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WORLD MODELS IN FORMALIZED SYSTEMS OF THEODICY

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From the point of view of formalized theodicy, the domain of research is a relational system $\langle X, R \rangle$, where nonempty X is a universe of objects or real beings and $R \subseteq X \times X$ is a binary relation. The relation R is expected to:

1⁰. be *transcendental*, so that the set of beings B is a subset of the field of the relation R : $B \subseteq PR$

2⁰. *existential*, in the sense that whenever xRy and y exists: Ey , also x exists: Ex .

The aim of such theories is to prove the existence of *extreme elements* of the relation R , i.e. its first or minimal element, its last or maximal element. The existence of these elements is deduced from the formal properties assigned to the relation under consideration.

The first formalizations by Jan Salamucha¹, Józef Bocheński², Leon Koj³, and Johannes Bendiek⁴ presented the world as a finite chain of moving and moved beings. The relation of moving was supposed to be, according to these authors, irreflexive, asymmetric, transitive

¹ J. Salamucha, *Dowód na istnienie Boga. Analiza logiczna argumentacji św. Tomasza z Akwinu*, *Collectanea Theologica* 15(1935), 53-92.

² J. I. M. Bocheński, *Compte rendu nr 935*, *Bulletin Thomiste* 12(1935), 601-603.

³ L. Koj, *Poglądy ks. Salamuchy na uściślenie filozofii*, manuscript, KUL, Lublin 1954.

⁴ J. Bendiek, *Zur logischen Struktur der Gottesbeweise*, *Franciskanische Studien* 38(1956), 1-38, 296-321.

and total. Salamucha claimed the existence of *primum movens*, i.e. the first element of the relation of moving, while Bocheński claimed that of *movens immobile*, i.e. the minimal element. The proof was after all based on a set-theoretical assumption that finite chains have a first element that is also the minimal element. Bendiek⁵ noticed a very important detail that in the “ways” of Aquinas the principle *non est procedere in infinitum* can not be about the prohibition of endless regress (*Unendlichkeit*) but rather is about the prohibition of beginningless regress (*Anfanglosigkeit*), for infinite chains can also have a beginning. On the other hand, Jan Salamucha⁶ himself commented on the linear model of reality, that “such a conception of the world is not very probable. More suggestive will be the grasp of the world as a bunch of series”. It is worth mentioning, however, that any maximal chain in a partial order relation can be chosen for consideration, and by means of the assumption that it has a minimal element it can be further claimed that the whole relation has a minimal element. Francesca Rivetti Barbò⁷ pointed out that obviously her predecessors’ assumption that the relation of moving is connected was false.

Another conception of linear order in the world was suggested by Peter Geach⁸. For him the world “is the whole composed of parts and [it] undergoes the process of change”, it is “a very great object”⁹. If every such aggregate of bodies as the world in every moment is, originates from the previous one and becomes the next, then this relation of becoming in the field of all stages of the world is a chain. This conception changes the aim of deduction significantly as it is no longer about discovering the first link in the chain. **Having established that no element of the chain contains in itself the sufficient reason, we must conclude that this reason is beyond the chain.**

⁵ Ibid., 10.

⁶ J. Salamucha, op. cit., 87.

⁷ F. Rivetti-Barbò, *La struttura logica della prima via per provare l'esistenza di Dio. Applicazioni di logica simbolica e nessi di contenuti*, Rivista di Filosofia Neoscolastica 52(1960), 241-320.

⁸ P. T. Geach, *Aquinas*, in: G.E.M. Anscombe, P. T. Geach, *Three Philosophers*, Basil Blackwell & Mott Ltd, Oxford 1963.

⁹ Ibidem, 140, 141.

In 1966 Bowman L. Clarke noticed the role of the Kuratowski – Zorn lemma in the proof of the existence of the first mover. The lemma was used in the formalization of the argument *ex motu* by Korneliusz Policki¹⁰. He assumes that the relation of being moved has a non-empty field, is irreflexive and transitive, yet not coherent. He defines the sum of the relation of being moved and identity and obtains a weak order: reflexive, antisymmetric and transitive. He adds the axiom that any two chains in the field of the relation have a common upper boundary. From this and from the Kuratowski – Zorn lemma he proves the existence of the one *primum movens immobile*. The lemma was used in a similar manner by Reihard Kleinknecht¹¹, and I have shown¹² how Policki's set-theoretical calculus can be expressed in first-order language.

The formalizations discussed above did not **define whether the domain** of the relations under consideration was discontinuous (discrete) or continuous (dense). However, it is possible to claim that they mainly concern the relations with a discontinuous field, since the formalizations accept the principle *non est procedere in infinitum*. The first to set it forth explicitly was Laurent Larouche¹³, who accepts the axiom that the relation of genidentity¹⁴ is a dense order¹⁵.

¹⁰ K. Policki, *W sprawie formalizacji dowodu 'ex motu' na istnienie Boga*, Roczniki Filozoficzne 23(1975), 19-30.

¹¹ R. Kleinknecht, *Auswahlaxiom und causa prima*, in: F. Ricken (ed.), *Klassische Gottesbeweise in der Sicht der gegenwärtigen Logik und Wissenschaftstheorie*, Kohlhammer, Stuttgart 1991, 111-123.

¹² E. Nieznański, *Formalizacyjne próby ustalenia logiko-formalnych podstaw stwierdzenia pierwszych elementów rozważanych w tomistycznej teodycei*, in: E. Nieznański (ed.), *W kierunku formalizacji tomistycznej teodycei*, Miscellanea Logica, t. 1, ATK, Warsaw 1980, 7-194; E. Nieznański, *Logical analysis of Thomism - The Polish programme that originated in 1930s*, in: J. Szrednicki (red.), *Initiatives in Logic*, Dordrecht 1987, 128-155.

¹³ L. Larouche, *Examination of the Axiomatic Foundations of a Theory of Change I*, Notre Dame Journal of Formal Logic 9(1968)4, 371-384; parts: II, III, IV resp. Notre Dame Journal of Formal Logic 10(1969)4, 277-284; vide supra, 385-409; vide supra 12(1971)3, 378-380.

¹⁴ 'x is genidentical with y', means that the "same" object is a temporary object x and then a temporary object y.

¹⁵ $x < y \rightarrow \exists z x < z < y$.

A new world model in the formalized systems of theodicy was introduced by Francesca Rivetti-Barbò¹⁶, who used the notion of ancestral closure. If an n -tuple relative product of the relation R is denoted by R_n , then a strong ancestral closure of the relation is defined as $R_{po} = \bigcup_{n=1}^{\infty} R^n$. Apart from the extreme elements of the relation R , the extreme elements of the relation R_{po} can be looked for. The new aim of deduction becomes to prove the existence of the relatively first element (first in regard to a specific element y belonging to the field of the relation R): $x \in IR/y \leftrightarrow x \in \text{Min}R \wedge xR_{po}y$. Another valuable solution offered by Rivetti-Barbò was the replacement of the coherence axiom (which had been rejected in regard to the relations under consideration: that the relation R is connected) by the axiom: $x=y \vee xRy \vee yRx \vee \exists z (zRx \wedge zRy)$. This approach was also adopted in the later formalizations by Rivetti-Barbò, Ivo Thomas¹⁷, and Wilhelm K. Essler¹⁸.

Heinrich Ganthaler and Peter Simons¹⁹ formalized Bernard Bolzano's cosmological proof²⁰ of the existence of God as follows: the primary (indefinable) relation is $xRy =: x$ conditions y (*bedingt*), B – a set of beings, $U = D_p R$ (the counterdomain of the relation R – conditioned beings), $P = B \cap U$ (contingent beings), $K = B - U$ (necessary beings). The fact that the set X is closed in regard to R – i.e. $X(R)$ – is defined as $X(R) \leftrightarrow \forall x \forall y (y \in X \wedge xRy \rightarrow x \in X)$. The adopted axioms are:

1. $B \neq \emptyset$
2. $B(R)$
3. $2^B - \{\emptyset\} \subseteq B$
4. $X(R) \rightarrow X \notin U$

Hence, we have the conclusion $K \neq \emptyset$.

In 1970 Kurt Gödel presented a formalized proof of the existence of *summum bonum*, that is, God. The calculus was modal, however

¹⁶ F. Rivetti-Barbò, op. cit.

¹⁷ I. Thomas, *Review of the Rivetti (1960)*, Journal of Symbolic Logic 25(1960), 347-348.

¹⁸ W. K. Essler, *Einführung in die Logik*, Stuttgart 1969.

¹⁹ H. Ganthaler, P. Simons, *Bernard Bolzanos kosmologischer Gottesbeweis*, Philosophia Naturalis 24(1987), 469-475.

²⁰ B. Bolzano, *Lehrbuch der Religionswissenschaft*, t. I, Salzburg 1834.

Wilhelm Essler²¹ regarded the modality as negligible. In Essler's simplified version, the model of reality is the relational system $\langle 2^B, \subseteq \rangle$. There are three axioms:

1. Supersets of positive classes are positive: $\forall F \forall G (F \subseteq G \wedge F \in Ps \rightarrow G \in Ps)$, where Ps denotes the family of positive classes;
2. Only a particular class or its complement is positive: $\forall F (F \in Ps \leftrightarrow \neg F \notin Ps)$;
3. The product of all positive classes is itself positive: $\bigcap Ps \in Ps$.

In the model, 2^B is Boole's algebra with the inclusion, Ps is a maximal filter (ultrafilter), and *summum bonum* $Gt = \bigcap Ps$. The existence and uniqueness of Gt are guaranteed by the property of the ultrafilter, i.e. that that all its generators are always singletons.

Aquinas's argument *ex possibile et necessario* was formalized by Anthony Kenny²². Let's assume the following, $Qx =: x$ has the *possibilitas* of not being, $Axt =: x$ exists at t ; $n =: \text{now}$. Kenny argues that $\sim \forall x Qx$, thus $\exists x \sim Qx$, because:

1. $\forall x (Qx \rightarrow \exists t \sim Axt)$
2. $\forall x Qx \rightarrow \exists t \forall x \sim Axt$
3. $\exists t \forall x \sim Axt \rightarrow \forall x \sim Axn$
4. $\sim \forall x \sim Axn$

Additionally, numerous formalized systems of theodicy referred to Leibniz, e.g. Krystyna Błachowicz²³ formalized the argument *ex motu* for the existence of God, which was presented by Leibniz in 1666 in *Demonstratio existentiae Dei ad mathematicam certitudinem exacta*.

²¹ W. K. Essler, *Grundzüge der Logik II*, Frankfurt 1987; W. K. Essler, *Gödels Beweis*, in: F. Ricken (ed.), *Klassische Gottesbeweise in der Sicht der gegenwärtigen Logik und Wissenschaftstheorie*, Kohlhammer, Stuttgart 1991, 140-152.

²² A. Kenny, *The five Ways. St. Thomas Aquinas' Proofs of God's Existence*, London 1969, 55.

²³ K. Błachowicz, *Leibniz's 'demonstratio existentiae Dei ad mathematicam certitudinem exacta'. A Logical Analysis*, in: J. Kopania (ed.), *Studies in Logic, Grammar and Rethoric II*, Papers of Warsaw University, Białystok 1982, 43-55; K. Błachowicz, *Leibniz, wczesne pojęcie substancji*, Idea, Białystok 1992.

Jerzy Perzanowski²⁴ reconstructs Leibniz's "empirical-analytical proof of the existence of God" as follows:

1. There would be nothing if God were impossible: $\sim M(G) \rightarrow \sim \exists x Ex$
2. Empirically, however, it is known that something exists: $\exists x Ex$
3. Thus, God is possible: $M(G)$
4. Nevertheless, God is ontologically perfect: $E(G) \leftrightarrow M(G)$
5. Thus, God exists: $E(G)$.

Also, the present author²⁵ formalized Leibniz's **proof of the existence** of an eternal being presented in *Neue Abhandlungen über den menschlichen Verstand* (1704, chapter X).

Referring to Leibniz's conception of the sufficient reason²⁶ and his principle *Nihil fit sine ratione sufficientis*²⁷ I formalized the proof of the existence of the Absolute *ex ratione sufficientis*²⁸. Let ρ denote the relation of the reason of existence and δ - of the sufficient reason. Then we define:

$$x\rho y \leftrightarrow \sim \diamond (Ey \wedge \sim Ex)$$

Hence, the following theses in the modal system T:

1. $\forall x x\rho x$
2. $\forall x \forall y \forall z (x\rho y \wedge y\rho z \rightarrow x\rho z)$
3. $\forall x \forall y (x\rho y \wedge Ey \rightarrow Ex)$

When $x\delta y \leftrightarrow x\rho y \wedge \forall z (z\rho x \rightarrow z=x)$ because of the principle of the sufficient reason: $\forall y (Ey \rightarrow \exists x x\delta y)$ it follows that the Absolute (α) exists. The existence is defined as:

$$\alpha x \leftrightarrow x\delta x$$

²⁴ J. Perzanowski, *Teofilozofia Leibniza*, in: G.W. Leibniz, *Pisma z teologii mistycznej*, Cracow 1994, 322.

²⁵ E. Nieznański, *Gründe, zureichende Gründe und Gottesbeweise ,ex contingentia mundi'*, in: F. Ricken (ed.), *Klassische Gottesbeweise in der Sicht der gegenwärtigen Logik und Wissenschaftstheorie*, Kohlhammer, Stuttgart 1991, 124-139, 125.

²⁶ G. W. Leibniz, *Wyznanie wiary filozofa, Rozprawa metafizyczna, Monadologia, Zasady natury i łaski oraz inne pisma filozoficzne* (1714), transl. by S. Cichowicz, Warsaw 1969, 289.

²⁷ *Ibid.*, 303.

²⁸ E. Nieznański, *Gründe, zureichende Gründe und Gottesbeweise ,ex contingentia mundi'*, op. cit.

When it comes to *ratio sufficientis* it is useful to differentiate between the sufficient reason and the adequate reason. The adequate reason is sufficient but not conversely. The sufficient reason of the being y is the last element in regard to y , $I\rho/y$, i.e. the minimal element in the relation ρ to y . However, there may be more such relatively first elements for the same being y , whereas none of them on its own is the adequate reason of existence of y . I defined the notion of sufficient reason in the sense of adequate reason (2004): $x\delta y \leftrightarrow \forall z (z\rho y \rightarrow x\rho z) \wedge \forall z (z\rho x \rightarrow x=z)$ ²⁹. If y is, for example, the present material world, then according to the above definition the principle *Nihil fit sine ratione sufficientis* directly determines the existence of the unique Absolute.

I prove the thesis: $C \cap PS = \emptyset \wedge C \subseteq B \wedge \forall x \forall b (x \in PS \wedge b \in C \rightarrow b\rho x) \wedge C \subseteq \text{Min}\rho \rightarrow \forall b [b \in C \rightarrow \alpha b \wedge b \in I\rho/PS \cup \{b\}]$, where S is the relation of becoming in the domain of material worlds (PS is the field of the relation S)³⁰. The Absolute is beyond this domain (and beyond time), and is sufficient reason of His own being and of the existence of every world.

Translated by Magdalena Tomaszewska

MODELE ŚWIATA W SFORMALIZOWANYCH SYSTEMACH TEODYCEI

Streszczenie

W artykule prezentuje się różne wizje świata opisywane przez wybrane sformalizowane systemy teodycei. Pierwsze z omawianych formalizacji rekonstruuje argument *ex motu* i prezentują rzeczywistość jako łańcuchową relację poruszania. Dowodzi się w tym przypadku *primum movens* lub *movens immobile*. Ideę łańcucha zakwestionowała Francesca Rivetti-Barbò w związku z fałszywym postulatem o spójności relacji poruszania. Peter Geach zasugerował jednak, że postulat ten da się utrzymać

²⁹ E. Nieznański, *Dowód na istnienie absolutu z zasady nieodzowności dostatecznej racji bytu*, Rocznik Naukowy Wyższej Szkoły Służby Społecznej (Suwałki) 6(2004), 13-23.

³⁰ E. Nieznański, *Philosophische Auswahl unter den möglichen Erklärungen der Wirklichkeit*, in: A. J. J. Anglberger, P. Weingartner (Hrsg.): *Wissenschaftlicher Atheismus kritisch betrachtet*, Frankfurt 2009, 225-234, 234.

w przypadku relacji stawiania się światów materialnych. Korneliusz Policki wyznaczył z kolei wizję świata, którą generuje częściowy porządek relacji poruszania. W swoim dowodzie istnienia *primum movens immobile* wykorzystał lemat Kuratowskiego-Zorna. Rivetti-Barbò i Ivo Thomas zaproponowali wizję świata opartą na quasi-półstrukturze multiplikatywnej. Kurt Gödel założył natomiast, że świat jest algebrą Boole'a, w której generatorem ultrafiltru własności pozytywnych jest *summum bonum*. Na koniec charakteryzuje się formalizacje prezentujące Leibniza koncepcję świata, w której kluczową rolę pełni zasada dostatecznej racji istnienia.

Słowa kluczowe: koncepcje Boga, teodycea formalna, dowody istnienia Boga