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Some equations to identify the threshold value in the DEMATEL method

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Abstract

DEMATEL technique is a graphical representation method to deal with complex systems. The final analyzed cause and effect categorization would be fundamentally dependent on the threshold value setting. This research is intended to present some mathematical models for calculating the threshold value in the DEMATEL method. The min(max) operator has been intentionally used for considering three equations to identify the threshold value. Additionally, the proposed mathematical equations are gradually developed to gain more useful data to yield a threshold value as well. Particularly, the expert's initial scoring for building the primary matrix would also be applied in one equation. Results show eliciting an expert's opinions regarding the value of a threshold value determination leads to setting relatively high thresholds. But, there would be an equation which takes advantage of more data derived from the total influence matrix T. Moreover, a span of different threshold values is gained by making use of the Hamacher t-conorms operator which especially would cause better complexity management of the final total matrix T based on expert's opinions. As a contribution to this research, threshold value determination is developed mathematically by making use of the direct data gained by the total matrix T. Besides combining data derived from total matrix T, the initial influence direct matrix given by experts, a simpler aggregating procedure and no need for statistical information compared to special Lenth's method hints at this research's novelty as well.

Keywords: *DEMATEL method, min(max) strategy, threshold value, mathematical model*

1. Introduction

Structural modelling has been considered as a view of a system that emphasizes the structure of the objects, including their different classes, relationships, attributes and operations. A structural model is inherently purposed to depict a graphical diagram which consists of a set of nodes and connections between the nodes. It should be noticed that in this diagram the nodes are usually considered as the main factors influencing the total operation of a system and besides the connections are shown by numbers or some directed vectors which are logically derived from the mathematical model deployed to simulate a real complicated problem. Generally speaking, structural diagrams are being used to understand

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some aspects of a system's complexity particularly the interrelationships between its components. In this sense, the casual inter-dependency and inter-relationship among a group of factors are the two vital and mandatory features which have been proposed by structural modelling diagrams. Briefly, we are keen to realize and visualize the critical cause-and-effect relationships between the factors/criteria influencing the whole system's performance for better-manipulating decisions and evaluating some major factors/criteria. Therefore a great deal of attention has been set by researchers to design useful techniques for presenting comprehensive approaches to scientifically investigate cause and effect pattern recognition using graphs and matrices.

Decision-making trial and evaluation laboratory (DEMATEL) introduced by Fontela and Gabus [4] as a kind of structural modelling has been widely used for analyzing the cause and effect relationships among elements of a system. The DEMATEL method has been also extended to delineate graphically the real-world systems, especially those associated with uncertainty and imprecise data involved in problems. Nowadays, the DEMATEL method is recognized as an effective structural technique due to its identification of cause-and-effect relationships among a group of known factors in a complex system. The DEMATEL method can convert the interrelations between factors into an intelligible structural model of the system and divide them into a cause group and an effect group [6].

Threshold value determination would be a significant part of the DEMATEL method. As a matter of fact, threshold value setting could cause a real complex problem to be more complex and unreachable or more simplified to discuss in management manipulating due to an unreasonable configuration. However, threshold value determination in the DEMATEL method can be addressed by some mathematical equations [5, 12] (for example the average method) in which experts are all satisfied or in some other ways in which experts clarified and come to a compromising point. On the other hand side, threshold value setting as an essential attribute of the DEMATEL method would be usually dependent on the existing level of management controllability of real problems. Moreover, there would be no evidence of proofed mathematical methodologies found to easily and preferably describe the threshold value determination. But, all studies related are being investigated due to the essence of problems and data being derived from the real problem structure.

2. Literature review

Over the past decades, the DEMATEL method and its related applications have been particularly applied and studied in a lot of scientific areas for better-making decisions. Finding the key barriers to the implementation of green supply chain management by Wang et al. [24], identifying critical risks in sponge city PPP projects using the DEMATEL method by Zhang et al. [25], evaluating the key factors affecting customer satisfaction in an internet banking system by Asad et al. [2], applying fuzzy the DEMATEL to explore the decisive factors of the auto lighting aftermarket industry in Taiwan by Li et al. [13] and identifying the key performance evaluation criteria for achieving customer satisfaction through balanced scorecard (BSC) by Pan and Nguyen [17] are typical applications of the DEMATEL method. Besides, some hybrid mathematical models including the DEMATEL method have been applied for industrial applications and decision-making problems. Suggesting a novel approach to group multi-criteria decision--making based on interval rough numbers using a hybrid model of the DEMATEL-ANP-MAIRCA by

Pamučar et al. [16], application of AHP and the DEMATEL methods in choosing and analyzing the measures for the distribution of goods in Szczecin region by Kijewska et al. [7], supplier selection using fuzzy the DEMATEL and fuzzy AHP by Ergun and Kuruoglu [22], a combination of the DEMATEL and BWM-based ANP methods for exploring the green building rating system in Taiwan by Liu et al. [15], structural modeling (ISM) and decision-making trail and evaluation laboratory (DEMATEL) method approach for the analysis of barriers of waste recycling in India by Chauhan et al. [3], a hybrid method of the Fuzzy DEMATEL/AHP/VIKOR approach to rank and select the best hospital nurses by Taati and Esmaili-Dooki [21], integrated fuzzy-DEMATEL and fuzzy-TOPSIS approaches for supplier selection by Agrawal and Kant [1] are some typical applications of hybrid models comprising the DEMATEL method.

Sheng-Li et al. [20] reviewed the DEMATEL methods and their applications comprehensively and technically. Their study showed vastly and pervasively the DEMATEL usability for different situations of decision-making and various industrial applications. Computer science, engineering, business and management, and decision sciences are the four major categories discussed in this study. Koca and Yıldırım [9] conducted a bibliometric analysis of the studies evaluated with the DEMATEL. As a special contribution, they showed the practical evolution of the DEMATEL in many different fields of study. In another study, Koca et al. [8] applied the DEMATEL method to evaluate the dimensions of the smart city concept. So, it would not be far from the truth to say that the DEMATEL has found its relevant utilization routes, while its basic mathematical approaches are still being improved.

Determination of the threshold value in the last step of the DEMATEL method would be a challenging part discussed by practitioners and experts to manage the complexity of a system. On the other hand, setting a reasonable level of threshold value would comprehensively result in delineating the causal effect of interrelationships being actually cleared in the final impact-relationship map. Although there have been some investigations and relevant studies [5, 12] discussing threshold value set to filter the total matrix T. The main goal of this study is to obtain threshold values by simply using data derived only from the total matrix T and the initial influence direct matrix. The easier calculating procedures (just applying min(max) operator and some usual arithmetic operators), particularly applying a few predefined parameters (directly derived from total matrix T) and initial influence direct matrix needed to yield threshold value would determine the novelty of our proposed equations.

The remaining part of this study consists of the following. Section 3 is devoted to reviewing the DEMATEL method and challenges accompanied by threshold value determination. Section 4 is designated to equation four models for threshold identification along with presenting some numerical studies. Discussion is brought in Section 5 and finally, the conclusion would also be expressed in Section 6.

3. DEMATEL method

As mentioned before, it would be clearly perceived that the DEMATEL method has been pervasively adopted in many areas of scientific interest. Accordingly, this Section is intended to review the classical DEMATEL method. Here are the following steps of the DEMATEL listed below:

Step 1. Building the primary relationship evaluation matrix M between the n-factors in a system (which was identified and averaged by some experts. The integer scale of no (0), low (1), medium

(2), high (3), and very high (4) influence is used to determine the group direct influence matrix $M = [m_{ij}]_{n\times n}$).

Step 2. Normalizing the direct influence matrix. The normalized direct influence matrix N can be achieved as follows:

$$N = M/k, \quad k = \max\left(\max_{1\le i\le n}\sum_{j=1}^{n} m_{ij},\ \max_{1\le j\le n}\sum_{i=1}^{n} m_{ij}\right) \tag{1}$$

where $\max_{1\le i\le n}\sum_{j=1}^{n} m_{ij}$ represents the total direct effect of criterion i applied on other criteria and $\max_{1\le j\le n}\sum_{i=1}^{n} m_{ij}$ represents the total direct effect which the criterion j receives from other criteria. Note that the normalized initial-direct relation matrix must have all the column sum of each column less than one as a sufficient condition for the DEMATEL feasibility [10].

Step 3. Calculate the total influence matrix $T = [t_{ij}]_{n\times n}$ by the following equation. The total influence matrix T is computed by summation of consecutive powers of the normalized matrix N

$$T = \lim_{k\to\infty}\left(N + N^2 + \cdots + N^k\right) = N(I-N)^{-1} \tag{2}$$

where I is the identity matrix with the same dimension of matrix N.

Step 4. Set a threshold value and draw the graphical influential relation map (GIRM) showing the final influential relationship between the criteria. In this step, we have the following values: r is the sum of the rows of the total influence matrix T, c – the sum of the columns of the total influence matrix T. Thus the cause and effect groups can be separated from each other by the element of $(r_i - c_i)$. The criterion i would be the effect factor if $(r_i - c_i)$ is negative and vice versa it would be the cause factor if $(r_i - c_i)$ is positive (for $i \in \{1, 2, \dots, n\}$). For filtering out the negligible effects in matrix T, a threshold value is determined to revise the matrix T.

Completing these steps resulted in depicting a valuable map as a structural explanation of prominent factors that can be used helpfully for decision-making.

3.1. Challenge with identification or setting the threshold value

As stated before, in the fourth step of the DEMATEL method, an appropriate threshold value must be set to filter out the numbers in matrix T in order to attain a suitable and reasonable map of critical relations (GRIM). However, this phase of the DEMATEL method is structurally used to simplify the visual map of the relationship as much as possible and also to achieve adequate information for further analysis and decision-making issues as well. But, the determination of threshold value has its own challenges due to the lack of unity of procedures expressed by some experts and it may differ from one study to another study. Emphatically, setting a threshold value can reduce the complexity of the structural model (GRIM) constructed by the matrix T. On the other hand, the complexity decrement of GRIM would be fundamentally dependent on the magnitude of a threshold value. If the threshold value is too low, many factors are included and the GRIM will be too complex to comprehend. In contrast, some important factors may be excluded if the threshold value is too high [20]. Generally, there are numerous ways of calculating the threshold value found in some studies done by a few scholars so far. Threshold value determination by experts [14, 20], using the maximum mean de-entropy technique [4, 12], the classical method of averaging of numbers in matrix T [18] and the Lenth's principles of distinguishing effect significance [11] are some familiar methods for filtering out the original matrix T in the DEMATEL method. As it is now clear, the DEMATEL method can assist to model the interrelationships among factors in a complex system, but still, there would be a vital challenge regarding identifying the threshold value for further analysis. More importantly, there have been no sturdy and easily judged mathematical comparative modifications using the different techniques to set a threshold value. However, it may be rooted in discovering the critical attributes and significant factors of a complex problem by the DEMATEL method. Strictly speaking, threshold value determination is practically dependent on the various fields of interest or study and also their inevitable implications.

Similarly to some pre-explained methods used for calculating threshold value in the DEMATEL method, in this research, we are interested in formulating three simple equations to obtain threshold value just only by applying data derived from total matrix T and the initial influence direct matrix given by experts. Comparatively, our suggested equations will pave the way to yield threshold values more easily with no computational complexity which are twined with those special procedures of Lenth's principles and the maximum mean de-entropy technique. Lenth's method is inherently based on the effect-sparsity assumption and regression coefficients. It must be noted that threshold value determination by Lenth's method relatively needs more information deploying the statistical t-test and f-test method and also threshold value determination by maximum mean de-entropy technique needs more complicated calculation due to exactly considering rearranging of total matrix T into ordered triplets set, the ordered dispatch-node set, receive-nodes and determining the mathematically de-entropy values. Besides the maximum mean de-entropy technique is an algorithmic method in 6 steps to reach the threshold value. For more information, we refer the reader to [12].

4. Proposed methods to set the threshold value (θ) in the DEMATEL method

4.1. Min(max) strategy

We make use of the $\min(\max)$ operator as a selection strategy of plausible decision payoffs. Each value in the ith row of matrix T displays the direct and indirect effects of each criterion exerting on the other criteria and similarly, each value in the jth column of matrix T shows the direct and indirect effects that the criterion C_j is receiving from other criteria. To easily interpret the outcome of matrix T as much as possible, it is interesting to decrease the internal complexity which does exist unavoidably in the total matrix T to a manageable level of a system. Therefore we are keen to set a threshold value that implies the maximum value of influences given from other factors and the maximum value of effects dispatching from one factor to the other factors.

The suggested equation phase of threshold value measurement is as follows:

$$\theta = \frac{\min(\max)_{C_j} + \min(\max)_{r_i}}{2} \tag{3}$$

$\min(\max)_{C_j}$ is the infimum amount of maximum numbers of each column in matrix T. $\min(\max)_{r_i}$ is the infimum amount of maximum numbers of each row in matrix T.

Now we are ready to use our proposed equation for solving some numerical cases. Moreover, alongside going through the DEMATEL method step by step, hereby the simplicity of our suggested equation for setting threshold value can be mathematically observed furthermore for each case, there will be ultimately a comparison of threshold value gained by the more often usually used method of averaging method.

Numerical study for more illumination and clarity

Case 1. Assume that an average initial intelligible relation matrix given by some experts as below

$$A = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \end{array} \begin{array}{c} \overbrace{\begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array}} \\ \begin{bmatrix} 0 & 2 & 1.67 & 2.33 \\ 1.67 & 0 & 2.67 & 2.67 \\ 0.67 & 2.33 & 0 & 1 \\ 2.67 & 2.67 & 2.67 & 0 \end{bmatrix} \end{array}$$

By finding the maximum of the row sums and column sums (8.01), the normalized initial direct-relation matrix would be calculated as follows

$$N = \begin{bmatrix} 0 & 0.25 & 0.208 & 0.291 \\ 0.208 & 0 & 0.333 & 0.333 \\ 0.084 & 0.291 & 0 & 0.125 \\ 0.333 & 0.333 & 0.333 & 0 \end{bmatrix}$$

Finally, the total matrix T is computed according to the DEMATEL procedure (Step 3).

$$T = \begin{bmatrix} 0.585 & 0.976 & 0.956 & 0.906 \\ 0.813 & 0.861 & 1.121 & 0.997 \\ 0.489 & 0.774 & 0.558 & 0.595 \\ 0.962 & 1.203 & 1.21 & 0.832 \end{bmatrix}$$

To set the threshold value θ by equation (3) proposed in the previous section, first, we pick up those numbers which are the maximum values of each row and column (Table 1). Thus, they are listed as follows:

Table 1. Maximum numbers of rows and columns in matrix T

Rows	0.976	1.121	0.774	1.21	min(maximum in rows) = 0.774
Columns	0.962	1.203	1.21	0.997	minimum(maximum in columns) = 0.962

θ would be consequently calculated by equation (3) as the 0.868. Now matrix T can be filtered out by eliminating the insignificant effects lower than the value based on a threshold value. So the final GRIM would be depicted by the matrix presented as follows:

$$\widetilde{T} = \begin{bmatrix} 0 & 0.976 & 0.956 & 0.906 \\ 0 & 0 & 1.121 & 0.997 \\ 0 & 0 & 0 & 0 \\ 0.962 & 1.203 & 1.21 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T as the threshold value, it can be seen that equation (3) yields a threshold value (0.868) greater than the threshold value obtained by calculating the average of matrix T numbers (0.865).

Case 2. Assume another average initial intelligible relation matrix given by some experts as below:

$$A = \begin{array}{c} \\ C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \end{array} \begin{array}{c} \overbrace{\begin{array}{cccccc} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array}} \\ \begin{bmatrix} 0 & 1.66 & 3.66 & 0.66 & 2.33 & 1.33 \\ 2.33 & 0 & 3.25 & 1.25 & 2 & 1.5 \\ 1.66 & 2.5 & 0 & 1.25 & 2.5 & 1.5 \\ 1.66 & 2.75 & 1.5 & 0 & 2 & 2 \\ 1.33 & 3 & 3.25 & 0.25 & 0 & 0.5 \\ 3 & 2.75 & 3 & 3.25 & 2.75 & 0 \end{bmatrix} \end{array}$$

Then the normalized initial direct-relation matrix N, by finding the maximum of the row sums and column sums (14.75) would be calculated as follows

$$N = \begin{bmatrix} 0 & 0.112 & 0.248 & 0.045 & 0.158 & 0.09 \\ 0.158 & 0 & 0.22 & 0.085 & 0.135 & 0.102 \\ 0.112 & 0.169 & 0 & 0.085 & 0.169 & 0.102 \\ 0.112 & 0.186 & 0.102 & 0 & 0.135 & 0.135 \\ 0.09 & 0.203 & 0.22 & 0.017 & 0 & 0.034 \\ 0.203 & 0.186 & 0.203 & 0.22 & 0.186 & 0 \end{bmatrix}$$

Eventually, the total matrix T is computed according to the DEMATEL procedure

$$T = \begin{bmatrix} 0.237 & 0.402 & 0.558 & 0.199 & 0.417 & 0.25 \\ 0.392 & 0.321 & 0.561 & 0.245 & 0.419 & 0.275 \\ 0.337 & 0.443 & 0.349 & 0.232 & 0.421 & 0.259 \\ 0.356 & 0.475 & 0.463 & 0.169 & 0.412 & 0.229 \\ 0.289 & 0.430 & 0.492 & 0.153 & 0.242 & 0.183 \\ 0.525 & 0.602 & 0.685 & 0.419 & 0.569 & 0.254 \end{bmatrix}$$

Now all the maximum values of each row and column in matrix T are listed in Table 2. The threshold θ is then calculated as 0.371 using equation (3).

Table 2. Maximum numbers of rows and columns in matrix T

Maximum in rows	0.558	0.561	0.443	0.475	0.492	0.685	min(maximum in rows) = 0.443
Maximum in columns	0.525	0.602	0.685	0.419	0.569	0.299	min(max in columns) = 0.299

Ultimately, matrix T can be filtered out by omitting the insignificant effects lower than the threshold value and the final GRIM would be depicted by the matrix presented as follows

$$\widetilde{T} = \begin{bmatrix} 0 & 0.402 & 0.558 & 0 & 0.417 & 0 \\ 0.392 & 0 & 0.561 & 0 & 0.419 & 0 \\ 0 & 0.443 & 0 & 0 & 0.421 & 0 \\ 0 & 0.475 & 0.463 & 0 & 0.412 & 0 \\ 0 & 0.430 & 0.492 & 0 & 0 & 0 \\ 0.525 & 0.602 & 0.685 & 0.419 & 0.569 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T as the threshold value, it can be got that our proposed equation (3) would yield a threshold value (0.371) greater than the threshold value obtained by calculating the average of matrix T numbers (0.368).

Case 3. In this point, it is aimed to use equation (3) for a real world initial average matrix A. This matrix has been used to evaluate the importance of the seven criteria determining the key success factors of hospital service quality in Taiwan [10, 19].

$$A = \begin{bmatrix} 0 & 1.5789 & 2.0526 & 1.7895 & 2.2632 & 2 & 1.3158 \\ 1.5263 & 0 & 2.0526 & 2.3684 & 2.3684 & 2.0526 & 1.6316 \\ 1.9474 & 1.9474 & 0 & 2.0526 & 2.4211 & 2.5263 & 1.9474 \\ 1.3684 & 2.2632 & 2.1053 & 0 & 2.2105 & 2.2632 & 1.5789 \\ 1.8421 & 2 & 2.2105 & 1.7895 & 0 & 2.2105 & 1.4737 \\ 2.0526 & 1.8421 & 2.1579 & 1.8421 & 2.2105 & 0 & 1.6842 \\ 1.0526 & 1.7368 & 1.8421 & 1.6316 & 1.5236 & 1.7368 & 0 \end{bmatrix}$$

when the normalized initial direct-relation matrix N is

$$N = \begin{bmatrix} 0 & 0.1215 & 0.1579 & 0.1377 & 0.1741 & 0.1538 & 0.1012 \\ 0.1174 & 0 & 0.1579 & 0.1822 & 0.1822 & 0.1579 & 0.1255 \\ 0.1498 & 0.1498 & 0 & 0.1579 & 0.1862 & 0.1579 & 0.1498 \\ 0.1053 & 0.1741 & 0.1619 & 0 & 0.1700 & 0.1741 & 0.1215 \\ 0.1417 & 0.1538 & 0.1700 & 0.1377 & 0 & 0.1700 & 0.1134 \\ 0.1579 & 0.1417 & 0.1659 & 0.1417 & 0.1700 & 0 & 0.1296 \\ 0.0809 & 0.1336 & 0.1417 & 0.1255 & 0.1174 & 0.1336 & 0 \end{bmatrix}$$

and the total matrix T is obtained as

$$T = \begin{bmatrix} 0.8298 & 1.0460 & 1.1475 & 1.0636 & 1.2031 & 1.1380 & 0.9013 \\ 0.9985 & 1.0119 & 1.2266 & 1.1717 & 1.2910 & 1.2199 & 0.9840 \\ 1.0425 & 1.1631 & 1.1137 & 1.1749 & 1.3181 & 1.2429 & 1.0213 \\ 0.9775 & 1.1462 & 1.2144 & 1.0035 & 1.2665 & 1.2168 & 0.9693 \\ 0.9882 & 1.1099 & 1.1988 & 1.1039 & 1.0989 & 1.1919 & 0.9453 \\ 1.0141 & 1.1165 & 1.2128 & 1.1224 & 1.2615 & 1.0634 & 0.9711 \\ 0.8035 & 0.9416 & 1.0115 & 0.9402 & 1.0306 & 0.9996 & 0.7115 \end{bmatrix}$$

the threshold value θ is eventually calculated as 1.026 by equation (3) according to the information in Table 3.

Table 3. Maximum numbers of rows and columns in matrix T

Max in rows	1.2031	1.2910	1.3181	1.2665	1.1988	1.2615	1.0306	min(max in rows) = 1.0306
Max in columns	1.0425	1.1631	1.2266	1.1749	1.3181	1.2429	1.0213	min(max in columns) = 1.0213

As a result, the total influence matrix T can be refined as follows by putting elements lower than the threshold value as zero.

$$\widetilde{T} = \begin{bmatrix} 0 & 1.0460 & 1.1475 & 1.0636 & 1.2031 & 1.1380 & 0 \\ 0 & 0 & 1.2266 & 1.1717 & 1.2910 & 1.2199 & 0 \\ 1.0425 & 1.1631 & 1.1137 & 1.1749 & 1.3181 & 1.2429 & 0 \\ 0 & 1.1462 & 1.2144 & 0 & 1.2665 & 1.2168 & 0 \\ 0 & 1.1099 & 1.1988 & 1.1039 & 1.0989 & 1.1919 & 0 \\ 0 & 1.1165 & 1.2128 & 1.1224 & 1.2615 & 1.0634 & 0 \\ 0 & 0 & 0 & 0 & 1.0306 & 0 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T to determine the threshold value, it can be seen that equation (3) yields a threshold value (1.026) lower than the threshold value obtained by calculating the average of matrix T numbers (1.080).

Case 4. Sixty-six educational experts were asked to specify the relationships between the seven measurement dimensions of the innovation support system in Taiwanese higher education. The initial direct relation matrix is obtained by averaging the matrices from the 66 experts, which is as follows [9, 23]

$$A = \begin{bmatrix} 0 & 0.12 & 1.31 & 1.62 & 0.27 & 0.33 & 0.03 \\ 1.24 & 0 & 2.33 & 0.57 & 1.13 & 0.06 & 0.71 \\ 3.91 & 3.76 & 0 & 2.97 & 1.19 & 0.23 & 0.04 \\ 3.29 & 0.24 & 0.26 & 0 & 0.3 & 1.75 & 1.22 \\ 1.07 & 2.93 & 3.35 & 1.1 & 0 & 3.63 & 1.32 \\ 3.01 & 1.25 & 2.63 & 2.77 & 1.29 & 0 & 1.1 \\ 2.98 & 3.03 & 3.42 & 2.2 & 3.78 & 3.89 & 0 \end{bmatrix}$$

The normalized initial direct-relation matrix N is calculated as

$$N = \begin{bmatrix} 0 & 0.006 & 0.069 & 0.084 & 0.014 & 0.017 & 0.002 \\ 0.064 & 0 & 0.121 & 0.029 & 0.059 & 0.003 & 0.037 \\ 0.203 & 0.195 & 0 & 0.154 & 0.062 & 0.012 & 0.002 \\ 0.170 & 0.012 & 0.013 & 0 & 0.016 & 0.091 & 0.063 \\ 0.055 & 0.152 & 0.174 & 0.057 & 0 & 0.188 & 0.068 \\ 0.156 & 0.065 & 0.136 & 0.144 & 0.067 & 0 & 0.057 \\ 0.154 & 0.157 & 0.177 & 0.114 & 0.196 & 0.202 & 0 \end{bmatrix}$$

The total influence matrix T would be then gained as follows

$$T = \begin{bmatrix} 0.049 & 0.035 & 0.092 & 0.112 & 0.129 & 0.038 & 0.014 \\ 0.137 & 0.059 & 0.169 & 0.086 & 0.089 & 0.043 & 0.054 \\ 0.294 & 0.241 & 0.085 & 0.218 & 0.099 & 0.064 & 0.036 \\ 0.237 & 0.059 & 0.079 & 0.064 & 0.053 & 0.128 & 0.081 \\ 0.221 & 0.257 & 0.289 & 0.180 & 0.077 & 0.249 & 0.109 \\ 0.282 & 0.149 & 0.223 & 0.233 & 0.119 & 0.069 & 0.090 \\ 0.363 & 0.301 & 0.344 & 0.273 & 0.277 & 0.303 & 0.066 \end{bmatrix}$$

Lastly, we have the threshold θ calculated by (3) using the data in Table 4 as approximately 0.111.

Table 4. Maximum numbers of rows and columns in matrix T.

Maximum in rows	0.112	0.169	0.294	0.237	0.289	0.282	0.363	min(max in rows) = 0.112
Maximum in columns	0.363	0.301	0.344	0.273	0.277	0.303	0.109	min(max in columns) = 0.109

Therefore the total influence matrix T is reformed as follows by putting the elements lower than the threshold value as zero

$$\widetilde{T} = \begin{bmatrix} 0 & 0 & 0 & 0.112 & 0 & 0 & 0 \\ 0.137 & 0.059 & 0.169 & 0 & 0 & 0 & 0 \\ 0.294 & 0.241 & 0 & 0.218 & 0 & 0 & 0 \\ 0.237 & 0.059 & 0 & 0 & 0 & 0.128 & 0 \\ 0.221 & 0.257 & 0.289 & 0.180 & 0 & 0.249 & 0 \\ 0.282 & 0.149 & 0.223 & 0.233 & 0.119 & 0 & 0 \\ 0.363 & 0.301 & 0.344 & 0.273 & 0.277 & 0.303 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T as the threshold value, it can be seen that our proposed equation (3) yields a threshold value (0.111) lower than the threshold value obtained by calculating the average of matrix T numbers (0.147).

Case 5. Here we are interested to examine equation (3) for a rather high dimensional complicated system to obtain the final total matrix. Thus the given initial average direct influence matrix D (Table 5) is as follows [7]

Table 5. Initial average direct influence matrix D

D	A1	A2	A3	F1	F2	T1	T2	T3	T4	P1	P2	P3	U1	U2	U3	U4
A1	0.00	3.69	3.5	3.65	3.26	2.85	2.62	2.82	2.58	4.23	4.02	4.45	4.58	4.62	4.29	4.34
A2	3.45	0.00	3.36	3.71	3.45	2.13	2.14	2.53	2.72	4.14	4.03	4.01	4.37	4.31	4.02	3.83
A3	3.23	3.46	0.00	3.45	3.56	1.65	1.78	1.43	1.65	4.04	4.23	4.34	3.56	3.65	3.52	3.84
F1	4.23	4.67	4.27	0.00	4.67	4.82	4.69	4.84	4.67	4.83	4.39	4.62	4.73	4.82	4.59	4.81
F2	4.35	4.73	4.12	4.56	0.00	4.62	4.84	4.67	4.04	4.69	4.74	4.59	4.83	4.69	4.82	4.79
T1	1.36	1.54	1.36	3.54	3.46	0.00	4.21	3.76	3.82	1.67	2.53	2.54	4.18	3.54	3.23	3.51
T2	1.43	1.23	1.75	2.95	2.57	2.51	0.00	2.91	3.23	2.54	2.74	1.72	2.54	2.86	3.53	2.32
T3	2.43	2.41	2.16	2.94	3.43	3.41	3.32	0.00	3.54	1.94	2.18	1.54	2.31	2.11	2.75	3.04
T4	1.31	1.67	1.87	2.95	2.92	3.04	2.87	2.72	0.00	2.43	1.65	2.72	2.11	2.98	3.35	2.59
P1	2.56	2.76	2.15	4.56	4.67	4.11	4.23	4.34	4.26	0.00	4.61	4.45	4.03	4.02	4.12	3.56
P2	2.45	2.34	2.45	4.86	4.43	4.62	4.34	4.12	4.69	4.51	0.00	4.48	3.65	3.23	3.54	3.76
P3	2.24	2.58	2.67	4.69	4.66	4.45	4.32	4.64	4.11	4.23	4.42	0.00	3.11	3.56	3.34	3.32
U1	3.45	3.62	3.72	4.45	4.76	4.12	3.87	3.93	3.85	2.56	2.19	2.64	0.00	3.23	3.54	3.02
U2	3.53	3.24	3.75	4.34	4.23	4.34	3.51	3.45	3.63	2.63	2.65	3.12	3.21	0.00	2.61	2.76
U3	3.43	3.65	3.54	4.27	4.28	4.62	3.56	3.95	4.21	2.89	2.57	3.32	3.57	3.26	0.00	2.49
U4	3.62	3.75	3.06	4.25	4.61	4.42	4.24	4.15	4.27	3.53	2.65	2.76	3.05	3.42	3.41	0.00

Going through the steps of the DEMATEL method, total matrix E (all values are multiplied by 10) is then obtained as shown in Table 6.

Table 6. Total matrix E

E	A1	A2	A3	F1	F2	T1	T2	T3	T4	P1	P2	P3	U1	U2	U3	U4
A1	1.26	1.86	1.79	2.24	2.19	2.06	1.99	2.01	2.01	2.07	2.01	2.11	2.20	2.22	2.19	2.15
A2	1.67	1.29	1.70	2.16	2.12	1.88	1.84	1.89	1.94	1.99	1.94	1.98	2.10	2.10	2.07	2.00
A3	1.55	1.67	1.15	2.00	2.02	1.70	1.68	1.63	1.684	1.87	1.87	1.92	1.88	1.90	1.89	1.89
F1	2.10	2.27	2.17	2.12	2.75	2.67	2.62	2.63	2.64	2.47	2.37	2.46	2.57	2.60	2.58	2.55
F2	2.11	2.27	2.14	2.72	2.11	2.64	2.63	2.59	2.55	2.44	2.41	2.44	2.57	2.57	2.60	2.54
T1	1.18	1.28	1.22	1.84	1.83	1.31	1.86	1.79	1.82	1.40	1.48	1.52	1.80	1.73	1.70	1.70
T2	1.04	1.07	1.11	1.56	1.51	1.46	1.09	1.48	1.54	1.36	1.34	1.24	1.40	1.45	1.55	1.36
T3	1.22	1.29	1.22	1.63	1.69	1.64	1.61	1.14	1.65	1.32	1.33	1.28	1.44	1.42	1.52	1.52
T4	1.05	1.16	1.16	1.60	1.60	1.57	1.52	1.50	1.14	1.36	1.23	1.40	1.38	1.51	1.56	1.56
P1	1.66	1.78	1.65	2.42	2.43	2.29	2.28	2.28	2.30	1.55	2.13	2.16	2.19	2.20	2.23	2.11
P2	1.61	1.70	1.66	2.43	2.37	2.33	2.26	2.22	2.32	2.13	1.48	2.13	2.11	2.07	2.13	2.10
P3	1.56	1.70	1.67	2.37	2.36	2.27	2.23	2.26	2.21	2.06	2.06	1.50	2.01	2.08	2.07	2.02
U1	1.66	1.77	1.74	2.23	2.27	2.12	2.06	2.06	2.07	1.76	1.68	1.78	1.50	1.95	2.01	1.89
U2	1.63	1.67	1.70	2.17	2.15	2.09	1.96	1.94	1.99	1.73	1.70	1.80	1.89	1.46	1.83	1.81
U3	1.66	1.78	1.72	2.23	2.22	2.20	2.03	2.07	2.13	1.82	1.74	1.88	2.00	1.97	1.53	1.84
U4	1.57	1.80	1.67	2.24	2.28	2.19	2.14	2.12	2.16	1.91	1.76	1.82	1.94	2.00	2.01	1.50

Now, we have the threshold θ calculated by equation (3) using the data in Table 7 as approximately 0.184.

As a comparison to the more often used method of an average of numbers in matrix E as the threshold value, it can be seen that our proposed equation (3) yields a threshold value (0.184) lower than the threshold value obtained by approximately calculating the average of matrix E numbers (0.189).

Table 7. Maximum numbers of rows and columns in matrix E

Max in rows	0.224	0.216	0.202	0.275	0.272	0.186	0.156	0.169	
Max in columns	0.211	0.227	0.217	0.272	0.275	0.267	0.263	0.263	
Max in rows	0.160	0.243	0.243	0.237	0.227	0.217	0.223	0.228	min(max in rows) = 0.156
Max in columns	0.264	0.247	0.241	0.246	0.257	0.260	0.260	0.255	min(max in columns) = 0.211

4.2. Modification of min(max) strategy to set the threshold value (θ)

The DEMATEL method is an intrinsically expert-based assessment. To assess the numeric relationship between n factors or criteria of a complex system, some experts are nominated to express their opinions about the direct influence of the ith factor on the jth factor. Hence, as a modification, it is intended moderately to apply the expert's points of view to equation (3). But, this modification would be modelled into equation (3) using the arithmetic average of the initial matrix which presents the primary degree of relationship among n factors. Thus, equation (3) can be expressed as follows

$$\theta = \mu \left(\frac{\min(\max)_{C_j} + \min(\max)_{r_i}}{2^{\mu}} \right) \tag{4}$$

where μ is an arithmetic mean of all the initial relationship matrix elements.

Given the proposed equation above, here we are interested to use it for solving some numerical cases and compare its result with the averaging method of obtaining threshold value.

Example 1. If we apply equation (4) for the same Case 1 with initial relation matrix μ as 1.564, the threshold value θ would be 0.9181. The final matrix $\widetilde{T}$ would be consequently filtered out

$$\widetilde{T} = \begin{bmatrix} 0 & 0.976 & 0.956 & 0 \\ 0 & 0 & 1.121 & 0.997 \\ 0 & 0 & 0 & 0 \\ 0.962 & 1.203 & 1.21 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T as the threshold value, it can be seen that our proposed equation (4) yields a threshold value (0.918) greater than the threshold value obtained by calculating the average of matrix T numbers (0.865).

Example 2. The threshold value θ is calculated as 0.387 by means of equation (4) for Case 2 problem with initial relation matrix μ as 1.733. Thus the final matrix $\widetilde{T}$ could be revised as follows

$$\widetilde{T} = \begin{bmatrix} 0 & 0.402 & 0.558 & 0 & 0.417 & 0 \\ 0.392 & 0 & 0.561 & 0 & 0.419 & 0 \\ 0 & 0.443 & 0 & 0 & 0.421 & 0 \\ 0 & 0.475 & 0.463 & 0 & 0.412 & 0 \\ 0 & 0.430 & 0.492 & 0 & 0 & 0 \\ 0.525 & 0.602 & 0.685 & 0.419 & 0.569 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T as the threshold value, it can be got that our proposed equation (4) would yield a threshold value (0.387) greater than the threshold value obtained by calculating the average of matrix T numbers (0.368).

Example 3. It is now purposed to examine equation (4) for Case 3 with initial relation matrix μ as 1.6422. The threshold value θ is calculated as 1.0796 using equation (4). So the final matrix T could be filtered out as follows

$$\widetilde{T} = \begin{bmatrix} 0 & 0 & 1.1475 & 0 & 1.2031 & 1.1380 & 0 \\ 0 & 0 & 1.2266 & 1.1717 & 1.2910 & 1.2199 & 0 \\ 0 & 1.1631 & 1.1137 & 1.1749 & 1.3181 & 1.2429 & 0 \\ 0 & 1.1462 & 1.2144 & 0 & 1.2665 & 1.2168 & 0 \\ 0 & 1.1099 & 1.1988 & 1.1039 & 1.0989 & 1.1919 & 0 \\ 0 & 1.1165 & 1.2128 & 1.1224 & 1.2615 & 0 & 0 \\ 0 & 0 & 1.0115 & 0 & 0 & 0 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T to determine the threshold value, it can be seen that equation (4) yields a threshold value (1.0796) almost equal to the threshold value obtained by calculating the average of matrix T numbers (1.080).

Example 4. The threshold value θ is also computed as 0.117 using equation (4) for Case 4 problem with initial relation matrix μ as approximately 1.503. Finally, the matrix T can be filtered out as shown below

$$T = \begin{bmatrix} 0 & 0 & 0 & 0.112 & 0 & 0 & 0 \\ 0.137 & 0 & 0.169 & 0 & 0 & 0 & 0 \\ 0.294 & 0.241 & 0 & 0.218 & 0 & 0 & 0 \\ 0.237 & 0 & 0 & 0 & 0 & 0.128 & 0 \\ 0.221 & 0.257 & 0.289 & 0.180 & 0 & 0.249 & 0 \\ 0.282 & 0.149 & 0.223 & 0.233 & 0.119 & 0 & 0 \\ 0.363 & 0.301 & 0.344 & 0.273 & 0.277 & 0.303 & 0 \end{bmatrix}$$

As a comparison to the more often used method of an average of numbers in matrix T as the threshold value, it can be seen that equation (4) results in a threshold value (0.117) lower than the threshold value obtained by calculating the average of matrix T numbers (0.147).

Example 5. The threshold value θ is also computed as 0.124 using equation (4) for Case 5 problem with initial relation matrix μ as approximately 3.272.

As a comparison to the more often used method of an average of numbers in matrix T to calculate the threshold value, it can be seen that equation (4) results in a threshold value (0.124) lower than the threshold value which obtained by calculating the average of matrix E numbers (0.189).

4.3. Modification B of min(max) strategy to set the threshold value (θ)

In this part, the main purpose is to concurrently use the sum of the direct and indirect effects dispatching from factor r_i being the sum of the rows derived from total influence matrix T, the prominence value which describes the strength of influences that are given and received of the factor $r_i + c_i$, c_i is the sum of the columns derived from total influence matrix T and the net effect that the factor contributes to the system $(r_i - c_i)$ to define an elastic function as follows

$$F_i = f\left(r_i, c_i\right) = \left[r_i(r_i + c_i)\exp\left(r_i - c_i\right)\right] \tag{5}$$

To explain more about the function F_i, it must be paid attention that F_i implicitly can shrink or expand the rectangular surface which is made of the elements of r_i and $r_i + c_i$. Illustratively, it can be imagined that there would be a rectangle supposed for every criterion and geometrically it can be reshaped by the particular factor of $\exp(r_i - c_i)$

$$\theta = W_n\left(\frac{\min(\max)_{c_i} + \min(\max)_{r_i}}{2^{W_n}}\right) \tag{6}$$

where W_n would be defined as

$$W_n = \frac{n}{2}\left(\frac{\max F_i \quad + \min F_i}{\sum\limits_{i=1}^{n} F_i}\right) \tag{7}$$

and n is the number of criteria.

By the suggested equation (6), here we want to use it for solving some numerical cases and compare its result with the averaging method of obtaining threshold value.

Numerical study

At this step, equation (6) has to be numerically examined for Cases 1–5. The data needed to calculate the threshold value θ are gathered in Table 8.

Table 8. Data derived from Cases 1–4 and those needed to calculate the threshold θ

Example	$\min(\max)_{c_i}$	$\min(\max)_{r_i}$	$\max F_i$	$\min F_i$	$\sum_{i=1}^{n} F_i$	W_n	n	θ
Case 1	0.962	0.774	76.275	3.269	146.314	1.092	4	0.889
Case 2	0.299	0.443	64.76	3.61	103.709	1.978	6	0.373
Case 3	1.0213	1.0306	176.366	53.491	832.96	0.966	7	1.015
Case 4	0.109	0.112	20.058	0.214	32.382	2.191	7	0.106
Case 5	0.211	0.156	50.895	4.091	372.434	1.181	16	0.191

Now for comparison, the threshold values determined by equation (6) for Cases 1–5 and those calculated by averaging the numbers of matrix T are shown in Table 9.

Table 9. Threshold values obtained from equation (6) and by the averaging method

Example	Equation (6)	Averaging
Case 1	0.889	0.865
Case 2	0.373	0.368
Case 3	1.015	1.080
Case 4	0.106	0.147
Case 5	0.191	0.189

4.4. Aggregating procedure to set the threshold value (θ)

In this part, we are interested to combine our predefined equations with an average method to upgrade the threshold values. The main reason which is based here would be the closeness of threshold values calculated by equations (3), (4) and (6) to those obtained by the averaging method. So, the Hamacher t-conorm operator (equation (8)) is specially used to aggregate the data. It is worth to mention deploying the Hamacher t-conorm would be a flexible mathematical aggregating operator for setting different values of threshold. This issue would be later shown in the numerical study.

$$S_\gamma (a, b) = \frac{a + b - (2 - \gamma)\, ab}{1 - (1 - \gamma)\, ab}, \quad \gamma \geq 0 \tag{8}$$

where S_γ is the Hamacher t-conorm operator, a is considered the threshold value obtained by equations (3), (4) or (6) and b is considered the threshold value computed by averaging method. Furthermore, by changing the value of γ, a span of different threshold values would be achieved.

Numerical study

This section is dedicated to examine a numerical case using our suggested aggregating procedure to set the threshold value in the DEMATEL method.

Example 1. For having a real case study, the initial average direct influence matrix X (Table 10) and the final total matrix T (Table 11) are given as follows [5].

Table 10. Initial average direct influence matrix X

X	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}
A_1	0	0	0	1	2	1	1	1	9	1	1	1	1
A_2	8	0	0	1	1	1	1	1	10	1	1	1	1
A_3	0	0	0	1	1	1	1	1	9	1	1	1	1
A_4	0	0	0	0	0	1	1	1	1	9	1	1	1
A_5	0	0	0	0	0	1	1	1	1	9	1	1	1
A_6	1	1	1	1	1	0	1	1	1	1	10	1	1
A_7	1	1	1	1	1	1	0	1	1	1	8	1	1
A_8	1	1	1	1	1	1	1	0	1	1	7	1	1
A_9	1	1	0	0	0	3	0	0	0	0	0	8	1
A_{10}	0	0	0	0	0	0	2	2	0	0	0	8	1
A_{11}	5	5	0	0	0	5	0	0	0	0	0	9	1
A_{12}	0	0	1	0	0	1	1	1	1	0	0	0	10
A_{13}	1	1	1	1	2	1	1	1	1	1	1	1	0

Table 11. The final total matrix T

T	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8	A_9	A_{10}	A_{11}	A_{12}	A_{13}
A_1	0.084	0.065	0.030	0.063	0.110	0.152	0.086	0.086	0.432	0.121	0.161	0.287	0.195
A_2	0.407	0.085	0.039	0.082	0.106	0.200	0.109	0.109	0.597	0.144	0.208	0.377	0.254
A_3	0.082	0.063	0.029	0.062	0.072	0.148	0.082	0.082	0.427	0.107	0.156	0.277	0.188
A_4	0.062	0.048	0.027	0.022	0.031	0.106	0.095	0.095	0.113	0.374	0.142	0.253	0.172
A_5	0.062	0.048	0.027	0.022	0.031	0.106	0.095	0.095	0.113	0.374	0.142	0.253	0.172
A_6	0.216	0.167	0.074	0.077	0.094	0.178	0.103	0.103	0.239	0.134	0.526	0.369	0.245
A_7	0.191	0.148	0.070	0.073	0.089	0.190	0.062	0.097	0.218	0.127	0.440	0.328	0.222
A_8	0.179	0.138	0.068	0.071	0.086	0.179	0.094	0.059	0.208	0.124	0.396	0.308	0.210
A_9	0.104	0.080	0.033	0.026	0.038	0.181	0.046	0.046	0.111	0.048	0.111	0.409	0.219
A_{10}	0.052	0.040	0.033	0.023	0.033	0.113	0.113	0.113	0.078	0.042	0.101	0.390	0.207
A_{11}	0.343	0.265	0.052	0.055	0.079	0.324	0.085	0.85	0.286	0.099	0.209	0.577	0.320
A_{12}	0.069	0.053	0.069	0.034	0.053	0.111	0.081	0.081	0.137	0.063	0.113	0.134	0.452
A_{13}	0.108	0.083	0.058	0.060	0.106	0.113	0.079	0.079	0.151	0.116	0.142	0.193	0.112

To set the threshold value θ by equation (3), first, we pick up those numbers which are the maximum values of each row and column in total matrix T (Table 12). Thus, they are listed as follows

Table 12. Maximum numbers of rows and columns in matrix T

Maximum in rows	0.432	0.597	0.427	0.374	0.374	0.526	0.44
Maximum in columns	0.407	0.265	0.074	0.082	0.11	0.324	0.113
Maximum in rows	0.396	0.409	0.39	0.577	0.425	0.193	min(max in rows) = 0.193
Maximum in columns	0.113	0.597	0.374	0.526	0.577	0.452	min(max in columns) = 0.074

Consequently, threshold value θ by using equation (3) would be calculated as 0.134. The threshold value by averaging (the elements in total matrix T) method is also obtained as 0.146 for this case study. Moreover, threshold value θ would also be gained as 0.142 applying equation (4) and the arithmetic mean of all numbers in initial matrix X as 1.3846. For setting threshold value θ by equation (6) for this case study we use data gathered in Table 13.

Table 13. Data needed to calculate the threshold

$\min(\max)_{c_i}$	$\min(\max)_{r_i}$	$\max F_i$	$\min F_i$	$\sum_{i=1}^{n} F_i$	W_n	n	θ
0.074	0.193	0.717	0.096	4.829	1.094	13	0.137

It should be mentioned that for this real case study, threshold value θ has been calculated as 0.310 using Lenth's method [5].

At this step, all mandatory data are now in hand to obtain a span of threshold value using equation (8) for this numerical case. Final threshold values are gathered in Table 14 shown below.

Calculating threshold value θ in Table (14) can be apparently continued using the aggregating procedure for γ greater than 15, but it must be noted that the determined threshold value would not exceed the maximum number in total matrix T. The maximum number in a total matrix for this case study is

Table 14. Threshold values computed by suggested equations (3), (4), (6), (8)

Proposed equations	θ	Proposed equations	θ	Proposed equations	θ
(3)	0.134	(4)	0.142	(6)	0.137
by averaging	0.146				
(3), (8), $\gamma = 0$	0.246	(4), (8), $\gamma = 0$	0.252	(6), (8), $\gamma = 0$	0.248
(3), (8), $\gamma = 1$	0.260	(4), (8), $\gamma = 1$	0.267	(6), (8), $\gamma = 1$	0.263
(3), (8), $\gamma = 2$	0.274	(4), (8), $\gamma = 2$	0.282	(6), (8), $\gamma = 2$	0.277
(3), (8), $\gamma = 3$	0.288	(4), (8), $\gamma = 3$	0.296	(6), (8), $\gamma = 3$	0.291
(3), (8), $\gamma = 4$	0.301	(4), (8), $\gamma = 4$	0.310	(6), (8), $\gamma = 4$	0.305
(3), (8), $\gamma = 5$	0.314	(4), (8), $\gamma = 5$	0.323	(6), (8), $\gamma = 5$	0.317
(3), (8), $\gamma = 6$	0.326	(4), (8), $\gamma = 6$	0.336	(6), (8), $\gamma = 6$	0.330
(3), (8), $\gamma = 7$	0.338	(4), (8), $\gamma = 7$	0.348	(6), (8), $\gamma = 7$	0.342
(3), (8), $\gamma = 8$	0.349	(4), (8), $\gamma = 8$	0.360	(6), (8), $\gamma = 8$	0.353
(3), (8), $\gamma = 9$	0.360	(4), (8), $\gamma = 9$	0.371	(6), (8), $\gamma = 9$	0.364
(3), (8), $\gamma = 10$	0.371	(4), (8), $\gamma = 10$	0.382	(6), (8), $\gamma = 10$	0.375
(3), (8), $\gamma = 11$	0.381	(4), (8), $\gamma = 11$	0.393	(6), (8), $\gamma = 11$	0.356
(3), (8), $\gamma = 12$	0.391	(4), (8), $\gamma = 12$	0.403	(6), (8), $\gamma = 12$	0.396
(3), (8), $\gamma = 13$	0.401	(4), (8), $\gamma = 13$	0.413	(6), (8), $\gamma = 13$	0.406
(3), (8), $\gamma = 14$	0.410	(4), (8), $\gamma = 14$	0.423	(6), (8), $\gamma = 14$	0.415
(3), (8), $\gamma = 15$	0.419	(4), (8), $\gamma = 15$	0.432	(6), (8), $\gamma = 15$	0.424
by Lenth's method	0.310				

0.597. Thus we have got several different threshold values depending on the expert's opinions (particularly including the complexity of the total matrix graph) which could be eventually picked up for decisive materials. It should be mentioned that our mean of complexity would point out to the number of arcs (remained after filtering out the total matrix T) between nodes of criteria. Accordingly, by aggregating procedure, we can have different threshold values among which there are some values very close to those obtained by Lenth's method. For a better understanding of this important issue threshold values gathered in Table 14 are graphically compared in Figure 1.

Example 2. To examine our proposed aggregating procedure, it is intended to test this procedure for another numerical case. So the information from Case 1 is used to calculate the different threshold values by equation 8. All the computed threshold values have been brought together in Table 15.

Table 15. Threshold values computed by suggested equations (3), (4), (6), (8)

Proposed equations	θ	Proposed equations	θ	Proposed equations	θ
(3)	0.868	(4)	0.918	(6)	0.889
by averaging	0.865				
(3), (8), $\gamma = 0$	0.928	(4), (8), $\gamma = 0$	0.946	(6), (8), $\gamma = 0$	0.935
(3), (8), $\gamma = 1$	0.982	(4), (8), $\gamma = 1$	0.989	(6), (8), $\gamma = 1$	0.985
(3), (8), $\gamma = 2$	0.989	(4), (8), $\gamma = 2$	0.994	(6), (8), $\gamma = 2$	0.992

Calculating threshold value θ in Table 15 can be clearly continued using the aggregating procedure for γ greater than 2, but it must be noted that the determined threshold value would not exceed the maximum number in total matrix T. The maximum number in the total matrix for this Case study is 1.21. Thus as stated before we have got several different threshold values which depending on the expert's opinions (particularly including the complexity of the total matrix graph) could be eventually picked up for decision materials.

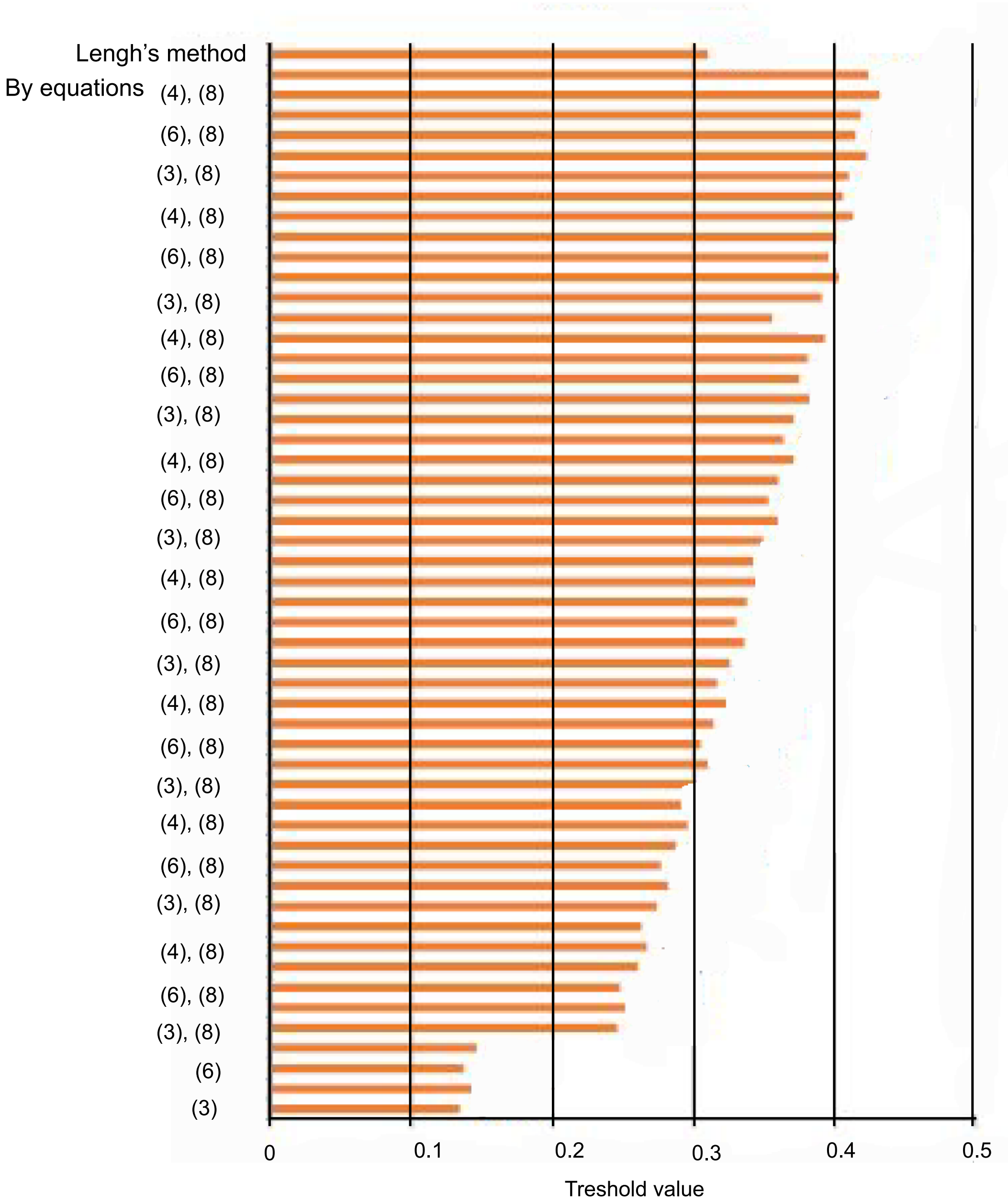

Figure 1. Graphical show of threshold values obtained by suggested equations and Lenth's method

5. Discussion

For the determination of the threshold value in the DEMATEL method, we present some mathematical models. Threshold values (for Cases 1–5) calculated by equations (3), (4) and (6) along with the calculating each step by which the expert's numerical opinions adopted to identify θ are compared in Table 16 as well. Comparatively, threshold values estimated by equation (4) are rather bigger than those estimated by equations (3) and (6) for Cases 1–4. It would be also noted that using the DEMATEL method accompanied with equation (4) to determine the threshold value is where the expert's direct opinions are counted on for two calculating steps. First, to obtain the initial relation matrix T by experts and second, apply the arithmetic mean of the initial relation matrix into equation (4). But, there is much more information taken from the total influence matrix T used to calculate the threshold value by equation (6).

Table 16. Proposed equations outputs for numerical cases

θ by Eq.	Case 1	Case 2	Case 3	Case 4	Case 5	Calculating steps by which expert's numerical opinions are adopted to identify θ
(3)	0.868	0.371	1.026	0.111	0.184	one step (building up the initial relation matrix)
(4)	0.918	0.387	1.0796	0.117	0.124	two steps (the initial relation matrix and its arithmetic mean applying into the equation)
(6)	0.889	0.373	1.015	0.106	0.191	one step (building up the initial relation matrix)

Additionally, in Table 17 and in Figure 2 threshold values calculated for Cases 1–5 are collected to have comparison views with those calculated by the more often used averaging technique. Taking a brief look at Table 17, it may be concluded that the suggested equations would yield us threshold values that are not too far from those calculated by averaging technique, however, it is known that averaging method would be a routine way of threshold calculating but there would be reasonable and evolutionary procedures in our proposed equations deploying the cause and effect numbers that derived from r_i and c_i elements. On the other hand, as it is clear that there is no mathematical proof relatively using existing threshold value techniques, thus as a core advantage of the suggested equations discussed in this study, their feasibility would be the most important feature that satisfies their design.

Table 17. Proposed equations output against averaging output

θ by Equation	Case 1	Case 2	Case 3	Case 4	Case 5
(3)	0.868	0.371	1.026	0.111	0.184
(4)	0.918	0.387	1.0796	0.117	0.124
(6)	0.889	0.373	1.015	0.106	0.191
By averaging	0.865	0.368	1.080	0.147	0.189

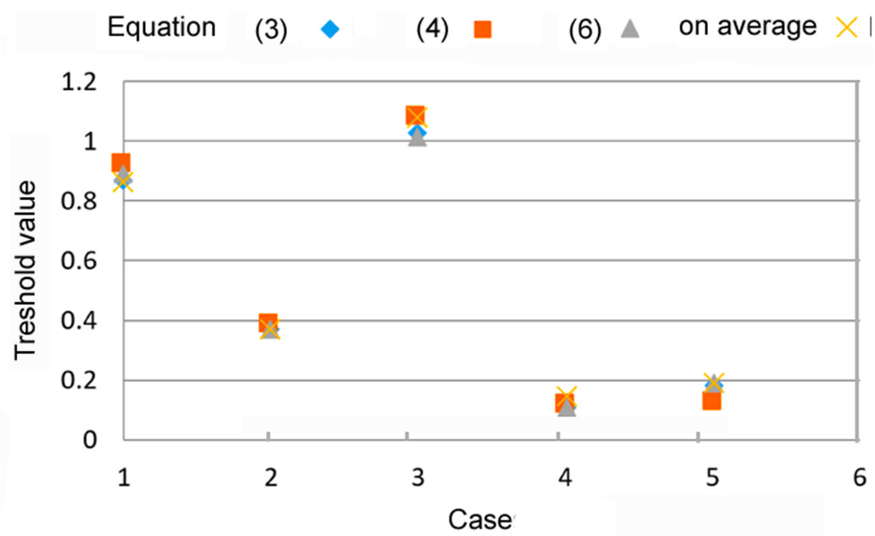

Figure 2. Threshold value calculated for each case of 1,2,3,4,5.

Additionally, the information gained by each equation (3), (4), (6) have been concurrently and deliberately combined by making use of an aggregating approach to reach some different threshold values. This span of threshold values gives more options for depicting the final graph of the total matrix T based on experts' opinions in the DEMATEL method. In particular, having achieved too many reasonable threshold values would manage the final total matrix's complexity according to how much extent the total matrix must be simplified.

Finally, for clarifying the usefulness of our proposed equations in this study, we make Table 18 showing the needed step-by-step information to set the threshold value.

Table 18. Type and number of data necessary for proposed equations

Equation	Described mandatory information needed to set the threshold value	Number of typical and initial data being used
(3)	maximum value of $\mathrm{R}_i$and C_j in total matrix T	2
(4)	maximum value of R_i and C_j in total matrix T and arithmetic mean of all the initial relationship matrix elements	3
(6)	maximum value of $\mathrm{R}_i$and C_j in total matrix T and number of criteria	3
(8), (3)	maximum value of R_i and C_j in total matrix T and average value of total matrix T elements	3
(8), (4)	maximum value of R_i and C_j in total matrix T and arithmetic mean of all the initial relationship matrix elements and the average value of total matrix T elements	4
(8), (6)	maximum value of R_i and C_j in total matrix T and number of criteria and average value of total matrix T elements	4

Accordingly, each of our suggested equations can be applied to real problems (such as the real problems stated and discussed before in our case studies) due to the dimension of the problem being studied and its complexity. It is worth noting that equations presented in this study try to make use of direct data being derived from total matrix T and then they are mathematically evolved so that other information (like μ in equation (4), number of criteria n in equation (6) and aggregating procedure combining of average method and our proposed equation (3), (4), (6)) are to be inserted to reshape previous equation. Of course, in this study, we are not interested to advise using a special threshold value setting equation but we are keen to apply the initial data which can be seen and derived easily from the total matrix T in the DEMATEL method. On the other hand, there is yet no proof or evidence established and particular method for threshold value determination which can be mathematically, identically and pragmatically applicable to all real-world problems. All the studies that have done before are to suggest some methods for threshold value determination [10, 12, 14, 23] and similarly, we do make offer some simpler equations for threshold value setting. Basically, threshold value configuration is absolutely a decisive resolution dependent on experts' opinions. They can optionally use the simple averaging method or other suggested procedures investigated before or our equations presented in this study.

6. Conclusion

Some strict methods to calculate the threshold value in the DEMATEL method have been discussed. Equations (3), (4), (6) and (8) were presented to distinguish the threshold value for filtering out negligible effects in the total influence matrix T. The min(max) operator is deployed as a main strategy to find the infimum element of each row and column of the total influence matrix T. Four mathematical models were suggested to set the threshold value (one base mathematical model, two its modifications, and an aggregating procedural method). Besides, it has been tried to develop the three proposed equations so as to gradually use the data which could be derived from the total influence matrix T.

As a compound approach, we have modelled the threshold value using the min(max) operator and using the arithmetic mean of the initial relation matrix provided by experts in equation (4) simultaneously. Especially in equation (6) as an enhanced model, the component of $\exp(r_i - c_i)$ has functioned as a shrinkage or an expander to describe the level of effect and prominence of a factor. On the other hand, in this research, we gradually make use of data which could be derived from the total matrix T in the last step procedure of the DEMATEL method. Furthermore, by applying an aggregating approach (making use of Hamacher t-conorm operator), the information derived by averaging method and equations (3), (4), (6) together was used to obtain a span of different threshold values, unlike the special Lenth's method which yields a unique threshold value. This span of several threshold values can more likely afford to manage the complexity of the final total matrix graph due to the expert's opinion.

Overall as has been stated before, the novelty of the proposed equations discussed here in this present study comes back to simplicity and more easily utilization of data derived from matrix T and the initial influence direct matrix which is applied to reach the threshold value. Clearly speaking, in this paper, we have proposed some mathematical equations for threshold configuration in the DEMATEL method which involve the data directly got from the final total matrix T. Additionally, these suggested equations set the threshold value without the need for statistical methods or other complicated mathematical solutions. But strictly speaking, identifying a consistent threshold value in the DEMATEL method to be congruent to all real-world problem situations, approaches and scientific areas will be open to examination by researchers. Definitely, this problematic part of the DEMATEL model will have absolutely remained a vital challenge and this is because of inherently expert-based structural modelling. For future study, it is suggested to modify and apply these equations to determine the threshold value in the fuzzy DEMATEL method.

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