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A FEW REMARKS ON THE STOCHASTIC STRUCTURE OF THE UNEMPLOYMENT RATE IN POLAND BY GENDER

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Abstract: The quarterly unemployment rate from the Labour Force Survey covering Poland's data from the first quarter 2005 to the third quarter 2019 was investigated. The issue was to reveal its stochastic structure as a trend, seasonality and disturbance and to make a prognosis. The analysed data comes from a survey based on rotational design, so the problem of possibly autocorrelated survey errors was taken into consideration. Following Harvey (2000), Pfeffermann, Feder, and Signorelli (1997), Yu and Mantel (1997) and Bell and Carolan (1998) it seemed to be of great importance to include the proper autocorrelation structure of the errors into a statistical treatment. It appeared that for Polish unemployment data that structure was not as it could have been expected. After the model was fitted to the data, a conclusion about the specificity of the unemployment rate with respect to gender was drawn. Unemployment forecast until 2020:Q4 is provided.

Keywords: Unemployment rate, rotational design, structural time series.

1. Introduction

The unemployment rate provides insights into the economy's spare capacity and unused resources. Traditionally, it is widely recognized as a key indicator of labour market performance. Unemployment tends to be cyclical and decreases when the economy expands. When the economy is in poor shape and jobs are scarce, the unemployment rate can be expected to rise. When the economy is growing at a healthy rate and jobs are relatively plentiful, it can be expected to fall. Unemployment usually increases when companies are trying to cut costs. Employees who are expected to work harder do not receive any additional compensation for the extra hours worked. Unemployment can also have a negative mental effect on those who are still working. They may become more concerned about losing their own jobs or be hesitant to look for something better.

There are some issues to be addressed. It is desirable to focus on the unemployment rate as a random variable which evolves stochastically over time. Therefore the issue was to reveal its stochastic structure as a trend, seasonality and disturbance. The data from the target series were from a survey. The question was whether sample surveys errors from a rotational design should be taken into account. It seemed to be of great importance to include the errors into a statistical treatment (cf. Harvey, 2000; Pfeffermann, Feder, and Signorelli, 1997; Yu and Mantel, 1997; Bell and Carolan, 1998).

2. Data

The quarterly unemployment rate from the Labour Force Survey was investigated. The data covered Poland's unemployment rate from the first quarter of 2005 to the third quarter of 2019. The rate is plotted in Figure 1. The measure of unemployment is based on the definition by the International Labour Organisation. The unemployment rate is the number of unemployed persons as a percentage of the labour force (the total number of people employed and unemployed). Unemployed persons comprise persons aged 15 to 74 who fulfil all three of the following conditions:

- they are without work during the reference week;
- they are available to start work within the next two weeks;
- they have been actively seeking work in the past four weeks or have already found a job to start within the next three months.

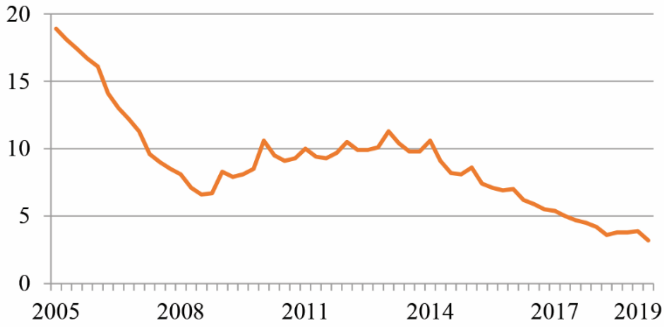


Fig. 1. Poland's unemployment rate

Source: own work.

The decomposition of the unemployment rate into trend, seasonality and irregularity is depicted in Figure 2. The decomposition was based on the state space form of structural time series (see Harvey (1989)). The question of how the irregular part was autocorrelated is important. The autocorrelation functions of that part and of the residuals are depicted in Figure 3. According to the underlying model, the two parts

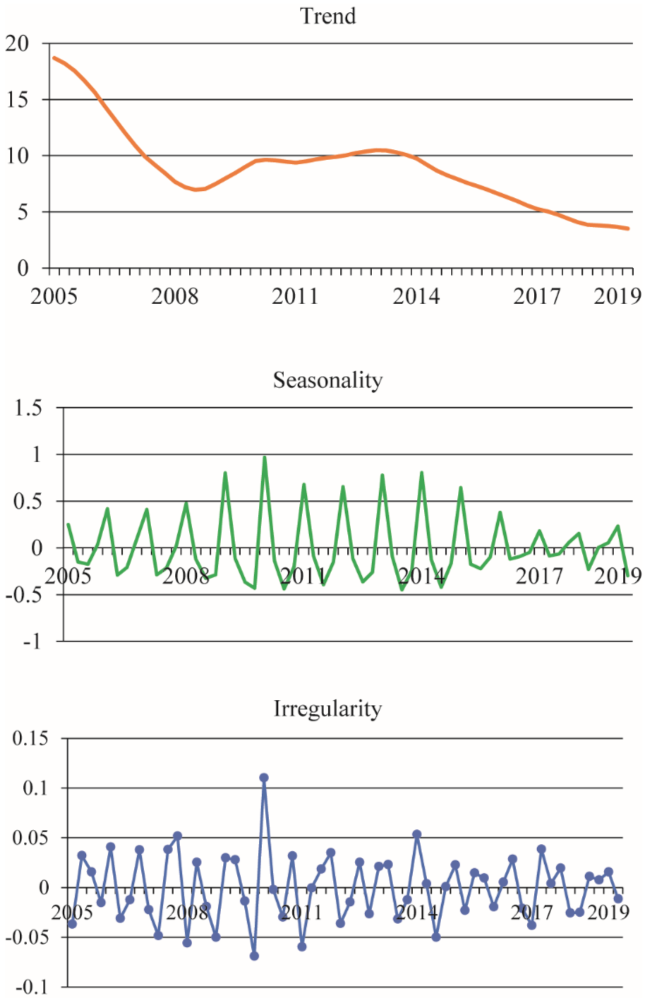


Fig. 2. Decomposition of unemployment rate into trend, seasonality and irregularity

Source: own work.

can not be autocorrelated. This is not the case, and thus the irregular part reflects the unknown systematic pattern of the unemployment rate which should be considered in the statistical treatment. Unfortunately, the pattern contradicts the expectations that come from surveys errors of a rotational design explained in the next section.

3. Rotational design

In the Labour Force Survey it takes three months to cover all the households in the sample. In the next quarter a half of the households are questioned in the same area as before and a quarter are replaced by new households, as illustrated in Table 1. For example in Year 2:Q4 households from sample 8; 9; 12; 13 are questioned. Sample 13 is new in Year 2 but sample 8 consists of the previously questioned households in Year 1: Q3, Q4 and Year 2: Q3.

Let e_t denotes sampling error from the rotational design and t time in quarters. Following Harvey (2000) and Pfeffermann et al. (1997), it can be assumed that

$$\rho(e_t, e_{t-1}) = \begin{cases} 0.50\phi & \text{for } m = 1 \\ 0 & \text{for } m = 2 \\ 0.25\phi^3 & \text{for } m = 3 \\ 0.50\phi^4 & \text{for } m = 4 \\ 0.25\phi^5 & \text{for } m = 5 \\ 0 & \text{for } m > 5 \end{cases},$$

where $\phi \in (-1,1)$ denotes serial correlation of a 1 month lag of sampling errors related to the same panel. This set-up assumes that there is no change from one time period to another, either in the composition of the panel or in the measurement of individual units.

As shown in Figure 3, neither of covariance functions confirm the assumption. This means that the seasonality pattern assumed in the applied statistical model is not good enough to capture seasonal variability.

Table 1. Rotational design of the Labour Force Survey in Poland

Sample	Year 1				Year 2			
number	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
1	x							
2	x	x						
3	–	x	x					
4	–	–	x	x				
5	x	–	–	x	x			
6	x	x	–	–	x	x		
7		x	x	–	–	x	x	
8			x	x	–	–	x	x
9				x	x	–	–	x
10					x	x	–	–
11						x	x	–
12							x	x
13								x

Source: own work.

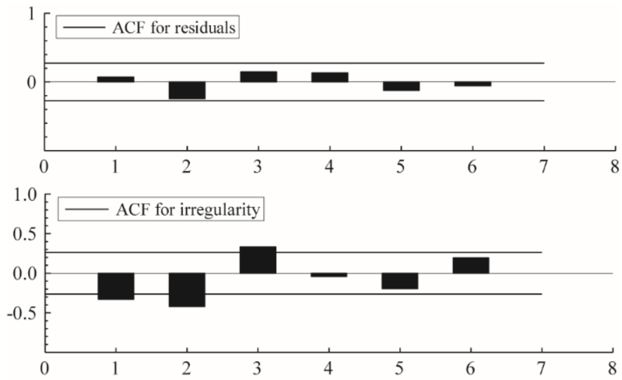


Fig. 3. Autocorrelation functions for residuals and irregularity

Source: own work.

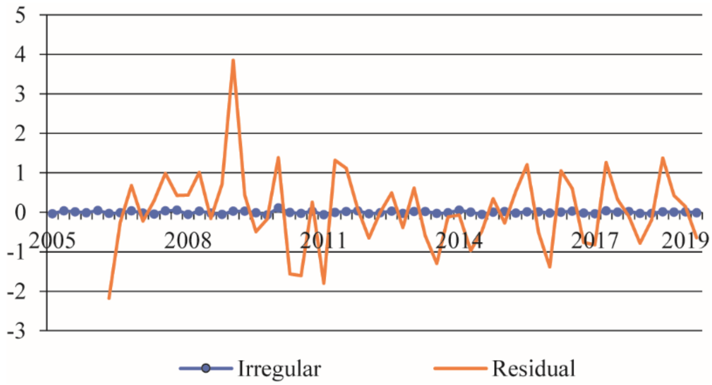


Fig. 4. Residuals and irregularity

Source: own work.

Therefore the seasonal component in the given model should be supplemented (details in Section 4). Residuals and irregularities are plotted in Figure 4. Note the difference between the variability of the plotted lines. The magnitude of the difference suggests that irregular components may be omitted from the model.

4. Univariate statistical model

Let y_t, $t = 1, \dots T$ denote quarterly observations of unemployment rate. A structural time series model for y_t is set up in terms of *trend* μ_t, *seasonal* γ_t, *cycle* c_t and *irregular* ϵ_t if

$$y_t = \mu_t + \gamma_t + c_t + \epsilon_t, \qquad \epsilon_t \sim NID(0, \sigma_\varepsilon^2), \qquad t = 1, \dots, T,$$

where state equations for these components are as follows

Trend

$$\begin{array}{ccc} \mu_{t+1} & = & \mu_t + \beta_t \\ \beta_{t+1} & = & \beta_t + \zeta_t,\ \ \zeta_t \sim NID(0,\sigma_\zeta^2)^{\cdot} \end{array}$$

Seasonality

$$\begin{array}{ccc} \gamma_t & = & \gamma_{1t} + \gamma_{2t} \\ \begin{bmatrix} \gamma_{1t+1} \\ \gamma_{1t+1}^{\star} \end{bmatrix} & = & \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \gamma_{1t} \\ \gamma_{1t}^{\star} \end{bmatrix} + \begin{bmatrix} \omega_{1t} \\ \omega_{1t}^{\star} \end{bmatrix}, \\ \gamma_{2t+1} & = & \gamma_{2t} + \omega_{2t} \end{array}$$

where $\omega_{1t}, \omega_{1t}^{\star}, \omega_{2t}$ are independent $N(0, \sigma_\omega^2)$ variables

Cycle

$$\begin{bmatrix} c_{t+1} \\ c_{t+1}^{\star} \end{bmatrix} = \rho_c \begin{bmatrix} cos\lambda_c & sin\lambda_c \\ -sin\lambda_c & cos\lambda_c \end{bmatrix} \begin{bmatrix} c_t \\ c_t^{\star} \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t^{\star} \end{bmatrix},$$

where $\rho_c \in \langle 0,1\rangle$ is a damping factor, $2\pi/\lambda_c$is the period of the cycle and $\omega_{1t},\ \omega_{1t}^{\star}$, ω_{2t} are independent $N(0,\sigma_c^2)$.

The notation $NID(0,\sigma^2)$ denotes a normally distributed, serially independent, random variables with mean zero and variance σ^2. A comprehensive treatment of the state space approach to time series analysis can be found in Harvey (1989) and Durbin et al. (2001).

5. Unemployment univariate time series analysis

Let us consider the model with trend, seasonal and cycle (with $2\pi/\lambda_c = 4$):

$$y_t = \mu_t + \gamma_t + c_t.$$

According to the remark in Section 3, irregular component $\epsilon_t$was omitted. The autocorrelation functions for residuals in the model are shown in Figure 5.

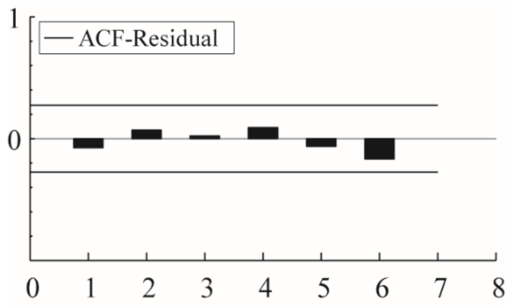


Fig. 5. Autocorrelation functions for residuals in the model with trend, seasonal and cycle components

Source: own work.

The residuals were not autocorrelated, hence these three components: trend, seasonal and cycle, explain the systematic variability of unemployment. It is interesting to compare the given model $y_t = \mu_t + \gamma_t + c_t$ with the model $y_t = \mu_t + \gamma_t + \epsilon_t$ used in Section 2. Component ϵ_t is replaced by c_t. The series $\{\epsilon_t\}_t$ can not be autocorrelated but not the series $\{c_t\}_t$ thus component c_t should explain better the shape of the autocorrelation function in Figure 3. Since c_t is a component with period 4 it can be perceived as a part of the seasonal variability of unemployment. This means that the seasonal effect was divided between components γ_t and c_t. The amplitude of γ_t in model $y_t = \mu_t + \gamma_t + \epsilon_t$ is approximately twice the amplitude of γ_t in model $y_t = \mu_t + \gamma_t + c_t$ (compare Figure 2 with Figure 6).

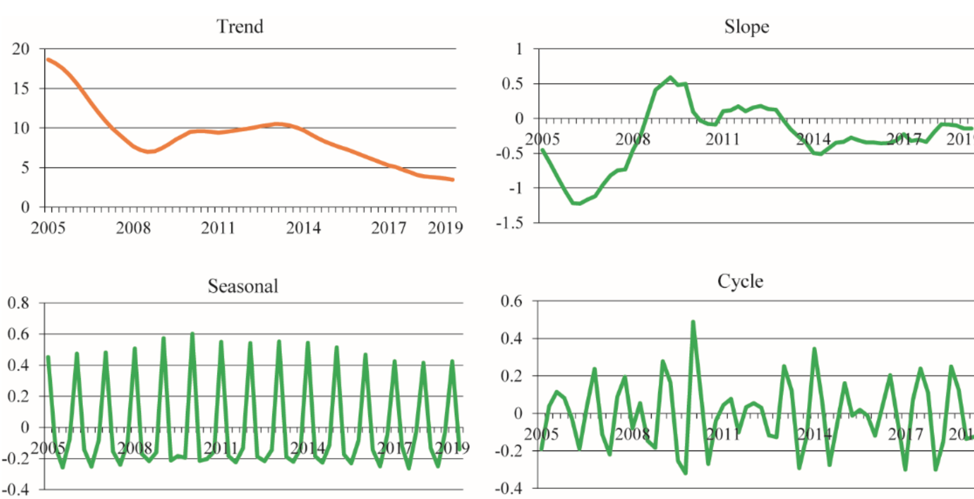
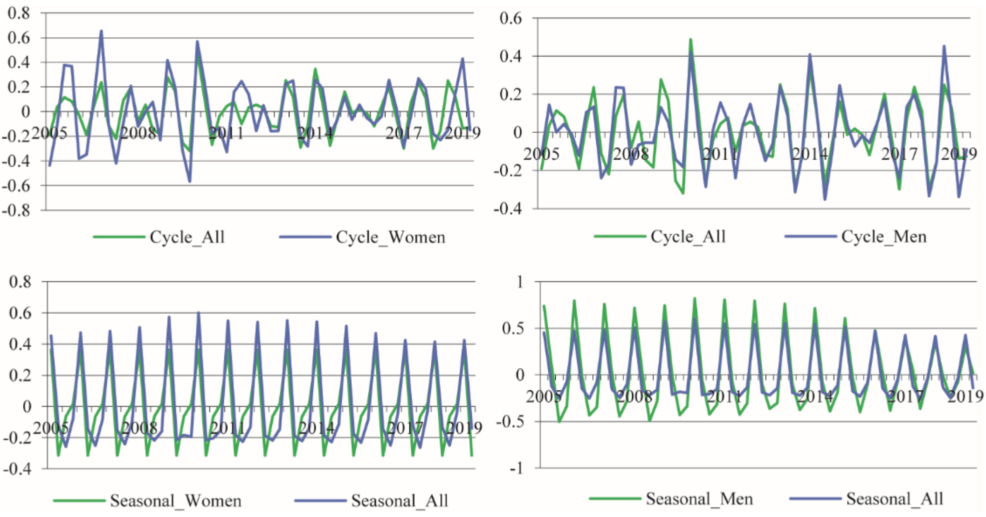


Fig. 6. Components of unemployment rate: trend, slope, seasonality and cycle

Source: own work.



_All – component for unemployed women and men, _Women – component for unemployed women, _Men – component for unemployed men. Estimation is based on univariate model $y_t = \mu_t + \gamma_t + c_t$ in every case

Fig. 7. Seasonality and cycle components

Source: own work.

The seasonal and cycle component exhibited in Figure 6 are much more characteristic for unemployed women than men. This is seen in Figure 7 where a comparison of the components with respect to gender is made. Consequently it is recommended that unemployment analysis should be based on two-dimensional time series of unemployed women and men in cases where the structure of seasonal effect is of main interest.

6. Two-dimensional statistical model

Let y_{1t} denotes an unemployment rate in time t for women and y_{2t} for men. A structural time series for $\boldsymbol{y_t} = [y_{1t}, y_{2t}]'$ is set up in terms of trend $\boldsymbol{\mu_t}$, seasonal $\boldsymbol{\gamma_t}$, cycle $\boldsymbol{c_t}$ and irregular $\boldsymbol{\epsilon_t}$ if

$$\boldsymbol{y_t} = \boldsymbol{\mu_t} + \boldsymbol{\gamma_t} + \boldsymbol{c_t} + \boldsymbol{\epsilon_t}, \ \ \boldsymbol{\epsilon_t} \sim N(0_2, \Sigma_\epsilon), \qquad t = 1, \dots, T.$$

Trend

$$\begin{aligned} \boldsymbol{\mu_{t+1}} &= \boldsymbol{\mu_t} + \boldsymbol{\beta_t} + \boldsymbol{\eta_t}, \quad \boldsymbol{\eta_t} \sim NID(0_2, \Sigma_\eta) \\ \boldsymbol{\beta_{t+1}} &= \boldsymbol{\beta_t} + \boldsymbol{\zeta_t}, \qquad\quad \boldsymbol{\zeta_t} \sim NID\left(0_2, \sigma_\zeta^2\right) \end{aligned}$$

Seasonality

Let $\boldsymbol{\Gamma_t} = [\boldsymbol{\gamma_{11t}},\ \boldsymbol{\gamma_{21t}},\ \boldsymbol{\gamma^{\star}_{11t}}, \boldsymbol{\gamma^{\star}_{21t}}, \boldsymbol{\gamma_{12t}}, \boldsymbol{\gamma_{22t}}]'$. Then

$$\begin{aligned} \boldsymbol{\gamma_t} &= \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \boldsymbol{\Gamma_t} \\ \boldsymbol{\Gamma_{t+1}} &= diag\left(\begin{bmatrix} cos\lambda & sin\lambda \\ -sin\lambda & cos\lambda \end{bmatrix} \otimes I_2, -1, -1\right) \boldsymbol{\Gamma_t} + \boldsymbol{\omega_t} \\ \boldsymbol{\omega_t} &\sim NID(0_6, I_3 \otimes \Sigma_\omega),\ \ \lambda = \frac{2\pi}{4}\ . \end{aligned}$$

Cycle

$$\begin{bmatrix} \boldsymbol{c_{t+1}} \\ \boldsymbol{c^{\star}_{t+1}} \end{bmatrix} = \left\{\rho_c \begin{bmatrix} cos\lambda_c & sin\lambda_c \\ -sin\lambda_c & cos\lambda_c \end{bmatrix} \otimes I_2 \right\} \begin{bmatrix} \boldsymbol{c_t} \\ \boldsymbol{c^{\star}_t} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\omega_t} \\ \boldsymbol{\omega^{\star}_t} \end{bmatrix}$$
$$\boldsymbol{\omega_{ct}} \sim NID(0_4, I_2 \otimes \Sigma_{\omega c}).$$

Matrices Σ_η, Σ_ζ, $\Sigma_{\omega c}$, Σ_ϵ are (2 × 2) dimensional, $\otimes$ denotes the Kronecker product, 0_n denotes n – dimensional vertical vector of zeros, I_n is a $(n \times n)$ unit matrix and $diag(A_1, \dots, A_n)$ denotes a square matrix such that the main-diagonal blocks are square matrices $A_1, \dots, A_n$ and all off-diagonal blocks are zero matrices.

7. Unemployment two-dimensional time series analysis

Let us consider the model

$$\boldsymbol{y_t} = \boldsymbol{\mu_t} + \boldsymbol{\gamma_t} + \boldsymbol{c_t}$$

with components defined as in Section 6. According to this model some conclusions can be drawn with respect to male and female unemployment rates:

- the rates decreased since 2005, although during the global economic crisis of 2008-2009 they increased and after 2013 decreased (Figure 8, left panel)
- the difference between male and female unemployment rates was less than 2% since 2006 and less than 1% since 2015 (Figure 8, right panel)
- the dynamics of the unemployment rates approached their stable state and are comparable with respect to gender (Figure 9)
- the seasonal effect for men has increased systematically since 2014; the seasonal effect for women was not as regular as for men and had no tendency to decrease (Figure 10).

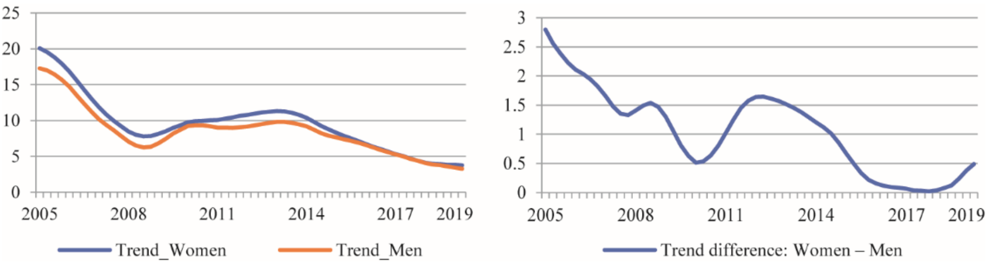


Fig. 8. Trend comparison: women vs men

Source: own work.

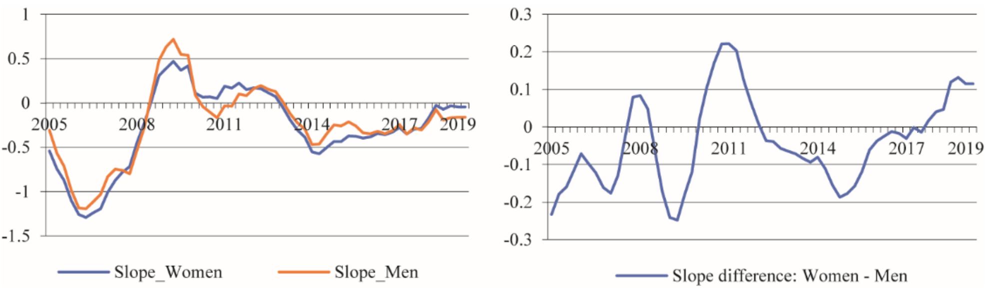


Fig. 9. Slope comparison: women vs men

Source: own work

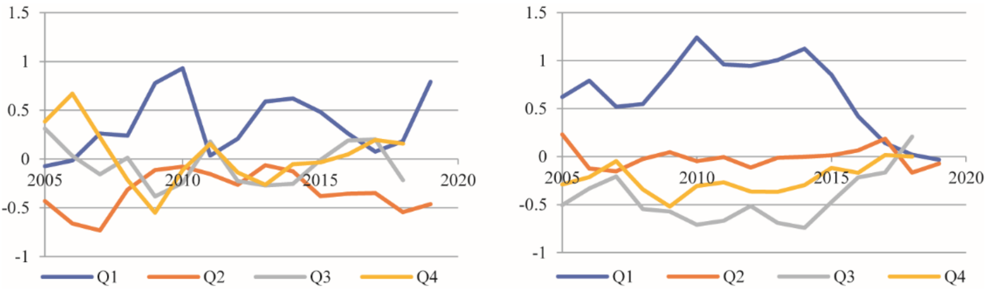


Fig. 10. Individual seasonal effect $\gamma_t + c_t$ for women (left side) and men (right side); Q_i – unemployment rate in i-th quartile, $i = 1,2,3,4$.

Source: own work

The deseasonalized unemployment forecast reflects the decreasing rate in the next year (see Table 2). If seasonality is taken into account the forecast is as in Table 3. Note that with respect to men there is a small difference between the two types of unemployment rate forecast. The reason is that the seasonal effect for male unemployment became insignificant in the last two years.

Table 2. Unemployment trend forecast for women and men

	2019:Q3	2019:Q4	2020:Q1	2020:Q2	2020:Q3	2020:Q4
Women	3.7%	3.7%	3.6%	3.6%	3.5%	3.5%
Men	3.1%	3.0%	2.8%	2.6%	2.5%	2.3%

Source: own work.

Table 3. Unemployment forecast for women and men

	2019:Q3	2019:Q4	2020:Q1	2020:Q2	2020:Q3	2020:Q4
Women	3.3%	3.8%	4.2%	3.3%	3.3%	3.5%
Men	3.1%	3.0%	2.9%	2.6%	2.3%	2.3%

Source: own work.

8. Conclusion

The quarterly unemployment rate from the Labour Force Survey was investigated. The data covered Poland's unemployment rate from the first quarter of 2005 to the third quarter of 2019. The unemployment analysis was expressed in terms of a trend, slope and seasonal structural time series model. The specificity of a rotational design was taken into consideration. It was revealed that sampling error of the Labour Force Survey in Poland was an insignificant component of the used model. The result was unexpected. During the analysis it appeared that it is recommended to investigate a two-dimensional time series analysis for male and female unemployment rates. The following interesting remarks were noted:

1. The unemployment rates increased during the 2008-2009 crisis then kept approximately steady at a level of about 10% until 2013. Then the rates were decreasing until 2019.
2. The unemployment rates are expected to decrease in 2020.
3. The dynamics of the unemployment rates approached their stable state.
4. The seasonal effect for men became insignificant in 2018.
5. The deseasonalized unemployment rate between women and men equalized in 2018 (see Figure 8).

The analysis of the Polish unemployment rate produced two unexpected results: the seasonal effect for men became insignificant and the difference between male and female unemployment rates equalized which is a new phenomenon in the Polish labour market. The seasonal effect for men has radically decreased since 2014 (see Figure 10, right panel) and it is interesting wheather this new phenomenon will hold in future. Unfortunately the difference between male and female is going to increase. This is undesirable since a large percentage of women is perceived as one of the many threats among the unemployed in Poland (Zdrojewski and Toszewska, 2010). It is interesting that the implied small difference between male and female unemployment rates, and what caused that difference, is expected to rise. However this goes beyond the scope of this work.

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KILKA UWAG O STOCHASTYCZNEJ STRUKTURZE STOPY BEZROBOCIA W POLSCE WEDŁUG PŁCI

Streszczenie: Analizowana jest kwartalna stopa bezrobocia według Badania Aktywności Ekonomicznej Ludności w zakresie od pierwszego kwartału 2005 r. do trzeciego kwartału 2019 r. Celem pracy jest dopasowanie strukturalnego modelu szeregu czasowego obejmującego trend, sezonowość oraz błąd, a także wyprowadzenie prognozy bezrobocia. Ponieważ analizowane dane zebrano według rotacyjnego planu losowania, błędy powinny być ze sobą skorelowane. Zgodnie z sugestią autorów, takich jak: Harvey (2000), Pfeffermann i in. (1997), Yu i in. (1997) oraz Bell i in. (1998), wzięcie tego faktu pod uwagę jest niezwykle ważne przy wyborze modelu statystycznego. Okazało się, że w przypadku analizowanych danych postać zależności błędów nie jest taka, jakiej by należało oczekiwać. Po uwzględnieniu struktury szeregu czasowego stopy bezrobocia dopasowano odpowiedni model. Na podstawie oceny składowych modelu porównano poziom bezrobocia wśród kobiet i mężczyzn oraz wyprowadzono prognozę do czwartego kwartału 2020 r.

Słowa kluczowe: poziom bezrobocia, plany rotacyjne, strukturalne szeregi czasowe.

Quote as: Jaworski, S. (2020). A few remarks on the stochastic structure of the unemployment rate in Poland by gender. *Econometrics. Ekonometria. Advances in Applied Data Analysis*, *24*(2).