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Asset liability management for the Bank of Uganda defined benefits scheme by stochastic programming

Herbert Mukalazi*[1] Torbjörn Larsson[2] Juma Kasozi[1] Fred Mayambala[1]

[1] *Department of Mathematics, Makerere University, Uganda*
[2] *Department of Mathematics, Linköping University, Sweden*
* *Corresponding author: hmukalazi@kyu.ac.ug*

Abstract

We develop a model for asset liability management of pension funds, which is solved by stochastic programming techniques. Using data provided by the Bank of Uganda Defined Benefits Scheme, which is closed to new members, we obtain the optimal investment policies. Randomly sampled scenario trees using the mean and covariance structure of the return distribution are used for generating the coefficients of the stochastic program. Liabilities are modelled by remaining years of life expectancy and guaranteed period for monthly pension. We obtain the funding situation of the scheme at each stage, and the terminal cash injection by the sponsor required to meet all future benefit payments, in absence of contributing members.

Keywords: closed scheme, finance, asset liability management, scenario generation, stochastic programming

1. Introduction

A pension is a term for single or periodic payments to a beneficiary, which replaces the income of an employee in case of reaching a certain age, or in the case of disability or death. A pension fund is considered to be an organisation, obliged with paying pensions and it has a task of making benefit payments to members who have ended their active working and earning careers. The payments are made to the retirees in accordance to a benefit formula that prescribes the flow of payments to which each member in the fund is entitled. The pension funds planning horizons stretch for several decades, while receiving contributions from active members and paying benefits to retirees. Hence the fund managers have a trade-off between long term gains and fulfilling short term solvency requirements, while anticipating future policy adjustments. According to [21], this setting is suited for a Stochastic Programming (SP) approach with dynamic portfolio strategies.

When modelling optimisation problems, the deterministic approach is used, where parameters are known at the time of making the decision, or stochastic optimisation in which the parameters are uncertain at the time of making the decision. The goal of stochastic optimisation is to find optimal decision

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policies in problems involving uncertainty. Programming refers to the fact that various parts of the problem can be modelled as linear or non-linear mathematical program [2]. In this paper, we implement stochastic optimisation using the SP technique, we refer readers not familiar with SP to [22, 26] for detailed explanations.

Asset Liability Management (ALM) for pension funds is a risk management approach, which takes into account the assets, liabilities, and different policies and regulations. The management of a pension fund should find acceptable policies that guarantee with a large probability that the solvency of the fund is sufficient during the planning horizon, and at the same time, all benefit payouts can be made. Management of assets involves decisions on the investment portfolio while the liability consists of future pension payments [16].

In several countries, mandatory public schemes are usually supplemented by occupational schemes. According to [25], access to public pension for the working population in emerging economies is limited to 10% to 25%. Occupational schemes are broadly categorised into Defined Contribution (DC) and Defined Benefit (DB) schemes. In DB schemes, a benefits formula linked to salary and years of service is used, while in DC schemes the amount contributed to the fund is specified. According to [15], most DB type Pay-As-You-Go (PAYG) systems, however, have no or little linkage between annual benefits and retirement age while funded DC plans are actuarially neutral, since conversion to an annuity takes place at actual retirement. These schemes give incentives that are of strong economic importance for their sustainability. The study by [7] constructed one of the leading models for ALM in financial decision making for the Russell (financial institution) - Yasuda (insurance organisation), using multistage SP. The model determines an optimal investment strategy that incorporates a multi-period approach and enables the decision makers to define risks in tangible operational terms. It also handles the complex regulations imposed by Japanese insurance laws and practices. The most important goal is to produce a high income return to pay annual interest on savings type insurance policies without sacrificing the goal of maximising the long term wealth of the firm. During the first two years of use, fiscal 1991 and 1992, the investment strategy devised by the model yielded extra income of 8.7 billion Yen or US$ 79 million.

Financial applications of SP and non-linear SP methods were described in detail by [23] and they setup a multistage SP model. A support model to sustain management of pension funds in strategic planning of available asset and liability policy instruments was described by [3]. The main characteristic of this approach is the modelling of relevant risk-drivers by scenarios rather than probability distributions. He described the scenario generation methodology and how scenarios are used by pension fund managers to simulate and improve ALM strategies. He described how the process of managerial learning can be improved by hybrid simulation-optimisation method which applies concepts of non-linear global optimisation to determine asset allocations for efficient frontier of contribution rates and downside insolvency risks. He concluded by showing that the application of the developed model to a particular fund leads to annulment of the infeasibility of current ALM policy and a reduction of the expected yearly contributions. Application of SP for ALM was formulated by [9] while [21] tested scenario generation methods and developed a new SP ALM model. To compare among several scenario generation methods, [28] summarised the applications of SP on asset allocation, fixed income securities management and ALM. In order to obtain a dynamic SP model for bond portfolio management, [27] applied the grid method to generate scenarios. Stochastic optimal control theory was employed by [8] to analytically solve the ALM problem under a mean-variance optimisation framework in continuous-time. Hibiki [17] applied simulated path structure substituting event trees to generate scenarios, which helped to avoid some computational difficulties and obtained more effective SP results. SP has been proven to be an efficient approach in designing effective strategies in wealth and ALM in practice [18]. Consiglio, Cocco and Zenios [10] developed a scenario optimisation model for integrative ALM, analysed the trade-offs in structuring such policies studied and alternative choices.

In pension funds, future asset returns, liabilities, streams of contributions and benefits are unknown. An application of SP means that these uncertainties are modelled as random parameters in a discrete time model with a finite planning horizon [13]. Scenario trees are used in SP to describe the uncertainty of parameters in the discrete-time setting. The scenario tree branches off every random parameter in each time stage. This approach requires a finite discrete distribution that is limited in the number of possible values of the random parameters. According to [21], the performance of SP can be improved by choosing an appropriate scenario generation method. Formal optimal decision approaches for a multi-period ALM model for a pension fund were studied by [4]. They used Conditional Value at Risk (CVaR) as a risk measure. The model is based on sample path formulation of the fund liabilities and returns of the financial instruments included in the portfolio. The same optimal decisions are made for sample path which exhibit similar performance characteristics. Compared to traditional SP algorithms for which problem dimension increases exponentially in number of time stages, their approach exhibits a linear growth dimension. Bai and Ma [1] designed a model for finding optimal contribution rates and portfolio allocations that takes into account the funding situation of the fund. Their objective function was a combination of [4, 21] SP ALM models using CVaR risk measure, the model was solved with dynamic SP techniques. They added CVaR constraints and considered the real situation of pension funds in China. A long term model of ALM for Tanzania pension funds by SP was presented by [19]. Their decisive factor for a long term ALM is that Tanzania pension funds face an increase of their members' life expectancy which will cause retirees to contributors' dependence ratio to increase. They presented a SP approach which allocates assets with best return to raise the asset value closer to the level of liabilities. Liabilities were modelled using number of years of life expectancy for monthly benefit. Scenario trees were generated using Monte Carlo simulation. Numerical results suggest that in order to improve long term sustainability of the Tanzania pension fund system, it is necessary to make reforms concerning contribution rate, investment guidelines and formulate target funding ratios to characterise the pension fund's solvency situation.

Some studies on Uganda's social security system include, [22] studied ALM for the Parliamentary Pension Scheme of Uganda by SP, the status of social security in Uganda by [6, 20] proposed adoption of a twin peak mechanism in the financial sector, [24] examined social, economic and demographic risk factors, [5] used PROST to analyse the future liabilities that the Ugandan Public Servants Pension Scheme might accumulate under the provisions of Cap 286, unless it is reformed. The latter recommended a hybrid reform option composed by a small DB scheme and a complementary DC scheme. The Ministry of Finance Planning and Economic Development through the Uganda Retirement Benefits Regulatory Authority (URBRA), set up the investment limits for all the different retirement benefits schemes in Uganda. However, the different retirement benefit schemes can set their strategic asset allocation limits which should not violate the limits set by URBRA. The strategic asset allocation limits for Bank of Uganda Defined Benefits Schemes (BoUDBS), and all the other asset allocation limits used in this study are given in Table 1.

Table 1. Asset investment limits

Assets	URBRA (%)		BoUDBS (%)		Modified (%)	
	Lower	Upper	Lower	Upper	Lower	Upper
Government securities	0.0	70.0	30.0	70.0	0.0	30.0
Corporate bonds	0.0	15.0	0.0	30.0	2.0	15.0
Term deposits	0.0	30.00	0.0	20.0	2.0	40.0
Investment property	0.0	10.0	0.0	20.0	5.0	15.0
Equity	0.0	85.0	5.0	50.0	5.0	40.0

The aim of this study is to develop a SP model for ALM of pension funds. As an application, we consider the financial planning problem of the BoUDBS. We find optimal investment policies, optimal contribution rates and funding status for the BoUDBS, a closed DB scheme. The multi-stage SP ALM model is done for a horizon of 30 years from 2018 to 2048. We use data from the scheme's annual reports and bio-data information. Established abridged mortality tables are used for future expectation of life and survival probabilities.

The remainder of the paper is organised as follows. In Section 2 we formulate the SP model for asset liability management of pension funds. In Section 3 we present the scenario generation methods for economic factors, liabilities and benefit payments. In Section 4 we present the demographic evolution of the BoUDBS scheme members, analyse results from application of our ALM model to the BoUDBS and Section 5 gives the conclusion.

2. Stochastic programming ALM model

We formulate a stochastic programming model for the asset liability management of the BoUDBS. The decisions are made for a planning horizon of 30 years, from 2018 to 2048. The different stages are indexed by $t = 0, \ldots, T$, with $t = 0$ as the start of the planning horizon, and $t = T$ denotes its end. The model is based on [21], the model by [11] includes chance constraints for solvency of pension funds which complicates the numerical solutions. Following [7], we penalise deficits in the objective function to avoid computational complications. The model is presented in compact form so that the structure of the scenario tree is not described by a set of constraints but is implicitly incorporated in the model. Hence we change notation from a set of scenarios $s \in \{1, \ldots, S\}$ to the nodes of the scenario tree $n \in \{1, \ldots, N_t\}$. A scenario s corresponds to the path from the root node to the terminal node. The realisations of random variables at different stages are represented by the nodes of the scenario tree, and N_t denotes the number of nodes of the scenario tree in stage t. If node $n \in \{1, \ldots, N_t\}$ at time t, then its predecessor at time $t - 1$ is denoted by $\tilde{n}$.

The asset liability management model is formulated as a linear multi-stage stochastic program. Decisions x_t are taken in time stages $t = 1, \ldots, T$. Hence the asset portfolio is not optimised at the beginning of the horizon. The model is introduced in terms of the objective function and constraints. We define the indices in Table 2, random parameters in Table 3, deterministic parameters in Table 4 and decision variables in Table 5.

Table 2. Indices

Index	Description
t	Time, $t = 0, \ldots, T$
i	Asset class, $i = 1, \ldots, I$
n	Node, $n = 1, \ldots, N_t$

Table 3. Random parameters

Parameter	Description
B_{tn}	Benefit payment at node n of stage t
L_{tn}	Liabilities at node n of stage t
S_{tn}	Total salaries of members at node n of stage t
r_{itn}	Return on asset category i at node n of stage t

Table 4. Deterministic parameters

Parameter	Description
X_i^0	Initial amount held in asset i
M_0	Initial cash position
c_L	Minimum contribution rate
c_U	Maximum contribution rate
Δc^L	Lower bound for decrease in contribution rate
Δc^U	Upper bound for increase in contribution rate
$F_{\min}$	Minimum funding ratio
F_T	Target funding ratio at end of planning horizon
w_L	Lower bound on proportion of asset mix
w_U	Upper bound on proportion of asset mix
γ_i^p	Transaction cost incurred in purchasing asset i
γ_i^s	Transaction cost incurred in selling asset i
λ	Risk aversion parameter

Table 5. Decision variables

Variable	Description
X_{itn}^h	Amount held in asset category i at node n of stage t
X_{itn}^p	Amount purchased of asset category i at node n of stage t
X_{itn}^s	Amount sold of asset category i at node n of stage t
A_{tn}	Asset value at node n of stage t
c_{tn}	Contribution rate at node n of stage t
H_{Tn}	Cash injection by sponsor at node n of the end of the horizon, required to attain F_T
D_{tn}	Deficit relative to the minimum funding ratio at node n of stage t

2.1. Objective

We adopt the objective function in [21], which minimises the overall contribution rate and risk, and modify its last term to cater for a closed scheme in absence of contributing members. Risk aversion is modelled by quadratic penalty on the deficits D_{tn}. To ensure solvency of the fund at the end of the planning horizon, the sponsor should make a cash injection H_{Tn} to achieve the target funding ratio of the pension fund F_T.

$$\min \sum_{t=0}^{T-1} \left(\sum_{n=1}^{N_t} \frac{c_{tn}}{N_t} \right) + \lambda \sum_{t=1}^{T} \left(\sum_{n=1}^{N_t} \frac{1}{N_t} \left(\frac{D_{tn}}{L_{tn}} \right)^2 \right) + \sum_{n=1}^{N_T} \frac{H_{Tn}}{N_T} \tag{1}$$

In the objective function, λ is the risk aversion penalty parameter, the first term is the sum of average contribution rates for every stage, the second term is the risk aversion, using square of the ratio of deficit to liability and the third term is the average cash injection by the sponsor at the end of the planning horizon, which ensures that the fund can clear all future benefits. The scheme sponsor wishes to minimise his overall contribution over the planning horizon while keeping the fund solvent.

2.2. Asset inventory constraints

These are the constraints that describe the dynamic change in asset investment portfolio at each stage. There is no rebalancing at the end of the horizon.

$$X_{i01}^h = X_i^0 + X_{i01}^p - X_{i01}^s \quad \text{for } i = 1, \ldots, I \tag{2}$$

$$X_{itn}^h = (1 + r_{itn})\, X_{i,t-1,\tilde{n}}^h + X_{itn}^p - X_{itn}^s \quad \text{for } n = 1, \ldots, N_t,\ t = 1, \ldots, T-1,\ i = 1, \ldots, I \tag{3}$$

Equation (2) describes the initial amount invested in each asset at the initial stage when $t = 0$.

2.3. Total asset value

At the end of each stage, the fund measures its total asset value to determine its solvency. The asset value at the end of a given period is the sum of the asset value at the beginning of the period and the returns on each asset during the period.

$$A_{tn} = \sum_{i=1}^{I} (1 + r_{itn})\, X^h_{i,t-1,\tilde{n}} \;\text{ for }\; n = 1, \ldots, N_t,\; t = 1, \ldots, T \tag{4}$$

2.4. Cash balance constraints

These constraints ensure that the cash inflow into the scheme is equal to cash outflow from the scheme. Cash inflow is due to contributions from the members and the selling of assets. The cash outflow is due to benefit payments to the retirees and purchase of assets. We incorporate the transaction costs incurred in buying and selling of assets on the asset prices. Ensuring that cash inflow is equal to cash outflow yields the following equations.

$$c_{01}S_{01} + M_0 + \sum_{i=1}^{I} (1 - \gamma_i^s)\, X^s_{i01} = B_{01} + \sum_{i=1}^{I} (1 + \gamma_i^p)\, X^p_{i01} \tag{5}$$

$$c_{tn}S_{tn} + \sum_{i=1}^{I} (1 - \gamma_i^s)\, X^s_{itn} = B_{tn} + \sum_{i=1}^{I} (1 + \gamma_i^p)\, X^p_{itn} \;\text{ for }\; n = 1, \ldots, N_t,\; t = 1, \ldots, T-1 \tag{6}$$

2.5. Goal constraints

The minimum funding ratio set by a pension fund becomes its goal. Deficits are registered whenever the funding ratio is less than F_{min}. These deficits are penalised in the objective function. To guarantee that there are no deficits at the end of the planning horizon, the sponsor should make a cash injection H_{Tn} into the fund which will result in a desired funding ratio F_T at the end of the planning horizon.

$$A_{tn} \geq F_{\text{min}} L_{tn} - D_{tn} \;\text{ for }\; n = 1, \ldots, N_t,\; t = 1, \ldots, T \tag{7}$$

$$A_{Tn} \geq F_T L_{Tn} - H_{Tn} \;\text{ for }\; n = 1, \ldots, N_T \tag{8}$$

$$D_{tn} \geq 0 \;\text{ for }\; t = 1, \ldots, T,\; n = 1, \ldots, N_t \tag{9}$$

$$H_{Tn} \geq 0 \;\text{ for }\; n = 1, \ldots, N_T \tag{10}$$

2.6. Short sales constraints

We do not consider short sales in this problem, hence amount of assets sold must be less than or equal to the amount of assets held in the previous time period.

$$X^s_{itn} \leq X^0_i \;\text{ for }\; i = 1, \ldots, I,\; t = 1,\; n = 1, \ldots, N_t \tag{11}$$

$$X^s_{itn} \leq X^h_{i,t-1,\tilde{n}} \;\text{ for }\; i = 1, \ldots, I,\; n = 1, \ldots, N_t,\; t = 2, \ldots, T \tag{12}$$

2.7. Contribution rate constraints

The level of contribution as well as the change in contribution rates are bounded and specified by the pension fund.

$$c_L \leq c_{tn} \leq c_U \;\text{ for }\; n = 1, \ldots, N_t,\; t = 0, \ldots, T-1 \tag{13}$$

$$\Delta c^L \leq c_{tn} - c_{t-1,\tilde{n}} \leq \Delta c^U \;\text{ for }\; n = 1, \ldots, N_t,\; t = 1, \ldots, T-1 \tag{14}$$

2.8. Asset weight mix boundaries

The asset weight mix is bounded through the investment limits, which are given in Table 1.

$$w_L \sum_{i=1}^{I} X_{itn}^h \leq X_{itn}^h \leq w_U \sum_{i=1}^{I} X_{itn}^h \text{ for } n = 1, \ldots, N_t,\ t = 0, \ldots, T-1,\ i = 1, \ldots, I \tag{15}$$

3. Scenario generation

A stochastic programming model requires scenarios of the possible realisations of stochastic elements. The random elements of the model include salaries and returns for all asset classes. Data on the actual values of the stochastic parameters becomes available in stages, and the decisions at every stage depend on the observations at that particular time and not on the future realisations.

3.1. Economic scenario generation

The asset return scenarios provide information about future asset returns so that we can evaluate possible investment policies for the pension fund. Since ALM focusses on strategic long term decisions, a small set of asset classes is sufficient. Each asset scenario should contain a time series of salary increases, to transform the real expected values of the benefits and liabilities into nominal values.

According to URBRA, the investment field of pension funds in Uganda is limited to cash and call deposits, term deposits, government securities, equity, investment property and a very small portion not exceeding 5% is allowed for investment in other financial products with good liquidity. The BoUDBS invests in five kinds of assets; government securities (treasury bonds and treasury bills), corporate bonds, equity, term deposits and investment property. We need to generate 6 kinds of economic scenarios, these are the total salaries of the scheme members at the beginning of year t, S_{tn} and return rates of the five kinds of assets r_{itn}, for $i = 1, 2, 3, 4, 5$. The scenarios for liabilities at the end of year t, L_{tn} and benefit payments in year t, B_{tn} will be obtained based on the economic scenario generation.

We need to forecast the future distribution and consider correlations among variables, in order to simulate the 6 kinds of economic scenario variables within the planning horizon of 30 years. To model asset returns, we generate the time series using a vector autoregressive model as applied in [3].

$$h_t = \kappa + \Omega h_{t-1} + \epsilon_t, \text{ where } \epsilon_t \sim N(0, \Sigma) \text{ for } t = 1, \ldots, T \tag{16}$$

$$h_{it} = \ln(1 + r_{it}) \text{ for } t = 1, \ldots, T,\ i = 1, \ldots, I \tag{17}$$

where I is the number of time series, r_{it} is the discrete rate of return of asset i in stage t. The returns on each asset are transformed to $\ln(1 + r_{it})$ to avoid heteroscedasticity, h_t is a $\{I \times 1\}$ vector of continuously compounded rates, κ is a $\{I \times 1\}$ vector of intercept terms, Ω is the $\{I \times I\}$ matrix of coefficients, $\epsilon_t = (\epsilon_{1t}, \ldots, \epsilon_{It})^T$ is a I dimensional vector of error terms, with $\mathbb{E}(\epsilon_t) = 0$, $\mathbb{E}\left(\epsilon_t \epsilon_t^T\right) = \Sigma$ and $\mathbb{E}\left(\epsilon_s \epsilon_t^T\right) = 0$ for $s \neq t$. The covariance matrix Σ is assumed to be non singular.

To obtain the simulated returns, we incorporate the following relation from [14] to adjust simulated returns for the length of each planning period.

$$r_{it} = (1 + \mu_i)^\tau + p_{it}\sigma_i\sqrt{\tau} - 1 \tag{18}$$

In equation (18), p_{it} is the rate of return produced by the vector autoregressive model, μ_i is the mean return of asset i, σ_i is the standard deviation in the return of asset i, and $\tau = 5$ is the length of each planning period. We construct the scenario tree for a planning horizon of 30 years, having a branching structure of 1–10–5–5–4–4–2, for $t = 0, 5, 10, 15, 20, 25, 30$ from 2018 to 2048. This tree has 13311 nodes and 8000 scenarios. The branching structure is not unique but is chosen in such a way that we reduce the growth rate in problem dimension in number of time stages. We use the software MatLab to simulate scenarios, with simulations in node n basing on data in the predecessor node $\tilde{n}$ to obtain all economic scenarios data at all nodes.

3.2. Liabilities and benefit payments scenario generation

Liabilities are the future benefits to be paid to members when they retire, and the value of the liabilities is the present value of the expected benefit payments. The effect trustee deed and rules of the BoUDBS is the deed of amendment dated April 2014, a member can retire on attaining the retirement age of 60 years subject to have been employed by the sponsor on permanent and pensionable terms aged between 18 and 45 years. Early retirement is allowed on attaining 55 years and ill-health retirement. When an employee leaves service of the sponsor, the benefits are deferred until attainment of the retirement age. The different kinds of benefits are; retirement (commuted benefit and monthly pension), retirement on health grounds, death of a member and deferred benefits. These benefits depend on the number of years that the member has been building rights, by contributing to the fund and the average annual salary in the last three years prior to retirement. In our analysis, we consider the commuted benefits and monthly pensions, these are the only benefits reflected in the payments from the fund. The deferred benefits are paid upon the member reaching the retirement age. The bank insures its employees, on death of a serving member the insurance company pays money to the fund which is then paid to the beneficiary.

The salary in year t, for a member who joined in year ν is given by

$$S_t = S_\nu \prod_{k=0}^{t-\nu} (1 + g_{\nu+k})$$

where $g_{\nu+k}$ is the stochastic annual salary growth rate. In our calculations, the final pensionable salary S_{final} of a member aged j years in year t whose normal retirement age is r_{age} is given by

$$S_{\text{final}} = \frac{S_t}{3} \left(\prod_{k=0}^{r_{\text{age}}-j-2} (1 + g_{\nu+k}) + \prod_{k=0}^{r_{\text{age}}-j-1} (1 + g_{\nu+k}) + \prod_{k=0}^{r_{\text{age}}-j} (1 + g_{\nu+k}) \right) \tag{19}$$

which is the average annual salary of the member in the last three years preceding a member's normal retirement date. The monthly pension which is given for life to a retiree and is guaranteed for a period of $\tau = 5$ years is given by

$$MB = \left(\frac{m_1}{60} + \frac{m_2}{65} \right) \times S_{\text{final}} \times 75\% \times \frac{1}{12} \tag{20}$$

where m_1 are years of pensionable service prior to 30^{th} June 2013, m_2 are years of pensionable service after 30^{th} June 2013, $1/60$ is accrual factor for benefits of the scheme prior to 30^{th} June 2013, $1/65$ is accrual factor for benefits of the scheme after 30^{th} June 2013, 75% is the commutation rate of the full annual amount of pension, S_{final} is the final pensionable salary introduced in equation (19). We convert this to annual benefit in equation (21) which we use in calculations that follow,

$$AB = MB \times 12 \tag{21}$$

The annual benefits increase at a constant rate of 5% per annum. The members of the scheme also contribute to the National Social Security Fund (NSSF), on retirement their NSSF benefits in respect of pensionable service are deducted from the commuted benefit. The net commuted benefit at the time of retirement is given by

$$CB = \left(\frac{m_1}{60} + \frac{m_2}{65} \right) \times S_{\text{final}} \times \kappa \times 25\% - \text{NSSF}_{\text{benefit}} \tag{22}$$

where κ is the commutation factor at retirement, 25% is the commutation rate of the full annual amount of pension, and $\text{NSSF}_{\text{benefit}}$ is the amount whose annuity value is the NSSF benefit in respect of pensionable service from date of entry to 30^{th} June 2013. From the bank of Uganda annual report 2017/2018, as at 30^{th} June 2018, the weighted duration of the defined benefit obligation was 15.4 years. Hence in our calculations, we use $\kappa = 15$ as the commutation factor. Using equations (21) and (22), the total benefit payouts BP_t in year t is given by

$$BP_t = NR_t \times CB + NP_t \times AB \tag{23}$$

where NR_t is the total number of members retiring in year t and NP_t is the total number of pensioners in the same year.

The total expected commuted benefit in year t for members of age j years is given by

$$CB_{tj} = P_t^{r_{\text{age}}-j} \times n_j \times \left(\left(\frac{m_1}{60} + \frac{m_2}{65} \right) \times S_{\text{final}} \times \kappa \times 25\% - \text{NSSF}_{\text{benefit}} \right)_{tj} \tag{24}$$

where $P_t^{r_{\text{age}}-j}$ is the probability that a member aged j years in year t lives for $r_{\text{age}} - j$ more years until the retirement age r_{age} and n_j is the number of members aged j years in year t.

The total expected yearly benefits in year t for members aged j years, with a guaranteed period of τ years after retirement is

$$AB_{tj} = P_t^{r_{\text{age}}-j} \times n_j \times \left(\frac{m_1}{60} + \frac{m_2}{65} \right) \times S_{\text{final}} \times 75\% \times \left(\tau + P_t^{r_{\text{age}}+\tau-j} \times \mathbb{E}P_{(r_{\text{age}}+\tau)_{tj}} \right) \tag{25}$$

where $P_t^{r_{\text{age}}+\tau-j}$ is the probability that a member aged j years in year t lives for $r_{\text{age}} + \tau - j$ more years after retirement, and $\mathbb{E}P_{(r_{\text{age}}+\tau)_{tj}}$ is the expected remaining life in year t for a member aged j years, when he reaches the age of $r_{\text{age}} + \tau$ years, these are shown in Table 16.

Total expected benefit in year t for members aged j years is

$$B_{tj} = CB_{tj} + AB_{tj}$$

The liability at time t is the discounted present value of expected total benefit. The total liability in year t is hence given by

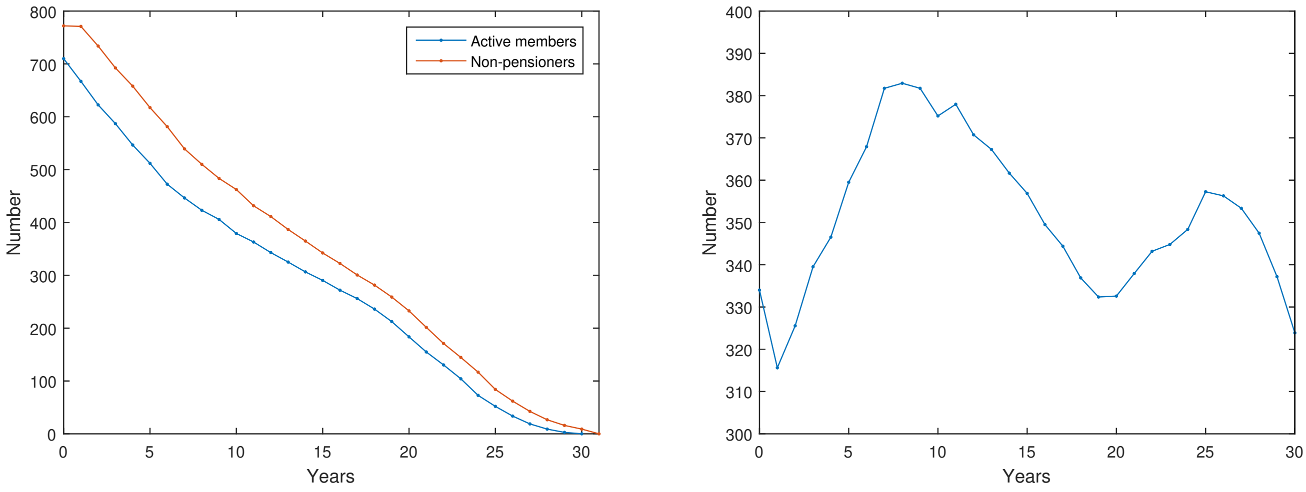
$$L_t = \sum_{j=j_0}^{r_{\text{age}}-1} \frac{B_{tj}}{(1+r)^{r_{\text{age}}-j}} \tag{26}$$

where j_0 is the minimum age of the active members and r is a discounting factor. Basing on data provided from the life tables and economic scenarios data at each node, we calculate scenario data at each node for liabilities L_{tn}.

4. Numerical results

4.1. Demographic status

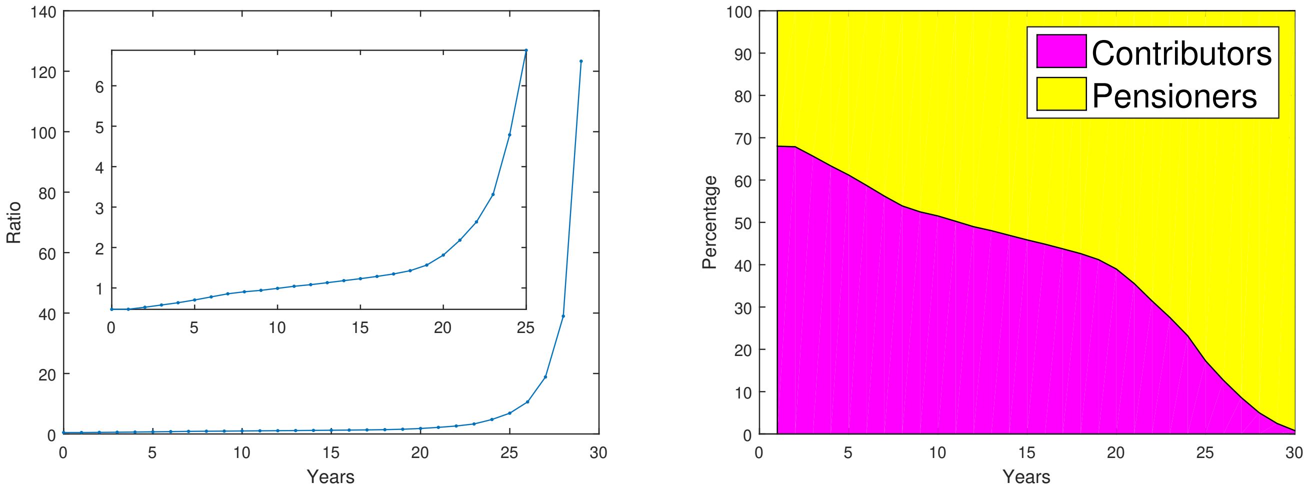
The future demographic status of the fund members in the different categories is modelled by a Markov model which uses state transition probabilities. Data from the BoUDBS was used to find the state transition matrices, the bio-data information used was from 1995 to 2018. The projection results are shown in Figure 1, with Figure 1a) for non-pensioners (active and deferred members) and Figure 1b) for pensioners. It is evident that the scheme will have no active members starting from 2048.



a) b)

Figure 1. Projected scheme populations, note the different scales: a) non-pensioners' population 2018–2048, b) pensioners' population 2018–2048

In Figure 2, we compare the contributing members and pensioners, the pensioners to active members' dependency ratio is shown in Figure 2a) while their percentage composition is given in Figure 2b). These figures show that there is gradual increase in pensioners' composition and hence dependence ratio in the first 20 years, and then rapid increase in the last 10 years. The scheme does not admit new members, hence the composition of pensioners increases until the time when there are no active members.



a) b)

Figure 2. Distribution of members: a) dependence ratios, b) percentage composition

From Figure 3, there is a big number of members going into deferment. This results from the fund using big discounting factors in valuing benefits of those leaving service of the sponsor. From the Bank of Uganda annual report 2017/2018, the discount rate is based on a 15–year government bond yield which was 15.98% in 2017 and 15.03% in 2018. The number of retiring members fluctuates depending on the age distribution of the non-pensioners. The number of non-pensioners dying reduces on the horizon due to reduction in their number and reduction in mortality. The dying pensioners remain quite stable on the horizon, despite the reduction in mortality the increase in their number keeps their deaths high.

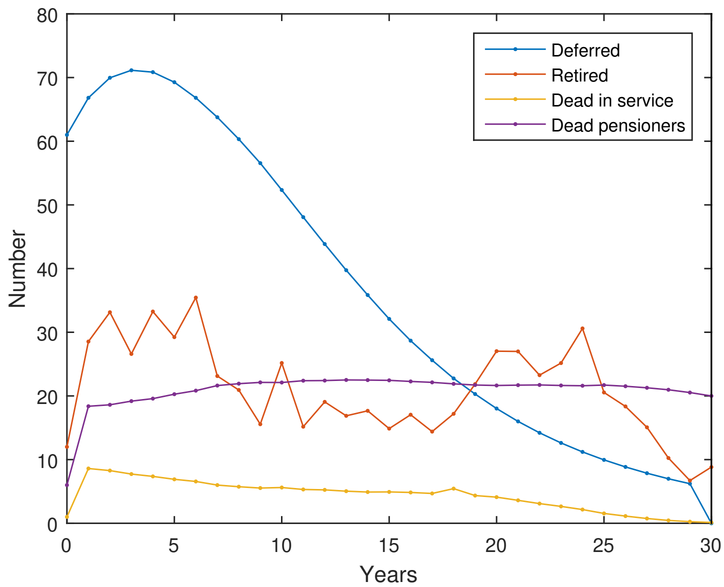


Figure 3. Smaller scheme populations

4.2. Statistics from historical data and model calibration

The assets considered in this study are government securities (Gs), corporate bonds (Cb), term deposits (Td), investment property (Ip) and equity (Eq). We use data from the scheme's annual reports about the asset returns and general salary increase from 2010 to 2018 to estimate the coefficients of the VAR model. The descriptive statistics of the time series are given in Table 6, and Table 7 gives the correlation matrix.

Table 6. Statistics, time series 2010–2018

	Mean	St. Dev
Sa	0.0564	0.0436
Gs	0.1256	0.0800
Cb	0.0801	0.0351
Td	0.1134	0.0655
Ip	0.0641	0.0045
Eq	0.1184	0.1561

Table 7. Correlations, annually 2010–2018

	Sa	Gs	Cb	Td	Ip	Eq
Sa	1					
Gs	-0.4905	1				
Cb	-0.1864	0.3497	1			
Td	0.0096	-0.1323	-0.1894	1		
Ip	-0.7396	0.2617	0.4319	-0.3837	1	
Eq	0.0308	-0.5734	-0.4116	-0.0208	0.2984	1

In specifying the VAR model, we do not use lagged terms in modelling returns of government securities, corporate bonds, investment property and equity as shown in Table 8. This is done to avoid having unstable and spurious predictability of returns. We model return on term deposits and rate of salary increase by a first order autoregressive model. We estimate the parameters of the VAR model using iterative least squares minimisation as discussed in [12]. The estimated correlation matrix of the residuals is shown in Table 9. Monte Carlo simulation and Cholesky decomposition are used to generate the scenario tree for the SP model. Cholesky decomposition is used to preserve the covariance structure of return rates. Future returns are obtained by sampling from the error distribution of the equations estimated in Table 8. The simulated returns are then used in equation (18) which accounts for the duration of each planning period, thus giving the simulated future returns at each node.

The R^2 value measures the percentage of variation in the values of the dependent variable that can be explained by the variation in the independent variable. The last column of Table 8 gives the R^2 values, a 0.6% R^2 value for salary growth means that 0.6% of variation in salary growth can be explained by variation in the lagged term and the remaining 99.4% is due to random variability. The rest of the R^2 values show that variability in all the returns is mainly due to random effects.

Table 8. Coefficients of the VAR model

							R^2
$\ln(1+\text{Sa}_t) =$	0.0489	+	$0.0801 \ln(1+\text{Sa}_{t-1})$	+	ϵ_{1t}	$\sigma_{1t} = 0.0442$	0.0060
$\ln(1+\text{Gs}_t) =$	0.1189	+	ϵ_{2t}			$\sigma_{2t} = 0.0756$	0.0005
$\ln(1+\text{Cb}_t) =$	0.0793	+	ϵ_{3t}			$\sigma_{3t} = 0.0336$	0.0020
$\ln(1+\text{Td}_t) =$	0.1053	+	$0.0159 \ln(1+\text{Td}_{t-1})$	+	ϵ_{4t}	$\sigma_{4t} = 0.0613$	0.0002
$\ln(1+\text{Ip}_t) =$	0.0637	+	ϵ_{5t}			$\sigma_{5t} = 0.0042$	0.0402
$\ln(1+\text{Eq}_t) =$	0.1046	+	ϵ_{6t}			$\sigma_{6t} = 0.1602$	0.0001

Table 9. Residual correlations of the VAR model

	Sa	Gs	Cb	Td	Ip	Eq
Sa	1					
Gs	0.4993	1				
Cb	0.6365	0.4109	1			
Td	0.0527	0.0683	0.0866	1		
Ip	0.4226	0.3061	0.3355	0.1214	1	
Eq	-0.3778	-0.5775	-0.4442	-0.4552	0.3343	1

4.3. Solution to the BoUDBS SP ALM model

In Section 2, we developed a SP model for ALM of pension funds. The stochastic program is based on a scenario tree, which describes the return distributions and evolution of the liabilities. In this section, we present results of the solution to the model. The SP model has been solved with a randomly sampled scenario tree as input. The size of the model formulated as a compact linear programming problem is 204696 constraints, 252906 variables and 1 objective. The SP model is solved with AMPL and Cplex. (Here, and in the following, all monetary values are given in billion (bn), Uganda Shillings, UGX.)

The model parameters are displayed in Table 10. The initial contribution rate and initial funding ratio obtained from historical data are 0.211 and 1.008 respectively, initial cash position is UGX 1.13 bn, initial total annual salary is UGX 79.31 bn and the initial asset value is UGX 420.41 bn. The minimum funding ratio is 0.80, whenever the funding ratio is less than this value, deficits are given a quadratic penalty in the objective with a risk aversion parameter of 4. The target funding ratio is set at 1.00, therefore, at the end of the planning horizon, the sponsor should make a cash injection H_{Tn} into the fund to lift the funding ratio to this target value. The upper and lower bounds on contribution rate are set at 0.04 and 0.75, respectively. The decrease and increase in contribution rate are bounded by -0.05 and 0.05, respectively. Based on historical data, transaction costs of 0.005 are incurred in buying and selling of assets.

Table 10. Parameters

M_0	c_L	c_U	Δc^L	Δc^U	$F_{\min}$	F_T	γ_i^p/γ_i^s	λ	S_{01}
UGX 1.13 bn	0.04	0.75	-0.05	0.05	0.80	1.00	0.005	4	UGX 79.31 bn

The initial asset mix consists of 61.79% government securities, 0.57% corporate bonds, 1.57% term deposits, 6.8% investment property and 29.27% equity as shown in Figure 4.

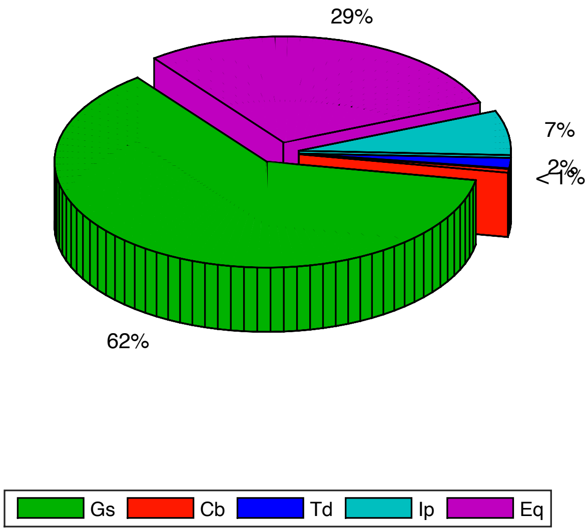


Figure 4. Initial asset mix

4.3.1. Optimal investment strategies

We present the optimal solutions under each of the asset allocation limits given in Table 1. The optimal objective values and terminal cash injection by the sponsor are given in Table 11.

Table 11. Objective and terminal cash injection

Limits	URBRA	BoUDBS	Modified
Objective value	1083.08	1083.39	3397.09
H_{Tn}	1081.00	1081.31	3394.39

The optimal solutions under URBRA and BoUDBS limits are very closely similar. Therefore, in the following results, we only present results under BoUDBS and modified investment limits. Table 12 gives the information about the optimal solution of the SP model, using BoUDBS asset allocation limits. The allocation to government securities remains constant at its upper limit of 0.7 from stage 2 to stage 5, and then slightly reduces to 0.6987 in stage 6. This results from the government securities having the highest returns with relatively low risk. The remaining portion of the portfolio is shared among term deposits and equity from stage 2 to stage 6. The share for equity reduces from 0.3 in stage 2 to 0.2557 in stage 6. Although equity gives higher returns than term deposits, it is more risky. Hence the allocation to term deposits increases from 0 in stage 2 to 0.0456 in stage 6, in a way that minimises risk of underfunding towards the end of the horizon. There is no allocation to corporate bonds and investment property from stage 2 to the end, because although they are less risky, their returns are very low compared to those of the other assets.

Table 12. Optimal investment strategy with BoUDBS limits

Variables	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
c_{tn}	0.211	0.261	0.311	0.361	0.411	0.461
X^h_{1tn}	259.08	519.52	900.68	1565.98	2715.76	4690.02
X^h_{2tn}	2.41	0.00	0.00	0.00	0.00	0.00
X^h_{3tn}	6.58	0.00	9.73	19.25	68.61	306.25
X^h_{4tn}	28.50	0.00	0.00	0.00	0.00	0.00
X^h_{5tn}	122.71	222.65	376.28	651.88	1095.28	1716.42
X^p_{1tn}	0.00	0.00	0.17	0.06	0.12	0.04
X^p_{2tn}	0.00	0.00	0.00	0.00	0.00	0.00
X^p_{3tn}	0.00	0.00	9.73	10.73	47.00	219.86
X^p_{4tn}	0.00	0.00	0.00	0.00	0.00	0.00
X^p_{5tn}	0.00	7.51	11.44	25.75	38.64	75.39
X^s_{1tn}	0.00	9.81	20.76	30.63	61.19	127.01
X^s_{2tn}	0.00	0.00	0.00	0.00	0.00	0.00
X^s_{3tn}	0.00	0.00	0.00	7.68	10.43	27.86
X^s_{4tn}	0.00	0.00	0.00	0.00	0.00	0.00
X^s_{5tn}	0.00	0.00	9.64	8.62	41.73	203.43
w_1	0.6179	0.7000	0.7000	0.7000	0.7000	0.6987
w_2	0.0057	0.0000	0.0000	0.0000	0.0000	0.0000
w_3	0.0157	0.0000	0.0076	0.0086	0.0177	0.0456
w_4	0.0680	0.0000	0.0000	0.0000	0.0000	0.0000
w_5	0.2927	0.3000	0.2924	0.2914	0.2823	0.2557
A_{tn}	420.41	744.47	1295.76	2247.51	3907.25	6775.70
D_{tn}	0.00	0.01	1.84	48.74	191.09	489.24

Table 13 gives the information about the optimal solution of the SP model, using the modified asset allocation limits. The allocation to government securities remains constant at its upper limit of 0.3 from stage 2 to the end, as earlier explained. The allocations to corporate bonds and investment property are constant at their lower limits of 0.02 and 0.05 respectively, as earlier explained. The remaining 0.63 of the portfolio is shared among term deposits and equity. The allocation to equity reduces from 0.3998 in stage 2 to 0.3604 in stage 6, as earlier explained. Hence the allocation to term deposits increases from 0.2302 in stage 2 to 0.2696 in stage 6.

4.3.2. Cost and risk

The average contribution rate and terminal cash injection represent the cost of the pension scheme. The risk term is the second downside moment of the funding ratio. The variation in cost and risk terms of the objective at all the stages is shown in Table 14. Costs under all asset allocation limits are the same in the first six stages and only differ in the final stage. There is increase in cost by 0.05 in the subsequent stages, which is the maximum increase in contribution rate allowed in the model. This is due to deficits which should be reduced by extra contributions by the sponsor. The government securities which give the highest return are given a big upper bound of 0.7 under URBRA and BoUDBS asset allocation limits. This allows for high growth rate in asset value, so that the sponsor incurs a smaller terminal cash injection.

In stage 1, there is no risk as the fund begins with no deficits. In stage 2, the risk is very low as small deficits begin to emerge. For the rest of the horizon, there is relatively high risk due to increase in deficits. Risk is higher under modified limits, where only a maximum of 0.3 of the portfolio is allocated to government securities compared to that 0.7 under BoUDBS limits. In addition, there is mandatory allocation of 0.07 of the portfolio to investment property and corporate bonds under modified investment limits, which are much less profitable.

Table 13. Optimal investment strategy with modified limits

Variables	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6
c_{tn}	0.211	0.261	0.311	0.361	0.411	0.461
X^h_{1tn}	259.08	214.17	358.18	600.84	1004.81	1674.91
X^h_{2tn}	2.41	14.28	23.88	40.06	66.99	111.66
X^h_{3tn}	6.58	164.35	283.52	483.76	833.10	1505.44
X^h_{4tn}	28.50	35.69	60.00	100.14	167.47	279.15
X^h_{5tn}	122.71	285.39	468.66	778.00	1276.99	2011.88
X^p_{1tn}	0.00	0.00	0.00	0.00	0.00	0.00
X^p_{2tn}	0.00	1.78	2.90	4.95	8.09	13.17
X^p_{3tn}	0.00	2.23	10.74	22.71	50.68	169.31
X^p_{4tn}	0.00	6.76	11.08	18.82	31.06	51.04
X^p_{5tn}	0.00	1.04	2.23	13.33	25.45	51.64
X^s_{1tn}	0.00	12.15	21.61	34.07	60.60	107.32
X^s_{2tn}	0.00	0.00	0.00	0.00	0.00	0.00
X^s_{3tn}	0.00	0.19	0.88	10.67	22.92	50.87
X^s_{4tn}	0.00	0.00	0.00	0.00	0.00	0.00
X^s_{5tn}	0.00	1.82	13.58	25.70	59.70	189.87
w_1	0.6179	0.3000	0.3000	0.3000	0.3000	0.3000
w_2	0.0057	0.0200	0.0200	0.0200	0.0200	0.0200
w_3	0.0157	0.2302	0.2375	0.2415	0.2487	0.2696
w_4	0.0680	0.0500	0.0500	0.0500	0.0500	0.0500
w_5	0.2927	0.3998	0.3925	0.3885	0.3813	0.3604
A_{tn}	420.41	716.22	1203.06	2013.42	3377.25	5645.95
D_{tn}	0.00	2.14	76.88	280.64	720.96	1618.99

Table 14. Variation in cost and risk

Term	Expression	Limits	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5	Stage 6	Stage 7
Costs	$\sum_{n=1}^{N_t} \frac{c_{tn}}{N_t}$	BoUDBS	2.11×10^{-1}	2.61×10^{-1}	3.11×10^{-1}	3.61×10^{-1}	4.11×10^{-1}	4.61×10^{-1}	–
		Modified	2.11×10^{-1}	2.61×10^{-1}	3.11×10^{-1}	3.61×10^{-1}	4.11×10^{-1}	4.61×10^{-1}	–
	$\sum_{n=1}^{N_T} \frac{H_{Tn}}{N_T}$	BoUDBS	–	–	–	–	–	–	1.08×10^3
		Modified	–	–	–	–	–	–	3.39×10^3
Risk	$\sum_{n=1}^{N_t} \frac{1}{N_t} \left(\frac{D_{tn}}{L_{tn}}\right)^2$	BoUDBS	–	1.25×10^{-10}	1.58×10^{-5}	6.82×10^{-4}	2.55×10^{-3}	4.99×10^{-3}	7.66×10^{-3}
		Modified	–	2.85×10^{-5}	3.79×10^{-3}	1.53×10^{-2}	3.13×10^{-2}	5.01×10^{-2}	7.07×10^{-2}

From Figure 5, under all asset allocation limits, the contribution rate increases by 0.05 at each stage, as earlier explained from the cost of the pension scheme. Also the number of active members reduces while that of pensioners increases, requiring for more contributions from the fewer remaining contributors. Although higher increases would be required, there is an upper bound of 0.05.

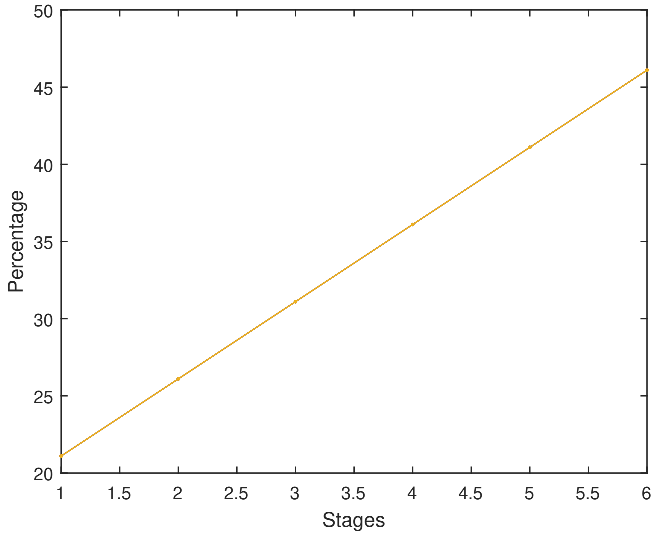


Figure 5. Average contribution rates

4.3.3. Average deficits

The deficits under each of the investment limits are given in Figure 6. Initially there are no deficits, from stage 1 to stage 3 deficits gradually increase, they further increase from stage 2 to stage 7, the increase being much higher under modified asset allocation limits, where there is a low upper bound on allocation to government securities, which give the highest returns. There are also restrictions to ensure that part of the asset is allocated to corporate bonds and investment property under modified limits, which give low returns. Deficits are calculated at the beginning of each planning period, before contributions for that period are received. In order to clear deficits in the final stage, we include additional stage 8. This is used to clear the deficits in stage 7 after the terminal cash injection by the sponsor is received.

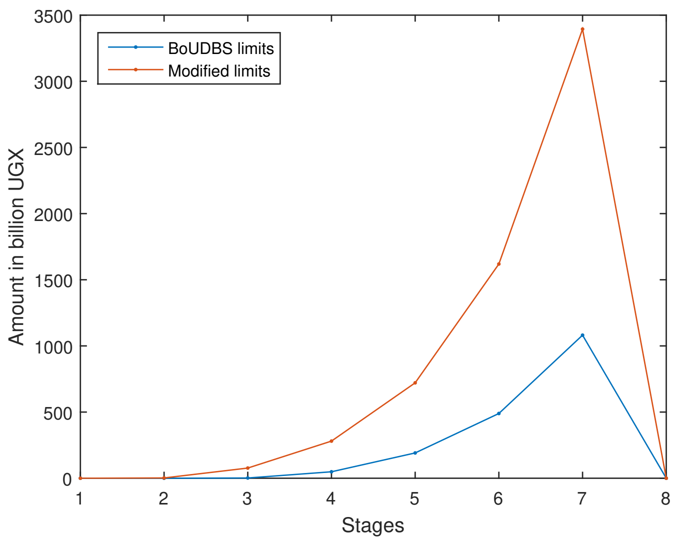


Figure 6. Average deficits

4.3.4. Average asset and liability values

The variation in average values of assets and liabilities at each stage is shown in Figure 7, in the subsequent stages, the sponsor pays extra contributions to reduce the gap between assets and liabilities. In the final stage, the value of assets and liabilities are equal due to cash injection by sponsor to meet the target funding ratio.

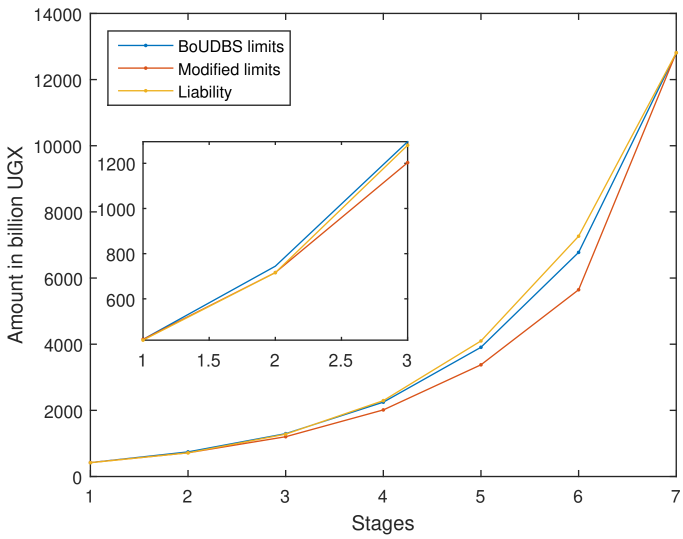


Figure 7. Average asset and liability values

4.3.5. Average funding ratios

The average funding ratios at each stage are shown in Figure 8, they are obtained from the equation

$$\bar{F}_t = \frac{\bar{A}_t}{\bar{L}_t} \tag{27}$$

where $\bar{A}_t$ and $\bar{L}_t$ are the average values for assets and liabilities, respectively in stage t. The SP model with BoUDBS limits outperforms the one with modified investment limits. Hence having the largest portion of the portfolio invested in government securities, and the remainder in equity and term deposits, and no consideration of corporate bonds and investment property greatly improves the funding status of the scheme. The final funding ratio is the same due to cash injection by the sponsor to meet the pre-specified target funding ratio.

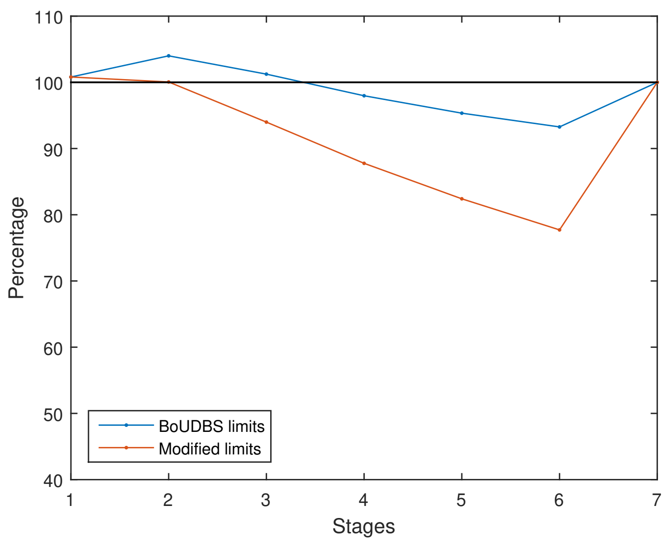


Figure 8. Average funding ratios

5. Conclusions

In this paper, we have developed a SP model for asset liability management of pension funds. We applied the model to the financial planning problem of the BoUDBS. The model was solved by SP techniques, to find optimal portfolio allocations and associated costs and risk. The model takes into account the funding situation of the fund at each stage. Randomly sampled scenario trees using the mean, and covariance structure of the return distribution were used for generating the coefficients of the stochastic program. Scenario trees were generated by Monte Carlo simulation. Liabilities were modelled by remaining years of life expectancy and guaranteed period for monthly pension. We calculated the average cost and risk of the SP policy under different asset investment limits, and studied the variation in optimal values of contribution rates, terminal cash injection, risk, deficits, assets, liabilities and funding ratios. Our results show that in order to keep the fund solvent, the sponsor should make remedial contributions at each stage. In the final stage where there are no contributing members, the sponsor should make a cash injection into the fund to ensure that all future benefits can be paid.

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A. Appendix

A.1. Mortality high income countries 2015–2070

These are given in Table 15 and 16.

Table 15. Probabilities of dying

Age	15/20	20/25	25/30	30/35	35/40	40/45	45/50	50/55	55/60	60/65	65/70
20	0.00306	0.00273	0.00258	0.00237	0.00221	0.00208	0.00196	0.00184	0.00172	0.00161	0.00151
25	0.00359	0.00350	0.00337	0.00321	0.00303	0.00286	0.00275	0.00266	0.00253	0.00240	0.00226
30	0.00412	0.00412	0.00399	0.00381	0.00365	0.00344	0.00326	0.00314	0.00303	0.00289	0.00273
35	0.00527	0.00515	0.00500	0.00471	0.00445	0.00421	0.00394	0.00371	0.00354	0.00338	0.00320
40	0.00747	0.00725	0.00692	0.00650	0.00610	0.00570	0.00535	0.00499	0.00467	0.00442	0.00418
45	0.01160	0.01103	0.01048	0.00976	0.00922	0.00860	0.00801	0.00751	0.00700	0.00653	0.00616
50	0.01845	0.01715	0.01613	0.01508	0.01418	0.01337	0.01249	0.01165	0.01092	0.01018	0.00950
55	0.02864	0.02644	0.02454	0.02281	0.02157	0.02022	0.01908	0.01781	0.01659	0.01556	0.01452
60	0.04244	0.03965	0.03663	0.03362	0.03168	0.02987	0.02798	0.02637	0.02456	0.02285	0.02145
65	0.05905	0.05599	0.05194	0.04741	0.04410	0.04138	0.03882	0.03630	0.03406	0.03180	0.02941
70	0.09327	0.08797	0.08227	0.07580	0.07069	0.06610	0.06182	0.05813	0.05456	0.05143	0.04762
75	0.15648	0.14885	0.14046	0.13087	0.12326	0.11626	0.10979	0.10399	0.09835	0.09319	0.08745
80	0.25361	0.24350	0.23211	0.21894	0.20846	0.19853	0.18947	0.18097	0.17259	0.16456	0.15663

Table 16. Expectation of life

Age	15/20	20/25	25/30	30/35	35/40	40/45	45/50	50/55	55/60	60/65	65/70
20	61.52	62.07	62.67	63.39	63.97	64.54	65.08	65.61	66.14	66.67	67.17
25	56.70	57.23	57.83	58.53	59.11	59.67	60.20	60.72	61.25	61.77	62.27
30	51.89	52.42	53.02	53.72	54.28	54.83	55.36	55.88	56.40	56.91	57.41
35	47.10	47.63	48.22	48.91	49.47	50.01	50.53	51.04	51.56	52.07	52.56
40	42.33	42.86	43.45	44.13	44.68	45.21	45.72	46.23	46.73	47.24	47.72
45	37.63	38.15	38.73	39.40	39.93	40.46	40.96	41.44	41.94	42.43	42.91
50	33.04	33.55	34.11	34.76	35.28	35.78	36.26	36.74	37.22	37.69	38.16
55	28.61	29.09	29.63	30.25	30.75	31.23	31.69	32.14	32.60	33.05	33.50
60	24.38	24.80	25.31	25.90	26.37	26.82	27.25	27.67	28.10	28.504	28.95
65	20.34	20.72	21.17	21.71	22.14	22.57	22.96	23.35	23.74	24.14	24.53
70	16.45	16.79	17.19	17.66	18.05	18.43	18.78	19.13	19.49	19.85	20.19
75	12.87	13.15	13.48	13.88	14.21	14.54	14.84	15.14	15.46	15.78	16.06
80	9.77	9.99	10.25	10.57	10.83	11.10	11.34	11.59	11.85	12.12	12.34