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David I. Spivak, *Category Theory for the Sciences*, The MIT Press Cambridge, Massachusetts London, England: 2014, pp. 486

The reviewed book – *Category Theory for the Sciences* – is a textbook for teaching category theory for students in field of science (e.g. physics, neuroscience, and computation) for the purposes of modelling language through the sciences. Furthermore, this book is a development of a textbook offered by Spivak: *Category Theory for the Scientist*. The intention of the book is to craft a bridge between the world of mathematical concepts that are commonly used by mathematicians and in the world of the sciences: the phenomena of studies, and our frameworks and models in scientific disciplines. The purpose is to describe the phenomena of the sciences with a universal language of category theory.

Category theory was invented by Samuel Eilenberg and Saunders Mac Lane in the early 1940's. The idea during that time was to create a link between two fields in mathematics: topology and algebra. However, in doing so, one unintended consequence was that the the idea worked out to be an adequate framework for not only topology and algebra, but many other mathematical fields as well. Soon after, Bill Lawvere offered a radical and ambitious idea about category theory: that it would serve as a new foundation – instead of set theory – as the foundation of mathematics. Lawvere allegedly showed that set theory was a category of sets and that category theory was more general. Moving to the 1980's Joachim Lambek used category theory in computer science.

The reaction to category theory among mathematicians was that it was far too abstract for them in the 1980's; but in the 21st century, category theory is now used – at least – by the community of pure mathematicians in the graduate level of American university. In fact, according to Spivak, it is the language of choice today for graduate level algebra and topology courses. It is a part of the contemporary mathematician's toolkit. It is important to note that, historically, category theory is not strictly limited to mathematicians – it is also a concern for logicians and computer scientists.

Moving to the book, Spivak extends category theory from being described as esoteric – a concern for only the theoretical sciences – to something practical. Meaning that these concepts are accessible to even those who only have a minimal mathematical background (declared to be linear algebra). Namely a mathematical conceptual structure for the the practical sciences.

The book is structured into seven chapters and additionally there is a reference and index section. Chapter one is an introduction to category theory. Items listed in this chapter is the brief history of category theory, the intention of the book, what is expected from the students reading the book, and additional literature on category theory. Chapter two is "The Category of Sets". Spivak defines for us the category of sets, functions, commutative diagrams, and ologs. "Ologs" – or ontology logs – are a new notion developed by Spivak; it consists of a series of operations in order to bridge mathematics with various conceptual landscapes. In the third chapter "Fundamental Considerations in Set", the goal is to teach how to combine sets to get new sets. Other parts of the chapter are concepts of products and coproducts, finite limits in the category of set, finite colimits in the category of set, and other notions in the category of set. Chapter four is called "Categories and Functors, without admitting it". The main concepts that are presented are monoids, groups, graphs, orders, and databases: scheams and instances. The goal of this chapter is to use what has been understood by "sets" to look at mathematical worlds which is organized by a certain kind of domain. The chapter discusses classical objects from mathematics until the concept of "database" is introduced. Database is a classical object from computer science and the rest of the chapter discusses computer science.

In the fifth chapter we finally confront category theory itself; as such the title is called "Basic Category Theory". The concepts that are covered are as follows: Categories and functors, common categories and functors from pure math, natural transformations and categories and schemas are equivalent. *Category C* is defined as follows: One announces some constituents (A. objects, B. morphisms, C. identities, D. compositions) and shows that they conform to some laws (1. identity law, 2. associativity law). Specifically, once announces:

A. a collection $\mathrm{Ob}(C)$, elements of which are called *objects*;
B. for every pair $x, y \in \mathrm{Ob}(C)$, a set $\mathbf{Hom}_c(x, y) \in \mathbf{Set}$; it is called the *hom-set from z to y*; its elements are called *morphisms from z to y*;
C. for every object $x \in \mathrm{Ob}(C)$, a specified morphism, denoted

$\mathbf{id}_x \in \mathbf{Hom}_c(x,x)$, and called *the identity morphism on x*;

D. for every three objects $x, y, z \in \mathrm{Ob}(C)$, a function
$\circ$: $\mathbf{Hom}_c(y, z) \times \mathbf{Hom}_c(y, x) \to \mathbf{Hom}_c(x, z)$
called *the composition formula*.

Given objects $x, y \in \mathrm{Ob}(C)$, we can denote a morphism $f \in \mathbf{Hom}_c(x, y)$ by f: $x \to y$; we say that x is the *domain* of f and that *y is the codomain* of f. Given also $g : y \to z$, the composition formula is written using infix notation, so $g \circ f : \mathrm{x} \to z$ means $\circ\,(g,f) \in \mathbf{Hom}_c(x, z)$.

One must then show that the following category laws hold:

1. For every $x, y \in \mathrm{Ob}(C)$ and every morphism $f : x \to y$, we have

$$f \circ \mathbf{id}_x = f \text{ and } \mathbf{id}_y \circ f = f.$$

2. If $w, x, y, z \in \mathrm{Ob}(C)$ are any objects, and $f : w \to x$, $\mathrm{g} : x \to y$, and $h : y \to z$ are any morphisms, then the two ways to compose yield the same element in $\mathbf{Hom}_c(w, z)$:

$$(h \circ g) \circ f = h \circ (g \circ f) \in \mathbf{Hom}_c(w,z).$$

After the given definition above there is a remark that includes a brief explanation as to why objects $\mathrm{Ob}(C)$ of C are said to be a collection rather than a set, the collection of sets is not a set itself: to avoid Russell's paradox. While the model is not given, it is citied as being modelled by Grothendieck's notion of expanding universes.

Chapter six is "Fundamental Considerations of Categories". The chapter is about the limits and colimits in a given category C. The chapter also covers categorical constructions such as the simple notion of opposite categories and the Grothendieck construction. In Chapter seven we find "Categories at Work". The last chapter covers the topics of adjoint functors (or adjunctions) which functions much like a dictionary translating between different categories. This is interesting because of problems that arise from the philosophy of language such as Quine's indeterminacy of translation. The rest of the chapter covers categories of functors, monads, and operads.

As for the general structure of the textbook it should be noted that while the exercises are plentiful and useful, it might be helpful for the student to try to apply more exercises to various definitions; furthermore, while it is easily accessible to obtain the proposed solution to the exercises, there is a pedagogical issue of easily tempting a frustrated student to look at the proposed solution. The problems are relevant to real world problems in the sciences. The sequencing and organization of the chapters is logical.

In summary, this textbook covers what it aims to teach. It is the first of its kind to try and teach category theory on a practical level to the uninitiated in advanced mathematics (of course this is not to say that you do not need mathematics – in fact there is a certain level of mathematics assumed on the part of the student, again linear algebra).

Even though this book is not primarily aimed at philosophical interests, it is worth commenting that that there are philosophically interesting aspects of the book such as assumed answers to problems in the philosophy of language. The book itself is ambitions for the domain it aims to cover; nevertheless, it could provide invaluable and robust tools for various scientist to communicate in.

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