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Modifications of order scales for assessing debtors

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Abstract

In previous research, the Extended Order Scale (EOS) dedicated to risk assessment was analysed. It was characterised by a Numerical Order Scale (NOS) evaluated by trapezoidal oriented fuzzy numbers (TrOFNs). However, the research showed that EOS with two-stage orientation phases, was too complicated. Therefore, the main aim of our paper is to simplify a Complete Order Scale (COS) to a zero- or one-stage order scale and a hybrid approach. For this purpose, a way to calculate the scoring function is presented. The results show that changes in the COS structure influence the values of a scoring function. Replacing just one linguistic indicator gives different results. Another finding of the research is the method's flexibility that allows an expert to individually choose the most suitable COS. The research proves that the boundary between various linguistic labels cannot be precisely defined. However, knowledge of a formal COS structure allows it to be transformed into a less complex one.

Keywords: *order scale, credit scoring, trapezoidal oriented fuzzy numbers*

1. Introduction

Insufficient means for the assessment of potential borrowers and credit scoring were major obstacles that led to banks struggling in the field of loans. The economic crisis in the USA in 2007-2008, which then became worldwide, proved that the quantitative methods of measuring credit risk used by banks in their evaluation, assessment, and granting of mortgages were highly ineffective and faulty. In many cases, the final outcome was left to the individual decisions of analysts, although the entities applying for funding did not satisfy all of the initial conditions fully. Within the legal framework, many of these applications were acceptable at a minimum level. However, this approach led to many defaults on loans, which consequently had a negative effect on the credit portfolios of the banks.

Understandably, one objective of every bank is to keep credit risk levels reasonably low. This should also be accompanied by an increasing (or at least non-decreasing) volume of operations. Unfortunately, the target of at least a non-decreasing volume of operations frequently puts pressure on analysts to grant loans and credit to borrowers with low or questionable creditworthiness. Therefore, the Basel Committee on Banking Supervision in its document regarding a standardised approach to assessing credit risk

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recognises the importance of mitigating credit risk by “improving the incentives for banks to manage credit risk in a prudent and effective manner” [2]. Also, the final Basel III framework will introduce more granular assessments of credit risk. By 2023 banks will need to follow new Standardised Credit Risk Assessment (SCRA) procedures to calculate the level of risk for unrated credit exposure, as well as exposure in countries that do not allow the use of external credit ratings. If, under the framework of Basel III, banks choose to utilise the internal models method, they do not have to use a single model and also no particular form of model is required. Analytical models, as well as simulation models, are acceptable as long as they are subject to supervisory review and meet all of the requirements [3].

When evaluating the credit risk of potential borrowers, analysts and experts must consider a very broad range of individual elements, which often belong to various groups of qualitative factors. The nature of these factors predispose them to be expressed by linguistic labels belonging to a given scale. These factors are innately inaccurate and therefore standard numerical methods do not fully apply in such an environment, as reducing the analysis to one final number deprives the decision-maker of a lot of information. Therefore, it must be stressed that inaccurateness and imprecision do not hamper assessment. Quite the contrary, it allows experts and decision-makers to incorporate their professional experience and preferences.

Determining, assessing, and constructing a template for evaluation is part of a standard procedure. It is one of the most crucial parts of the assessment process. Consequently, if the assessment is positive, it allows for financial means to be granted [8]. The template must be detailed enough to adequately reflect possible options, which will determine the acceptable space of any operation. A given template should also be cohesive. It should take into account the professional and educational background of the experts. It must perceive each factor differently, based on the experts’ knowledge and experience, as they are the ones making decisions under different economic and practical circumstances. Furthermore, multi-criteria methods can be useful, as they enable the determination of an appropriate scoring function [15]. Eventually, they enable a final decision to be made on the basis of imprecise measures. Therefore, oriented fuzzy numbers can be used in such evaluation processes in an attempt to minimise the loss of information.

In previous work, [21], [22], various linguistic evaluation order scales (OS) were proposed. The implementation of such scales involves imprecise phrases used in the process of evaluating credit risk, based on the assessments of experts. The Extended Order Scale (EOS), applied to the assessment of credit risk, is related to a Numerical Order Scale (NOS) determined by trapezoidal oriented fuzzy numbers. The proposed EOS used two-stage orientation phrases. Unfortunately, in the course of consultation with experts, it appeared that such a solution is too detailed for many bank analysts. Hence, the main aim of this paper is to present other ways of simplifying EOSs to a one-stage or zero-stage order scale.

Furthermore, we need to stress the existence of a problem related to the lack of associativeness when summing trapezoidal oriented fuzzy numbers (TrOFNs). Therefore, the paper presents a course of action which enables unambiguous determination of the scoring function and avoids the above-mentioned obstacles.

The paper is organised as follows. An introduction and credit risk review are followed by Section 3 in which a brief overview of Trapezoidal Oriented Fuzzy Numbers is presented. Section 4 provides a review of Multi-Criteria Decision Making approaches, focusing primarily on their application to financial problems. Section 5 presents an Order Scale (OS) dedicated to the assessment of credit risk. Section 6 defines the scope of the work on scoring functions for assessing potential borrowers and the main elements of the methodology. In Section 7 the proposed approach is implemented in a real-case study and the results of simplifying Complete Order Scales (COS) by utilising various sets of perception indicators are presented. Finally, conclusions are drawn along with possible directions for future research.

2. Credit risk - framework

Classification of debtors into appropriate groups is subject to frameworks defined by financial advisory institutions, for instance the Basel Committee on Banking Supervision, and to European regulations. The significance of the solutions presented in Basel II and Basel III has been acknowledged by the European Union legislators, as many of the regulations found their way into EU law, for instance in Capital Requirements Regulation (EU, No. 575/2013), Capital Requirements Directive IV (Directive 2013/36/EU - CRD IV) and the Bank Recovery and Resolution Directive (Official Journal of June 7, 2019). Also, the European Banking Authority recognises that it is crucial, across the EU, to consistently implement regulations related to topics such as credit risk adjustments, definition of default, permission to use a standardised or IRB approach, appropriateness of risk weights or credit risk mitigation techniques [1].

One priority of the banking sector is to efficiently manage base capital, evaluate exposure to possible risks, and decisions regarding potential loans. The need to implement Basel II standards [2] followed from numerous factors, e.g. market liberalization, establishing the financial stability of the entire banking system and, most of all, the advancement of supervision over potential risks and the effectiveness of loan granting. Our research concentrates on the adoption of appropriate practice regarding the importance of not only quantitative, but also qualitative, standards concerning the assessment and classification of the creditworthiness of potential borrowers. The opportunity to use either a standard method or an IRB approach in deciding whether to give credit lays a foundation for the development and modification of approaches using parametric methods, such as linear discriminant analysis, regression analysis and credit scoring, as well as non-parametric methods, such as neural networks, expert systems, support vector machines, machine learning. These approaches include qualitative methods in which expert knowledge is recognised as important to define a neural network, fuzzy logic systems, prediction matrices etc. [18], [19].

In decisions regarding the granting of credit based on Basel II and Basel III, we can clearly see that what is of most importance to the banking industry is not just expressible by numbers, but also by knowledge and experience. More stress is put on a company's ability to meet its obligations on a regular basis than the level of debt exposure that the company must incur. When determining ratings based on Basel II and Basel III, the following parameters concerning a potential debtor can be taken into account: analysis of the relevent sector(s), a company's development plan, its liquidity, characteristics of its management, quality of suppliers/customers, risk levels in the country/region, the quality of an enterprise, its budget, the level of interest paid, compliance with the terms of repayment, credit threshold, the breakdown of debts between short, medium and long term, the level of stocks and commercial credit held by a company, current accounts which are inactive or have a negative balance, and many others. As these factors are fuzzy (imprecise) in nature, the bank, through the knowledge of its experts, compiles all the accessible information and expresses an opinion on the reliability and creditworthiness of the company. This opinion can come in the form of a rating. Obviously, despite the fact that various banks or agencies analyse similar sets of information, a rating of a single potential borrower can vary. Fuzzy sets (FS) allow the utilisation of the knowledge and experience of experts, together with innately fuzzy factors, to classify debtors.

3. A brief overview od oriented fuzzy numbers

The symbol $F(\mathbb{R})$ denotes the family of all fuzzy subsets on the real line $\mathbb{R}$. A fuzzy number (FN) is usually defined to be a fuzzy subset of the real line $\mathbb{R}$. The most general definition of a FN was formulated by Dubois and Prade [7]. The set of all FNs is denoted by the symbol F. The notion of an ordered FN was introduced by Kosiński [10], and Kosiński et al. [11]. For formal reasons, Kosiński's theory was revised in [13]. In this revised theory, the notion of an ordered FN is simplified to the notion of an oriented FN (OFN). On the other hand, arithmetic operations on OFNs have a very high level of complexity [14]. Moreover, various research on the utilisation of linguistic labels and OFNs shows that the concepts of different levels of labels are usually modelled as Gauss functions. As a trapezoid function seems to be the best mathematical approximation of Gauss-like functions, Trapezoidal Oriented Fuzzy Numbers (TrOFNs) seem to be the most appropriate for practical use. TrOFNs are also easy to identify and interpret. For this reason, we restrict our considerations to the case of TrOFNs defined as follows [13]:

Definition 1. For any monotonic sequence $(a, b, c, d) \subset \mathbb{R}$, a TrOFN $\overleftrightarrow{Tr}(a, b, c, d) = \overleftrightarrow{\boldsymbol{T}}$ is the pair of orientation $\overrightarrow{a, d} = (a, d)$ and a given FN $\boldsymbol{T} \in \mathbb{F}$ described by the membership function $\mu_{\boldsymbol{T}}(\cdot|a, b, c, d) \in [0, 1]^{\mathbb{R}}$ given by the identity

$$\mu_T(x) = \mu_{T_r}(x|a, b, c, d) = \begin{cases} 0 & \text{if } x \notin [a, d] \equiv [d, a] \\ \frac{x-a}{b-a} & \text{if } x \in [a, b[\equiv]a, b] \\ 1 & \text{if } x \in [b, c] \equiv [c, b] \\ \frac{x-d}{c-d} & \text{if } x \in]c, d] \equiv [c, d[\end{cases} \tag{1}$$

Remark 1. The identity (1) additionally leads to a modified notation of intervals that is used in OFN theory. The notation $\mathcal{J} \equiv \mathcal{K}$ means that "the interval $\mathcal{J}$ may be equivalently replaced by the interval $\mathcal{K}$".

The symbol $\mathbb{K}_{Tr}$ denotes the space of all TrOFNs. If $a < d$, then the TrOFN $\overleftrightarrow{Tr}(a, b, c, d)$ has a positive orientation $\overrightarrow{a, d}$, which informs us about the possibility of an increase in the approximated number. The space of all positively oriented TrOFNs is denoted by the symbol $\mathbb{K}^+_{Tr}$. If $a > d$, then the OFN $\overleftrightarrow{Tr}(a, b, c, d)$ has a negative orientation $\overrightarrow{a, d}$, which informs us about the possibility of a decrease in the approximated number. The space of all negatively oriented TrOFNs is denoted by the symbol $\mathbb{K}^-_{Tr}$. If $a = d$, then the OFN $\overleftrightarrow{Tr}(a, a, a, a) = [[a]]$ describes the real number $a \in \mathbb{R}$. Let the symbol $*$ denote any arithmetic operation defined in $\mathbb{R}$. By the symbol $\boxed{*}$ we denote an extension of the arithmetic operation $*$ to $\mathbb{K}_{Tr}$. Kosiński [10] has proposed to define arithmetic operators on $\mathbb{K}_{Tr}$ in such a way that subtraction is the inverse operator to addition. According to Kosiński's approach, we can extend basic arithmetic operators to the case of $\mathbb{K}_{Tr}$ in such a way that for any pair $\left(\overleftrightarrow{Tr}(a, b, c, d)\right), \overleftrightarrow{Tr}(p - q, q - b, r - c, s - d) \in \mathbb{K}^2_{Tr}$ and $\beta \in \mathbb{R}$, the arithmetic operations of an extended sum $\boxplus$ and dot product $\boxdot$ are defined as follows [13]:

$$\begin{aligned} &\overleftrightarrow{Tr}(a, b, c, d) \boxplus \overleftrightarrow{Tr}(p - q, q - b, r - c, s - d) \\ &= \begin{cases} \overleftrightarrow{Tr}(\min\{p, q\}, q, r, \max\{r, s\}) & \text{if } (q < r) \vee (q = r \wedge p \leq s) \\ \overleftrightarrow{Tr}(\max\{p, q\}, q, r, \min\{r, s\}) & \text{if } (q > r) \vee (q = r \wedge p > s) \end{cases} \end{aligned} \tag{2}$$

$$\beta \boxdot \overleftrightarrow{Tr}(a, b, c, d) = \overleftrightarrow{Tr}(\beta \cdot a, \beta \cdot b, \beta \cdot c, \beta \cdot d) \tag{3}$$

In general, the addition of TrOFNs is not associative. Moreover, for any pair

$$\left(\overleftrightarrow{Tr}(a, b, c, d), \overleftrightarrow{Tr}(e, f, g, h)\right) \in \left(\mathbb{K}^+_{Tr} \cup \mathbb{R}\right)^2 \cup \left(\mathbb{K}^-_{Tr} \cup \mathbb{R}\right)^2$$

we have [14]

$$\overleftrightarrow{Tr}(a, b, c, d) \boxplus \overleftrightarrow{Tr}(e, f, g, h) = \overleftrightarrow{Tr}(a + e, b + f, c + g, d + h) \tag{4}$$

4. Multi-criteria decision making approach – a review

There are many approaches in Multi-Criteria Decision Making (MCDM) to determining financial decisions and solving problems from other disciplines (e.g. medicine, human resources, energy management, marketing, supply chains, location etc.). However, focusing solely on financial decisions, we can find examples of MCDM being used in portfolio analysis, assessing sovereignty risk or ranking credit risk algorithms. In [23] a method based on PROMETHEE II, using agreement and disagreement indexes, was presented to predict the probability of an entity going bankrupt. The main goal was to define the degree to which each alternativel outranked or was outranked by the other alternatives in all available categories. An attempt to analyze mortgage risk was made in [12]. This was conducted by defining a synthetic risk index using a participatory process, in order to support an operation to restructure debts.

Another example of using MCDM in finance is presented in [24]. The proposed model involves a methodology that combines Group Decision Making (GDM), fuzzification, and techniques from Artificial Intelligence (AI). The model was applied to assess the credit risk of loans offered by a financial entity to potential debtors. The main goal was to classify potential debtors into two groups: those that should be granted a loan and those that should not. A slightly different MCDM approach was used in [5] to classify potential clients according to their creditworthiness. Moreover, in [25] a multicriteria optimization method involving a penalty approach was also implemented for assessing credit risk. In [6], a multicriteria classification model using reference alternatives was proposed to allocate sovereign credit securities into three categories of risk. Another approach to assessing credit risk with the use of the PROMETHEE method and Analytic Hierarchy Process (AHP) can be found in [17]. However, not all of these methods implement linguistic elements and, as indicated in [16], including qualitative factors in MCDM increase the reliability of the final decision. More on MCDM can be found in [4], [23], [20].

5. Order scale defined for assessing credit risk

Linguistic decision analysis is used to solve decision-making problems with linguistic information [9]. Such linguistic information should be ordered according to an appropriate scale. The starting point for determining any order scale is to define the Tentative Order Scale (TOS) with the use of linguistic variables. A TOS is defined to be a sequence

$$\overline{TOS} = (X_i)_{i=1}^{n}$$

of linguistic labels X_i. The order of the linguistic labels is then determined by the order of the sequence $\overline{TOS}$. Any TOS can also be enhanced by intermediate values, which are obtained using a sequence of perception indicators (PIs) given in the form of the sequence $\overline{PI}$

$$\overline{PI} = (Y_j)_{j=-m}^{j=m} .$$

The order of the PIs is determined by the order of the sequence $\overline{PI}$. The Cartesian product of the sets $\overline{TOS}$ and $\overline{PI}$ forms an Extended Order Scale (EOS) defined as a lexicographically ordered set

$$\begin{aligned}\overline{EOS} &= \overline{TOS} \times \overline{PI} = \left\{(X_i, Y_j) : i = \overline{1,n}, j = \overline{-m,m}\right\}\\ &= \left\{Z_{(2\cdot m+1)\cdot(i-1)+m+1+j} : i = \overline{1,n}, j = \overline{-m,m}\right\} = (Z_k)_{k=1}^{n\cdot(2\cdot m+1)}\end{aligned}$$

of order labels Z_k. Such an EOS is called *m*-stage one. For the convenience of further considerations, TOS and EOS might also be characterised by a Numerical Order Scale (NOS). Imprecise understanding of the meaning of order labels results in the fact that an NOS should be expressed by a fuzzy number (FN). Any triple order (*TOS, EOS, NOS*) is called a Complete Order Scale (COS).

The main subject of consideration in this paper will be the two-stage COS proposed in [20] as a support tool for assessing potential debtors. The proposed COS contains an NOS defined by TrOFNs. This COS will be called COS1. COS1 is presented in Table 1.

Table 1. Complete Order Scale COS1

TOS	EOS	Symantic Meaning	NOS
	C - -	much below bad	$\overleftrightarrow{Tr}\left(1,1,\frac{3}{4},\frac{1}{4}\right)$
	C -	below bad	$\overleftrightarrow{Tr}\left(\frac{5}{4},1,\frac{3}{4},\frac{2}{4}\right)$
	C $\sim$	around bad	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
C		bad	$\overleftrightarrow{Tr}\left(1,1,1,1\right)$
	C +	somewhat better than bad	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$
	C + +	better than bad	$\overleftrightarrow{Tr}\left(1,1,\frac{5}{4},\frac{7}{4}\right)$
	B - -	below average	$\overleftrightarrow{Tr}\left(2,2,\frac{7}{4},\frac{5}{4}\right)$
	B -	somewhat below average	$\overleftrightarrow{Tr}\left(\frac{9}{4},2,\frac{7}{4},\frac{6}{4}\right)$
	B $\sim$	around average	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
B		average	$\overleftrightarrow{Tr}\left(2,2,2,2\right)$
	B +	somewhat above average	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$
	B + +	above average	$\overleftrightarrow{Tr}\left(2,2,\frac{9}{4},\frac{11}{4}\right)$
	A - -	below good	$\overleftrightarrow{Tr}\left(3,3,\frac{11}{4},\frac{9}{4}\right)$
	A -	below good	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$
	A $\sim$	around good	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
A		good	$\overleftrightarrow{Tr}\left(3,3,3,3\right)$
	A +	somewhat above good	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$
	A + +	above good	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$

6. Scoring function for assessing borrowers

Each credit application A is evaluated by experts from the point of view of a criteria set

$$\Phi = \{\mathcal{C}_l : l = 1, 2, \ldots,\ p\}$$

described in Table 2. The outcome of this assessment is to ascribe a set of partial assessments A

$$\Psi\left(A\right) = \left\{\overleftrightarrow{Tr}\left(A, C_l\right) = \overleftrightarrow{Tr}\left(a_l, b_l, c_l, d_l,\right) : l = 1, 2, \ldots, p\right\}$$

to each credit application.

This assessment is based on an expert method using EOS and NOS as presented in Table 1. The value of the scoring function is determined as the average of the assessments in set Ψ. However, here we encounter a specific obstacle. As we know, the addition of TrOFNs is not associative. Thus, the sum of a multiple number of terms depends on the order of the summands. This implies that a scoring function, given as the average sum of assessments $\overleftrightarrow{Tr}\left(A, C_l\right)$, is not uniquely determined. Assuming a natural order determined by the sequence $\left(C_l\right)_{l=1}^{p}$ would result in the fact that the sets of partial assessments ascribed to individual credit applications would vary according to the permutation of positively and negatively oriented TrOFNs. This prevents the reliable comparison of scores ascribed to individual credit applications. Therefore, in the case considered, any method of calculating the scoring function should be accompanied by a suitable method for ordering the components of a portfolio.

The guarantee of an unambigous score is ensured by implementing the following method for ordering criteria, applied to each credit application A separately. At the outset, we define the set

$$\Phi^{+}\left(A\right) = \left\{C_l : \overleftrightarrow{Tr}\left(A, C_l\right) \in \mathbb{K}_{Tr}^{+}, l = 1, 2, \ldots, p\right\}$$

of criteria evaluated by positively oriented TrOFNs. The set $\Phi^{-}\left(A\right) = \Phi \setminus \Phi^{+}\left(A\right)$ contains all the criteria evaluated by negatively oriented TrOFNs.

In the next step, using (4) we calculate the following partial sums for the scoring function:

$$\overleftrightarrow{S}^{+}(A) = \boxplus_{C_l\in\Phi^{+}(A)}\overleftrightarrow{Tr}(A, C_l) = \overleftrightarrow{Tr}\left(\sum_{C_l\in\Phi^{+}(A)} a_l,\ \sum_{C_l\in\Phi^{+}(A)} b_l,\ \sum_{C_l\in\Phi^{+}(A)} c_l,\ \sum_{C_l\in\Phi^{+}(A)} d_l\right)$$

$$\overleftrightarrow{S}^{-}(A) = \boxplus_{C_l\in\Phi^{-}(A)}\overleftrightarrow{Tr}(A, C_l) = \overleftrightarrow{Tr}\left(\sum_{C_l\in\Phi^{-}(A)} a_l,\ \sum_{C_l\in\Phi^{-}(A)} b_l,\ \sum_{C_l\in\Phi^{-}(A)} c_l,\ \sum_{C_l\in\Phi^{-}(A)} d_l\right)$$

Finally, we calculate the value $\overleftrightarrow{S}^{-}(A)$ of the scoring function. From (2) and (3), we get

$$\overleftrightarrow{S}(A) = p^{-1} \boxdot \left(\overleftrightarrow{S}^{+}(A) \boxplus \overleftrightarrow{S}^{-}(A)\right)$$

Tables 2 and 3 present partial evaluations prepared by an expert using COS1. The tables also present the values of a scoring function for this example.

7. Simplification of complete order scales

In the research presented in [21] the experts used COS1 utilising a set of perception indicators {much below, below, about, above, much above}. The experts were given credit applications. Each of the experts evaluated the credit applications individually. The assessments made by one of the experts is given in the COS1 and COS2 columns (EOS, NOS) of Table 2.

Table 2. An expert's evaluations using different Complete Order Scales (COS1, COS2)

		COS1		COS2	
No.	Criteria	EOS	NOS	EOS	NOS
1	Prospects of business	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4}, 1, \frac{5}{4}, \frac{6}{4}\right)$	C + +	$\overleftrightarrow{Tr}\left(1, 1, \frac{5}{4}, \frac{7}{4}\right)$
2	Board members' experience	A + +	$\overleftrightarrow{Tr}\left(3, 3, \frac{13}{4}, \frac{15}{4}\right)$	A + +	$\overleftrightarrow{Tr}\left(3, 3, \frac{13}{4}, \frac{15}{4}\right)$
3	Chairperson's experience	A + +	$\overleftrightarrow{Tr}\left(3, 3, \frac{13}{4}, \frac{15}{4}\right)$	A + +	$\overleftrightarrow{Tr}\left(3, 3, \frac{13}{4}, \frac{15}{4}\right)$
4	Range of regional operations	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4}, 1, \frac{5}{4}, \frac{6}{4}\right)$	C + +	$\overleftrightarrow{Tr}\left(1, 1, \frac{5}{4}, \frac{7}{4}\right)$
5	Range of international operations	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$
6	Risk associated with market	B + +	$\overleftrightarrow{Tr}\left(\frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}\right)$	B + +	$\overleftrightarrow{Tr}\left(\frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}\right)$
7	Risk associated with trade	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}\right)$	B + +	$\overleftrightarrow{Tr}\left(\frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}\right)$
8	Risk associated with suppliers	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$
9	Risk associated with customers	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$
10	Diversification—products	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4}, 2, 2, \frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4}, 2, 2, \frac{10}{4}\right)$
11	Diversification—sales markets	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4}, 1, 1, \frac{6}{4}\right)$	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4}, 1, 1, \frac{6}{4}\right)$
12	Diversification—supply market	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4}, 2, 2, \frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4}, 2, 2, \frac{10}{4}\right)$
13	Quality of suppliers	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$
14	Quality of customers	C + +	$\overleftrightarrow{Tr}\left(1, 1, \frac{5}{4}, \frac{7}{4}\right)$	C + +	$\overleftrightarrow{Tr}\left(1, 1, \frac{5}{4}, \frac{7}{4}\right)$
15	Clean criminal record of board members	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4}, 3, \frac{11}{4}, \frac{10}{4}\right)$
16	Clean criminal record of the chairperson	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}\right)$	B + +	$\overleftrightarrow{Tr}\left(\frac{7}{4}, 2, \frac{9}{4}, \frac{10}{4}\right)$
	Score		$\overleftrightarrow{Tr}\left(\frac{97}{48}, \frac{104}{48}, \frac{107}{48}, \frac{118}{48}\right)$		$\overleftrightarrow{Tr}\left(\frac{98}{48}, \frac{104}{48}, \frac{107}{48}, \frac{119}{48}\right)$

After completing this part of the study, the experts agreed, as mentioned before, that the proposed method (COS1) is too complex to be used in practice. In this case, the construction of a two-stage COS1 enables experts to apply the following simplified forms of COS:

- one-stage COS2 using a set of perception indicators {much below, about, much above}; COS2 is derived from COS1 by replacing the indicators {below, above} by indicators {much below, much above}, respectively

- one-stage COS3 using a set of perception indicators {below, about, above}; COS3 is derived from COS1 by replacing the indicators {much below, much above} by indicators {below, above}, respectively
- zero-stage COS4 using a set of perception indicators {about}; COS4 is derived from COS1 by replacing the indicators {much below, below, about, above, much above} by the indicator {about},
- mixed-stage (a hybrid approach) COS5 using a set of perception indicators {much below, below, about, above}; COS5 is derived from COS1 by replacing the indicator {much above} by the indicator {above},
- mixed-stage (a hybrid approach) COS6 using a set of perception indicators {below, about, above, much above}; COS6 is derived from COS1 by replacing the indicator {much below} by the indicator {below}.

COS3 and COS4 are presented in Table 3 [21].

Table 3. An expert's evaluations with the use of different Complete Order Scales (COS3, COS4)

No.	Criteria	COS3 EOS	COS3 NOS	COS4 EOS	COS4 NOS
1	Prospects of business	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
2	Board members' experience	A +	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$	A ∼	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
3	Chairperson's experience	A +	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$	A ∼	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
4	Range of regional operations	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
5	Range of international operations	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A ∼	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
6	Risk associated with market	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
7	Risk associated with trade	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
8	Risk associated with suppliers	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A ∼	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
9	Risk associated with customers	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A ∼	$\overleftrightarrow{Tr}\left(\frac{10}{4},3,3,\frac{14}{4}\right)$
10	Diversification—products	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
11	Diversification—sales markets	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
12	Diversification—supply market	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
13	Quality of suppliers	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
14	Quality of customers	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
15	Clean criminal record of board members	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
16	Clean criminal record of chairperson	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
	Score		$\overleftrightarrow{Tr}\left(\frac{95}{48},\frac{104}{48},\frac{107}{48},\frac{116}{48}\right)$		$\overleftrightarrow{Tr}\left(\frac{80}{48},\frac{104}{48},\frac{104}{48},\frac{128}{48}\right)$

COS5 and COS6 (mixed approach) are presented in Table 4.

For the purpose of comparison, the expert's assessments were made according to COS1. Using the appropriate transformations, these assessments were translated into assessments according to COS2, COS3, COS4, COS5 and COS6. All the transformed evaluations are shown in Table 2, Table 3 and Table 4. The last line of each table presents the values of a scoring function calculated according to the individual types of COS. These results indicate that the choice of COS has an impact on the value of the scoring function. As the final recommendation depends on this score, it can be influenced by the method applied.

The significant advantage of COS1 lies in the fact that its implementation enables each expert to individually choose the most suitable type of COS. From a formal point of view, it is acceptable to allow experts working in one team to use different types of these COSs. The only necessary condition is that each individual expert consistently uses his/her chosen COS scale.

Table 4. An expert's evaluations with the use of different Complete Order Scales (COS5, COS6)

No.	Criteria	COS5 EOS	COS5 NOS	COS6 EOS	COS6 NOS
1	Prospects of business	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$
2	Board members' experience	A +	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$	A + +	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$
3	Chairperson's experience	A +	$\overleftrightarrow{Tr}\left(\frac{11}{4},3,\frac{13}{4},\frac{14}{4}\right)$	A + +	$\overleftrightarrow{Tr}\left(3,3,\frac{13}{4},\frac{15}{4}\right)$
4	Range of regional operations	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$
5	Range of international operations	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$
6	Risk associated with market	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B + +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$
7	Risk associated with trade	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$
8	Risk associated with suppliers	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$
9	Risk associated with customers	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$
10	Diversification—products	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
11	Diversification—sales markets	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$	C ∼	$\overleftrightarrow{Tr}\left(\frac{2}{4},1,1,\frac{6}{4}\right)$
12	Diversification—supply market	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$	B ∼	$\overleftrightarrow{Tr}\left(\frac{6}{4},2,2,\frac{10}{4}\right)$
13	Quality of suppliers	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$
14	Quality of customers	C +	$\overleftrightarrow{Tr}\left(\frac{3}{4},1,\frac{5}{4},\frac{6}{4}\right)$	C + +	$\overleftrightarrow{Tr}\left(1,1,\frac{5}{4},\frac{7}{4}\right)$
15	Clean criminal record of board members	A - -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$	A -	$\overleftrightarrow{Tr}\left(\frac{13}{4},3,\frac{11}{4},\frac{10}{4}\right)$
16	Clean criminal record of the chairperson	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$	B +	$\overleftrightarrow{Tr}\left(\frac{7}{4},2,\frac{9}{4},\frac{10}{4}\right)$
	Score		$\overleftrightarrow{Tr}\left(\frac{96}{48},\frac{107}{48},\frac{111}{48},\frac{120}{48}\right)$		$\overleftrightarrow{Tr}\left(\frac{95}{48},\frac{104}{48},\frac{108}{48},\frac{112}{48}\right)$

8. Conclusion

This paper has presented a formal structure for defining a COS. Knowledge of this structure enables the transformation of a given two-stage COS to less complex structures. It was noticed that a change in the COS structure influences the value of a scoring function.

Even replacing only one linguistic indicator (COS5, COS6) can bring different conclusions. This means that despite experts' opinions and claims that they are not affected by the set of linguistic indicators used, this set can affect the outcome of an application for credit.

Therefore, further examination of this influence should be a direction for future research. This, in turn, requires collecting a sufficient amount of data to provide a representative statistical sample. Obtaining a statistically sufficient sample is one of the limitations of the proposed method, along with the variability of the results mentioned above regarding the inability of experts to precisely capture the boundaries between the linguistic labels proposed.

The application of FNs when defining NOS always leads to an imprecise scoring function. The phenomenon of imprecision has already been broadly presented in the literature. However, future studies investigating the imprecision of scoring functions should still be conducted. The problem of the non-associativeness of summing trapezoidal oriented fuzzy numbers can be overcome by the proposed approach, but other possible solutions should be investigated.

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