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FORECASTING THE CRITICAL POINTS OF STOCK MARKETS' INDICES USING LOG-PERIODIC POWER LAW

Abstract: This article presents Log-Periodic Power Law and considers its usefulness as a forecasting tool on the financial markets. One of the estimation methods of this function was presented and six models were built, based on time series of the DJIA and the WIG20. Estimated models were utilized to predict crashes of those indices. The variations between the actual values of analyzed indices observed in the forecasted period and values observed in the actual period of their downturn were assembled to assess the results. In three cases, relative errors were below 5%; and in three cases, they were higher than 15%.

Keywords: Log-Periodic Power Law, critical point of stock market index, forecast.

1. Introduction

In 2002, Didier Sornette published his controversial book [Sornette 2002] which, apart from the central thesis, proposed a suggestion that physicists are better-equipped to make forecasts in relation to financial markets than the experts working in this particular field of economics. Soon, similar opinions were voiced by others, also in Poland, e.g. by Professor S. Drożdż in an interview dated July 31, 2003[1]. Those claims made by physicists or, more precisely, the proponents of econophysics, were based on the positive *ex post* evaluation of stock index forecasts made using the Log-Periodic Power Law (LPPL). In theory, the function was seen as a useful and accurate instrument of market crash prediction.

Taking into account the fact that the majority of individuals, social groups, companies, governments and other actors participate to a certain extent in the exchange on the financial markets, either directly or indirectly (e.g. by participation in investment funds), there is a need for the accurate prediction of potential market crashes. Since the aforementioned experts postulate a method for addressing this issue, it is only natural that this method be examined and put to the test. Hence, the

[1] http://www.forum.gpwinfo.pl/showthread.php?t=2227, 01.02.2008 (accessed 30.08.2012).

main purpose of this paper is to present the function under study as well as the verification of its use as a valid tool for forecasting changes of selected indices that describe the economic situation on stock exchange markets.

The economic situation is a concept used in several contexts, depending on the subject being studied. In general, as it relates to the economy as such, the economic situation represents a set of indices of economic activities, such as production, employment, inflation, and so on, that are used to describe the current state of the economy [Lubiński 2004, p. 11]. Taking the discussion to the level of the stock exchange market, a good economic situation would be related to an increase of long position stock prices. Professional literature often relates the economic situation directly to business (economic) cycles.

Economic cycles are characterized, among other determinants, by critical points, marking the transition from one phase to the other. As such, they can be further divided into:

– upper turning points *(upturns)* – marking the highest point of economic prosperity (expansion), to be followed by a decline (recession),
– lower turning points (*downturns*) – marking the shift from decline (depression) to a period of growth (recovery).

The main purpose of economic cycle forecasts is to provide the answer to "the question whether the emergence of a critical point is probable in the near future and what fluctuations can be expected in the basic macroeconomic indices […]?" [Lubiński 2004, p. 237]. In the case of stock exchange markets, this may also involve the prediction of potential turning points in the graph of a stock market index.

2. The log-periodic function and estimation of its parameters

The graph of the log-periodic function is a product of physicists' research on the model of power function as an earthquake predictor. One of the forms of this function, as postulated by D. Sornette and C. G. Sammis as a potential model for the above research, is described as follows [Sornette, Sammis 1995]:

$$y(t) \approx A + B(t_c - t)^{\beta} + C\,(t_c - t)^{\beta}\cos(\omega\, log(t_c - t) + \varphi)\,, \qquad (1)$$

where: $y(t) > 0$ – real value of the variable under study, e.g. the value (or value logarithm) of the index over time *t*,

$A > \mathbf{0}$ – parameter interpreted as a theoretical value of the variable at critical point ($\hat{y}_{t_c}$),

$B < \mathbf{0}$ – parameter interpreted as the step of the variable if *C* approaches zero,

C – parameter characterizing the proportion of oscillation weigh around the exponential growth,

$t_c > \mathbf{0}$ – critical time (point) – *time of crash*,

$t < t_c$ – any time before the critical point,
β – parameter of exponential growth,
ω – parameter describing the oscillation frequency,
φ – shift parameter [Brée, Joseph 2010, p. 4].

For the purpose of this study, the critical point should be identified with a sudden drop of the index, i.e. a strong, incessant downward trend, typically observed over the course of several days. The downward trends of this type can be identified using 0.995 quantile of ε index, calculated using the following formula [cf. Jacobsson 2009, p. 7–17]:

Let: $r_i = \log v(t_i) - \log v(t_{i-1})$, where $[v(t]_i)$ – index value at time t_i, then ε index will be $\sum r_i$ for consecutive $|r_i| < s \wedge r_i \leq 0$ where s is a standard deviation of r_i value progression. Critical points are described by pairs of $\{t_{j=1}, v(t_{j=1})\}$ of all ε 0.995 quantile observations.

In 1995, J.A. Feigenbaum and P.G.O. Freund concluded in their pioneering study of the capital market as a complex system, postulating that a critical point of such a system (here – a turning point in the economic cycle) can be predicted from the monitoring of exponential, log-periodic fluctuations [Feigenbaum, Freund 1996, p. 4]. The postulated applicability of this function in the form of (1) was based on the observation of log-periodic oscillations accompanying previous stock exchange crashes. The oscillations in question are identified as:

– exponential trends (*divergence*);
– self-similarity of the function's graphs (repeating geometric patterns observed in the periods preceding the crash [Sornette 2002, p. xvii,]) – as illustrated in Figure 1.

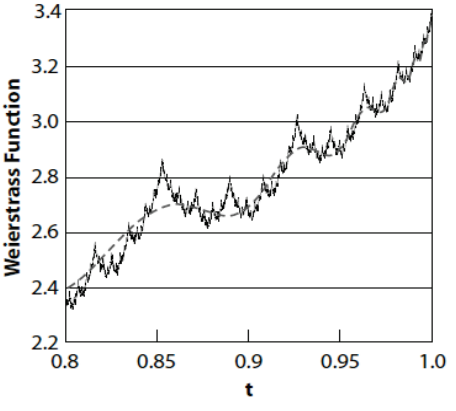

Figure 1. Graph bearing a log-periodic signature, on the example of Weierstrass function

Source: [Sornette 2002, p. 197].

For one to apply the log-periodic function means to reject the hypothesis of rational expectations which holds that economic entities (investors), while fully aware of its long-term impact on the economy, base their reaction in part on altruism, jealousy, panic and other emotions which are not readily quantifiable.

D. Sornette points out that economists should not focus on the apparent irrationality of investors' choices, but rather study the way that those irrational behaviors aggregate into complex, repeatable patterns of collective behavior in the long-term perspective. The market may provide mechanisms guarding against the impact of collective behavior on the index value. If such mechanisms are not present, the irrational behavior of investors may influence the index, resulting in the formation of "speculative bubbles" [Sornette 2002, p. 138]. If the identified formation is in fact an example of a speculative bubble, then the crash of the index is a probable scenario. The index value turning point is referred to as the critical point (time) t_c, and can be forecasted on the basis of log-periodic function properties. The most frequently adopted method of parameter estimation in this case is the estimation via adjusting the function (1) against data sets.

The log-periodic function (1) is described by seven parameters. However, for the purpose of adjustment, it may be simplified by reducing the number of free parameters to four [Johansen, Ledoit, Sornette 2000, p. 16]. Hence, the equation (1) can be expressed as:

$$y(t) \approx A + Bf(t) + Cg(t), \tag{2}$$

where:

$$f(t) = (t_c - t)^{\beta} \quad : t < t_c, \tag{3}$$

$$g(t) = (t_c - t)^{\beta} \cos(\omega \log(t_c - t) + \varphi) \quad : t < t_c. \tag{4}$$

For each set of four nonlinear parameters $\theta = (t_c, \omega, \beta, \varphi)$, one can calculate values of linear parameters $b = (A, B, C)$ using the least squares method (LSM), as follows:

$$\begin{pmatrix} \sum_{i=1}^{n} y(t_i) \\ \sum_{i=1}^{n} y(t_i)f(t_i) \\ \sum_{i=1}^{n} y(t_i)g(t_i) \end{pmatrix} = \begin{pmatrix} N & \sum_{i=1}^{n} f(t_i) & \sum_{i=1}^{n} g(t_i) \\ \sum_{i=1}^{n} f(t_i) & \sum_{i=1}^{n} f(t_i)^2 & \sum_{i=1}^{n} f(t_i)g(t_i) \\ \sum_{i=1}^{n} g(t_i) & \sum_{i=1}^{n} f(t_i)g(t_i) & \sum_{i=1}^{n} g(t_i)^2 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix}. \tag{5}$$

Thus:

$$X'y = (X'X)b, \tag{6}$$

where:

$$X = \begin{pmatrix} 1 & f(t_1) & g(t_1) \\ \vdots & \vdots & \vdots \\ 1 & f(t_n) & g(t_n) \end{pmatrix} \text{ and } b = \begin{pmatrix} A \\ B \\ C \end{pmatrix}.$$

The solution of the above equation (9) results in a vector of parameter estimations:

$$\hat{b} = (X'X)^{-1}X'y. \tag{7}$$

The available empirical studies of log-periodic function show that the calculated parameters θ may be further limited (cf. [Jacobsson 2009, p. 23]) to facilitate optimization, i.e.:

– $\omega \in \langle 6; 15 \rangle$,
– $\beta \in \langle 0{,}1; 0{,}9 \rangle$,
– $\varphi \in \langle 0; 2\pi \rangle$.

For the purpose of calculation, the assumption of $t_c > t$ can be elaborated in the form of $t_c > \max(t)$.

Moreover, it is expected that the parameter A will describe the forecasted value of the index or price at the moment of crash, i.e. $A = p(t_c)$, with parameter $B < 0$ (in the case of "speculative bubble"), and parameter $C \neq 0$, to satisfy the requirement of log-periodic oscillation.

Taking into account the above restrictions, the goal function is construed, minimizing the sum of square errors (SSE). Thus:

$$\min_\theta F(\theta) = \sum_{i=1}^{n} \left(y_\theta(t_i) - \hat{y}_\theta(t_i)\right)^2, \text{ where } \theta = (t_c, \omega, \beta, \varphi). \tag{8}$$

The domain of function (8) includes many local minimums of similar value. The crucial research problem here is to find a method for the calculation of the integral minimum.

Figure 2 presents changes in log-periodic function values (the LPPL axis) for cases when two parameters are floated, with the remaining two set at constant value as calculated for an exemplary case of the Dow Jones Industrial Average index for a set of data preceding the Great Depression by two years. The plane determined by variable parameters $\{\beta, t_c\}$ and the remaining parameters stabilized does not show local minimums in the estimated model. For the remaining pairs of variable values, the local optima appear periodically. Moreover, visual analysis of the graphs shows that the increase of β parameter value corresponds with an increase of LPPL function value in the case of variable φ and ω, and a decrease for floated t_c value. For floated pair of parameters $\{\varphi, \omega\}$, the value of LPPL oscillates around a set level. The increase of t_c will result in a general drop of log-periodic function value for floated φ and ω. The above analysis shows that it is impossible to predict an exact weight of impact of a given parameter upon the value of the LPPL function when the remaining parameters are floated. The log-periodic function domain includes a number of local minimums and maximums which can be determined, by approximation, using suitable methods of approximation.

Initially, the researchers tried to adjust the log-periodic function to data using such methods as *Simplex* and *Quasi-Newton*, but those algorithms were often trapped in the local minimum. This is due to the fact that the algorithms in question are able

to determine minimum values, but only in the vicinity of the procedures' starting points. For the purpose of this paper, the genetic algorithm (GA) was used. The advantage of this method comes from the fact that it does not require additional information on the shape of the calculated plane (i.e. inclination or local folds). The goal function does not need to be continuous nor smooth. Moreover, the search for solutions runs in parallel, with each element of domain population exploring the plane in multiple directions. GA implementation shows the method to be relatively quick for the purpose of determining solutions close to the optimum value, even with large deviation of parameters and in complicated applications.

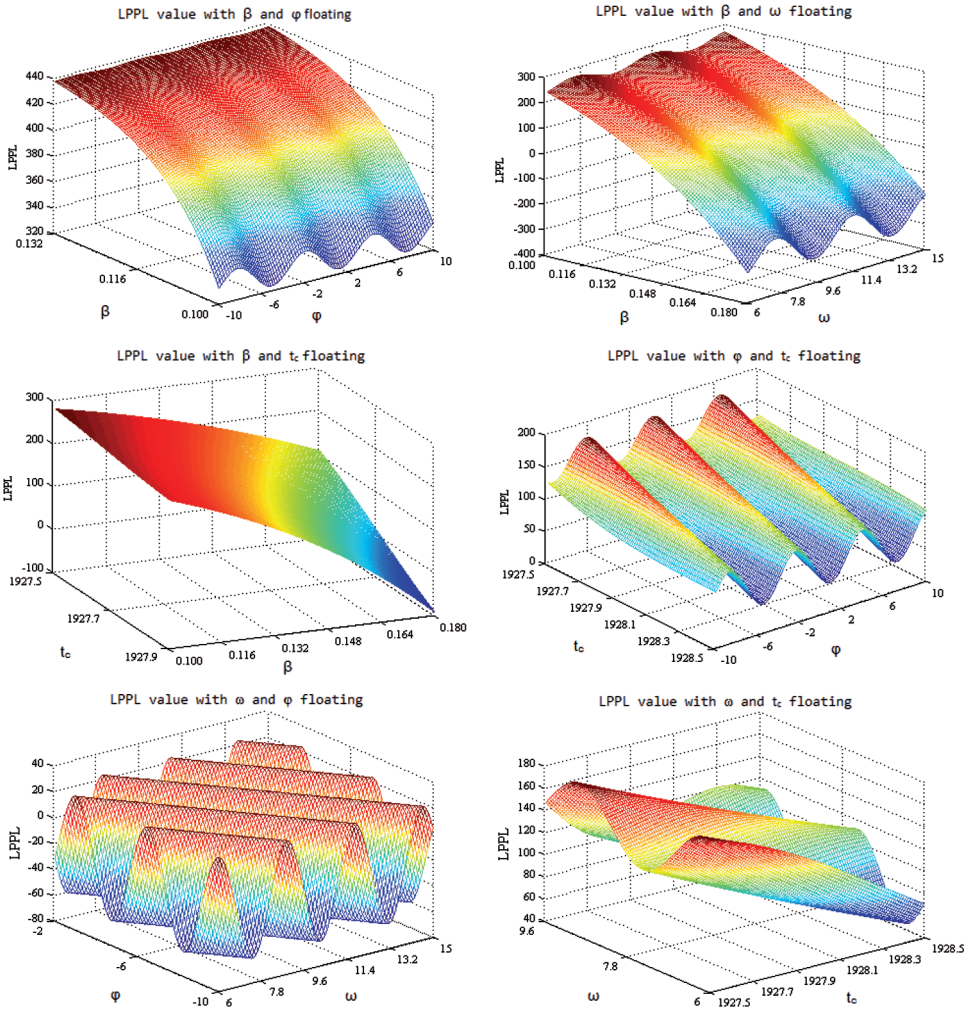

Figure 2. Graphs of log-periodic function after floating two of the four parameters

Source: own research.

The research on log-periodic function typically involves its adjustment to fuzzy time series (i.e. time series characterized by irregular time intervals). This is due to the fact that the function is usually employed to the study of share prices, with time intervals made irregular due to the constraints of the stock exchange working week. One of the simplest, although analytically suboptimal, ways to deal with this problem of irregularity is to convert the time domain index into a series of natural numbers, with the analysis result then converted back to the proper time format consistent with the input data. Another, more appropriate solution, also used by other researchers in the study of log-periodic function (cf. [Sornette 2002] and [Jacobsson 2009]), is to convert the domain elements, expressed in the format of *DD/MM/YYYY*, by means of the following equation:

$$t = YYYY + \frac{1}{12} * (MM - 1) + \frac{1}{365} * (DD - 1). \tag{9}$$

Thus, the time notation of *01/01/2011* is expressed in the form of t = 2011, the date of *2011/07/01* represents the year's 'midpoint', while the last day of the year (*31/12/2011*) is represented by t = 2011.9988586. This method helps retain the proper time scale and the results can be easily converted back from the time format (t) to the date notation format.

3. Forecast of selected indices

The log-periodic function presented above was used as the basis for the task of forecasting two arbitrarily selected stock exchange indices, namely the DJIA index of the New York Stock Exchange and the WIG20 index of the Warsaw Stock Exchange.

The DJIA forecasts were constructed using data from three separate periods:

- 17/Sept/1985 – 25/Aug/1987 (period I),
- 04/Aug/2006 – 09/Oct/2007 (period II),
- 09/Mar/2009 – 03/May/2011 (period III).

The selection of the two former periods was dictated by their potential, as the applicability of the estimated function for admonitory crash forecasting purposes could be easily verified using the actual index crashes of 1987 and 2008. The third period was selected on the basis of the observed log-periodic signature in the registered flow of data.

Table 1 presents parameter estimations and function adjustments determined on the basis of the selected time scales. The resulting coefficients of determination show the good adjustment of the constructed models. At the statistical significance level of 0.05, the most significant were the estimates of linear parameters $\{A, B, C\}$ of the models used.

Comparing the forecasted time of critical point occurrence against the historical record of the DJIA crash (Table 2), it is apparent that the relations between those two values are not uniform. In one case (for the II period data), the forecast falls earlier

Table 1. Estimates of parameters of log-periodic models of the DJIA index for the selected periods of analysis

Period	R^2	A	B	C	t_c	ω	β	φ
I	98.68%	3808.17	–1768.21	–102.74	1987.98	8.27	0.37	0.53
II	93.73%	15842.69	–2838.96	–272.31	2008.36	13.46	0.75	4.74
III	95.85%	14148.36	–2164.88	–338.55	2012.20	8.26	0.90	2.95

Source: own calculations based on daily closing quotations of the Dow Jones Industrial Average index [http://www.djaverages.com/].

than the historical occurrence, while the remaining two periods show the opposite relation, i.e. the historical occurrence took place earlier than predicted by the forecast model. There are also significant differences in time intervals between the forecasted and historical occurrence for each of the cases under study.

Table 2. Actual vs. theoretical time of the DJIA crash for selected periods of analysis

Period	Theoretical critical point		Date of historical critical point occurrence t_c	$t_c - \hat{t}_c$
	$\hat{t}_c$	In date format		
I	1987.98	1987-12-23	1987-10-14	–70
II	2008.36	2008-05-09	2008-10-01	145
III	2012.20	2012-03-12	2011-07-22	–234

Source: own calculations based on daily closing quotations of the Dow Jones Industrial Average index [http://www.djaverages.com/].

How to interpret the accuracy of the resulting forecast? For the forecast to be deemed accurate, the forecasted time of crash occurrence should fall slightly earlier than the actual crash occurrence. This was not the case in the periods under study. For a potential investor, the value of such a forecast can be measured by the resulting benefit; thus, it seems that the evaluation of forecast accuracy can be based on the deviation between DJIA values observed at the point forecasted vs. the historical occurrence of index crash (Equation 10). The results presented in Table 3 show that the deviations between those two values were less than 5% only in one case. For the remaining two periods, the deviations were relatively large, i.e. more than 15%.

$$\varphi = \frac{|y(t) - y(t_c)|}{y(t)} \cdot 100. \tag{10}$$

Similarly to the method used in the evaluation of the DJIA data, the evaluation of the WIG20 index was based on estimates of log-periodic functions for three distinct data periods:

– 09/May/2004 – 5/Jul/2007 (period I),
– 12/Jul/2006 – 5/Jul/2007 (period II),
– 19/May/2010 – 16/Jun/2011 (period III).

Table 3. Values of the DJIA index in the forecasted and historical point of index crash, and their relative deviation, based on the models for selected periods

Period	DJIA value at the point of		Relative deviation in %
	$\hat{t}_c$	t_c	
I	2005.64	2412.70	16,9
II	12745.88	10831.07	–17,7
III	12959.71	12587.42	–2,2

Source: own calculations based on daily closing quotations of the Dow Jones Industrial Average index [http://www.djaverages.com/].

For each of the selected periods, the data was characterized by a log-periodic signature. The results of model parameter estimations are shown in Table 4. As in the case of the DJIA models, the models used for the WIG20 estimation were well-adjusted to empirical data and showed good significance of the linear parameters $\{A, B, C\}$, i.e. the statistical significance level of 0.05.

Table 4. Estimations of the parameters of log-periodic models of the WIG20 index for selected periods

Period	R^2	A	B	C	t_c	ω	β	φ
I	97.51%	3799.66	–812.12	–79.19	2007.55	6.84	0.90	1.08
II	91.23%	4116.31	–1078.96	–115.15	2007.57	8.00	0.47	1.37
III	94.02%	3019.46	–484.32	9.49	2011.63	7.67	0.90	2.34

Source: own calculations based on daily closing quotations of the Warsaw Stock Exchange Index 20 [http://stooq.pl/q/d/?s=wig20].

The times of the WIG20 crashes forecasted using the log-periodic models fell consistently earlier than the actual, historical occurrences (Table 5). Compared with the DJIA estimates, the time intervals between the forecasted and historical crash point occurrences were significantly shorter.

Table 5. Actual vs. theoretical time of the WIG20 crash for selected periods

Period	Theoretical critical point		Date of historical critical point occurrence t_c	$t_c - \hat{t}_c$
	$\hat{t}_c$	In date format		
I	2007.55	2007-07-19	2007-07-23	4
II	2007.57	2007-07-27	2007-08-13	17
III	2011.63	2011-08-19	2011-07-27	23

Source: own calculations based on daily closing quotations of the Warsaw Stock Exchange Index 20 [http://stooq.pl/q/d/?s=wig20].

As was the case for the DJIA, the evaluation of forecast accuracy was measured by the relative deviation of the WIG20 index values at the forecasted and historical occurrence of index crashes (Table 6). As shown in Table 6, in two cases the relative deviation between these values was less than 5%, and considerably large in one period under study, in line with the results obtained for the DJIA index estimates.

Table 6. Values of the WIG20 index in the forecasted and historical point of index crash, and their relative deviation, based on the models for selected periods

Period	WIG20 value at the point of		Relative deviation in %
	$\hat{t}_c$	t_c	
I	3861.02	3890.49	0,8
II	3697.31	3557.95	–3,9
III	2252.40	2689.21	16,2

Source: own calculations based on daily closing quotations of the Warsaw Stock Exchange Index 20 [http://stooq.pl/q/d/?s=wig20].

4. Conclusions

The aim of this study was to analyze the utility of log-periodic functions which, according to some researchers, can be used to good effect for the forecast of turbulent changes of certain phenomena, such as the changes observed on the financial markets. The evaluation of this method's accuracy was based on six log-periodic models constructed for the evaluation of stock exchange index behavior – 3 for the DHIA index, and 3 for the WIG20 index. The forecast objective was the prediction of the time of the index crash occurrence. The study shows that the models used were well-adjusted to historical data and that their linear parameters were statistically significant. The estimated models were then used to predict the date of the index crash for each period under study. The forecast results were evaluated using relative deviation between the forecasted and historically recorded index values at the crash points. The forecast errors were relatively small (less than 5%) for one of the three periods analyzed in relation to the DJIA index, and in two of the three periods analyzed in relation to the WIG20 index. Despite the relatively small number of constructed models and the resulting forecasts of index crash points, it seems that log-periodic functions cannot be deemed accurate for the purpose of stock market crash predictions, contrary to the opinion presented by researchers of certain scientific disciplines.

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PROGNOZOWANIE PUNKTÓW ZWROTNYCH INDEKSÓW GIEŁDOWYCH PRZY UŻYCIU FUNKCJI LOG-PERIODYCZNEJ

Streszczenie: Celem artykułu jest prezentacja funkcji log-periodycznej oraz próba oceny jej przydatności jako narzędzia prognozowania na rynkach finansowych. Przedstawiono jeden z możliwych sposobów szacowania parametrów tej funkcji oraz zbudowano sześć modeli na podstawie szeregów czasowych dwóch indeksów giełdowych DJIA oraz WIG20. Wyznaczone modele posłużyły do budowy prognoz załamania tych indeksów. Uzyskane rezultaty poddano ocenie za pomocą odchyleń między wartościami rozważanych indeksów, jakie zanotowano w przewidywanym i rzeczywistym czasie ich gwałtownej zmiany. Nie wszystkie prognozy były trafne. W trzech przypadkach otrzymane błędy względne były mniejsze niż 5%, oraz w trzech wyniosły ponad 15%.

Słowa kluczowe: funkcja log-periodyczna, załamanie indeksu, prognoza.