QUANTITATIVE METHODS IN ECONOMICS
Vol. XIII, No 2, 2012, pp. 7 – 21

RADAR COEFFICIENT OF CONCENTRATION

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Abstract: In the following work we have described a process of using radar charts to measure concentration of a distribution. The process utilises the idea of Gini index based on a Lorenz curve as well as a method presented by the authors in [Binderman, Borkowski, Szczesny 2010]. The presented technique can also be used by analysts to create new coefficients of concentration based on measures of similarity and dissimilarity of objects so that from the set of constructed coefficients one that best fulfils the required criteria of sensitivity can be chosen.

Keywords: Gini index, Schutz's measure, radar coefficient of concentration, radar method, radar measure of conformability, measure of similarity, synthetic measures, classification, cluster analysis

INTRODUCTION

One of task that are given to analysts is to present concentration (non-uniformity in terms of possession) of a "resource" and the level of its changes in a given time frame in a clear and simple manner. For example it can be the change of concentration of accrued gains for clients of a commercial bank, non-uniformity of salary in a corporation, the level of concentration of land ownership by private household in Poland or, by expanding the definition of non-uniformity, presentation of a demographical structure valuation on a given geographical area. Analysts often do not possess data on a level of a single object. On the other hand, they do have access to data in tabular form. Which means performing analysis based on aggregated data, essentially using data in a form of a set of vectors, which coordinates describe directly or indirectly the structures in question. Bibliography in the field of measurement of similarity or dissimilarity of structures provides a rich set of instruments. The most important Polish publications are [Chomątowski, Sokołowski 1978, Kukuła 1989, 2010, Strahl 1985, 1996, Strahl (red.) 1998, Walesiak 1983, 1984 et al]. However, only few of them could have been inspired by visualization, i.e. graphical representation of structures (see. {Binderman, Borkowski, Szczesny 2009, 2010a, 2010b, 2010c; Borkowski Szczesny 2002, 2005; Binderman, Szczesny 2009, 2011; Binderman 2011; Ciok 2004; Ciok, Kowalczyk, Pleszczyńska, Szczesny 1995]). Moreover, not every visualization technique is easily applicable when representing a larger number

of structures. Additionally, it is worth mentioning, that the consumer of the analysis is most often expecting conclusions supported by values of appropriate measures having straightforward interpretation but also intuitive charts. However, present day, basic office tools allow to relatively easily implement simple methods of measuring structures' similarity as well as visualization thereof. Only very complex techniques require support from specialized equipment to perform measurement and visualization.

Authors have been engaged in the research on measuring similarity or dissimilarity of structure, especially in the field of economical-agricultural studies, in both static and dynamic approaches (see [Binderman, Borkowski, Szczesny, Shachmurove 2012, Binderman, Borkowski, Szczesny 2008, 2009, 2010b,c; Borkowski Szczesny 2002]). Bibliography in this fields provides a rich set of instruments.

The word “structure” can have multiple meanings depending on context, i.e. an economical structure, agricultural structure and so on. An in-depth analysis of the term structure in relation to economical studies was performed in [Kukuła 2010, Malina 2004].

Let

$$\Re_+^n := \{\mathbf{x} = (x_1, x_2, \ldots, x_n) : x_i \geq 0, i = 1, 2, \ldots, n\}, n \in \mathbb{N},$$

$$\Omega := \left\{\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \Re_+^n : \sum_{i=1}^{n} x_i = 1\right\}.$$

In the following work the elements of set Ω will be called *structural vectors* or *structures* for short.

Let X denote any non-empty set, function $\mathrm{d}: X \times X \to \Re := (-\infty, +\infty)$ for any two elements *x, y* from X fulfills the following:

1. $\mathrm{d}(x, y) \geq 0$,
2. $\mathrm{d}(x, x) = 0$,
3. $\mathrm{d}(x, y) = \mathrm{d}(y, x)$.

Function d(x, y) can be treated as a *measure of dissimilarity* of elements x and y. In literature it is often called distance. However, it needs to emphasized that this is not a metric. Naturally, every metric is a distance. A diameter of set X will be equal to: .

$$\rho_X := \sup_{x,y \in X} \mathrm{d}(x, y).$$

We will say a function $\mathrm{s}: X \times X \to [0,1]$ is a *measure of similarity* when for any two elements x and y from set X it fulfills:

1. s(x,x)=1,
2. s(x,y)=s(y,x).

Let the diameter of set **X** - $\rho_X > 0$ be finite. Let us notice that using the measure of dissimilarity of elements d we can define *the measure of similarity* by equation:

$$\mathrm{s}_{\mathrm{d}}(x,y) = 1 - \frac{\mathrm{d}(x,y)}{\rho_X}.$$

In a special case when $\rho_X = 1$ the above equation takes form:

$$\mathrm{s}_{\mathrm{d}}(x,y) = 1 - \mathrm{d}(x,y).$$

With the development of techniques of visualization analysts started to utilize heuristic measures, which are intuitive and seem to be a promising path of advance, in order to compare ordering of objects. Visualization of objects, which have many features, based on polygons (i.e. radar charts from MS EXCEL) is one of such techniques. Authors have dedicated a few works to this problem [see Binderman, Borkowski, Szczesny 2008, 2010a, 2011]. Let us define a synthetic pseudo-radar measure of vector $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in [0;1]^n$ as [por. Binderman, Borkowski, Szczesny 2008]:

$$\mathrm{R}(\mathbf{x}) = \sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i x_{i+1}}, \quad x_{n+1} := x_1 \tag{1}$$

This measure is normalized (i.e. it takes values from interval [0, 1]) and allows to define in various ways the function of dissimilarity (distance) of two given objects $\mathbf{x} = (x_1, x_2, \ldots, x_n)$, $\mathbf{y} = (y_1, y_2, \ldots, y_n) \in \Omega$. For example:

$$d_1(\mathbf{x}, \mathbf{y}) = |R(\mathbf{x}) - R(\mathbf{y})|, \quad d_2(\mathbf{x}, \mathbf{y}) = R(|\mathbf{x} - \mathbf{y}|), \tag{2}$$

where $|\mathbf{x} - \mathbf{y}| := (|x_1 - y_1|, |x_2 - y_2|, \ldots, |x_n - y_n|)$.

The above “distances” induce measures of similarity of structures:

$$\mathbf{x} = (x_1, x_2, \ldots, x_n), \mathbf{y} = (y_1, y_2, \ldots, y_n) \in \Omega:$$

$$s_{d_1}(\mathbf{x},\mathbf{y}) = 1 - d_1(\mathbf{x},\mathbf{y}), \quad s_{d_2}(\mathbf{x},\mathbf{y}) = 1 - d_2(\mathbf{x},\mathbf{y}) \cdot \tag{2'}$$

Example 1. Let $\mathbf{x} = \left(\frac{1}{2}, 0, \frac{1}{2}\right)$, $\mathbf{y} = \left(\frac{1}{2}, \frac{1}{2}, 0\right)$, then $|\mathbf{x} - \mathbf{y}| = \left(0, \frac{1}{2}, \frac{1}{2}\right)$

$\mathrm{R}(\mathbf{x}) = \mathrm{R}(\mathbf{y}) = R(|\mathbf{x} - \mathbf{y}|) = \frac{1}{2\sqrt{3}}$, $\quad d_1(\mathbf{x}, \mathbf{y}) = 0$, $d_2(\mathbf{x}, \mathbf{y}) = \frac{1}{2\sqrt{3}}$, which implies $s_{d_1}(\mathbf{x},\mathbf{y}) = 1$, $\quad s_{d_2}(\mathbf{x},\mathbf{y}) = 1 - \frac{1}{2\sqrt{3}}$,

where measures d_1, d_2, s_1, s_2 are defined as in (2), (2'), respectively.

MEASUREMENT OF CONCENTRATION

Economic inequality was for a long time in the center of attention of both, sociologists and economists. However, the meaning of that term is not precisely defined. Naturally, it is easy to differentiate between a state of equality and inequality, but given two non-uniform distributions of a resource it is non-trivial to determine which of the two is “more” unequal. In general it is accepted that a distribution where each household possesses the same income is called an egalitarian distribution, one that is void of any inequality. When studying inequality one measures the degree to which the studied distribution differs from an egalitarian one. To measure the degree of dissimilarity (concentration) one must decide on a particular measure. However, a choice of a measure in practice means a decision on how to specifically define inequality/concentration.

As mentioned in the introduction we will limit ourselves to aggregated data, which means henceforth $\mathbf{x}, \mathbf{y} \in \Omega$ denote two structures, where y denotes a structure of objects divided into quintile groups, meaning having uniform coordinates equal to 1/n, while x denotes a structure of a resource associated with those n groups of objects contained in structure y. This does not decrease the level of generality of our analysis as a population of size n can be defined by two structures with n coordinates.

We will call the following a cumulation of a vector $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \Omega$

$$\mathbf{cum}(\mathbf{x}) := \left(\sum_{i=1}^{1} x_i, \sum_{i=1}^{1} x_i, \ldots, \sum_{i=1}^{1} x_i, 1 \right) = \mathbf{x}^{\wedge} = \left(x_1^{\wedge}, x_2^{\wedge}, \ldots, x_n^{\wedge} \right).$$

Practicians who study levels of differentiation of income or other resources possessed by a given group of objects most often present the following postulates about coefficient d(x, y) used for measurement (which deals in a case of aggregated data with dissimilarity between structures of entities and resources possessed by those entities):

- coefficient assumes the value of 0 if the resource is uniformly distributed across all objects (the structures are identical x = y);
- values of the coefficient are consistent with principle of transfers, which states that any transfer of resources between a "poorer" object to a "richer" one increases the non-uniformity in the population (which means that transfers between components of the structure, x_i and x_{i+s} increases the values of d(x, y));
- transfer sensitivity axiom : the influence a transfer from a "poor" object to a "poorer" one has on the value of the coefficient, when the value of the transfer is constant, is greater the richer the giving object is (which means that the farther away the giving object is from the receiving one, the greater the change of the value of dissimilarity should be);
- coefficient d(x, y) assumes its maximum when all the resources are possessed by a single object (in case of dissimilarity of two structures when, for example, x = (0, 0, …, 0, 1));
- scale invariance axiom means that the value of the coefficient does not change when the values of resources experience proportionate changes.

Naturally, the fourth postulate can be omitted because it follows from the second postulate.

The most popular coefficient used to measure the level of concentration (dissimilarity) of distribution, which fulfills the above postulates, is the Gini index, defined as doubled area between the Lorenz curve and the diagonal of a unit square (see [Barnett 2005, Hoffmann and Bradley 2007]).

In order to present the construction of a basic coefficient of dissimilarity (concentration) of distribution based on radar charts, let us inscribe a regular n-gon $\mathbf{F}_n$ into a unit circle with a radius of 1 and centered at the origin of in the Cartesian coordinate system in the Euclidean plane $(z,w)=(0,0)$. Let us connect the vertices of the n-gon with the origin of the coordinate system. We will denote the resulting line segments of length 1 as O_1, O_2, …, O_n, starting with the segment covering the vertical axis *w*.

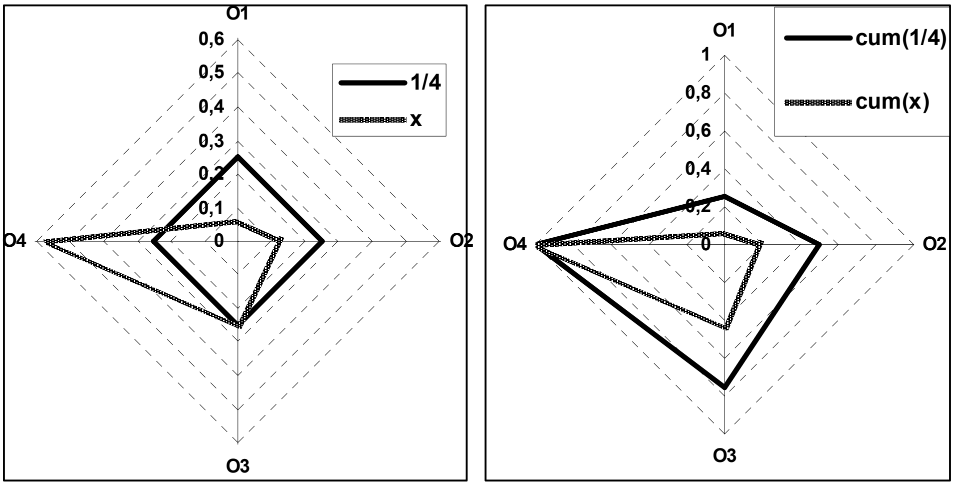
If the features of object $\mathbf{x}=(x_1,x_2,\ldots,x_n)$ assume unit values from the interval <0, 1>, that is $\mathbf{0} \le \mathbf{x} \le \mathbf{1} \equiv 0 \le x_i \le 1$, i=1,2,...n,, where $\mathbf{0} = (0, \ldots, 0)$ and $\mathbf{1} = (1, \ldots, 1)$, then we can present the values of features of this object on a radar chart. To do this, let us denote by x_i a point on O_i, which was constructed by intersecting the segment O_i with a circle of radius x_i and centered at the origin of the coordinate system, for i = 1, 2, …, n. By connecting x_1 with x_2, x_2 with x_3, …, x_{n-1} with x_n and x_n with x_1 we will construct a polygon W_n.

In the following figure 1 (radar chart) we find illustrations for vectors representing structures:

$$\mathbf{y} = \mathbf{1/4} = \left(\frac{1}{4},\frac{1}{4},\frac{1}{4},\frac{1}{4}\right), \quad \mathbf{x} = \left(\frac{1}{16},\frac{2}{16},\frac{4}{16},\frac{9}{16}\right) \tag{3}$$

and their respective vectors representing those structures when they are in cumulative form: **cum**(**1/4**)=(*1/4*,*2/4*,*3/4*,*1*) i **cum**(**x**)=(*1/16*,*3/16*,*7/16*,1).

Figure 1. Left: illustration of structures defined as in (3). Right: illustration of structures as defined in (2) in cumulative form.



Source: own research

Let us notice that polygon representing **cum(x)** is contained within a polygon induced by vector **cum(1/4)**. Let vector $\mathbf{x}' = (x_1', x_2', \ldots, x_n')$ denote any structure ($\mathbf{x}' \in \Omega$) which coordinates fulfill the condition : $x_1' \le x_2' \le \ldots \le x_n'$. It can be proved that a polygon designated by **cum(x')** is contained within a polygon designated by **cum(1/n)** for n >= 4.

THEOREM 1.

Let a vector $\mathbf{x} = (x_1, x_2, \ldots, x_n) \in \mathbf{\Omega}$, $n \in \mathbb{N}$, the structure $\mathbf{x}' = (x_1', x_2', \ldots, x_n')$ means the vector, created by the permutation of the coordinates of the vector **x**, that its coordinates satisfy the condition: $x_1' \le x_2' \le \ldots \le x_n'$. We denote by

$\mathbf{x}^{\wedge} = \left(x_1^{\wedge}, x_2^{\wedge}, ..., x_n^{\wedge}\right)$ the cumulation of the vector $\mathbf{x}'$ i.e. $\mathbf{x}^{\wedge}$=**cum**($\mathbf{x}'$). If the radar polygons $\mathcal{W}_{\mathbf{x}^{\wedge}}, \mathcal{W}_{\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}}$ are generated by vectors $\mathbf{x}^{\wedge}$ i $\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}$, respectively, then $\mathcal{W}_{\mathbf{x}^{\wedge}} \subset \mathcal{W}_{\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}}$.

Proof. Let us suppose that the assumption of the theorem are satisfied but $\mathcal{W}_{\mathbf{x}^{\wedge}} \supset \mathcal{W}_{\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}}$ and $\mathcal{W}_{\mathbf{x}^{\wedge}} \neq \mathcal{W}_{\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}}$. This means that there exists *k*∈ {*1,2, ..., n-1*} such that $x_k^{\wedge} > \frac{k}{n}$. The last inequality and the definition of the vector $\mathbf{x}'$ together imply

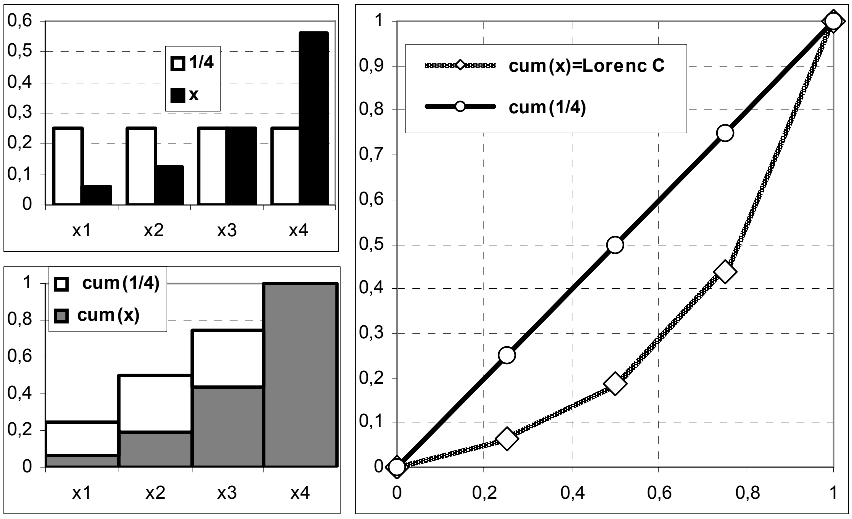
$$\sum_{i=1}^{k} x_i' > \frac{k}{n},\ x_k' > \frac{1}{n} \ \text{and}\ \mathrm{x}_j' > \frac{1}{n}\ \text{for}\ j = k+1,\ k+2, ..., n$$

Hence $\mathrm{x}_j^{\wedge} > \frac{k}{n}$ for $j = k, k+1, ..., n$. In particular, $\mathrm{x}_n^{\wedge} > \frac{n}{n} = 1$, which contradicts the assumption. Thus $\mathcal{W}_{\mathbf{x}^{\wedge}} \subset \mathcal{W}_{\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}}$ for all $\mathbf{x} \in \mathbf{\Omega}$. The last inequality follows from the turn that $\mathrm{x}_j^{\wedge} > \frac{k}{n}$ dla $j = k, k+1, ..., n$. In particular, that $\mathrm{x}_n^{\wedge} > \frac{n}{n} = 1$. But with the notion we have that $\mathrm{x}_n^{\wedge} = 1$, this contradicts our assumption, therefore, that $\mathcal{W}_{\mathbf{x}^{\wedge}} \supset \mathcal{W}_{\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}}$ i $\mathcal{W}_{\mathbf{x}^{\wedge}} \neq \mathcal{W}_{\frac{\mathbf{1}}{\mathbf{n}}^{\wedge}}$.

Which is similar to the situation when a polygon designated by the abscissa and the Lorenz curve is contained within any triangle of a unit square. More precisely, a polygon designated by the abscissa and a cumulated structure of a resource is contained within a polygon designated by the abscissa and a cumulated specialization of structure of a resource, which is identical with the structure of objects – meaning when the resource is uniformly distributed across all objects.

For the considered example of vectors **x** and **y** in the following figure 2, we have presented a structure of a resource defined by vector x compared against an egalitarian structure (one with uniform coordinates) in both forms, normal and cumulated (both as column charts – left part of figure 2) as well as in the form of a Lorenz curve (right part of figure 2). It can be easily seen that in this case the Lorenz curve is identical to with the so called curve of cumulated frequency of a resource placed on four intervals of equal length into which the interval [0, 1] was divided. Let us notice that the classic Gini index in this example is equal to the complement to 1 for the ratio of two areas: one underneath the Lorenz curve and other beneath **cum(1/4)**. We will denote this coefficient as $\mathcal{G}$. Using the remaining two geometrical interpretations (radar polygon and column chart for cumulated structure) in a similar manner, we arrive at two coefficients $\mathcal{G}$R and $\mathcal{G}$S that measure the non-uniformity of the distribution. It can be easily show that in the case of structure (1/16, 2/16, 4/16, 9/16) we have $\mathcal{G}$=0,40625, $\mathcal{G}$R=0,6041(6), $\mathcal{G}$S=0,3250.

Figure 2. Presentation of structures (2) in normal and cumulated form



Source: own research

However, it needs to be mentioned that both coefficients $\mathcal{G}$ and $\mathcal{G}$S, when there is a low amount of objects (in this case a low amount of coordinates of vector **x**), meaning when all of the resource in in the possession of a single object, assume values far removed from 0. Specifically, for **x** = (0, 0, 0, 1) we have symbol $\mathcal{G}$=0,75, $\mathcal{G}$S=0,60 i $\mathcal{G}$R=1,0. After introducing normalizing factors (meaning after dividing by 0,75 and 0,6, respectively), for the previously considered structure (1/16, 2/16, 4/16, 9/19) we receive values $\mathcal{G}$=0,40625/0,75 = 0,541(6) = $\mathcal{G}$S=0,3250/0,60.

In general, the area S_1 of a radar polygon induced by vector $\mathbf{x}=(x_1,x_2,...,x_n) \in [0,1]^n$ is defined as follows [Binderman, Borkowski, Szczesny 2008]:

$$S_1 = \sum_{i=1}^{n} \frac{1}{2} x_i x_{i+1} \sin\frac{2\pi}{n} = \frac{1}{2}\sin\frac{2\pi}{n}\sum_{i=1}^{n} x_i x_{i+1}, \quad \text{gdzie } x_{n+1} := x_1.$$

Which means is can be shown that area S_0 of a radar polygon $\mathbf{F}_n$, induced by vector **cum(1/n)** = (1/n, 1/n, …, 1/n) is defined by:

$$S_0 = \frac{1}{2}\sin\frac{2\pi}{n}\left(\sum_{i=1}^{n-1}\left(\sum_{j=1}^{i} x_j\right)\left(\sum_{j=1}^{i+1} x_j\right)+\frac{1}{n}\right) = \frac{1}{2}\sin\frac{2\pi}{n}\left(\sum_{i=1}^{n-1}\frac{i(i+1)}{n^2}+\frac{1}{n}\right) =$$

$$= \frac{2n^2+4}{12n}\sin\frac{2\pi}{n}$$

It can be easily proved that if $\mathbf{x}' = (x_1', x_2', \ldots, x_n') \in \Omega$ has this : $x_1' \le x_2' \le \ldots \le x_n'$ property, that its coordinates fulfill then radar polygon W_n induced by vector **cum(x')** is contained in radar polygon $\mathbf{F}_n$, induced by vector **cum(1/n)**. The ratio of areas of those polygons S_1/S_0 can be assumed to be the measurement of similarity of a given structure (distribution of a resource) to a uniform structure (egalitarian distribution) and a coefficient defined as:

$$\mathcal{G}R = 1 - \frac{S_1}{S_0} = 1 - \frac{6n}{2n^2+4}\left[\sum_{i=1}^{n-1} \hat{x}_i \hat{x}_{i+1} + x_1\right], \text{ gdzie } \hat{x}_i := \sum_{j=1}^{i} x_j' \qquad (4)$$

can be assumed to be a measurement of concentration/non-uniformity of distribution of a resource set by structure **x'**. It is easy to show, that measure $\mathcal{G}R$(**1/n**)=0, $\mathcal{G}$R((0,…,0,1))=1.

DEFINITION

A measurement defined by equation (4) will be called a radar measure of concentration (non-uniformity of income).

The radar measure fulfills the 5 previously mentioned postulates set by practicing. Let us notice that Gini index, fulfilling the postulates, has this property that $\mathcal{G}$(**1/n**)=0 i $\mathcal{G}$((0,…,0,1))=1-1/n. However, if we desire for it to assume a value of one for the structure (0, 0, …, 1), we can multiply it by n/(n – 1).
Using the same idea of a geometrical interpretation we can transform (symbol) the equation for the measure when we are using a column chart to:

$$\mathcal{G}S = 1 - \frac{\left[\sum_{i=1}^{n}\min[\hat{x}_i), y_i\,]\right]}{\left[\sum_{i=1}^{n} y_i\right]} = 1 - \frac{2}{n+1}\sum_{i=1}^{n}\min(\hat{x}_i, y_i)\ , \qquad (5)$$

gdzie $\hat{\mathrm{x}}_i = \sum_{j=1}^{i} x_j',\ y_{\mathrm{i}} = \frac{i}{n},\ i = 1,\ldots,n$

Naturally, coefficient $\mathcal{G}$S also fulfills the conditions postulated by practicians, but for the structure (0, 0, …, 1) is assumes a value of (n – 1)/(n + 1). However, after normalization (meaning multiplying by a factor of (n + 1)/(n – 1)) it is equal to the value of a normalized Gini index. This is why we will not be considering this coefficient any more.

Another means of creating a measure of concentration is by using measures of dissimilarity of structures in cumulated form and the same idea that was behind the Gini index (where "distance" is the area between a Lorenz curve and the diagonal). Meaning, by using the technique of radar coefficients it can be shown that a coefficient defined as:

$$W_k = \frac{d_k[(\frac{1}{n},\frac{2}{n},...,1),\mathbf{cum}(\mathbf{x})]}{d_k[(\frac{1}{n},\frac{2}{n},...,1),(0,0,...,0,1)]},\ k=1,2, \tag{6}$$

where d_k is defined by equation (2). Those coefficients also fulfill the previously mentioned postulates. Overall, an analyst can create many such coefficients.

COEFFICIENTS' SENSITIVITY TO CHANGES

Whenever we are faced with a problem of comparing non-uniformity of distribution of a resource between objects in multiple populations or in one population but in multiple time periods, there is a risk that it can't be done by visualization alone. We need to possess a non-uniformity coefficient which is sensitive to that special type of changes of non-uniformity that interest the researcher/analyst. Because the most popular Gini index may prove to be unresponsive to the aspect of changes that the analyst wants to study. Naturally, the study of sensitivity of various coefficients requires an appropriate mathematical workshop. However, today, with the ubiquitous computer tools, it can be achieved by utilizing simple office tools. Let us show this on an artificial example. Let us assume we are interested in the disappearance of the so called middle class and we want to test whether the coefficient $\mathcal{G}$R is more sensitive to that change than Gini index. In table 1 we can see changes of fictitious structure of, for example, salaries in a big corporation in various time periods or, perhaps, the changes of the structure of income from all possible sources in a given society. For simplification purposes, let us assume our data is aggregated to decile (nie jestem pewien czy to jest dobre tłumaczenie) groups. In Table 2 we present the values of six coefficients of concentration. The first three are the well-known coefficients based on the Lorenz curve: Gini, Schutz and L=(l-$\sqrt{2}$)/(2-$\sqrt{2}$), where l the length of the Lorenz curve (see Barnett R. 2005, Hoffmann and Bradley 2007, Kakwani 1980, Lamber 2001, Rosenbluth 1951). The latter three are based on visualization methods that use radar charts. Coefficients $\mathcal{G}$r1 and $\mathcal{G}$r2 were created by applying formula (6) to equation (2).

The data was compiled in such a manner that we begin with a structure that possesses a large middle class, composing 50% of the whole population and owning 80% of the resources. Afterwards, we add the rich class. During the studied period there is a large outflow of resources from the middle class to the rich class and a small outflow from the middle class to the poor one. We are interested in such a coefficient that would signalize those changes by increasing its value.

Table 1. Fictitious structures: egalitarian (T0), and during seven periods (T1, …, T7)

	d1	**d2**	**d3**	**d4**	**d5**	**d6**	**d7**	**d8**	**d9**	**d10**
T0	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1	0,1
T1	0,00000	0,01000	0,02000	0,02000	0,05000	0,18000	0,18000	0,18000	0,18000	0,18000
T2	0,00889	0,01000	0,02000	0,02000	0,05000	0,16000	0,18000	0,18000	0,18000	0,19111
T3	0,00889	0,02000	0,04000	0,05000	0,05000	0,06000	0,18000	0,18000	0,18000	0,23111
T4	0,00889	0,02625	0,06000	0,06000	0,06000	0,06000	0,08000	0,18000	0,18000	0,28486
T5	0,00889	0,02750	0,07000	0,07000	0,07000	0,07000	0,07000	0,08000	0,18000	0,35361
T6	0,00989	0,04262	0,07000	0,07000	0,07000	0,07000	0,07000	0,07000	0,07000	0,45749
T7	0,05311	0,05311	0,05311	0,05311	0,05311	0,05311	0,05311	0,05311	0,05311	0,52200

Source: own research

Table 2. Coefficients of concentration during the studied periods

	Lorenz Curve			Radar's diagram		
	Gini	**Schutz**	**L**	**$\mathcal{G}$R**	**$\mathcal{G}$r1**	**$\mathcal{G}$r2**
T1	0,4220	0,4000	0,2247	0,4698	0,2718	0,4834
T2	0,4220	0,3911	0,2126	0,4799	0,2788	0,4820
T3	0,4220	0,3711	0,1803	0,5194	0,3067	0,4714
T4	0,4220	0,3449	0,1670	0,5572	0,3346	0,4626
T5	0,4220	0,3336	0,1736	0,5872	0,3575	0,4549
T6	0,4220	0,3575	0,2010	0,6216	0,3849	0,4538
T7	0,4220	0,4220	0,2327	0,6277	0,3898	0,4528

Source: own research

Table 2 shows that the most popular Gini index is not sensitive to those changes in the structure, that are defined in Table 1, while radar coefficients $\mathcal{G}$R and $\mathcal{G}$r1 clearly show that changes towards increasing the level of concentration are happening. On the other hand, coefficient $\mathcal{G}$r2 indicates that the level of concentration is decreasing. Schutz and L coefficients are behaving in a similar fashion, but only during periods T1 – T5. We leave the decision which of those coefficients is best at picking up changes in times of increasing globalization. Naturally, such a decision requires defining which features are preferable.

In order to present in a more intuitive manner the idea of sensitivity of those coefficients to changes, we will consider the initial structure of resources **s0** defined in Table 3 and we will assume that further changes to it will involve transferring 0.01 of a resource from group **d1** to groups **d2**, **d3**, …, **d10**. We will denote structure created by these transfers as **s1**, …, **s9**. The values of the six chosen coefficients are present in Table 4, while the values of deltas of them are in Table 5 and Figure 3.

Table 3. Exemplary initial structure of resources for the purposes of the simulation

	d1	d2	d3	d4	d5	d6	d7	d8	d9	d10
s0	0,019	0,021	0,04	0,06	0,08	0,1	0,12	0,14	0,16	0,26

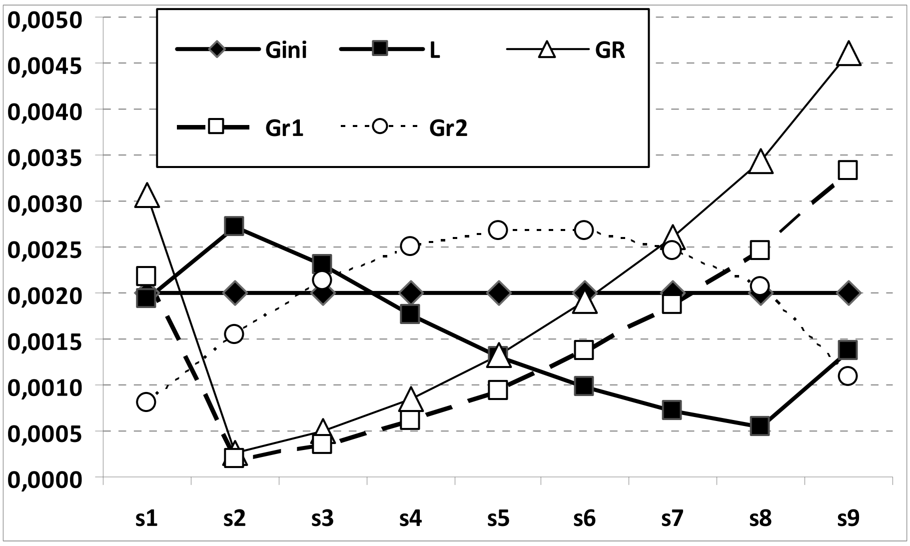
Source: own research

Table 4. Values of the chosen coefficients of concentration for the structure defined in Table 3 and its nine subsequent modifications involving transfers of resources from group **d1** to other decile groups

	s0	s1	s2	s3	s4	s5	s6	s7	s8	s9
Gini	0,3842	0,3862	0,3882	0,3902	0,3922	0,3942	0,3962	0,3982	0,4002	0,4022
Schutz	0,2800	0,2800	0,2800	0,2800	0,2800	0,2900	0,2900	0,2900	0,2900	0,2900
L	0,1350	0,1369	0,1396	0,1419	0,1437	0,1450	0,1459	0,1467	0,1472	0,1486
$\mathcal{G}$R	0,4998	0,5029	0,5032	0,5037	0,5045	0,5058	0,5077	0,5104	0,5138	0,5184
$\mathcal{G}$r1	0,2928	0,2949	0,2951	0,2955	0,2961	0,2970	0,2984	0,3003	0,3027	0,3060
$\mathcal{G}$r2	0,4192	0,4200	0,4215	0,4237	0,4262	0,4288	0,4315	0,4340	0,4360	0,4371

Source: own research

Figure 3. Increases of values of coefficients from Table 3. detailed information can be found in Table 5.



Source: own research

It is clear in the figure that the increase of Gini index is constant and equal to 0,002. However, individual increases of other coefficients have differed substantially. Radar coefficient $\mathcal{G}$R reacts more strongly than Gini index to transfers from **d1** to **d2** or **d10**, while experiencing lower changes when transfers happen from **d1** to **d2 – d6**. Which means that is displays a “sharper” reaction to creation of rich and poor groups.

Table 5. Changes (increases) in values of coefficients of concentration from Table 3.

	s0	s1	s2	s3	s4	s5	s6	s7	s8	s9
Gini	x	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020	0,0020
Schutz	x	0,0000	0,0000	0,0000	0,0000	0,0100	0,0000	0,0000	0,0000	0,0000
L	x	0,0019	0,0027	0,0023	0,0018	0,0013	0,0010	0,0007	0,0005	0,0014
$\mathcal{G}$R	x	0,0031	0,0003	0,0005	0,0009	0,0013	0,0019	0,0026	0,0034	0,0046
$\mathcal{G}$r1	x	0,0022	0,0002	0,0004	0,0006	0,0009	0,0014	0,0019	0,0025	0,0033
$\mathcal{G}$r2	x	0,0008	0,0015	0,0021	0,0025	0,0027	0,0027	0,0025	0,0021	0,0011

Source: own research

SUMMARY

In this work we have presented two approaches to creating coefficients of concentration as well as basic technique for verification of fitness for purpose of the created coefficients, which can be easily performed with standard office applications. Naturally, a more elegant approach is to deduce the properties of constructed coefficients by means of instruments provided by higher level mathematics. However, performing numerous well-planned simulations can not only simplify that process but also replace it altogether. Results that we have got for fictitious data show the strong suits of methods that use radar charts. Authors intend to verify their presented conceptions in their next work by using real data.

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