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EFFICIENCY AND RETURNS TO SCALE – A CONCEPT OF USING DETERMINISTIC APPROACH[1]

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Abstract: In the paper the author considered estimation of efficiency, which measures the ability of the company to obtain the maximum output from given inputs. The analysis has been carried out by using deterministic method (on the example of the DEA method, Data Envelopment Analysis). The two-step procedure was proposed, namely the returns to scale were specified in the sample and, basing on that, the efficiency of individual enterprises was assessed. In the paper the data from the companies of a key food processing sector in Poland, namely the meat processing, was used. The analysis covered the period 2006–2011, the sample covered from 195 up to 210 enterprises (depending on the analyzed year).

Keywords: efficiency, economies of scale, the DEA method, food processing sector

INTRODUCTION

The aim of the article was to propose an approach to measuring the basic economic category, namely efficiency, by using a non-parametric DEA method. A two-step procedure was applied. Firstly, the returns to scale were specified in the sample, and then the efficiency of individual enterprises was assessed using a model that takes into account the previously determined returns to scale. The discussed method has been applied to the meat processing industry in Poland. This sector was chosen due to the large size of the sample, as well as the strategic

[1] Praca naukowa finansowana ze środków na naukę w latach 2011-2013 jako projekt badawczy nr 2011/01/B/HS4/02612 *Stopień integracji w łańcuchu dostaw a efektywność przedsiębiorstw przetwórstwa rolno-spożywczego.* Projekt został sfinansowany ze środków Narodowego Centrum Nauki przyznanych na podstawie decyzji numer DEC-2011/01/B/HS4/02612.

importance and significant contribution to the production of the entire agri-food sector. Furthermore, with respect to the meat processing sector there are no comparative analyzes carried out, which justifies the need for their conduction.

In the literature, there are concepts of economic efficiency that determines the ratio of outputs achieved and inputs used. T.J. Coelli, D.S.P. Rao, Ch.J. O`Donnell and G.E. Battese, that refer to the dual approach to the issue in their researches on the efficiency, argue that the efficiency ratio increases by maximizing outputs with given inputs (an output-oriented approach), or by minimizing inputs with given outputs (an input-oriented approach). A company uses materials, labour and capital (inputs) in order to manufacture the final product (output), on the basis of which the authors define efficiency of companies as their ability to transform inputs into outputs.[2]

In the discussion on the efficiency the returns to scale are an important aspect. A manufacturer gains increasing returns to scale, if he achieves more than a proportional increase in production as a result of an increase in involved production factors. If this increase is proportional, we are dealing with constant returns to scale. In the case of an increase in production that is smaller than an increase in involved production factors one can say about decreasing returns to scale.[3]

THE DEA METHOD FOR ASSESSING THE RETURNS TO SCALE

The purpose of the appliance of the DEA method was to determine the efficiency for individual enterprises. The assumptions of this method were presented in this part of the paper. Due to the adoption of the two-step procedure for the efficiency assessment, an approach for determining the returns to scale using the DEA method was included in the content of this chapter.

The nonparametric DEA method was developed in 1978 by Charnes, Cooper, Rhodes.[4] The DEA is a method which assumes no random component and does not require functional form relating inputs to outputs for each of the analyzed DMUs[5].[6] The efficiency score is calculated by using the Debreu-Farrell measure. According to definition of Debreu and Farrell, the measurement of technical efficiency is the

[2] Coelli T.J., Rao D.S.P., O`Donnell Ch.J., Battese G.E. (2005) An introduction to efficiency and productivity analysis, 2. Edition, Springer, New York.

[3] Rembisz W. (2011) Analityczne właściwości funkcji produkcji rolniczej, Komunikaty, Raporty, Ekspertyzy, nr 544, Wyd. IERiGŻ-PIB, Warszawa, p. 18.

[4] Charnes, A., Cooper, W., Rhodes A. (1978) Measuring the Efficiency of Decision Making Units. „European Journal of Operational Research", 2 (6), p. 429.

[5] Decision making units (DMUs) are known as the objects of analysis, see: Cooper W., Seiford L., Tone K. (2007) Data Envelopment Analysis. A comprehensive text with models, applications, references, 2. Edition, Springer-Verlag, Berlin, p. 6-12.

[6] Cooper W., Seiford L., Tone K. (2007) Data Envelopment Analysis. A comprehensive text with models, applications, references, 2. Edition, Springer-Verlag, Berlin, p. 13.

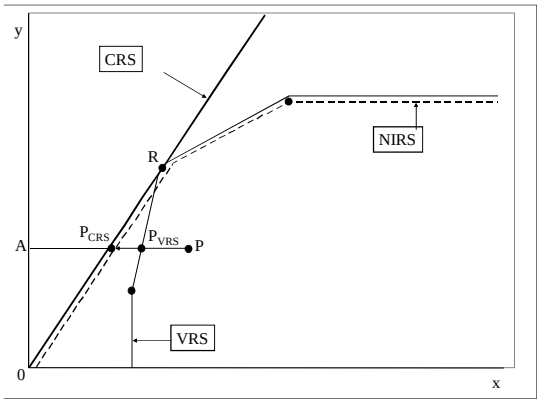
difference between one and the maximal possible reduction of inputs, while production of a certain volume of inputs is technologically possible.[7]

Depending on what is the aim of the analyzed DMUs, minimizing inputs or maximizing outputs, one can calculate the input-oriented technical efficiency or the output-oriented technical efficiency. Besides the input- and output-oriented DEA model, there is a further division including returns to scale. There are following models distinguished:

- CRS – Constant Returns to Scale
- VRS – Variable Returns to Scale
- NIRS – Non-Increasing Returns to Scale[8]

In 1984, Banker, Charnes and Cooper proposed a development of the model assuming constant returns to scale to the model with variable returns to scale. However, it is not possible to determine whether a tested DMU shows increasing or decreasing returns to scale. In 1985, Färe, Grosskopf and Lovell modified the BCC model, complementing it with the additional assumption of convexity,[9] which resulted in the creation of the model assuming non-increasing returns to scale (*NIRS*).

Figure 1. The DEA models (VRS, CRS and NIRS) and returns to scale



Source: own work based on Coelli T.J., Rao D.S.P., O`Donnell Ch.J., Battese G.E. (2005) An introduction to efficiency and productivity analysis, 2. Edition, Springer, New York, p. 174.

[7] Fried H.O., Lovell C.A.K., Schmidt S.S. (1993) The Measurement of Productive Efficiency Techniques and Applications. Oxford University Pres, New York, Oxford, p. 10.

[8] The NIRS includes the decreasing (DRS, Decreasing Returns to Scale) or constant returns to scale (CRS, Constant Returns to Scale). More about the DEA models in: Jarzębowski S. (2011) The efficiency of grain milling companies in Poland and in Germany- application of DEA method and Malmquist index, Universität Bonn-ILB Press, Bonn.

[9] Coelli T.J., Rao D.S.P., O`Donnell Ch.J., Battese G.E. (2005) An introduction..., op. cit.

Under the assumption of constant returns to scale, efficient DMUs create the CRS efficiency frontier, however, assuming that returns to scale are variable, the VRS efficiency frontier is created (see figure 1, where x – set of inputs, y – set of outputs).

Depending on which assumption of returns to scale will be adopted, technical efficiency is equal to[10]:[11]

- the ratio AP_{CRS}/AP for constant returns to scale (the ratio is equal to the technical efficiency at constant returns to scale - TE_{CRS});
- the ratio AP_{VRS}/AP for variable returns to scale (the ratio is equal to the technical efficiency at variable returns to scale - TE_{VRS}).

The ratio of AP_{CRS}/AP_{VRS} determines the efficiency of scale. The efficiency of scale can be (see figure 1) interpreted graphically as the ratio of the mean score of a given DMU in point P_{VRS} to the mean score in point R (point of technically optimal production scale).[12] The value, however, does not indicate whether the given DMU is in the area of increasing or decreasing returns to scale. The nature of returns to scale[13] (caused by both increasing and non-increasing returns to scale) can be determined by analyzing the results of the technical efficiency obtained in the NIRS and VRS model. If these values are not equal, it indicates the presence of increasing returns to scale in the DMU. However, if these values are equal, the DMU is characterized by decreasing returns to scale (if additionally the values obtained in the VRS and CRS models are equal, the DMU shows constant returns to scale).[14]

THE RETURNS TO SCALE AND EFFICIENCY OF ANALYSED ENTERPRISES

The study was carried out on the basis of data collected from meat processing enterprises across Poland, for which financial statements were available (panel data for the period 2006–2011). The sample covers from 195 up to 210 companies, depending on the analyzed year (including micro, small, medium and large enterprises). The production data is reported as revenue/expenditure denominated in PLN in constant prices. The production frontiers are fitted for a single output and two inputs. The inputs are: value of fixed assets (x_1), operating costs (x_2), and the output is net revenues from sales of goods and materials (y).[15]

[10] The technical efficiency was determined on the example of point P. The point is not neither on the CCR efficiency frontier nor on the VRS efficiency frontier, therefore the DMU is considered to be inefficient.

[11] Coelli T.J., Rao D.S.P., O`Donnell Ch.J., Battese G.E. (2005) An introduction..., op. cit., p. 173.

[12] Ibidem, p. 173.

[13] Increasing returns to scale occur when outputs increase faster in relation to growth of used inputs. Decreasing returns to scale occur, when outputs increase slower in relation to growth of used inputs. Constant returns to scale occur when outputs increase proportionally to growth of used inputs. Non-increasing returns to scale occur when outputs increase slower or proportionally to used inputs.

[14] Ibidem, p. 174.

[15] The source of data was the MONITOR POLSKI B where financial statements are published.

Specification of the DEA model

The DEA method (Data Envelopment Analysis) is a relatively modern tool based on a non-parametric approach to the creation the efficiency frontier.[16] In the DEA method, a system of linear equations is being solved, usually using the Simplex method. [17] With this technique, one can simultaneously process a large number of variables, taking into account the internal relations in an enterprise with the assumed objective function.[18] In the case of the DEA method it is possible to use several optimization models. They differ among others in assumptions about the returns to scale in the sample. While choosing a model one bases on the expertise or practices used by other research groups (similarly to the choice of variables), nevertheless the model selection affects the value of the obtained efficiency ratios. As shown in the previous part of the article, the calculation of three models, namely VRS, NIRS and CRS, is needed in order to determine the returns to scale.[19] An algebraic form of the models was followed after J. Zhu and W.D. Cook.[20]

The CRS model is presented in equations 1-4:

$$\max_{\phi_k, \lambda_k} \phi_k \tag{1}$$

$$\phi y_k \leq \sum^{I} \lambda_{ik} y_i \tag{2}$$

$$x_{nk} \geq \sum_{i=1}^{I} \lambda_{ik} x_{ni} \tag{3}$$

$$\lambda_{ik} \geq 0, \tag{4}$$

Equations 5-9 include the NIRS model:

$$\max_{\phi_k, \lambda_k} \phi_k \tag{5}$$

$$\phi y_k \leq \sum_{i=1}^{I} \lambda_{ik} y_i \tag{6}$$

[16] Rembisz W., Sielska A., Bezat A. (2011): Popytowo uwarunkowany model wzrostu produkcji rolno-żywnościowej, Wyd. IERiGŻ-PIB, Warszawa, p. 108.

[17] The linear programming has been commonly used since the 60's, when the Simplex algorithm was applied for problems of farmers, see: Zapf R. (1965) Zur Anwendung der linearen Optimierung in der landwirtschaftlichen Betriebsplanung, Berichte über Landwirtschaft..

[18] Steffen G., Born D. (1987) Betriebs- und Unternehmensführung in der Landwirtschaft, UTB für Wissenschaft, Stuttgart.

[19] No assumptions regarding returns to scale were made in advance. It was assumed that the functions are homogeneous of degree +1 for $\lambda > 0$. A set of observations is convex if for any two points in the set, all weighted average of these two points are also points in the same set.

[20] Zhu J., Cook W.D. (2007) Rank Order Data in DEA, Interval and Ordinal Data, [w:] Zhu J., Cook W.D. (red.): Modeling Data Irregularities and Structural Complexities in Data Envelopment Analysis. Springer, New York, p. 13-62.

$$x_{nk} \geq \sum_{i=1}^{I} \lambda_{ik} x_{ni} \tag{7}$$

$$\lambda_{ik} \geq 0, \tag{8}$$

$$\sum_{i=1}^{I} \lambda_{ik} \leq 1 \tag{9}$$

The VRS model is presented in equations 10-14:

$$\max_{\phi_k, \lambda_{ik}} \phi_k \tag{10}$$

$$\phi y_k \leq \sum_{i=1}^{I} \lambda_{ik} y_i \tag{11}$$

$$x_{nk} \geq \sum_{i=1}^{I} \lambda_{ik} x_{ni} \tag{12}$$

$$\lambda_{ik} \geq 0, \tag{13}$$

$$\sum_{i=1}^{I} \lambda_{ik} = 1 \tag{14}$$

where:

k – index indicating an analyzed object,

ϕ_k – multiplier of output level for an object k,[21]

i – index indicating next object $i=1,\ldots,I$, where I is a number of objects in the sample,

y_i – output of an object i,

n – index of next input,

x_{ni} – input n used by an object i,

λ_{ik} – coefficients of linear combination between objects i and k.

For all models the output-orientation was adopted, which was expressed in the objective function of the optimization problem (equations: 1, 5, 10). The output-orientation means that by a given level of input higher and higher level of output may be achieved. On the basis of the literature review it was stated that the purpose of the business activity is the profit maximization (output of the activity), which confirms the correctness of the use of the DEA method with output-oriented models. The evaluation was conducted by using the EMS software.[22]

[21] This is the inverse of the efficiency coefficient.

[22] EMS, Efficiency Measurement System, Scheel H. (2000): Software Version 1.3, University Dortmund [July 2013].

Evaluation of the returns to scale and efficiency in the analyzed enterprises

The DEA method was used for the means of the article, as it provides a lot of detailed information on the individual objects in the sample, among others on returns to scale for individual objects.

Table 1 includes the synthetic results of the analysis of the returns to scale for individual enterprises (calculated using the procedure presented in the paper). The percentage share of the companies characterized by decreasing, increasing and constant returns to scale was determined. The results are summarized by year. The "DRS" (Decreasing Returns to Scale) indicates that companies are characterized by decreasing returns to scale, the "IRS" (Increasing Returns to Scale) – companies with the increasing returns to scale, and the "CRS" (Constant Returns to Scale) – companies characterized by the constant returns to scale. The statement about returns to scale was made basing on the Fig. 1.

On the basis of the conducted analysis it was observed that the majority of the companies operating in the meat processing sector is characterized by decreasing returns to scale (within the period 2006-2011).

Table 1. Companies of meat processing sector with the returns to scale within the period 2006-2011

Year / returns to scale	2006	2007	2008	2009	2010	2011
DRS	86%	86%	86%	85%	85%	60%
IRS	12,5%	13%	13%	13%	13%	38,5%
CRS	1,5%	1%	1%	2%	2%	1,5%

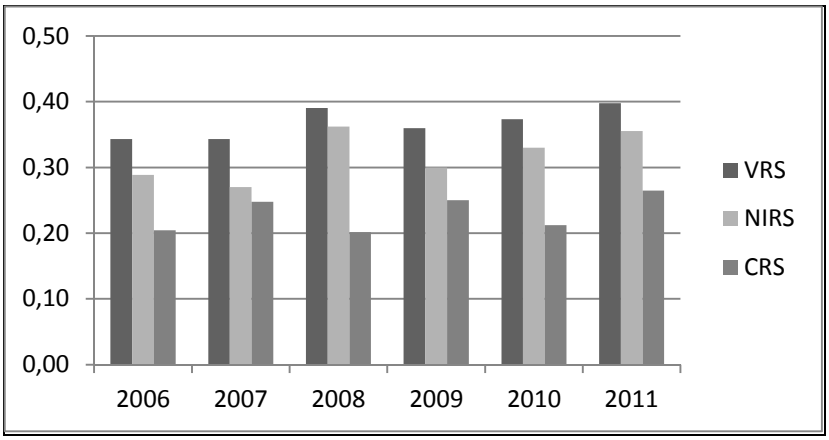
Source: own calculations using EMS software

Hence, most of the analyzed companies does not achieve the returns to scale, so the obtained increase in production is less than the increase in involvement of the production factors. According to R.D. Banker, W.W. Cooper, L.M. Seiford, R.M. Thrall, J. Zhu,[23] the presence of non-increasing returns to scale was assumed. This was the basis for the model specification.

In order to determine the efficiency on the basis of the returns to scale, the NIRS model (presented in equations 6-10) was adapted for the analyzed sample. The average efficiency ratios obtained using the NIRS model were presented in Figure 2. For a comparison, the efficiency ratios that would be obtained using the VRS and CRS models were also included in the analysis. The achieved efficiency ratios were presented by year (Figure 2).

[23] Banker R.D., Cooper W.W., Seiford L.M., Thrall R. M., Zhu J. (2004) Returns to scale in different DEA models, European Journal of Operational Research, Vol. 154, p. 359.

Figure 2. The average efficiency ratio calculated with use of the DEA method within the period 2006-2011[24]



Source: own calculations using EMS software

On the basis of the results showed in Figure 2 it can be stated that the average value of the efficiency ratio in the analyzed period was different depending on the model used. For the NIRS model with non-increasing returns to scale, the average efficiency ratio varied from 0,27 in 2007 to 0,36 in 2008 and 2011. The ratios obtained using the VRS model (Variable Returns to Scale) would be on average 15% higher compared to the results of the NIRS model. While using the CRS model (Constant Returns to Scale), the efficiency ratios would be on average 25% lower than the results of the NIRS model. The differences in the results are determined by assumptions of the DEA method, which was explained in the privies part of the paper. Technical efficiency score for constant returns to scale is not equal to efficiency score for variable returns to scale (see figure 1).

In the context of the conducted analysis, the proposed approach, according to which the returns to scale dominated in the sample are determined and then the DEA model specification is prepared for the efficiency measurement, seems to be appropriate. The proposed way of the model selection allows making a choice of the adequate model for a given sample with regards to the returns to scale and as a result it allows to a proper interpretation of the results. Basing on the conducted analysis it was claimed that in the analyzed sector in each year there is a place and necessity for efficiency improvement through effectively used manufacturing techniques reflecting use of inputs in order to manufacture the output.

[24] Due to the fact that while using the DEA method the relative efficiency is determined, there is no possibility to compare the results between years, see: Bezat A. (2012) Efficiency of Polish grain trade companies: an integrated application of SFA and DEA methods, Universität Bonn-ILB Press, Bonn.

SUMMARY

A company uses inputs in order to manufacture the output, on the basis of which the authors define efficiency of companies as their ability to transform inputs into outputs. A manufacturer gains increasing returns to scale, if he achieves more than a proportional increase in production as a result of an increase in involved production factors. If this increase is proportional, we are dealing with constant returns to scale. In the case of an increase in production lower than an increase in involved production factors one can say about decreasing returns to scale.

The deterministic tools, which analytical background is an optimization problem, (e.g. the DEA method, Data Envelopment Analysis) are methods for assessing the returns to scale, as well as efficiency. The methods require all decision making units to have comparable inputs and outputs and can handle multiple input and multiple output models.

The DEA method was applied for the assessment of the returns to scale of enterprises operating in the meat processing sector. The proposed approach involves determining the returns to scale that dominate in the sample, then the specification of the DEA model was prepared in order to assess the enterprises' efficiency. As demonstrated within the framework of the conducted analysis, the proposed way of model selection allows making a proper model specification and achieving right results and formulate reliable conclusions. It was also indicated in the article that basing – while determining the returns to scale in a given sample – on the expertise or practices used by other research groups affects the value of the obtained efficiency ratios. Hence the article contributes to the discussion on the methodological considerations about the key economic issues of efficiency measurement.

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