theoretical model proposed by Boyle and Schwartz is applied to Romanian unit-linked insurance contracts.

First we consider ***a single premium unit-linked contract*** offering investment guarantees upon death or at maturity of the contract. This situation corresponds precisely to that of a combination of a call option on a non-dividend paying stock plus the payment of the fixed amount g. If the value of the reference portfolio is denoted by x, the benefit can be expressed as follows:

$$Benefit = \max(g,x) = g + \max(x-g,0) = x + \max(g-x,0) \tag{2}$$

The value of the put option for a contract maturing with certainty at time T and hence the value of the guarantee is Π_T:

$$\Pi_T = ge^{-rT}N(-d_2) - xN(-d_1) \tag{3}$$

For the calculations of Π_T the following indicators were chosen:

[9] **P.P. Boyle**, **E.S. Schwartz**, *Equilibrium Prices of Guarantees…*, pp. 1–9.

TABLE 2: *The characteristics of unit-linked insurance contracts with investment guarantees issued by a life insurance company*

Financial indicators	Values		
Investment component (LEI)	7240		
Guaranteed component (LEI)[a)]	7240		7620
Risk-free rate return (%)[b)]	6.99		13.98
Volatility of the returns of the underlying stock (%)[c)]	13.72	27.43	54.86
Maturity (Years)	30		

a) First the guaranteed amount is equal with the investment component and then the guaranteed amount increases with 5%.

b) First we took into consideration the current rate on 10 year Treasury Bills and then r doubles its initial value.

c) We took into consideration the volatility of the returns of the underlying stock (27.43%) and then σ decreases with 50% and also the volatility doubles its initial value.

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

The data of the study consists of daily closing values of BET index. BET is the reference index for the Bucharest Stock Exchange (BVB) market. The daily data is taken from October 6, 2004 to October 6, 2014. We have calculated the daily standard deviation of returns series using software Eviews: σ_{daily} = 1,73% and then we have obtained the annual volatility for the Romanian stock market:

$$\sigma_{annual} = \frac{\sigma_{daily}}{\sqrt{N}} \tag{4}$$

The values of r and σ correspond to annual rates under the assumption of continuous compounding. The mortality component is ignored in this research.

TABLE 3: *Values of the premium (in Romanian currency – LEI) to cover maturity guarantee in single premium contracts when I (Investment component) = g (Guaranteed amount)*

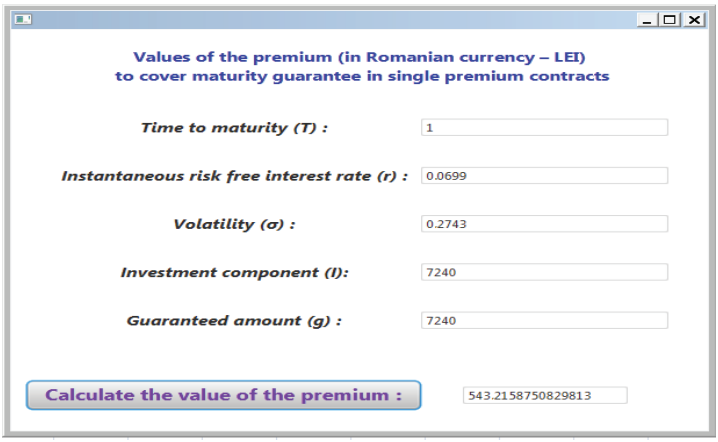
I	**7240**					
g	**7240**					
r	**0.0699**			**0.1398**		
σ	**0.1372**	**0.2743**	**0.5486**	**0.1372**	**0.2743**	**0.5486**
T	**Đ$_T$**					
1	186.71	543.22	1279.621	74.32	359.07	1036.49
2	180.26	634.43	1602.431	40.76	333.23	1152.67
3	159.97	661.33	1763.303	21.63	284.15	1142.88

I	**7240**					
g	**7240**					
r	**0.0699**			**0.1398**		
σ	**0.1372**	**0.2743**	**0.5486**	**0.1372**	**0.2743**	**0.5486**
T	**Đ$_T$**					
4	137.97	660.06	1842.819	11.43	235.41	1084.14
⋮						
10	49.28	498.41	1704.748	0.26	65.85	597.10
15	20.21	355.89	1381.438	0.01	21.98	324.99
20	8.29	247.35	1069.509	–	7.33	171.32
25	3.41	169.77	808.096	–	2.45	88.87
30	1.41	115.75	601.431	–	0.82	45.66

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

As it is shown in Figure 1, Figure 2, Figure 3, Figure 4, Figure 5, Figure 6 for calculating the values of the premiums to cover investment guarantees in single premium contracts the authors made an implementation of algorithms that describe the whole process in Java programming language.

FIGURE 1: *Computing the values of the premium (in Romanian currency – LEI) to cover maturity guarantee in single premium contracts when I = g*



S o u r c e: authors' processing.

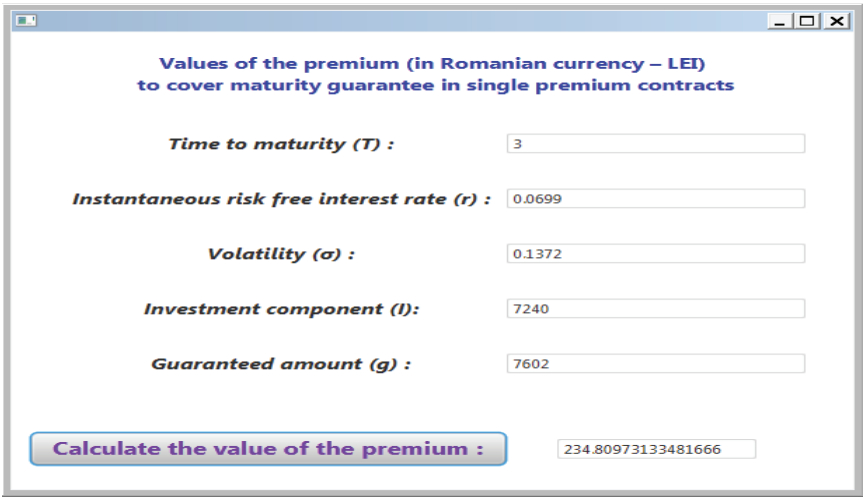
TABLE 4: *Values of the premium (in Romanian currency – LEI) to cover maturity guarantee in single premium contracts when I < g*

I	**7240**					
g	**7602**					
r	**0.0699**			**0.1398**		
σ	**0.1372**	**0.2743**	**0.5486**	**0.1372**	**0.2743**	**0.5486**
T	**Đ$_T$**					
1	320.84	708.23	1474.05	144.04	481.45	1201.94
2	277.89	778.91	1789.59	71.51	421.26	1295.99
3	234.81	790.35	1942.46	36.34	349.96	1267.53
4	196.67	776.24	2013.63	18.72	285.49	1192.67
⋮						
10	65.59	564.18	1827.81	0.40	77.05	644.52
15	26.35	398.10	1472.68	0.02	25.44	348.76
20	10.68	274.75	1136.38	–	8.43	183.22
25	4.36	187.70	856.74	–	2.81	94.83
30	1.79	127.53	636.64	–	0.94	48.64

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

Table 3 and 4 present the values of the guarantee under two different assumptions concerning the fixed amount g. According to the empirical results, the value of the guarantee is particularly sensitive to the variance of the reference portfolio, and also the value of the guarantee does not necessarily decrease with increasing maturity (it increases only in the first years). For the higher values of the variance the value of the guarantee may increase over the first years (up to 5 years) before it begins to decrease. However for the various combinations of r and σ the value of the guarantee is zero after 15 years. Most insurers do not issue contracts with maturity less than 7 years.

FIGURE 2: *Computing the values of the premium (in Romanian currency – LEI) to cover maturity guarantee in single premium contracts when I <g*



S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

Another situation is when the guaranteed amount consists of the gross premiums, respectively the guarantee is the sum of the investment component plus the additional loadings, including the premium for the guarantee.

So the guaranteed amount will be g' = g+ Π_T . As would be expected the new values of Π_T presented in Table 5 are higher than the corresponding values in Table 4.

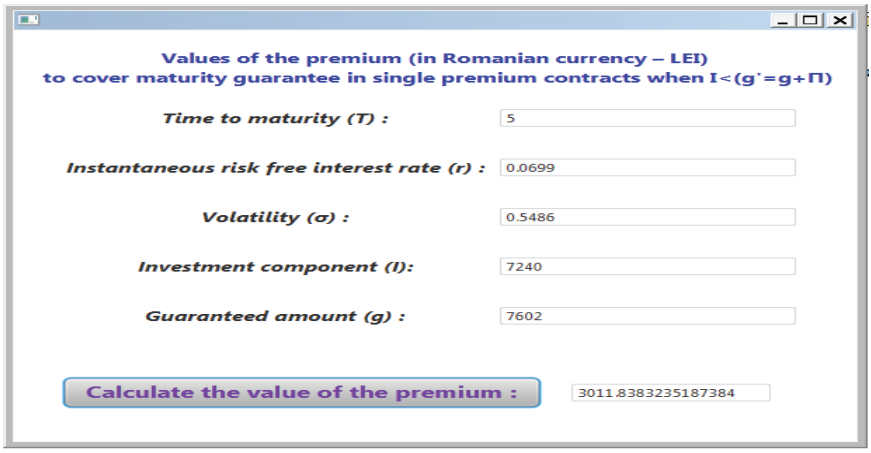
TABLE 5: *Values of the premium (in Romanian currency – LEI) to cover maturity guarantee in single premium contracts when I < (g' = g+ Π_T)*

I	**7240**					
g	**7602**					
r	**0.0699**			**0.1398**		
σ	**0.1372**	**0.2743**	**0.5486**	**0.1372**	**0.2743**	**0.5486**
T	**Đ$_T$**					
1	478.32	1096.96	2375.149	181.56	677.18	1825.54
2	372.20	1141.00	2815.442	79.23	539.26	1861.36
3	293.98	1111.97	2991.473	38.16	421.38	1739.82
4	234.68	1056.52	3038.494	19.18	329.05	1574.58

I	**7240**					
g	**7602**					
r	**0.0699**			**0.1398**		
σ	**0.1372**	**0.2743**	**0.5486**	**0.1372**	**0.2743**	**0.5486**
T	**Đ$_T$**					
⋮						
10	68.90	674.12	2473.895	0.40	79.56	731.34
15	26.84	446.91	1853.135	0.02	25.69	372.04
20	10.76	296.37	1349.727	–	8.46	189.31
25	4.38	197.28	973.1486	–	2.81	96.40
30	1.80	131.79	699.0312	–	0.94	49.04

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

FIGURE 3: *Computing the values of the premium (in Romanian currency – LEI) to cover maturity guarantee in single premium contracts when I < (g' = g+ Π_T)*



S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

Secondly the authors consider ***regular premium contracts*** which offer investment guarantees. Initially the assumption is made that there is a constant rate of investment of D per unit time in the reference portfolio.

TABLE 6: *The characteristics of unit-linked insurance contracts with investment guarantees issued by a life insurance company*

Financial indicators	Values	
Investment component (LEI)	724	
Guaranteed component (LEI)	724	
Risk-free rate return (%)[a)]	6.99	13.98
Volatility of the returns of the underlying stock (%)[b)]	27.43	54.86
Maturity (Years)	20	

a) First we took into consideration the current rate on 10 year Treasury Bills and then r doubles its initial value.

b) We took into consideration the volatility of the returns of the underlying stock (27.43%) and then σ decreases with 50% and also the volatility doubles its initial value.

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

The annual premium Π_n is calculated as follows:

$$\frac{ge^{-rn} + W - D\sum_{T=0}^{n-1} e^{-rT}}{\sum_{T=0}^{n-1} e^{-rT}} \tag{5}$$

Table 7 presents the various values of Π_n. Similar with the case of the single premium insurance, the value of the guarantee increases dramatically when variance increases. An insurance company can reduce the investment risk associated with granting these types of guarantees by selecting those reference portfolios which have the most stable returns.

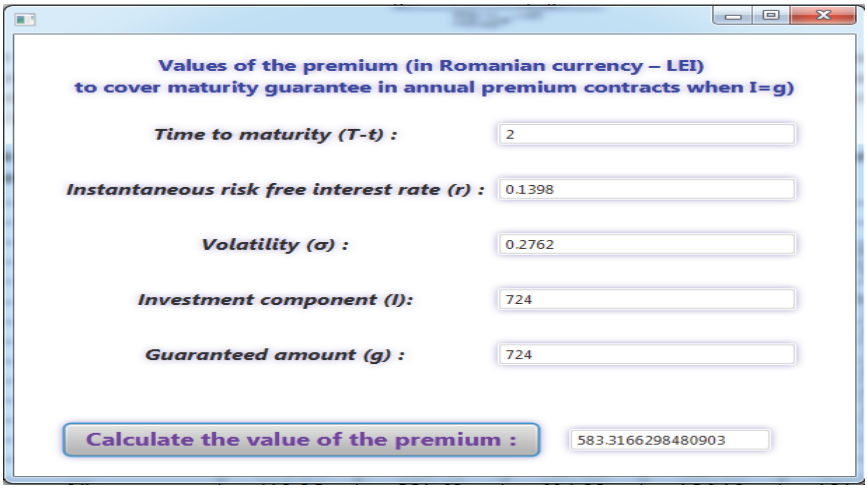
TABLE 7: *Values of the premium (in Romanian currency – LEI) to cover maturity guarantee in annual premium contracts when I = g*

I	**724**					
g	**724**					
r	**0.0699**	**0.1398**	**0.1398**	**0.0699**	**0.1398**	**0.1398**
σ	**0.2762**	**0.2762**	**0.5486**	**0.2762**	**0.2762**	**0.5486**
T	**W**			**Π_n**		
1	103.20	130.37	198.11	962.72	997.13	1137.58
2	157.90	209.92	291.86	542.07	582.83	654.32
3	203.09	276.43	362.30	392.54	432.14	474.68

I	**724**					
g	**724**					
r	**0.0699**	**0.1398**	**0.1398**	**0.0699**	**0.1398**	**0.1398**
σ	**0.2762**	**0.2762**	**0.5486**	**0.2762**	**0.2762**	**0.5486**
T	**W**			**Π_n**		
4	242.60	333.66	418.53	313.78	350.84	377.73
⋮						
15	505.85	637.27	667.57	115.09	137.89	139.92
16	520.54	648.44	675.32	109.69	132.67	134.83
17	534.20	658.18	681.99	104.88	128.18	130.49
18	546.93	666.67	687.72	100.57	124.29	126.77
19	558.79	674.08	692.67	96.69	120.93	123.57
20	569.84	680.53	696.93	93.19	118.02	120.81

S o u r c e: Authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

FIGURE 4: *Computing the values of the premium (in Romanian currency – LEI) to cover maturity guarantee in annual premium contracts when I = g*



S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

6. Investment Strategies

The model presented by Boyle and Schwartz suggests a dynamic approach to the investment of premiums on unit-linked contracts. The optimal investment strategy involves that not all the investment component be invested in the reference portfolio.

To examine the operation of this strategy the authors considered the following elements:

TABLE 8: *The characteristics of unit-linked insurance contracts with investment guarantees issued by a life insurance company*

Financial indicators	**Values**	
Type of the policy	Single policy	Annual policy
Investment component (LEI)	7240	724
Guaranteed component (LEI)	7240	724
Risk-free rate return (%)	6.99	
Volatility of the returns of the underlying stock (%)	27.43	
Maturity (Years)	10	

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

Table 9 shows that the equilibrium value of the maturity guarantee is 498.41 LEI. Maturity of the contract is certain at the end of the 10 years and the value of the reference portfolio 7240 LEI. The policy owner is assumed to pay 7738.41 LEI. After the premium is paid, the insurer's liabilities consist of the units in the reference portfolio credited to the policyholder and valued at 7240 LEI, and a put option on the reference portfolio. So the amount invested in the reference portfolio is S = 6461.19 LEI and the amount invested in the risk free asset is R = 1277.22 LEI. In table 9 the authors have presented the amounts to be invested in the reference portfolio and the risk free asset at yearly intervals for different values of the reference portfolio. The investment is heaviest in the risk free asset when the value of the reference portfolio is low, but for high values of the reference portfolio most of the financial funds are invested in the reference portfolio.

TABLE 9: *Investment strategy for the single premium contract*

Value of the reference portfolio (x)		*Time to maturity (T)*				
		1	*2*		*9*	*10*
1448	***S***	0.00	0.24		315.28	389.49
	R	6751.21	6295.20		3649.13	3350.57
2896	***S***	4.62	102.30		1520.11	1658.48
	R	6746.92	6206.91		2996.88	2710.24
4344	***S***	307.29	968.20		3086.91	3224.82
	R	6477.60	5508.57		2337.06	2108.24
5792	***S***	1950.09	2847.74		4733.27	4848.04
	R	5108.84	4146.53		1803.80	1634.85
7240	***S***	4723.84	5142.78		6372.58	6461.19
	R	3059.38	2731.65	…	1397.23	1277.22
8688	***S***	7425.34	7360.16		7983.13	8048.86
	R	1465.02	1650.97		1091.37	1008.31
10136	***S***	9601.20	9349.92		9563.29	9610.75
	R	603.69	948.18		860.85	804.76
11584	***S***	11379.82	11135.67		11117.05	11150.48
	R	226.43	529.43		685.79	649.10
13032	***S***	12958.70	12781.19		12649.33	12672.22
	R	80.30	291.59		551.56	528.72
14480	***S***	14454.57	14340.83		14164.60	14179.65
	R	27.60	159.87		447.59	434.57

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

This investment strategy can be extended to include regular (annual) premium policies as shown in Table 10. This extension involves the derivation of the investment strategy for the regular premium contract. In this case the annual investment component is 724 LEI and the guaranteed amount payable at death or maturity is the sum of the investment components. So in Table 10, the authors have presented the amounts to be invested in the risk free asset and in the reference portfolio to maintain the hedged position.

FIGURE 5: *Investment strategy for the single premium contract*

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

TABLE 10: *Investment strategy for the regular (annual) premium contract*

Value of the reference portfolio (x)		Time to maturity (T)				
		1	2		9	10
145	S	40.44	0		0	0
	R	297.37	1393.41		7229138827.14	105123358074.76
290	S	80.89	0		0	0
	R	594.73	2786.82		14458277654.28	210246716149.52
434	S	121.05	0		0	0
	R	890.05	4170.62		21637560351.58	314645085547.91
579	S	161.50	0		0	0
	R	1187.42	5564.03		28866699178.73	419768443622.67
724	S	201.94	0	...	0	0
	R	1484.78	6957.45		36095838005.87	524891801697.43
869	S	242.38	0		0	0
	R	1782.15	8350.86		43324976833.01	630015159772.20
1014	S	282.83	0		0	0
	R	2079.52	9744.27		50554115660.15	735138517846.96
1158	S	322.99	0		0	0
	R	2374.83	11128.07		57733398357.45	839536887245.34

Value of the reference portfolio (x)		Time to maturity (T)				
		1	2		9	10
1303	S	363.44	0		0	0
	R	2672.20	12521.48		64962537184.59	944660245320.10
1448	S	403.88	0		0	0
	R	2969.57	13914.89		72191676011.73	1049783603394.87

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

FIGURE 6: *Investment strategy for the regular premium contract*

S o u r c e: authors' processing based on the Annual Reports of National Bank of Romania and the Bucharest Stock Exchange Database.

7. Conclusions

Unit-linked life insurance contracts are popular and widely used on the insurance market. The benefits are linked to an underlying asset with or without certain guarantees so that the policyholders have the opportunity to participate in the financial market and (eventually) be protected from the downside development of the financial market[10].

[10] **J. Li**, **A. Szimayer**, *The Uncertain Mortality Intensity Framework: Pricing and Hedging Unit-Linked Life Insurance Contracts*, Bonn Econ Discussion Papers, University of Bonn, 2011, p. 2.

Therefore this model provides a theoretical framework for valuing these investment guarantees. According to the empirical results, the value of the guarantee is particularly sensitive to the variance of the reference portfolio, and that the value of the guarantee does not necessarily decrease with increasing maturity (it increases only in the first years). In the case of the single premium policy, for higher values of the variance the value of the guarantee may increase over the first years (up to 5 years) before it begins to decrease. However for the various combinations of r and σ the value of the guarantee is zero after 15 years. Most insurers do not issue contracts with maturity less than 7 years. For the annual premium insurance, the value of the guarantee increases dramatically when variance increases. Nevertheless, the investment is heaviest in the risk free asset when the value of the reference portfolio is low, but for high values of the reference portfolio most of the financial funds are invested in the reference portfolio.

Also the authors propose some legislative changes in the Romanian legislation regarding unit-linked life insurance market that may authorize the Romanian insurers to offer unit-linked contracts with and without investment guarantees.

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ROZWÓJ ALTERNATYWNYCH STRATEGII DLA EMITENTÓW POLIS UBEZPIECZENIOWYCH TYPU UNIT-LINKED Z GWARANCJĄ INWESTYCYJI

(Streszczenie)

Jedną z konsekwencji rozwoju rynków finansowych oraz pośrednio rynku ubezpieczeń było pojawienie się w ciągu ostatnich dwóch dekad ubezpieczeń na życie typu *unit-linked*. Umowy *unit-linked* są polisami na życie z komponentem inwestycyjnym. Ze względu na niestabilność finansową spowodowaną światowym kryzysem i wzmocnienie konkurencji rynkowej, ubezpieczyciele na rynkach międzynarodowych załączają gwarancje typu *unit-linked* do oferowanych produktów. Polisa *unit-linked* z gwarancją wkładu jest polisą, w której świadczenie na wypadek śmierci lub terminu zapadalności składa się z kwoty większej niż gwarantowana oraz wartości jednostek uczestnictwa w funduszach inwestycyjnych. Kwestię problematyczną stanowi minimalna wysokość świadczenia gwarantowanego na wypadek śmierci lub terminu zapadalności oraz ustanowienie odpowiednich rezerw dla tych gwarancji. Chociaż rumuńskie ustawodawstwo umożliwia rumuńskim ubezpieczycielom oferowanie umów typu *unit-linked* bez gwarancji inwe-

stycyjnych, niniejszy artykuł przedstawia propozycję wyceny polis ubezpieczeniowych *unit-linked* na gruncie teorii i badań empirycznych. Zaproponowany model uwzględnia ramy teoretyczne dla wyceny wartości gwarancji. Wyniki wskazują, że polityka inwestycyjna w zakresie tego rodzaju produktów jest zoptymalizowana i pozwala rumuńskim ubezpieczycielom na ograniczenie ryzyka inwestycyjnego związanego z gwarantowaniem świadczeń.

Słowa kluczowe: produkty *unit-linked*; gwarancja inwestycji; model Blacka–Scholesa–Mertona; teoria opcji; cena równowagi rynkowej