

# The validity and time-horizon of the Fed model for equity valuation: a co-integration approach

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Received: 22 May 2014 / Revised: 30 November 2014 / Accepted: 15 January 2015 / Published online: 26 March 2015

## ABSTRACT

Investors do arbitrage between bonds and stocks. The so-called “Fed model” asserts that comparing the level of the earnings yields of stocks to the nominal government bond yields is relevant when assessing the relative values of the two asset classes, and their prospective returns. A conceptual problem with this model is that it compares a real quantity, the earning yield, to a nominal one, the government bond yield, thus implying that economic agents suffer from money-illusion. The merits of the Fed model as an indicator of stock returns is still very controversial. In this article we try to quantify the scope of the Fed model by employing appropriate techniques of co-integration to validate, or invalidate, the Fed model. More precisely, we study the validity of the model geographically and using different frequencies in order to determine its potential time horizon. We obtain the following results. First, the Fed model is very limited in scope and in time: of the 21 pairs of countries and stock exchange indices tested, only three are potential candidates for the Fed model: the US, Italy and Mexico; in the US, the Standard and Poor’s 500 confirms the model, but only from 1980 to 2000. Second, for the Standard and Poor’s 500 from 1980 to 2000 the validity of the Fed model is confirmed, for a time horizon of one week or more for predicting the earning yield on stocks and a time horizon of one month or more for the nominal yield on bonds. Third, from 2000 onwards the long-term relationship between earning yields and nominal bond yields becomes inverted, and the Fed long-term relationship does not help predict any of the two variables compared to a simple vector autoregressive model (VAR). Overall, the evidence for the relevance of a linear long-term relationship between nominal US bonds yields level and the earnings ratio of broad stock indexes appears very weak, even when this relationship is allowed to vary over time, with a structural break somewhere in 2000 with an inversion of the relationship. In most cases, assuming and estimating a possible long-term relationship between earning yields and nominal bond yields does not improve the forecasts as short-term dynamics dominate.

*JEL classification:* G1 ; C1 ; G02; G14

*Keywords:* arbitrage; stock; earning yields; Fed model

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The author would like to thank Marianne Guille and Ali Skalli for their remarks and suggestions.

## 1. INTRODUCTION

The so-called Fed model for valuing stocks, which became widely popular after the release of the Humprey-Hawkins report to the Board of Governors of the Federal Reserve System (1997), has been and remains a widely controversial model among both scholars and practitioners. This simple model stipulates that comparing the ratio of the earnings over the price of the stock market to the yield of long-term government bonds indicates whether the stock market is overvalued or undervalued. The main idea is that bonds and stocks being competitive investments, their expected return – at least when adjusted for the perceived risk of each asset class – should be set equal by profit-maximizing investors. We will, throughout this article, denote by  $Y$  the yield on long-term government bonds and by  $E/P$  the earning yield of stocks, that is, the ratio of earnings  $E$  divided by the price level of the stock market  $P$ . Let us make the bold assumption, for simplicity of illustration, that both asset classes are perceived as incurring the same risk by market participants. Then the Fed model argues that the two quantities  $Y$  and  $E/P$  will be set equal by rational, return-oriented investors. For example, if it happens that  $Y < E/P$ , it would mean that stocks are cheap (relative to bonds), and investors will sell bonds and buy stocks, pushing the price  $P$  of the stocks higher until the  $E/P$  ratio is again set to the level of  $Y$ . We will refer to the Fed model that asserts a strict equality of  $Y$  and  $E/P$  as to the “strict” Fed model. Adjusting for perceived risks or other factors can yield other Fed models: the common idea of those “general” Fed models is that the position of  $E/P$  relative to  $Y$  is a pertinent indicator for valuing stocks. Such models can include adjustments for perceived risks, as the one proposed by Asness (2003) which uses prior realized volatility on stocks and bonds as a proxy for the risk perceived by investors.

From a theoretical perspective, it can be objected that  $E/P$  is not the return for investing in stocks, nor is the yield to maturity the return for investing in bonds (except if the bond is held to maturity and if all coupons can be reinvested at the same rate). But the main criticism made to the Fed model is that it compares a real quantity,  $E/P$ , to a nominal quantity, the government bond yield  $Y$ , and thus assumes that economic agents are victims of money-illusion, not being able to properly distinguish between real and nominal values. Indeed, stocks are claims on real assets, which are expected to appreciate with inflation, whereas the government bond yield is clearly a nominal quantity. Hence not only the Fed model would lack any firm theoretical grounding, but it is in flagrant contradiction with the hypothesis of efficient markets. One of the most vivid criticisms of the Fed model is certainly the one made by Asness (2003). Asness asserts the Fed model is correct as a descriptive tool – documenting how investors set the prices of stocks – but performs poorly as a predictive tool of subsequent returns. According to him investors are consistently making the same error, and the Fed model documents this error. This seems a paradox: how can a model which correctly describes the level to which investors set the price  $P$  of the stock market be denied any merit as a trading strategy? One element of answer could be the time horizon. It would be possible that the Fed model allows predicting correctly the future level of prices  $P$  of the stock, but without specifying the time horizon correctly. For example, and to sustain Asness claim, it could be that the time horizon is excessively short compared to the longer term horizon of the buy and hold strategy which Asness assumes when he shows that investing according to the signal offered by the Fed model actually incurs huge losses compared to, for example, a passive “buy and hold” investment strategy in bonds. Also consistent with this explanation are the findings of ap Gwilym et al. (2001). These authors study the historical in-sample performance of the Fed model as an tactical asset allocation tool for six different countries (the United States, the United Kingdom, France, Germany, Switzerland and Japan) and conclude that for short-term tactical allocation, the Fed model has some merit as a relative valuation tool, whereas over a 5 year horizon it underperforms the more traditional indicators, like the  $E/P$  ratio, in each country. Together with Asness’ own long-term study of the returns this suggests that the time horizon of the Fed model should be less than 5 years. Interestingly, no study trying to

estimate which minimum time period is required for applying the Fed model with success has been carried out to our knowledge. In this paper we propose a methodology based on in and out of sample comparisons of different model predictions to do so. The existence and predictability of a horizon for the Fed model would have huge practical and theoretical consequences. We use daily, end of the day, weekly and monthly, either end of the period or average over the sampling frequency period, data to assess the possible time horizon of the Fed model over a longer period and for different countries.

Because the Fed model posits the existence of a linear relationship between the earning yield and the nominal bond yield, linear regressions would be the natural candidate for evaluating the Fed model, as used by Asness (2003). Nevertheless, unit-root tests indicate that the series are non-stationary (independently of the frequency). Hence, a full co-integration approach would be more methodologically robust. More precisely, in Section 2 we explain why non-stationarity of what we call “Fed model indicators” implies that the Fed model cannot hold. This is because, when these indicators are not stationary, any deviations to their assumed fair level do not decrease over time. Because the stationarity of these indicators depends on the time-horizon and on the time-period, stationarity tests on the Fed indicators are carried out at different frequencies and for different sub-samples. Section 3 studies further the predictive ability of the model for the United States and the Standard & Poor’s 500 index (S&P 500). It uses both the Engle-Granger and the Johansen co-integration framework to build Vector Error Correcting (VEC) models. To compare the predictive value of the linear, long-term part of the model, it is compared in terms of performance to a simple Vector Auto-Regressive (VAR) model of similar size in terms of explanatory variables. Because the only difference between the two models is the long-term relationship (the “error-correcting term”) when the VAR outperforms the VEC according to different out-of-sample measures, it shows that this long-term linear relationship does not have any ability to predict future yields. It is also quite possible that, for example, the VAR would outperform the VEC for only one of the two variables forecast. For example it could be that the VAR outperforms the VEC for the nominal government yield but not for the earning yield. In that case we will conclude that the Fed model only has predictive power for the earning yield, and such results depend on the frequency at which the analysis is performed. Section 4 tries to determine the time horizon of the Fed model, also for the United States and the Standard and Poor’s, and uses Engle-Granger methodology and the same methodology as Section 3, while Section 5 concludes.

## 2. INTERNATIONAL EVIDENCE FOR THE STRICT FED MODELS

The data used in this section consists of the ten year generic government bond yields of different countries, as reported by Bloomberg, and of the earnings ratio E/P where E is defined as the trailing 12 months earnings of the enterprises quoted in some stock index of the country under study, and P is the price level of this index. List 1 in the Annex indicates the stock exchanges price-earning ratios studied. Admittedly, the choice of the trailing earnings for E is rather arbitrary. The source of our data is Bloomberg, and our dataset extends from 1 January 1963 to 24 March 2011 included. The frequencies of the data used here are daily (end of the day), weekly, and monthly (both end of the period and daily average of the period).

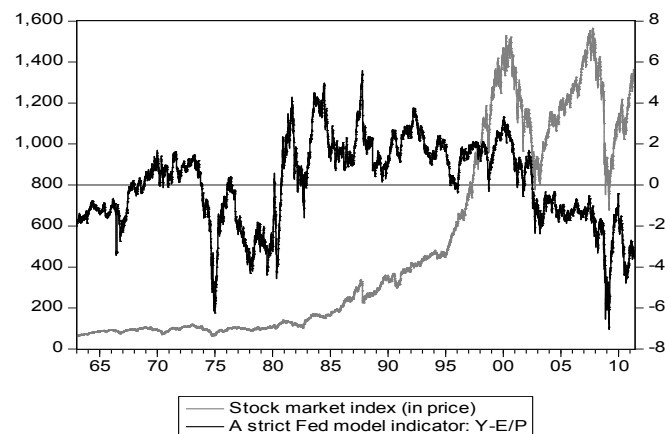
### Evaluating the strict Fed relationship

What we defined in the introduction as the *strict Fed model* asserts equality between the earning yield and the government yield. If the strict Fed model is right and this is indeed the long-term relationship between the two quantities, then taking the difference or the log-difference of the two series should result in a stationary series. Indeed if the resulting series has a unit root, then

the equilibrium is meaningless, since any disequilibrium of the relation  $Y = E/P$  does not tend to correct over time, but exhibits the highest degree of persistency. We can thus define different strict *Fed model indicators* and assess their stationarity. The most obvious indicator is the simple difference indicator  $Y - E/P$ . When it has a large negative sign, it means that stocks are cheap relative to bonds, hence rational investors seeking to maximize returns will bid up stock prices and the indicator will return towards 0. If the indicator is positive, then stocks are too expensive relative to bonds and the inverse will happen. Figure 1 below draws this indicator together with the stock index level<sup>2</sup>.

**Figure 1**

Simple strict Fed indicator for the S&P 500



For the United States and the S&P 500, the indicator did give a strong buying signal in the 1973 to 1980 period, which preceded the formidable increase in stock prices of the 1980 to 2000 period, but it also gave a strong selling signal from 1980 – or at least, from 1985, to 1995. An investor making investment decisions accordingly would have missed the run-up in equities of that period. Since 2002 the indicator has consistently given a buying signal, thus being unable to predict the stock market crash of 2008. Still, visual inspection also reveals it has correctly started giving a buying signal in 2002, precisely when the stock market had bottomed out of its previous 2000 crash. It also gave a strong correct buying signal in 2009, but failed to predict the 2007 crash. We leave it to the reader to try to draw eventual long-term trading rules from the data presented in Figure 1, but our overall sentiment is that visual inspection does not seem to indicate a strong and timely predictive power for this simple strict Fed indicator. Other possible strict Fed model indicators include the log difference  $\log(Y) - \log(E/P) = \log(Y/(E/P))$ , and the simple ratio  $Y/(E/P)$ . Similar observations can be drawn for these other strict Fed indicators. Of course, one could always try to design trading rules, and see if it incurs any more gains compared to some other strategies. Since any Fed indicator assumes the relevance of looking at both the government bond yield and the earning yield while assessing the level of stock prices, such a trading strategy should, in any study, be compared to the one using only the earning yield  $E/P$  as an indicator, since it is well known that the earning yield does have some predicting power for future stock returns. The problem is that there are an infinite number of possible trading strategies, depending on what we define to be a trading signal from the indicator, and the time-horizon to move in and out of the stock market<sup>3</sup>. Hence we need a formal test that would confirm or invalidate the relevance of strict

<sup>2</sup> The graph of this indicator for each other pair of country and stock market studied can be found in the document “Additional supporting evidence related to the article: ‘the validity and time-horizon of the Fed model: a co-integration approach’”, available from the author on request. The least we can say is that evidence is not overwhelming.

<sup>3</sup> This is a non-trivial problem: because of this infinite number of possible trading rules, one will always end up finding a rule which will work in-sample (that is, ex-post). To conclude that the Fed model is valuable on the sole basis of such a finding would thus be methodologically incorrect.

Fed model indicators. As mentioned earlier, a necessary condition is certainly the *stationarity* of the indicator. An indicator whose realization points towards a unit-root process is certainly not relevant: it means there are no forces that push the indicator back to equilibrium, and thus past shocks have a permanent effect on its level: not very pleasant since our trading signal involves comparing the indicator to the 0 level (or to any bands around 0, depending on how one defines a trading signal). If past shocks have a permanent effect on the indicator, then comparing it to any fixed band is irrelevant.

For each of the three different strict Fed model indicators mentioned we perform Augmented Dickey-Fuller (ADF) tests and Philipps-Perron tests with daily and monthly data, for each of our 24 samples. When the time-series is long enough, we also perform ADF tests on sub-samples. Tables 1, 2 and 3 in the Annex recapitulate the results, reporting the MacKinnon p-values corresponding to the t-statistics<sup>4</sup>.

The results are that the null of a unit root is rejected, at a 10% significance level and for all three indicators, for only four out of the 24 samples: the United States (S&P 500), Mexico, Italy and Australia. In the most recent period, from 1993 to 2011, none of the three indicators pass the test for the United States and the S&P 500: the tests indicate, for this subsample, very large p-values of more than 0.40. Not surprisingly, when looking at monthly data it is precisely the same countries which pass the tests. With only one exception (Canada), the *relative* strict Fed model indicator, that is, the second and third one, which involves the *quotient* of Y by E/P in log form or in simple form<sup>5</sup>, passes much more often the stationarity tests than the first absolute indicator, which assumes stationarity of the absolute change level in yields Y-E/P.<sup>6</sup>

**To conclude, for most markets, including the United States (S&P 500) from 1993 to March 2011, the Dow Jones Industrial and the NASDAQ, the strict Fed model fails to pass the most basic test: the so-called Fed indicator has probably a stochastic trend, hence its current level, partly due to the accumulation of *past* previous shocks, is irrelevant to assess the *current* over or under-evaluation of the stock market relative to bonds. Our results are thus in line with those of Gwilym et al. (2001) and Asness (2003). The (strict) Fed model does not travel well, nor does it fit the United States (S&P 500) for most recent periods, nor does it seem relevant for the other US stock indexes.**

Now the strict Fed model indicator seems stationary for the United State and S&P 500 for the whole sample period, for Mexico, Italy and Australia. We can assess the stationarity of the strict Fed indicator Y-E/P more precisely on each sub-sample by looking at rolling ADF tests. The statistical results are detailed in the supporting documentation can be obtained from the author on request<sup>7</sup>.

**They indicate that for some five-year periods, in particular the 2002 to 2008 period, the simple strict Fed indicator Y-E/P is not stationary, suggesting it cannot have been a good indicator during that period. The test with the relative strict Fed indicator gives similar results.**

<sup>4</sup> The lag for all the ADF tests was chosen so as to maximize the Schwarz Information Criteria, with a maximum of up to 25 lags. Philipps-Perron tests were performed using the Newey-West Bandwidth to select the bandwidth. Table 1 presents the result of the tests using daily, end of the day data, whereas Table 2 and Table 3 present the monthly, end of the month, and average of the month results. Notice that looking at both daily and monthly frequency adds really a new dimension to the problem, since the maximum lags used in ADF tests at daily frequency being 25, these daily stationarity tests necessarily have a shorter-time horizon than the monthly ones, for which the time lag is at minimum one month, and at maximum 25 months.

<sup>5</sup> Of course, since  $(Y-E/P)/(E/P) = Y/(E/P) - 1$  and that adding a constant does not change the results of the stationarity tests, this indicates that changes relatively to the current level of earning yields are usually more stationary than changes in the absolute levels.

<sup>6</sup> Indeed, at both monthly and daily frequency the third Fed indicator is stationary for the United Kingdom, Germany, and Japan, whereas the first indicator fails to satisfy stationarity in these three countries. All in all, it is now 7 out of 24 samples which pass the basic test of stationarity at both the daily and monthly frequency if we look at the third indicator instead of the first. This still leaves 17 samples in which the null of a unit-root process is not rejected, that is, the indicator seems to have a stochastic trend, hence behaving more like a random walk, with no economic forces pushing it back to 0 when shocks make it deviate from this assumed “fair” value.

<sup>7</sup> The document “Additional supporting evidence related to the article: ‘the validity and time-horizon of the Fed model: a co-integration approach’”, is available from the author on request.

### 3. ESTIMATED LINEAR FED MODELS: PERFORMANCE EVIDENCE

#### Outline of the methodology

As coined in the introduction, a general Fed model long-term relation could have any other form than a strict equality, in so far as it relies on comparing the earnings-to-price ratio  $E/P$  to the earning yields  $Y$ . A particular, yet more general case than a *strict* equality between the two ratios would be *linear* general Fed models, in which the assumed long-term relationship is:

$$Y = b * E/P + b_2, \text{ where } b \text{ and } b_2 \text{ stand for two constants, and } b > 0$$

This linear relation could be interpreted in a way similar to those of the general Fed model coined in the introduction which takes into account perceived relative risks between bonds and stocks: for example, the above relation with  $b_2 = 0$  would mean that stocks are deemed riskier than bonds if  $0 < b < 1$ , and less risky than bonds if  $b > 1$ . A negative  $b$  would be contradictory to the Fed model as it implied opposite dynamics for the earning yields and the government yields.

We study in this section the relevance of the linear general Fed model for the United States and the Standard and Poor's, using data from different frequencies, and within a co-integration theory framework as defined by Engle and Granger (1987) and using maximum likelihood methods as defined by Johansen, Stock and Watson (1995). This allows to build various Vector Error Correcting (VEC) models, whose merits as a quantitative version of the Fed model we assess by comparing the results of their predictions to those of similar models without error-correcting terms, like those of a simple Vector Autoregressive Models, in levels or in first difference, or to the naïve predictions of the explained variable lagged one period. The idea is to disentangle how much the Fed indicator actually improves forecasts when compared to these different alternative models which do not contain any long-term relationship between yields, earnings and stock market prices.

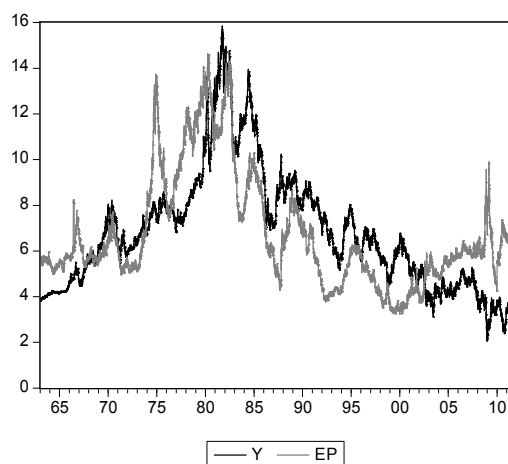
#### Visual inspection: a changing relation

The graphs below depict the very strong (visual) relation between the government yield and the earning yield of the S&P 500 over the whole sample period (graph on the left), and the price level of the S&P 500 (graph on the right).

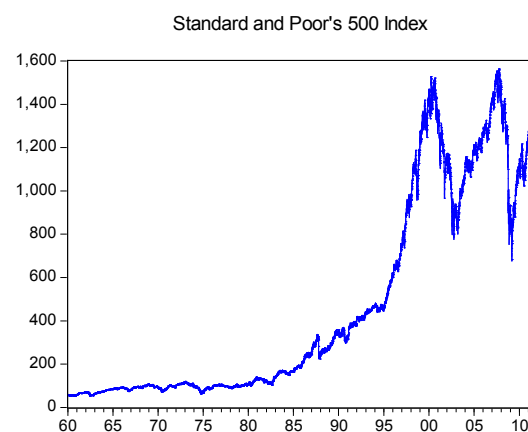
**Figure 2**

Time series of  $Y$ ,  $E/P$  and  $P$  for the US and the S&P 500

**Fig 2A.** US government yields and S&P 500



**Fig 2B.** S&P 500 in level

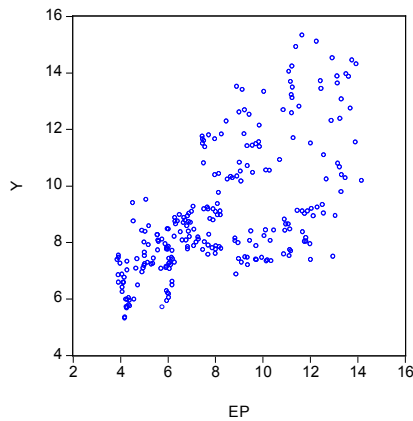


A first remark is that by looking at shorter sub-sample, the relationship between the two series is far from obvious. The graphs below represent scatter plots of the 10 year government bond yield against the earnings ratio of the S&P 500, for six different periods. They were obtained by using monthly data corresponding to the average of the daily observations of each month. For some periods, as for the 1985 to 1995 period, the two series seem to move together in the same direction, whereas for other periods, for example the 1975 to 1985 period, there does not seem to be any clear pattern. For the 2000 to 2011 period, the two series even seem inversely correlated, in plain contradiction with what the Fed model would predict, but in accordance with the “flight to safety” phenomenon, very strong in that period: afraid of stock returns, investors would flee the stock market and bid government bond prices up.

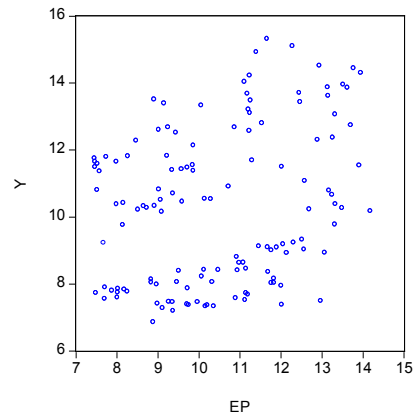
**Figure 3**

Scatter plots of  $Y$  and  $E/P$  for the US and the S&P 500 on different sub-samples

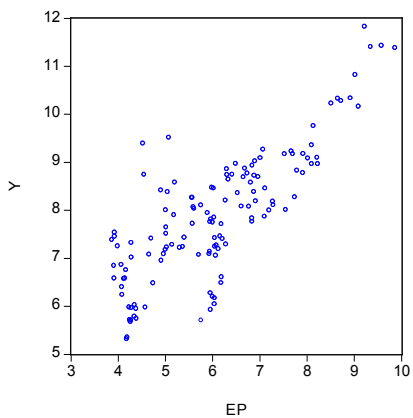
**Fig 3A. 1963–1975**



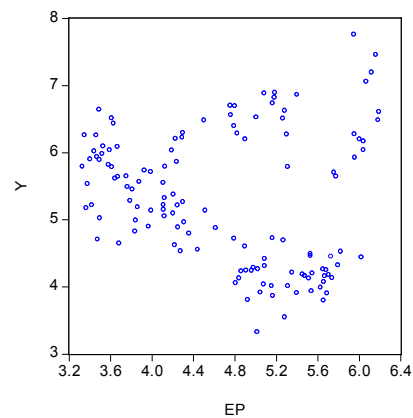
**Fig 3B. 1975–1985**



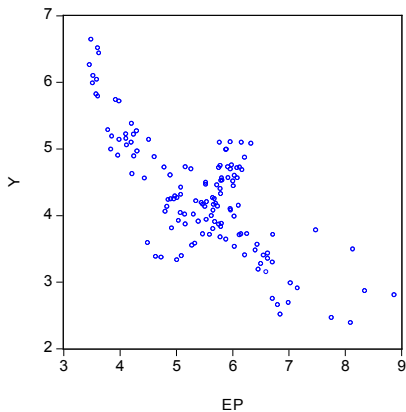
**Fig 3C. 1985–1995**



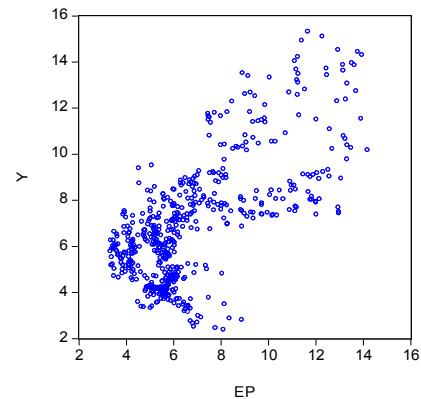
**Fig 3D. 1995–2005**



**Fig 3E. 2000–2011**



**Fig 3F. 1963–2011**



## Unit root tests for the government yield and the earning yield series

Johansen co-integration tests assume that the involved series are integrated of order one, hence we have to make sure this is the case, otherwise the tests are meaningless. We thus apply unit root tests to each of the two series  $Y$  and  $E/P$ . The detailed statistics results of these routine tests can be obtained from the author on request<sup>8</sup>.

### Tests at different frequencies, and for different sub-periods

For the sample from 1 January 1963 to 24 May 2011, Johansen tests performed with either daily, weekly or monthly data do not reject the null of no co-integration at the 10% significance level, for any lag length selected between 1 and 25.<sup>9</sup> The table below reports the Trace statistics and the Maximum Eigenvalue statistics of this test for the whole sample, together with the corresponding MacKinnon-Haug-Michelis (1999) p-values:

**Table 1A**

Johansen Co-integration Rank Test: Trace statistics

Number of co-integration relationships assumed in the null hypothesis of the test	Trace statistics	0.05 critical value	p-value
None	11.12764	15.49471	0.2038
At most 1	1.807713	3.841466	0.1788

**Table 1B**

Johansen Co-integration Rank Test: Maximum eigenvalue statistics

Test of the null of $r$ co-integrating relationships against the alternative of $r + 1$ co-integrating relationships	Trace statistics	0.05 critical value	p-value
None	9.319930	14.26460	0.2605
At most 1	1.807713	3.841466	0.1788

Dismissing the results of the tests and using co-integration techniques (Engle-Granger two steps method and Johansen maximum likelihood method) gives, unsurprisingly, inconsistent and unstable results<sup>10</sup>.

Now looking at the trends for the two series (see first chart of Figure 2) it is clear that we can separate our sample into two sub-samples so as to allow the Johansen test to capture the two deterministic trends better: the first linear deterministic trend is positively sloping from 1963 to 1980 whereas the second, from 1980 to 2011, has a negative slope.

We get, for the same tests as previously – in particular, using daily data – the following results for the two periods:

<sup>8</sup> These results are reported in Annex 4 of the document quoted in footnote 5, page 6.

<sup>9</sup> For example, for daily data and allowing 4 lags, an intercept but no deterministic trend in the co-integrating relation and a trend in the VAR give a Trace statistics of 11.127 corresponding to a p-value of 0.2039 for the null hypothesis of no co-integrating relationship.

<sup>10</sup> See the document quoted in footnote 5, page 6.



**Table 2A**

Johansen Co-integration Rank Test for the 1963 to 1980 period: Trace statistics

Number of co-integration relationships assumed in the null hypothesis of the test	Trace statistics	0.05 critical value	p-value
None	8.220717	15.49471	0.4421
At most 1	0.104091	3.841466	0.7470

**Table 2B**

Johansen Co-integration Rank Test from 1963 to 1980 period: Maximum eigenvalue statistics

Test of the null of $r$ co-integrating relationships against the alternative of $r + 1$ co-integrating relationships	Trace statistics	0.05 critical value	p-value
None	8.116626	14.26460	0.3670
At most 1	0.104091	3.841466	0.7470

**Table 3A**

Johansen Co-integration Rank Test for the 1980 to 2011 period: Trace statistics

Number of co-integration relationships assumed in the null hypothesis of the test	Trace statistics	0.05 critical value	p-value
None	15.99375	15.49471	0.0420
At most 1	1.100870	3.841466	0.2941

**Table 3B**

Johansen Co-integration Rank Test for the 1980 to 2011 period: Maximum eigenvalue statistics

Test of the null of $r$ co-integrating relationships against the alternative of $r + 1$ co-integrating relationships	Trace statistics	0.05 critical value	p-value
None	14.89288	14.26460	0.0397
At most 1	1.100870	3.841466	0.2941

Results for both periods are robust relative to the frequency used. The difference between the two sub-samples is striking: in the first sub-sample period the null of no co-integration is accepted with a high p-value of 0.44, whereas in the second the null of no co-integration is rejected at the 5% significance level, for daily data, and at a 10% significance level for weekly and monthly data.

The tables below detail the results for the 1980 to 2011 period, presenting the Trace statistics and the Maximum Eigenvalue statistics at each different frequency<sup>11</sup>.

<sup>11</sup> The p-values suggest there is more evidence of co-integration at the daily level than at lower frequency levels. This is in line with the results of the study of the strict Fed indicators indicated in the Annexes, where the p-value for stationarity of the strict Fed indicators were, most of the time, higher for daily frequencies (Table 1) than for monthly (Table 2).

**Table 4A**

Johansen Co-integration Rank Test for the 1980 to 2011 period: Trace statistics

Number of co-integration relationships assumed in the null hypothesis of the test	Frequency of data used	Trace statistics	0.05 critical value	p-value
None	daily end of the day	15.99375	15.49471	0.0420
	weekly end of the week	14.29764	15.49471	0.0751
	weekly, daily average	14.01039	15.49471	0.0827
	monthly end of the month	15.46071	15.49471	0.0506
	monthly, daily average	15.83884	15.49471	0.0444
At most 1	daily end of the day	1.100870	3.841466	0.2941
	weekly end of the week	1.915946	3.841466	0.1663
	weekly, daily average	2.079140	3.841466	0.1493
	monthly end of the month	1.910101	3.841466	0.1670
	monthly, daily average	1.947226	3.841466	0.1629

**Table 4B**

Johansen Co-integration Rank Test for the 1980 to 2011 period: Maximum eigenvalue statistics

Number of co-integration relationships assumed in the null hypothesis of the test	Frequency of data used	Trace statistics	0.05 critical value	p-value
None	daily end of the day	14.89288	14.26460	0.0397
	weekly end of the week	14.29764	15.49471	0.0751
	weekly, daily average	11.93125	14.26460	0.1133
	monthly end of the month	13.55061	14.26460	0.0646
	monthly, daily average	13.89161	14.26460	0.0572
At most 1	daily end of the day	1.100870	3.841466	0.2941
	weekly end of the week	1.915946	3.841466	0.1663
	weekly, daily average	2.079140	3.841466	0.1493
	monthly end of the month	1.910101	3.841466	0.1670
	monthly, daily average	1.947226	3.841466	0.1629

So far, we made certain that our two series, the government bond yield and the earnings ratio, were integrated of order one, using both augmented Dickey-Fuller tests and Phillips and Perron tests. We now see there is also strong evidence of co-integration for the 1980 to 2011 period, in particular at the daily and at the monthly frequency. We are thus in a position to follow the Johansen method for writing down a Vector Error Correction model for the two variables  $Y$  and  $E/P$ . We chose to build a VEC with 4 lags<sup>12</sup>, in order to get both a parsimonious model and uncorrelated residuals.

<sup>12</sup> The co-integration relation seems rather stable with respect to the number of lags selected. For example, writing a VEC with 25 lags gives the long term relationship:  $Y = 1.84161717598 * E/P + 4.70640290327$

The Johansen maximum likelihood methodology gives the following co-integration relationship:

$$Y = 1.8519020924 * E/P + 4.77205658219$$

Interestingly – and surprisingly – following Engle-Granger method for the same period gives a different co-integrating relationship:

$$Y = 0.900439413701 * E/P + 1.30149920728$$

Hence the two usual methods for estimating a VEC do not, in our case, give comparable results: the maximum likelihood method to estimate the long-term relationship used in the Johansen approach gives radically different results than the ordinary least square method used by Engle and Granger.

Now, the historical stability of the relation is another problem, and so is its economic interpretation. Concerning the Johansen estimation of the long-term relationship, it is rather dubious that in the long term an increase of 1 unit of the earning to price ratio would be followed by an increase of 1.84 units of the ten-year government yield. The Engle-Granger method<sup>13</sup> makes more sense economically: an increase of one unit of the earnings ratio would translate, at equilibrium, into an increase of 0.9 units for the yield Y. If we restrict our sample from 01/05/2011 to 25/05/2011, estimating the VEC using the Johansen approach gives the following relation:

$$Y = -0.775192725401 * E/P - 8.54556842774$$

Whereas the Engle-Granger approach gives the following:

$$Y = -0.601155146767 * E/P + 7.59160572346$$

The change of sign shows how much the “long-term relationship” depends on the sample. This result confirms our visual inspection of the scatter plots in the beginning of this section. Of course, there could be many possible structural factors explaining this change of the relationship. But maintaining that there is a stable relationship between the earnings ratio and the government bond yield, as does the Fed model, is contrary to evidence. Moreover, the sign is the opposite of what the Fed model suggests. This could be the effect of the so-called “flight to safety” where investors sell stocks to buy bonds, and hence, if earnings are more or less constant, E/P would go up while Y would go down. But if this explanation could hold for relatively short periods of stock market distress, it is hard to see why it would be the case for the whole first January 2000 to 24 March 2011 period.

### Historical stability of the long-term relationship

As noted earlier through visual inspection of the scatter plots, and in the previous paragraph, the long-term relationship does not seem particularly stable from a historical perspective. Table 5 below indicates the different long-term relationship of VEC models with 9 lags estimated during the mentioned period (the results are quite stable with respect to the number of lags):

<sup>13</sup> Recall the first step of Engle-Granger is a simple ordinary least square regression, as opposed to Johansen maximum likelihood approach.

**Table 5**

Co-integration relationship estimations, with associated speed of adjustment

Series	Co-integrating relation $Y = -b.E/P - b2$		Speed of adjustment*	
	b	b2	of Y	of E/P
1/01/1980 to 1/01/1985	-0.273985	-9.400664	-0.002821	0.007262
1/01/1985 to 1/01/1990	-1.427755	1.260989	0.001691	0.004768
1/01/1990 to 1/01/1995	-1.129606	-1.494971	-0.003522	0.001464
1/01/1995 to 1/01/2000	0.029319	-6.182875	-0.004146	0.001351
1/01/2000 to 1/01/2005	1.087241	-9.775998	-0.008214	-0.004656
1/01/2005 to 1/01/2010	1.078404	-10.68921	-0.004732	-0.002102
1/01/2005 to 24/05/2011	1.521183	-13.30656	-0.002583	-0.003555
1/01/1980 to 24/05/2011	-1.922247	5.221106	0.000127	0.000876

\* The speed of adjustment is the coefficient in front of the long term relationship in the VEC obtained following Johansen methodology. The higher it is, the more impact the long-term relationship has on the involved variable compared to the short-term dynamics.

The “long-term” relationship is very dependent on the sample selected, and in some cases the series involved does not even error-correct in the estimated VEC. A possible way to visually examine the stability (or the instability) of such a changing relationship is to plot the coefficients of successive (rolling) estimations of the long-term relationship<sup>14</sup>.

**We conclude that contrary to what asserts the Fed model, the long-term relationship is not stable. In particular the sign of b, for the most recent periods, is the opposite of what theory underlying the Fed model would suggest.**

Hence there is no evidence of a stable relationship between earnings yield and bond yields. We now turn to the evaluation of the general linear Fed model as a predictive tool. Could the Fed model, despite the evidence of instability in the relationship, be used as a predictive tool for timing the stock market? We will, in particular, look if allowing the long-term relationship to change continually gives better forecasts than those of other, simpler models.

### In-sample evaluation of the historical predictive power of the model

We evaluate, on historical data, the predictive power of the above VEC model for the earning price ratio and the government yield. To do so we compare our VEC model with other models, in particular with the simple model obtained by lagging the explained variable itself, which corresponds to naïve predictions one step ahead.

There is no unanimity as to whether or not VAR models can be written in level for non-stationary models. Sims (1980) and Sims, Stocks and Watson (1990) recommended against differencing even if the variables contain a unit root. We compare our VEC model with such a VAR in level, with the same number of lags. We use Root Mean Square Errors (RMSE) and the number of correctly predicted directions of change to compare these models.

<sup>14</sup> See the document quoted in footnote 5, page 6.

Now there are two possibilities for comparing these models on different sub-samples: one can either estimate the models a single time for the whole sample, from 1 January 1980 to 24 March 2011 and then look at RMSE and correctly predicted changes for different sub-samples (Table 6), or re-estimate the coefficients of the models for each sub-sample under study (Table 8).

**Table 6**

Models were estimated using the whole 01/01/1980 to 24/05/2011 period

Series: Y	VEC model	VAR model in level	Lagged variable	VEC model	VAR model in level	Lagged variable
Indicator:	RMSE	RMSE	RMSE	Correct changes	Correct changes	Correct changes
1/01/1980 to 1/01/1985	<b><u>0.198081</u></b>	0.198116	0.198154	623	629	<b><u>786</u></b>
1/01/1985 to 1/01/1990	0.197379	<b><u>0.197364</u></b>	0.197453	602	605	<b><u>759</u></b>
1/01/1990 to 1/01/1995	0.203514	<b><u>0.203502</u></b>	0.203507	619	641	<b><u>772</u></b>
1/01/1995 to 1/01/2000	<b><u>0.205790</u></b>	0.205798	0.205822	632	630	<b><u>768</u></b>
1/01/2000 to 1/01/2005	<b><u>0.204308</u></b>	0.204312	0.204301	633	624	<b><u>755</u></b>
1/01/2005 to 1/01/2010	0.182645	<b><u>0.182641</u></b>	0.182695	636	632	<b><u>786</u></b>
1/01/2005 to 24/05/2011	0.160218	<b><u>0.160216</u></b>	0.160260	801	788	<b><u>984</u></b>
1/01/1980 to 24/05/2011	<b><u>0.067971</u></b>	0.067979	0.068068	3910	3917	<b><u>4822</u></b>
Series: E/P	VEC model	VAR model in level	Lagged variable	VEC model	VAR model in level	Lagged variable
Indicator:	RMSE	RMSE	RMSE	Correct changes	Correct changes	Correct changes
1/01/1980 to 1/01/1985	<b><u>0.203602</u></b>	0.203670	0.203885	648	641	<b><u>786</u></b>
1/01/1985 to 1/01/1990	0.203703	<b><u>0.203656</u></b>	0.203876	596	595	<b><u>739</u></b>
1/01/1990 to 1/01/1995	0.203752	0.203743	<b><u>0.203729</u></b>	575	578	<b><u>748</u></b>
1/01/1995 to 1/01/2000	0.203760	0.203748	<b><u>0.203739</u></b>	641	645	<b><u>789</u></b>
1/01/2000 to 1/01/2005	0.203672	<b><u>0.203634</u></b>	0.203675	608	609	<b><u>747</u></b>
1/01/2005 to 1/01/2010	<b><u>0.203720</u></b>	0.203868	0.204408	571	573	<b><u>735</u></b>
1/01/2005 to 24/05/2011	<b><u>0.180216</u></b>	0.180354	0.180758	742	750	<b><u>961</u></b>
1/01/1980 to 24/05/2011	<b><u>0.081274</u></b>	0.081321	0.081684	3810	3818	<b><u>4768</u></b>

We can see that the Fed model performs poorly compared to the lagged variable or a simple VAR in level. We would like to be more precise and assess the usefulness of *the co-integrating relationship* in helping to improve the forecast. A simple idea would be to compare our VEC model with a VAR *in difference* with the same number of lags to assess if the error correcting term really brings any additional information to the forecast. Hence, the only explanatory variable that the VEC contains and that the VAR does not contain is the error-correcting term, that is, the estimated linear long-term Fed relationship. The following table, for each sample period, presents the obtained results:

**Table 7**

Models were estimated using the whole 01/01/1980 to 24/05/2011 period

Series	Y		E/P	
	VEC	VAR in difference	VEC	VAR in difference
1/01/1980 to 1/01/1985	<b><u>0.198081</u></b>	0.198084	<b><u>0.203602</u></b>	0.203689
1/01/1985 to 1/01/1990	<b><u>0.197379</u></b>	0.197382	<b><u>0.203703</u></b>	0.203744
1/01/1990 to 1/01/1995	<b><u>0.203514</u></b>	<b><u>0.203514</u></b>	0.203752	<b><u>0.203744</u></b>
1/01/1995 to 1/01/2000	<b><u>0.205790</u></b>	0.205791	0.203760	<b><u>0.203744</u></b>
1/01/2000 to 1/01/2005	0.204308	<b><u>0.204305</u></b>	<b><u>0.203672</u></b>	0.203689
1/01/2005 to 1/01/2010	0.182645	<b><u>0.182644</u></b>	<b><u>0.203720</u></b>	0.203744
1/01/2005 to 24/05/2011	0.160218	<b><u>0.160217</u></b>	<b><u>0.180216</u></b>	0.180224
1/01/1980 to 24/05/2011	<b><u>0.067971</u></b>	0.067972	<b><u>0.081274</u></b>	0.081326

If we use the RMSE criteria to assess performance, then the VEC model outperforms both the VAR in level and the VAR in difference for the whole sample period (see last column of Table 6 and 7). Nevertheless we note that on sub-samples it only outperforms the VAR in about half of the sub-samples. Moreover, the amount by which it outperforms the VAR on the whole period does not seem very large.

**Overall, the Fed model performs poorly compared to similar models. The changing nature of the VEC long-term relationship could explain such poor performance.** If we re-estimate the models for each of the sub-sample period, we find the results presented in Table 8 and 9:

**Table 8**

Models were estimated on each sub-sample period

Series: Y	VEC model	VAR model in level	Lagged variable	VEC model	VAR model in level	Lagged variable
Indicator:	RMSE	RMSE	RMSE	Correct changes	Correct changes	Correct changes
1/01/1980 to 1/01/1985	<b><u>0.197088</u></b>	0.197144	0.197368	610	613	<b><u>786</u></b>
1/01/1985 to 1/01/1990	0.196417	<b><u>0.196394</u></b>	0.196677	598	598	<b><u>759</u></b>
1/01/1990 to 1/01/1995	<b><u>0.202589</u></b>	0.202594	0.202636	666	656	<b><u>772</u></b>
1/01/1995 to 1/01/2000	<b><u>0.204806</u></b>	0.204822	0.204891	620	625	<b><u>768</u></b>
1/01/2000 to 1/01/2005	<b><u>0.203380</u></b>	<b><u>0.203380</u></b>	0.203437	632	628	<b><u>755</u></b>
1/01/2005 to 1/01/2010	<b><u>0.181964</u></b>	0.182077	0.182209	642	636	<b><u>786</u></b>
1/01/2005 to 24/05/2011	<b><u>0.159562</u></b>	0.159652	0.159784	802	800	<b><u>984</u></b>
1/01/1980 to 24/05/2011	0.067971	0.067979	0.068068	3910	3917	4822
Series: E/P	VEC model	VAR model in level	Lagged variable	VEC model	VAR model in level	Lagged variable
Indicator:	RMSE	RMSE	RMSE	Correct changes	Correct changes	Correct changes
1/01/1980 to 1/01/1985	<b><u>0.202566</u></b>	0.202639	0.203120	644	640	<b><u>786</u></b>
1/01/1985 to 1/01/1990	<b><u>0.202722</u></b>	0.202725	0.203124	588	594	<b><u>739</u></b>
1/01/1990 to 1/01/1995	<b><u>0.202787</u></b>	0.202794	0.202859	592	590	<b><u>748</u></b>
1/01/1995 to 1/01/2000	0.202772	<b><u>0.202770</u></b>	0.202798	650	641	<b><u>789</u></b>
1/01/2000 to 1/01/2005	<b><u>0.202737</u></b>	0.202746	0.202808	605	604	<b><u>747</u></b>
1/01/2005 to 1/01/2010	<b><u>0.202977</u></b>	0.203157	0.203974	583	559	<b><u>735</u></b>
1/01/2005 to 24/05/2011	<b><u>0.179558</u></b>	0.179729	0.180335	746	743	<b><u>961</u></b>
1/01/1980 to 24/05/2011	<b><u>0.081274</u></b>	0.081321	0.081684	3810	3818	<b><u>4768</u></b>

Our Vector Error Correcting model, with a different, changing “long-term” relationship for each subsample, now performs better than the VAR in level for most of the time. Yet, it is important to note that the lagged variable better predicts the *direction changes* of the explained variable than both models. Comparing our VEC to a VAR in first difference gives the following results:

**Table 9**

Models were estimated on each sub-sample period

Series	Y		E/P	
	VEC	VAR in difference	VEC	VAR in difference
1/01/1980 to 1/01/1985	<b><u>0.197088</u></b>	0.197107	<b><u>0.202566</u></b>	0.202783
1/01/1985 to 1/01/1990	<b><u>0.196417</u></b>	0.196427	<b><u>0.202722</u></b>	0.202842
1/01/1990 to 1/01/1995	<b><u>0.202589</u></b>	0.202602	<b><u>0.202787</u></b>	0.202789
1/01/1995 to 1/01/2000	<b><u>0.204806</u></b>	0.204825	<b><u>0.202772</u></b>	0.202775
1/01/2000 to 1/01/2005	<b><u>0.203380</u></b>	0.203404	<b><u>0.202737</u></b>	0.202745
1/01/2005 to 1/01/2010	<b><u>0.181964</u></b>	0.181985	<b><u>0.202977</u></b>	0.203013
1/01/2005 to 24/05/2011	<b><u>0.159562</u></b>	0.159576	<b><u>0.179558</u></b>	0.179617
1/01/1980 to 24/05/2011	<b><u>0.067971</u></b>	0.067972	<b><u>0.081274</u></b>	0.081326

Nevertheless, the amount by which the VEC, and hence the Fed model, outperforms a simple VAR in difference seems small and *possibly only due to the fact that in the OLS used to estimate the VEC we have one more explanatory variable than in the corresponding VAR (the error correcting term)*. One should try to factor out this purely mechanical effect of more explanatory variables before asserting the estimated long-term relationship improves the forecasts.

To detangle the real effect of the error correcting term from the purely mechanical effect of improving the forecast by having one explanatory variable more in the VEC than in the VAR, it could be useful to allow one more lag to the VAR. Unfortunately this solution increases by *two* the number of explanatory variables of the VAR, and not by one. Hence the results would be now biased toward finding that the VAR gives better prediction. A way to diminish that bias would be to allow more lags for both models, which will make the one more explanatory variable of the VAR less important. Hence we allow 10 lags in the VAR and only 9 in the VEC, which implies the OLS for the VAR has one more explanatory variable than for the VEC. Table 10 below reports the results:



**Table 10**  
Comparison between VEC and VAR

Series	Y		E/P	
	VEC	VAR in difference	VEC	VAR in difference
1/01/1980 to 1/01/1985	<b><u>0.197166</u></b>	0.197184	<b><u>0.202755</u></b>	0.202957
1/01/1985 to 1/01/1990	<b><u>0.196685</u></b>	0.196692	<b><u>0.202942</u></b>	0.203016
1/01/1990 to 1/01/1995	<b><u>0.202686</u></b>	0.202699	0.202800	<b><u>0.202794</u></b>
1/01/1995 to 1/01/2000	<b><u>0.203944</u></b>	0.203963	<b><u>0.201959</u></b>	<b><u>0.201959</u></b>
1/01/2000 to 1/01/2005	<b><u>0.203228</u></b>	0.203250	0.202602	<b><u>0.202598</u></b>
1/01/2005 to 1/01/2010	<b><u>0.181109</u></b>	0.181127	0.202043	<b><u>0.201959</u></b>
1/01/2005 to 24/05/2011	<b><u>0.158848</u></b>	0.158854	0.178792	<b><u>0.178775</u></b>
1/01/1980 to 24/05/2011	0.067889	<b><u>0.067887</u></b>	<b><u>0.081196</u></b>	0.081244

For each subsample period, the VEC model is still the best for predicting the earning to price ratio E/P, but the evidence is much more mixed than in the previous table. After all, the Fed model asserts that the long-term relationship (which is allowed to vary over time on each of the 7 subsamples indicated here) is relevant for predicting the E/P ratio, and consequently also the prices of stocks, given that the concept of earnings we are using here as a proxy for future earnings are trailing earnings (12 months). But what we actually observe is that the VAR in first difference forecasts the E/P ratio as well as the VEC and its long-term relationship. And although the Fed model seems to better forecast the yield Y on each sub-sample (because the long-term relationship is re-estimated on each sample period here), this is not the case for the whole sample<sup>15</sup> (suggesting, again, an instable relationship over longer horizons).

**We conclude that, at a daily frequency, the VAR in first difference or the VEC are about equivalent models. At the daily frequency the error correcting term does not seem to add much to the provision of the forecast. Moreover, to predict the direction of the changes in the variables (increase or decrease), both models do worse than the lagged variable itself.**

### Out-of-sample evaluation of the historical predictive power of the model

Now all the previous historical assessments of the models were done *in-sample*: although no information available at time  $t$  or after is given to the model to predict values at time  $t$ , the coefficients of the co-integrating relationship and of the VEC model itself are *estimated* using the *whole* sample period (that we occasionally also changed to get the results for Table 8 and 9). Hence these are *in-sample* predictions. How does the model perform for *out-of-sample* predictions?

To get an idea of the predictive power, out-of-sample, of a VEC model between the Y and E/P series we obtained Tables E and F.

<sup>15</sup> In other words, a single estimate of the long term relationship completely deprive the Fed model of its predictive power: the relationship needs to be estimated more often, on a smaller sub-sample.

To get the first line of our tables, we first estimate the two models from the date in the first column, which is 01/01/1980, to 1/1/1990. Then we use the model to predict out-of-sample estimates for the next date, 2/1/1990. We then re-estimate the model from 01/01/1995 to 2/1/1990 to get next forecast, for 3/1/2000. We are thus assured not to “peer” into the future, and that, at the same time, to use all the information available up to and including time  $t-1$  to make our predictions for time  $t$ . These can thus translate into practical, concrete models for predicting future movements of the explained variables. We proceed similarly with the other dates, always starting our estimation on the first date for a 10 year period before starting to predict for the next (daily) period.

Here too, we can define the residuals as the difference between the realized value and the predicted value and compare them with the residuals obtained from a “naïve” forecast (the variable itself lagged one period), or to those of a VAR, either in level or in first difference. The results are presented in Tables 11 and 12 below:

**Table 11**

Out-of-sample results with re-estimation of all models at each step

Series: Y	VEC model	VAR model In level	Lagged variable	VEC model	VAR model In level	Lagged variable
Indicator:	RMSE	RMSE	RMSE	Correct changes	Correct changes	Correct changes
Since 1/01/1980	0.052259	<b>0.052204</b>	0.052249	2700	2696	<b>3274</b> / 7817
Since 1/01/1990	<b>0.069018</b>	0.069090	0.069254	1454	1434	<b>1735</b> / 4164
Since 1/01/2000	<b>0.254042</b>	0.254046	0.254086	161	149	<b>194</b> / 512
Series: E/P	VEC model	VAR model In level	Lagged variable	VEC model	VAR model In level	Lagged variable
Indicator:	RMSE	RMSE	RMSE	Correct changes	Correct changes	Correct changes
Since 1/01/1980	<b>0.067341</b>	0.067411	0.067686	2555	2571	<b>3241</b>
Since 1/01/1990	<b>0.091924</b>	0.092067	0.092802	1343	1363	<b>1705</b>
Since 1/01/2000	<b>0.264602</b>	0.264565	0.264299	171	174	<b>223</b>

**Table 12**

Out-of-sample results with re-estimation of all models at each step

Series	Y		E/P	
	VEC	VAR in difference	VEC	VAR in difference
from 1/01/1980	0.052259	<b>0.052258</b>	0.067341	<b>0.067247</b>
from 1/01/1990	<b>0.069018</b>	0.069021	<b>0.091924</b>	0.091987
from 1/01/2000	0.254042	<b>0.254041</b>	<b>0.264602</b>	0.264618

Of course, this could also be done for other sub-samples<sup>16</sup>.

**Estimating the long-term relationship over the lagged 5 year period seems to give better results for predicting the earning yield at a daily frequency, but does worse for the bond**

<sup>16</sup> See Annex 7 of the document quoted in footnote 5, page 6.

**yield. Overall, at daily frequencies, the Fed model does not perform better than a simple VAR in first difference, in particular when the mechanical effect of a better fit due to one more explanatory variable is factored out by the addition of one more lag for the VAR than for the Fed model.**

### **3. THE TIME-HORIZON OF THE LINEAR FED MODEL: HOW SHORT IS THE LONG-RUN<sup>17</sup>**

In this section we first determine the period where a linear “Fed relationship” seems stable enough, and then try to determine the frequency at which the Fed model is relevant using daily, weekly and monthly data. The same methodology, which consists in comparing results obtained from a VEC model to a similar VAR model in first difference, is used. Because the methodology is similar to the one already described in Section 3 we do not fully describe the different statistics and steps again. The interested reader can find a full description on the author’s webpage<sup>18</sup>, Annex 6. Hence this section only recapitulates the main findings and the references to the additional documents of the detailed study.

Estimating a linear relationship between earning yields and bond yields for the US indicates that the relationship is highly unstable over the 1980 to present day period. More precisely, it seems to be highly unstable since 2001 (part A of Annex 6).

First, a long-term linear relationship between earning yield and bond yield does not exist besides the 1980 to 2000 period. For example, in the most recent period (from 2001 onward) there is no evidence of co-integration between the two variables. Nevertheless, at a monthly frequency the short-term interactions are significant and help improve the goodness-of-fit as evaluated by RMSE, MSAV or the number of correct changes, compared to the lagged variable (part D of Annex 6).

Restricting the time span to the 1980 to 2000 period, the Fed model cannot be observed at a daily frequency as a “long-term relationship”, although short-term dynamics between the earning yield and the bond yield exists (part B of Annex 6). At the weekly frequency, the Fed model is only useful for explaining the earnings ratio, and not the bond yield (Part E of Annex 6). At the monthly frequency, and for the 1980 to 2000 period, there is indeed evidence that assuming and estimating a long-term relationship between the earning yield and the bond yield does improve the forecasts (Part C of Annex 6).

### **4. CONCLUSION AND DIRECTION FOR FURTHER RESEARCH**

In accordance with other studies, we found that the general linear version of the Fed model is based on rather limited empirical evidence. Concerning its **geographical scope**, it does not “travel” well. Even restricted to the US, it does not work for other stock indexes than the S&P 500 and the relevant **historical scope** is confined within the 1980 to 2000 period. In particular, although co-movements in the earning yields and the bond yield exist in the 2001 to 2011 period, these movements are short-term in nature, in the sense that they are captured by immediate feedback when a VEC model is estimated, and do not contribute to the (long-term) error correcting term of the VEC in any meaningful way. In particular, building a VEC gives less precise results for forecasting than a VAR in first difference: the long-term relationship is a hindrance more than a useful tool.

<sup>17</sup> See Annex 7 of the document quoted in footnote 5, page 6.

<sup>18</sup> See the document quoted in footnote 5, page 6.

Concerning its **frequencies**, we found that the Fed model, during the 1980 to 2000 period, does not have any predictive power at the daily frequency. It starts having some predictive power for the earning yield (but not for the bond yield) at the weekly frequency, and for both the earning yield and the bond yield at monthly frequency. Although we use different sub-samples as a robustness-check for checking the better goodness-of-fit of the Fed model compared to simple similar models which do not contain a long-term relationship, a word of caution is warranted by the very sample-dependence observed of the Fed model in general.

A possible avenue for further research is the study of a *non-linear* relationship between equity yields and nominal bond yields, instead of a general linear relationship as studied in this paper. Could a more general, non-linear relationship be found between equity yields and nominal bond yields for more countries and stock exchange indices than the US and the Standard and Poor's? Moreover, the example of the US and the Standard and Poor's studied in depth in this article shows that this non-linear relationship, if it exists, will also have to describe fully the US data for the whole period from 1960 to the present, to be alternatively increasing or decreasing depending on time. Switching between the increasing or decreasing version of the model could potentially be done continuously, through the use of appropriate covariates of which GDP growth or unemployment are likely candidates, or through the use of Markov switching regimes, with an appropriate covariate as a trigger. In case no appropriate covariate is found, then one way of research could be the use of hidden Markov chains, in which the state of the economy – here the monotonicity of the relationship – depends on an unobservable state estimated from the data.

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## ANNEXES

## LIST 1

## Stock exchange indices description

<b>INDU Index (US):</b>	The Dow Jones Industrial Average is a price-weighted average of 30 blue-chip stocks that are generally the leaders in their industry.	01/01/1963
<b>SPX Index (US):</b>	The Standard and Poor's 500 Index is a capitalization-weighted index of 500 stocks. The index is designed to measure performance of the broad domestic economy through changes in the aggregate market value of 500 stocks representing all major industries.	02/05/1993
<b>CCMP Index (US):</b>	The NASDAQ Composite Index is a broad-based capitalization-weighted index of stocks in all three NASDAQ tiers: Global Select, Global Market and Capital Market.	01/01/1995
<b>SPTSX Index (Canada):</b>	The S&P/Toronto Stock Exchange Composite Index is a capitalization-weighted index designed to measure market activity of stocks listed on the TSX.	01/06/1994
<b>MEXBOL Index (Mexico):</b>	The Mexican IPC index (Indice de Precios y Cotizaciones) is a capitalization-weighted index of the leading stocks traded on the Mexican Stock Exchange.	01/08/2001
<b>IBOV Index (Brazil):</b>	The Bovespa Index is a total return index weighted by traded volume and is comprised of the most liquid stocks traded on the Sao Paulo Stock Exchange.	01/01/2007
<b>SXXE Index (Euro Area):</b>	The EURO STOXX (Price) Index is a capitalization-weighted index which includes countries that are participating in the EMU.	01/01/2002
<b>SXXP Index (Euro Area):</b>	The STOXX Europe 600 (Price) Index is a broad-based capitalization-weighted index of European stocks designed to provide a broad yet liquid representation of companies in the European region.	01/01/2002
<b>SX5E Index (Euro Area):</b>	The EURO STOXX 50 (Price) Index is a free-float market capitalization-weighted index of 50 European blue-chip stocks from those countries participating in the EMU. Each component's weight is capped at 10% of the index's total free-float market capitalization.	01/05/2001
<b>UKX Index (UK):</b>	The FTSE 100 Index is a capitalization-weighted index of the 100 most highly capitalized companies traded on the London Stock Exchange. The equities use an investibility weighting in the index calculation.	01/01/2002
<b>CAC Index (France):</b>	The CAC-40 Index is a narrow-based, modified capitalization-weighted index of the 40 highest market caps on the Paris Bourse.	01/06/2001
<b>DAX Index (Germany):</b>	The German Stock Index is a total return index of 30 selected German blue-chip stocks traded on the Frankfurt Stock Exchange. The equities use free-float shares in the index calculation.	01/04/1997
<b>IBEX Index (Spain):</b>	The IBEX 35 is the official index of the Spanish Continuous Market. The index is comprised of the 35 most liquid stocks traded on the Continuous market. It is calculated, supervised and published by the Sociedad de Bolsas.	01/04/1993
<b>FTSEMIB (Italy):</b>	The Index will consist of the 40 most liquid and capitalized stocks listed on the Borsa Italiana. In the FTSE MIB Index foreign shares will be eligible for inclusion.	01/01/2004
<b>AEX Index (Sweden):</b>	The AEX-Index is a free-float adjusted market capitalization weighted index of the leading Dutch stocks traded on the Amsterdam Exchange. The index was adjusted to the Dutch Guilder fixing rate.	01/01/2004

<b>OMX (Sweden):</b>	The OMX Stockholm 30 Index is a capitalization-weighted index of the 30 stocks that have the largest volume of the trading on the Stockholm Stock Exchange. The equities use free-float shares in the index calculation.	01/01/1994
<b>SMI Index (Switzerland):</b>	The Swiss Market Index is a capitalization-weighted index of the 20 largest and most liquid stocks of the SPI universe. It represents about 85% of the free-float market capitalization of the Swiss equity market.	01/01/2002
<b>NKY Index (Japan):</b>	The Nikkei-225 Stock Average is a price-weighted average of 225 top-rated Japanese companies listed in the First Section of the Tokyo Stock Exchange. The Nikkei Stock Average was first published on May 16, 1949, when the average price was ¥176.21 with a divisor of 225.	01/02/2000
<b>TPX Index (Japan):</b>	The TOPIX, also known as the Tokyo Stock Price Index, is a capitalization-weighted index of all companies listed on the First Section of the Tokyo Stock Exchange. The index is supplemented by the sub-indices of the 33 industry sectors.	01/04/1993
<b>SHASHR Index (China):</b>	The Shanghai A-Share Stock Price Index is a capitalization-weighted index. The index tracks the daily price performance of all A-shares listed on the Shanghai Stock Exchange that are restricted to local investors and qualified institutional foreign investors.	01/06/2005
<b>AS51 Index (Australia):</b>	The S&P/ASX 200 measures the performance of the 200 largest index-eligible stocks listed on the ASX by float-adjusted market capitalization. Representative, liquid and tradable, it is widely considered Australia's preeminent benchmark index. The index is float-adjusted. The index was launched in April 2000.	01/01/2000

NB: The unavailability of data in the Bloomberg series often came from the earning yield. The availability of the dividend yield, or of the dividend payout ratio, is even worse.

**TABLE 1**  
ADF and Philipps-Perron tests\* with daily data for Fed indicators

	Sample starts in	Y-E/P		Log(Y)-log(E/P)		Y/(E/P)	
		ADF	P-P	ADF	P-P	ADF	P-P
US and SPX	01/01/1963	-2.96 0.03	-2.87 0.04	-2.44 0.13	-2.42 0.13	-2.68 0.07	-2.65 0.08
US and SPX	01/01/1993	-1.51 0.53	-1.50 0.53	-1.41 0.57	-1.43 0.56	-1.64 0.45	-1.55 0.51
US and SPX	01/01/2001	-2.08 0.25	-2.09 0.24	-1.99 0.29	-2.03 0.27	-2.27 0.18	-2.21 0.20
US and INDU	02/05/1993	-2.38 0.15	-1.93 0.31	-2.21 0.20	-1.95 0.30	-2.07 0.25	-1.92 0.32
US and CCMP	01/01/1995	-2.03 0.27	-2.04 0.26	-2.11 0.24	-3.28 0.01	-11.77 0.00	-1.87 0.00
CA and SPTSX	01/06/1994	-2.52 0.11	-2.42 0.13	-2.05 0.26	-1.97 0.29	-2.37 0.14	-2.28 0.17
MX and MEXBOL	01/08/2001	-4.35 0.00	-4.00 0.00	-4.38 0.00	-4.14 0.00	-4.44 0.00	-4.29 0.00

\* The first line of each cell is the t-statistics (either from an ADF tests whose lag is selected by optimizing the Schwarz Information Criterion, with a maximum lag of 25, or a Phillips-Perron test, using the Neweast-West Bandwidth test). Data used is daily, end-of-the day data (closing prices).

BR and IBOV	01/01/2007	-2.06 0.26	-1.95 0.31	-1.78 0.38	-1.74 0.40	-1.61 0.47	-1.57 0.49
EU and SXXE	01/01/2002	-2.01 0.28	-1.91 0.32	-1.98 0.29	-1.92 0.31	-2.62 0.08	-2.57 0.09
EU and SXXP	01/01/2002	-2.08 0.24	-1.98 0.29	-1.97 0.29	-1.91 0.32	-2.39 0.14	-2.28 0.17
EU and SX5P	01/05/2001	-1.79 0.38	-1.58 0.48	-1.69 0.43	-1.67 0.44	-2.96 0.03	-2.96 0.03
UK and UKX	01/01/2002	-2.54 0.10	-2.29 0.17	-2.31 0.16	-2.13 0.23	-2.85 0.05	-2.61 0.08
FR and CAC	01/06/2001	-2.70 0.07	-2.44 0.13	-2.44 0.12	-2.26 0.18	-2.53 0.10	-2.39 0.14
DE and DAX	01/02/1997	-2.30 0.16	-2.30 0.16	-2.22 0.19	-2.24 0.19	-3.42 0.01	-3.52 0.00
DE and DAX	01/01/2001	-2.15 0.22	-2.16 0.22	-2.14 0.22	-2.15 0.22	-3.17 0.02	-3.26 0.01
ES and IBEX	01/04/1993	-2.87 0.04	-2.95 0.03	-2.06 0.26	-1.91 0.32	-2.65 0.08	-2.38 0.14
ES and IBEX	01/01/2001	-1.98 0.29	-1.71 0.42	-2.15 0.22	-1.93 0.31	-2.59 0.09	-2.49 0.11
IT and FTSEMIB	01/01/2004	-2.91 0.043	-2.88 0.047	-3.32 0.0139	-3.28 0.0158	-4.06 0.0011	-4.05 0.0012
SE and OMX	01/01/1994	-2.95 0.039	-2.91 0.04	-1.92 0.32	-1.92 0.32	-2.18 0.21	-2.17 0.21
CH and SMI	01/01/2002	-1.76 0.39	-1.75 0.40	-2.21 0.20	-2.04 0.26	-3.22 0.018	-3.27 0.016
JP and NKY	01/02/2000	-2.51 0.11	-2.38 0.14	-2.78 0.06	-2.82 0.055	-3.41 0.0104	-3.46 0.0089
JP and TPX	01/04/1993	-1.67 0.44	-1.57 0.49	-2.92 0.04	-2.94 0.04	-7.21 0.00	-7.99 0.00
CN and SHASHR	01/06/2005	-1.2 0.67	-1.30 0.62	-1.13 0.70	-1.25 0.65	-1.13 0.70	-1.19 0.67
AU and AS51	01/01/2000	-3.26 0.016	-3.19 0.02	-3.07 0.02	-2.98 0.03	-3.27 0.016	-3.15 0.02

**TABLE 2**

ADF and Philipps-Perron tests\* with monthly (end of the month) data for Fed indicators

	Sample starts in	Y-E/P		Log(Y)-log(E/P)		Y/(E/P)	
		ADF	P-P	ADF	P-P	ADF	P-P
US and SPX	01/01/1963	-2.90 0.04	-2.81 0.05	-2.51 0.11	-2.35 0.15	-2.73 0.06	-2.12 0.23
US and SPX	01/01/1993	-1.65 0.45	-1.62 0.46	-1.59 0.48	-1.55 0.50	-1.61 0.47	-1.59 0.48
US and SPX	01/01/2001	-2.21 0.20	-2.22 0.20	-2.18 0.21	-2.16 0.22	-2.22 0.19	-2.05 0.26
US and INDU	02/05/1993	-1.53 0.51	-1.85 0.35	-1.52 0.52	-1.84 0.35	-2.02 0.27	-1.79 0.38
US and INDU	01/01/2001	-1.87 0.34	-1.72 0.41	-1.73 0.41	-1.73 0.40	-2.06 0.26	-1.80 0.37
US and CCMP	01/01/1995	-2.16 0.21	-2.15 0.22	-1.40 0.57	-1.40 0.57	-0.94 0.77	-1.68 0.43
US and CCMP	01/01/2001	-1.48 0.54	-1.48 0.54	-1.44 0.55	-1.44 0.56	-1.90 0.33	-1.92 0.32
CA and SPTSX	01/06/1994	-2.83 0.05	-2.65 0.08	-1.81 0.37	-2.15 0.22	-2.18 0.21	-2.38 0.14
MX and MEXBOL	01/08/2001	-4.04 0.0017	-4.01 0.0017	-4.10 0.0014	-3.97 0.0021	-4.14 0.0012	-4.10 0.0014
BR and IBOV	01/01/2007	-1.81 0.37	-2.00 0.28	-1.62 0.46	-1.88 0.38	-1.51 0.51	-1.90 0.32
EU and SXXE	01/01/2002	-1.85 0.35	-1.82 0.36	-1.90 0.32	-1.80 0.37	-1.76 0.39	-2.32 0.16
EU and SXXP	01/01/2002	-1.92 0.31	-1.90 0.33	-1.89 0.33	-1.90 0.32	-2.67 0.08	-2.19 0.20
EU and SX5P	01/05/2001	-1.08 0.72	-1.39 0.58	-1.13 0.70	-1.56 0.49	-1.88 0.33	-2.84 0.054
UK and UKX	01/01/2002	-2.15 0.22	-2.02 0.27	-1.94 0.30	-2.30 0.17	-3.59 0.0069	-2.97 0.03
FR and CAC	01/06/2001	-2.23 0.19	-2.37 0.15	-2.08 0.25	-2.24 0.19	-2.21 0.20	-2.30 0.17
DE and DAX	01/02/1997	-2.03 0.27	-2.28 0.17	-2.14 0.22	-2.48 0.12	-3.27 0.0178	-3.64 0.0059
DE and DAX	01/01/2001	-1.87 0.34	-2.03 0.27	-2.05 0.26	-2.05 0.26	-3.04 0.0337	-3.33 0.015
ES and IBEX	01/04/1993	-1.53 0.51	-1.62 0.46	-1.72 0.41	-1.71 0.42	-1.97 0.29	-1.86 0.34

\* The first line of each cell is the t-statistics (either from an ADF tests whose lag is selected by optimizing the Schwarz Information Criterion, with a maximum lag of 25, or a Phillipps-Perron test, using the Neweast-West Bandwidth test). Data used is daily, end-of-the day data (closing prices).



ES and IBEX	01/01/2001	-1.61 0.47	-1.68 0.43	-1.93 0.31	-1.89 0.33	-2.04 0.26	-2.11 0.23
IT and FTSEMIB	01/01/2004	-2.56 0.104	-2.65 0.08	-2.65 0.08	-2.77 0.65	-2.85 0.054	-2.92 0.046
SE and OMX	01/01/1994	-2.31 0.16	-3.00 0.03	-1.80 0.37	-1.97 0.29	-2.11 0.23	-2.27 0.18
CH and SMI	01/01/2002	-1.55 0.50	-1.84 0.35	-1.85 0.35	-1.86 0.34	-1.84 0.35	-2.47 0.12
JP and NKY	01/02/2000	-2.34 0.15	-2.41 0.14	-2.98 0.039	-3.05 0.03	-5.29 0.00	-3.58 0.007
JP and TPX	01/04/1993	-1.50 0.52	-1.53 0.51	-1.85 0.35	-2.48 0.12	-6.10 0.00	-7.92 0.00
CN and SHASHR	01/06/2005	-1.91 0.32	-1.79 0.37	-1.44 0.55	-1.87 0.34	-1.43 0.55	-1.84 0.35
AU and AS51	01/01/2000	-3.22 0.020	-3.29 0.017	-3.51 0.009	-3.35 0.014	-4.18 0.001	-3.57 0.007

**TABLE 3**

ADF and Philipps-Perron tests\* with monthly (average of the month) data for Fed indicators

	Sample starts in	Y-E/P		Log(Y)-log(E/P)		Y/(E/P)	
		ADF	P-P	ADF	P-P	ADF	P-P
US and SPX	01/01/1963	-2.90 0.04	-2.81 0.05	-2.51 0.11	-2.35 0.15	-2.73 0.06	-2.12 0.23
US and SPX	01/01/1993	-1.58 0.48	-1.60 0.47	-1.55 0.50	-1.54 0.50	-1.52 0.52	-1.56 0.50
US and SPX	01/01/2001	-2.26 0.18	-2.23 0.19	-2.23 0.19	-2.17 0.21	-2.29 0.17	-2.04 0.26
US and INDU	02/05/1993	-1.53 0.51	-1.85 0.35	-1.52 0.52	-1.84 0.35	-2.02 0.27	-1.79 0.38
US and INDU	01/01/2001	-1.84 0.35	-1.79 0.38	-1.70 0.42	-1.70 0.42	-1.67 0.44	-1.76 0.39
US and CCMP	01/01/1995	-2.18 0.21	-1.94 0.30	-1.24 0.65	-1.29 0.62	-1.22 0.66	-1.87 0.34
US and CCMP	01/01/2001	-1.34 0.60	-1.43 0.56	-1.40 0.57	-1.40 0.57	-1.81 0.37	-1.80 0.37
CA and SPTSX	01/06/1994	-2.89 0.04	-2.57 0.10	-2.43 0.13	-2.12 0.23	-2.46 0.12	-2.00 0.28
MX and MEXBOL	01/08/2001	-3.42 0.0118	-3.42 0.0118	-3.50 0.0094	-3.56 0.0080	-3.61 0.0068	-3.67 0.0057

\* The first line of each cell is the t-statistics (either from an ADF tests whose lag is selected by optimizing the Schwarz Information Criterion, with a maximum lag of 25, or a Phillips-Perron test, using the Neweast-West Bandwidth test). Data used is daily, end-of-the day data (closing prices).

BR and IBOV	01/01/2007	-2.16 0.22	-1.97 0.29	-2.03 0.27	-1.85 0.35	-2.73 0.07	-1.85 0.34
EU and SXXE	01/01/2002	-1.77 0.39	-1.84 0.35	-1.77 0.39	-1.77 0.39	-1.76 0.39	-2.12 0.23
EU and SXXP	01/01/2002	-1.85 0.35	-1.92 0.32	-1.79 0.38	-1.88 0.33	-2.10 0.24	-2.08 0.25
EU and SX5P	01/05/2001	-1.31 0.62	-1.41 0.57	-1.28 0.63	-1.47 0.54	-2.20 0.20	-2.66 0.08
UK and UKX	01/01/2002	-2.12 0.23	-2.18 0.21	-2.89 0.04	-2.36 0.15	-3.95 0.0020	-2.87 0.05
FR and CAC	01/06/2001	-2.14 0.23	-2.37 0.15	-1.99 0.28	-2.22 0.19	-2.45 0.12	-2.29 0.17
DE and DAX	01/02/1997	-1.55 0.50	-1.73 0.41	-1.78 0.38	-2.06 0.26	-2.65 0.08	-3.14 0.025
DE and DAX	01/01/2001	-1.55 0.50	-1.73 0.40	-1.78 0.38	-2.06 0.26	-2.65 0.08	-3.14 0.02
ES and IBEX	01/04/1993	-2.40 0.14	-2.58 0.09	-1.72 0.41	-1.86 0.34	-2.11 0.23	-2.17 0.21
ES and IBEX	01/01/2001	-1.53 0.51	-1.62 0.46	-1.72 0.41	-1.71 0.42	-1.97 0.29	-1.86 0.34
IT and FTSEMIB	01/01/2004	-2.56 0.10	-2.65 0.08	-2.65 0.08	-2.77 0.065	-2.85 0.054	-2.92 0.046
SE and OMX	01/01/1994	-1.9 0.33	-2.61 0.09	-2.25 0.18	-1.92 0.31	-2.46 0.12	-2.18 0.21
CH and SMI	01/01/2002	-1.48 0.53	-1.78 0.38	-1.74 0.40	-1.77 0.39	-2.55 0.105	-2.51 0.11
JP and NKY	01/02/2000	-2.17 0.21	-2.30 0.17	-2.74 0.06	-2.91 0.04	-5.08 0.00	-3.70 0.005
JP and TPX	01/04/1993	-1.45 0.55	-1.50 0.52	-1.83 0.36	-2.24 0.19	-10.74 0.00	-10.93 0.00
CN and SHASHR	01/06/2005	-2.31 0.16	-1.85 0.34	-2.21 0.20	-1.84 0.35	-2.18 0.21	-1.81 0.36
AU and AS51	01/01/2000	-2.98 0.039	-3.14 0.02	-3.37 0.013	-3.19 0.02	-3.92 0.00	-3.39 0.013